Interference of light waves possessing sub-Poissonian statistics and the sensitivity of laser gravitational observations

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(Submitted 5 October 1985)
Zh. Eksp. Teor. Fiz. **90**, 1889–1899 (June 1986)

A study is made of the quantum fluctuations in the case of photodetection of light in an interference experiment when the dividing device (interferometer, semitransparent mirror, etc.) is illuminated by two radiation fluxes possessing sub-Poissonian statistics. It is established what physical requirements must be satisfied by the spectral and time characteristics of the incident waves and the observation procedure to ensure that the natural (shot) fluctuations of the photodetection of the secondary waves are suppressed. A mixing of light beams in an interference device subject to an external perturbation is of interest in connection with laser gravitational observations, and also as a method of controlling light that, under certain conditions, does not alter the quantum statistics of the primary light waves.

1. INTRODUCTION

In recent years, there has been great interest in obtaining coherent light fields in quantum states, i.e., states that do not have a classical analog (see the review of Ref. 1, and also Refs. 2–5), and also in the properties of such fields. Below, for brevity, we shall speak of quantum fields, and, in the important special case of sub-Poissonian statistics of the photons, of sub-Poissonian fields. The interest is due not only to the essential novelty of physical phenomena of this kind but also to the possibilities opened up for reducing the natural noise of light in high-precision optical measurements and in optical communication.

In this paper, we discuss the possibilities for suppressing observation noise in an interference experiment in which two light waves are mixed by means of an interferometer, by a semitransparent dividing plate, or by some other device and one of the secondary waves is then sent to a photodetector with high quantum efficiency. This problem has a direct bearing on the problem of the noise of a laser detector of gravitational waves, the sensitive element of which is an interferometer.[6,7] Interference mixing is also of interest as a method of controlling quantum light fields without (under certain conditions) introducing additional fluctuations. It is well known that light losses due to absorption, diffraction, nonideality of the photodetector, etc., increase the observation noise of the quantum light field. It can be shown that physical methods of influencing a light field such as, for example, amplitude and frequency modulation can cause losses of the radiation energy and therefore generate noise. This will occur in the case of amplitude modulation because of variable absorption or deflection of part of the light flux; in the case of frequency detection, losses are introduced by the detector (there is a frequency-dependent transmission).

At the same time, it was shown recently (see Refs. 7–10) that in the case of interference of two quantum light fields in an ideal divider the separation of part of the light into one of the scattering channels (the one in which the light is not detected) influences the statistics of the light in the observation channel quite differently from, for example, absorption losses. If one mixes waves in a squeezed[11] state, the fluctuations of the photon number in the secondary waves can actually be less than in the incident waves. Study of the phenomenon of antibunching[12] leads to similar conclusions. Quantum phenomena were investigated in the case of the interference of weak fields in Ref. 13.

Although the studies of Refs. 7–10 were specifically devoted to photodetection of quantum light fields in an interference experiment and in the case of heterodyning, they adopted a description of the noise associated with photon detection that is fundamentally simplified in the case of stationary (unpulsed) light fields. In particular, in the framework of the approach of these studies the important physical question of the part played by the spectral and time scales of the light fields, the interference device, and the observation procedure in the suppression of the natural fluctuations of the detection was not even posed.

The aim of the present paper is to elucidate the physical conditions under which there can be an appreciable reduction of the natural noise of observation in the case of stationary interference of quantum light fields (for example, in a laser gravitational experiment). We discuss the case of sub-Poissonian light waves and use for the description of the photodetection Glauber's well-known and universal photon-counter model.

As will be seen from the results, the spectral and time behavior of the quantum light fields is so important for noise reduction that the observation procedure must be matched to this behavior. It follows from the discussion in Sec. 5 that if the radiation of two sub-Poissonian lasers is mixed under favorable conditions (we have in mind here the conditions of generation in the sources, matching of the phases of the waves, and a high quantum efficiency of the detector), then the natural fluctuations of photodetection can be suppressed arbitrarily far below the shot level. However, for this it is necessary that the characteristic time of accumulation of photocounts be much greater than the relaxation time of the amplitude fluctuations in the light sources. In other words, it is the noise of the low-frequency components of the photo-

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flux that is suppressed.

The fluctuations of the photoflux at high frequencies have a shot nature for any photon statistics of the mixed waves, including quantum statistics.

2. FLUCTUATIONS IN THE OBSERVATION OF THE INTENSITY OF QUANTUM FIELDS

As measure of the noise of stationary photodetection one often (see Refs. 7–10 and many other studies) uses the rms fluctuation $\langle \Delta n^2 \rangle^{1/2}$ of the number of photons in the wave incident on the photodetector. From this point of view, significant suppression of the natural fluctuations of observation requires that the statistics of the photons in the incident wave be deeply sub-Poissonian, i.e., the condition $\langle \Delta n^2 \rangle \ll \langle n \rangle$ must hold. If waves in the squeezed state are mixed, this leads to the requirement of deep squeezing in at least one of the primary waves.[7–10]

However, it can be shown that in a typical proposed source of a stationary coherent quantum field (see Ref. 1) in which the resonator contains an active medium interacting parametrically with the light, there is escape of radiation from the resonator[1)] and relaxation in the medium, the necessary deep squeezing is not achieved, and the mean square of the fluctuation of the photon number is reduced compared with the Poisson value by not more than a factor of two. It would seem that in this and similar cases the natural noise of the photodetection must remain appreciable, i.e., comparable with the shot level. Nevertheless, in a study by Golubev and one of the authors of the present paper,[3] it was shown that the low-frequency noise of photodetection can in fact still be suppressed in the absence of deeply sub-Poissonian statistics of the radiation.

The point is that in photodetection the number of photons observed in the light wave does not correspond to a stationary distribution, but is the number $N(t)$ of photoelectrons produced by the light during an accumulation time $\tau_0 \equiv \gamma^{-1}$. Depending on the method of accumulation, $N(t)$ is some integral over the time of the photocurrent $i(t)$. If the source sends light directly to the photodetector, the dimensionless field strength $a(t)$ in the source resonator, the current of the detector, and its correlation function are related as follows:

$$\langle i(t)\rangle = C_p \langle a^+(t) a(t)\rangle = C_p \langle n(t)\rangle,$$

$$\tfrac{1}{2}\langle \{i(t_1), i(t_2)\}_+\rangle = \delta(t_1-t_2)\langle i(t_1)\rangle + C_p^2\{g(t_1,t_2)\Theta(t_2-t_1) + (t_2 \leftrightarrow t_1)\},$$

$$g(t_1,t_2) = \langle a^+(t_1) a^+(t_2) a(t_2) a(t_1)\rangle,$$

where the constant C_p relates the mean current to the mean number of photons in the resonator (see Ref. 3), $\Theta(t)$ is the step function, and $\{\ldots\}_+$ is the anticommutator. These relations are known from the theory of a Glauber photon counter (for nonfree fields, see Refs. 17 and 18). It can be seen that the characteristic fluctuation in the number of photocounts depends on the value of $g(t_1, t_2)$ at arbitrary times, i.e., on the dynamics of the fluctuations of the light intensity and, ultimately, on the spectral and time properties of the quantum field. It is this circumstance that is not taken into account in the simplified description.

3. INTERFERENCE EXPERIMENT WITH SUB-POISSONIAN LIGHT FIELDS

Suppose plane waves are mixed in the case of illumination of a twin-wave or multiwave interferometer, dividing plate, etc., such that the wave fronts coincide in the formation of the secondary waves. We assume that the interference device is a fast device—that the characteristic time of establishment of the field in it is short compared with the reciprocal half-width Γ^{-1} of the spectrum of the amplitude fluctuations of the light (see below) and compared with the accumulation time τ_0. Then, for a convenient choice of the wave phases, the dimensionless strengths $b_{1,2}$ in the scattering channels are $b_1 = ca_1 + sa_2$ and $b_2 = -sa_1 + ca_2$, where c and s are the real amplitude transmission coefficients of the divider, and $c^2 + s^2 = 1$ (there is no loss in the divider).

The interference mixing of fields in the squeezed state in an interference device disturbed by a gravitational wave has been considered by Caves[7] but he did not take into account the dynamics of the field fluctuations and the fields were assumed to be in a given state. The measure of the observation noise was assumed in Ref. 7 to be $\langle \Delta n^2 \rangle^{1/2}$ (see above), and therefore the remarks made in the previous section apply to the results of Ref. 7.

In this paper, we use the physical model of a sub-Poissonian laser source proposed in Ref. 3. It can be shown that the suppression of the shot noise of the excitation of the medium (by the choice of the method of pumping) leads under optimal conditions to a sub-Poissonian statistics of the radiation. Smirnov and Troshin[19] have recently shown that cooperative processes in the active medium of the laser can play an analogous role.

We introduce a representation with respect to coherent states in which the density matrix $\rho(t)$ of the incident light fields corresponds in the interaction representation to a weight function of the form $\Phi(\alpha_1, \alpha_2, t) = \langle \alpha_1, \alpha_2 | \rho(t) | \alpha_1 \alpha_2 \rangle$ (see Refs. 20 and 21), this also being smooth in the case of a quantum state of the field oscillators (in contrast to the diagonal weight in the Glauber-Sudarshan representation). By virtue of the uncertainty relation, this weight function, like any other weight function, is not a statistical distribution in the field amplitude and phase variables or in quadrature components; however, it gives a certain picture of the behavior of the corresponding fluctuations, and we shall use this to obtain clear explanations.

We define the region of uncertainty for the incident field $i (i = 1, 2)$, i.e., the region on the $(\mathrm{Re}\,\alpha_i, \mathrm{Im}\,\alpha_i)$ plane in which the weight function is essentially nonzero (if the waves are independent, the wave function factorizes). In the case of a large number of photons, the difference of the region of uncertainty for a sub-Poissonian field from the case of a field in a coherent Glauber state consists of the lesser extension with respect to the radial variable and greater extension with respect to the phase variable. Suppose for simplicity that $c = s = 2^{-1/2}$, i.e., in the scattering channel 1 the intensities of the primary waves are added. Figure 1a, which shows the regions of uncertainty for the sub-Poissonian fields 1 and 2 (with corresponding indices), shows that the

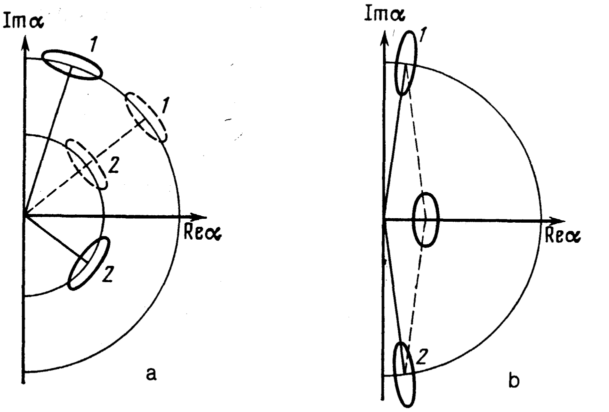

FIG. 1. Regions of quantum uncertainty for the incident waves: a) sub-Poissonian fields, b) quantum distributions for incident waves squeezed with respect to the phase variable.

field amplitude in the scattering channel 1 has minimal uncertainty if the mean phases of the incident waves are equal or differ by π, the phase uncertainty of the initial distributions is not too large, and the total mean intensity is not fortuitously equal to zero. For such matching of the wave phases (dashed line), the intensity of the scattered light responds to only the small change in the amplitudes of the contributions ca_1 and sa_2, while the small change in their phases does not play any role. Therefore, the interferometer must be adjusted in such a way that the investigated useful signal influences the moduli of the transmission coefficients c and s.

Suppose the frequencies of the waves are equal. After adjustment of the system to a low-noise measurement, i.e., after matching of the initial phases of the fields, conditions for measurement exist during a time Δt such that $D\Delta t \ll 1$, where D is the rate of diffusion of the difference phase in the system of the two independent laser sources. The duration of the expected signal τ_s and the accumulation time τ_0, which is chosen to be $\sim\tau_s$ (in the language of electronics, this means matching of the frequency band of the signal and the detecting system), must be sufficiently small, $\tau_0, \tau_s \ll D^{-1}$.

Since the phase diffusion is a slow process, the condition $\tau_s \ll D^{-1}$ can usually be readily satisfied and in principle permits retention of the difference phase by means of a feedback, the "fast" useful signal being unaffected by this.

4. DEVELOPMENT OF THE RADIATION FIELD IN THE INTERFERENCE PATTERN

It is convenient to introduce variables v_i and φ_i such that $\alpha_i = \exp(i\varphi_i)(u_i + v_i)^{1/2}$. It is here assumed that in the case of stationary lasing the weight function for the source i has a maximum with respect to the amplitude variable at $|\alpha_i| = (u_i)^{1/2}$, while v_i is a small deviation and intimately related to the fluctuation in the number of photons in the source resonator. Since from the time of establishment of the initial difference phase the sources generate light independently, the equation of motion of the total field density matrix has the form

$$\dot{\Phi} = (L_1 + L_2)\Phi, \tag{1}$$

where L_i, the development operator for field i, is found for a laser sub-Poissonian source in Ref. 3. After linearization with respect to the small deviation v_i and with allowance for the damping of the field in the resonator, we can obtain

$$L_i = \Gamma_i\left\{\frac{\partial}{\partial v_i} v_i + u_i(2+\xi_i)\frac{\partial^2}{\partial v_i^2}\right\} + D_i\frac{\partial^2}{\partial \varphi_i^2}. \tag{2}$$

The half-width Γ_i of the spectrum of the amplitude fluctuations depends on the rate of damping C_i of the field energy in the resonator and on the dimensionless intensity I_i: $\Gamma_i = C_i I_i(1 + I_i)$.

The statistics parameter ξ_i is defined in Ref. 3 both for a source of a quantum field in which the excitation shot noise is eliminated and for an ordinary laser with homogeneous gain profile. Sub-Poissonian statistics corresponds to the interval of values $-\frac{1}{2} < \xi_i < 0$; if $\xi_i > 0$, the radiation field has excess amplitude noise. To be definite, we assume that the fields a_i are generated under identical conditions, $C_i = C$ and $\Gamma_i = \Gamma$, but that the statistics may be different, $\xi_1 \neq \xi_2$.

Provided the initial matching of the wave phases is made with an uncertainty much greater than the quantum limit, it does not disturb the stationary (Gaussian) distribution in the variable v_i for each wave. It is convenient to assume that the distribution formed initially with respect to the difference phase $\varphi_{12} = \varphi_1 - \varphi_2$ is also Gaussian with central value $\varphi_{12}(0)$ and standard deviation $\langle\varphi_{12}^2\rangle^{1/2}$. The mean phase $\varphi_m = (\varphi_1 + \varphi_2)/2$ is not manifested in an experiment, and with respect to it we assume equidistribution. To such an initial condition there corresponds the weight function

$$\Phi(v_1, v_2, \varphi_1, \varphi_2, 0) = (2\pi)^{-1}\Phi_1^{(0)}(v_1)\Phi_2^{0}(v_2)\Psi_p(\varphi_{12}), \tag{3}$$

where

$$\Phi_i^{(0)}(v_i) = [2\pi u_i(2+\xi_i)]^{-1/2}\exp\{-v_i^2[2u_i(2+\xi_i)]^{-1}\}. \tag{4}$$

In (3), the Gaussian distribution on the interval $-\infty < \varphi_{12} < \infty$ of the form

$$\Psi(\varphi_{12}) = (2\pi\langle\Delta\varphi_{12}^2\rangle)^{-1/2}\exp\{-[\varphi_{12} - \varphi_{12}(0)]^2/2\langle\Delta\varphi_{12}^2\rangle\} \tag{5}$$

is transformed into a periodic distribution:

$$\Psi_p(\varphi_{12}) = \sum_{m=-\infty}^{\infty} \Psi(\varphi_{12} + 2\pi m). \tag{6}$$

To be specific, we assume that the accumulation (electron counting) is done by means of an integrating circuit, as is often the case in an experiment. Then the mean number of photoelectrons $\langle N_i(t)\rangle$ produced by the field in the scattering channel i in the time interval $(t - \tau_0, t)$ where $t \gtrsim \tau_0$, $\tau_0 = \gamma^{-1}$, and the mean square of the fluctuation, are

$$\langle N_i(t)\rangle = C_p \int_{-\infty}^{t} dt_1 \exp\{-\gamma(t-t_1)\}\langle b_i^+(t_1) b_i(t_1)\rangle,$$

$$\begin{aligned}\langle \Delta N_i^2\rangle = {}^1/_2\langle N_i(t)\rangle + C_p^2 \iint_{-\infty}^{t} dt_1\, dt_2 \exp\{-\gamma(t-t_1) - \gamma(t-t_2)\} \\ \times\{[G_{iiii}(t_1, t_2)\Theta(t_2 - t_1) + (t_2 \leftrightarrow t_1)] \\ - \langle b_i^+(t_1) b_i(t_1)\rangle\langle b_i^+(t_2) b_i(t_2)\rangle\}.\end{aligned} \tag{7}$$

By G_{ijlm} we denote the correlation function of the intensities of the scattered fields, $G_{ijlm} = \langle b_i^+(t_1) b_j^+(t_2) b_l(t_2) b_m(t_1)\rangle$. We denote the analogous correlation function for the intensities of the incident fields by g_{ijlm}, where $i, j, l, m = 1, 2$. The shot contribution to the square of the fluctuation has acquired the factor 1/2, which is due to the accumulation method (the high frequencies are cut off in the current noise). To find the correlation function $G_{iiii}(t_1, t_2)$ of the photocounts in the scattering channel i, we must express the scattered fields in terms of the incident fields, and this leads to calculation of the complete set of correlation function $g_{ijlm}(t_1, t_2)$ (this is the computational part of the work). We express explicitly the time dependence of the Heisenberg operators of the intensities of the incident waves:

$$g_{ijlm}(t_1, t_2) = \mathrm{Sp}\{a_l e^{-iH(t_2-t_1)} a_m e^{-iHt_1} \rho(0) e^{iHt_1} a_i^+ e^{iH(t_2-t_1)} a_j^+\}, \tag{8}$$

where E^{-iHt} is the evolution operator of the field, Applying $e^{iHt_1} \ldots e^{iHt_1}$ to the weight function $\Phi(\alpha_1, \alpha_2, 0)$, we obtain the development from $t = 0$ to $t = t_1$ of the initial condition (3) in accordance with Eq. (1), and then the weight function is transformed in accordance with the rule

$$a_m \ldots a_i^+ \to (\alpha_m + \partial/\partial\alpha_m^*)(\alpha_i^* + \partial/\partial\alpha_i) \ldots,$$

etc. The trace is taken by means of $\int d^2\alpha_1 d^2\alpha_2$. In the calculation, the results of which are given in the Appendix, it was found to be convenient to expand the weight function by making use of the property of orthogonality with a weight for eigensolutions of the Fokker-Planck equation.[22]

5. SUPPRESSION OF PHOTO-OBSERVATION NOISE IN THE SCATTERED LIGHT

Using the explicit form of the correlation functions of the intensities of the incident waves (see the Appendix), we find the mean value and the mean square of the fluctuation of the number of photocounts in the scattered wave 1. Under the condition $\tau_0 \lesssim t \ll D^{-1}$, $\langle \Delta\varphi_{12}^2\rangle^{1/2} \ll 1$ the result has the form

$$\langle N_1(t)\rangle = \frac{C_p}{\gamma}\{c^2 n_1 + s^2 n_2 + 2cs(n_1 n_2)^{1/2}\langle\cos\varphi_{12}\rangle_\varphi\}, \tag{9}$$

$$\begin{aligned}\langle \Delta N_1^2(t)\rangle = \Bigg[&\frac{C_p}{2\gamma}\left(1 + 2\frac{C_p}{\gamma+\Gamma}(c^2\xi_1 + s^2\xi_2)\right)\\ &\times(c^2 n_1 + s^2 n_2 + 2cs(n_1 n_2)^{1/2}\langle\cos\varphi_{12}\rangle_\varphi)\\ &-\frac{C_p^2}{\gamma(\gamma+\Gamma)}c^2 s^2 (n_1\xi_2 + n_2\xi_1)\langle\sin^2\varphi_{12}\rangle_\varphi\Bigg]\\ &+2\frac{C_p^2}{\gamma^2}c^2 s^2 n_1 n_2 (1 - \cos 2\varphi_{12}(0))\\ &\left(2Dt + \langle\Delta\varphi_{12}^2\rangle - \frac{1}{2n_1} - \frac{1}{2n_2}\right).\end{aligned} \tag{10}$$

Here and in what follows $D = D_1 + D_2$, and by n_i we denote the mean number of photons in the incident wave, $n_i = u_i - 1$. The expressions are abbreviated by the introduction of averaging, denoted by $\langle \ldots \rangle_\varphi$. By this we mean that the quantity within the angular brackets is averaged with respect to the difference phase φ_{12} with the Gaussian weight that arises through the angular diffusion; this weight is described by central value $\varphi_{12}(0)$ and mean square deviation $2Dt + \langle\Delta\varphi_{12}^2\rangle$. The contribution to the square of the fluctuations in the square brackets arises from the shot noise and the amplitude fluctuations, while the following contribution arises from the fluctuations of the phase (we denote these, respectively, by $\langle\Delta N_1^2(t)\rangle^{(a)}$ and $\langle\Delta N_1^2(t)\rangle^{(ph)}$). Low-noise measurement is possible, as noted above, in the case of matching of the wave phases, $\varphi_{12}(0) = 0, \pi$, it being necessary that the mean contributions to the intensity of the scattered wave from the incident waves not mutually cancel each other. In this case, we find

$$\begin{aligned}&\langle\Delta N_1^2(t)\rangle^{(a)} \approx \frac{1}{2}\langle N_1(t)\rangle\left(1 + 2\frac{C_p}{\gamma+\Gamma}(c^2\xi_1 + s^2\xi_2)\right),\\ &\langle\Delta N_1^2(t)\rangle^{(ph)} \approx 0.\end{aligned} \tag{11}$$

The importance of quantum phenomena (ordering in the photon flux) and the excess fluctuations relative to the shot fluctuations is determined by the ratio $C_p/(\gamma + \Gamma)$. Since $C_p = Cq$, where q is the relative loss of the light energy in transmission (in the absence of the interferometer) and in detection, and the half-width Γ of the spectrum of the amplitude fluctuations tends in the case of a strong field to the damping rate C of the field energy in the resonator, in the optimal case Γ, $C_p \to C$.

The possiblity of low-noise measurement is then determined by the spectral-time factors. If the counting time is short, $\tau_0 \ll C^{-1}$, i.e., $\gamma \gg C$, then independently of the statistics of the light the shot contribution to the fluctuation is dominant. The statistical properties of the radiation are important in the opposite case, $\tau_0 \gg C^{-1}$, $\gamma \ll C$, i.e., when the counting time is much greater than the characteristic correlation time of the intensity. Complete suppression of the observation shot noise occurs if there is interference between sub-Poissonian fields with maximal (for our physical model) degree of antibunching and squeezing of the photon number distribution, $\xi_i = -\frac{1}{2}$. The statistics parameters ξ_i of the primary waves occur in (11) with weights equal to the energy coefficients of transmission into the given scattering channels. Therefore, observation noise is also suppressed in the case of mixing of a "good" wave, $\xi_i \to -\frac{1}{2}$, with one into which more noise has been introduced but which has been significantly attenuated by the divider (but this latter wave can still remain strong). An analogous conclusion was drawn earlier in the case of the simplified description of the photodetection fluctuations in Refs. 8–10, in which it was proposed to detect a quantum field by mixing it with a wave described by a coherent Glauber state.

In the case of optimal phase matching, the fluctuation of the observable does not depend on small fluctuations of the phase. Let us estimate the admissible error $\delta\varphi$ in the establishment of the initial difference phase relative to the optimal values 0 and π. It should be noted that the signal change in the observable $N_1(t)$ produced by external disturbance of the interferometer with duration $\tau_s \sim \tau_0$ is masked, not by the total fluctuation of the observable that arises in the ensemble of measurement systems from the initial time, but by the increment of the fluctuation $\delta N_1(t) = N_1(t) - N_1(t-\tau_0)$ during the signal detection time τ_0. Since the random motion of the phase is slow and has the nature of diffusion,[2)] we can assert that

$$\langle \delta N_1^2(t) \rangle^{(ph)} \sim \langle \Delta N_1^2(t) \rangle^{(ph)} - \langle \Delta N_1^2(t-\tau_0) \rangle^{(ph)}.$$

Phase diffusion does not hinder quantum ordering in the flux of the scattered photons, provided $\langle \Delta N_1^{2\,(a)} \gg \langle \delta N_1^2 \rangle^{(ph)}$; this condition can be reduced to the form

$$\left[1+2\frac{C_p}{\gamma+\Gamma}(c^2\xi_1+s^2\xi_2) \right] \gg N_1\tau_0 D(1-\cos 2\delta\varphi). \quad (12)$$

Here, we have used the fact that there is no random suppression of the waves in the scattering channel, i.e., the condition $\langle b_1^+ b_1 \rangle \sim n_1, n_2$ is satisifed. In the case of scattering of classical waves without excess fluctuations, $\xi_i = 0$, the restriction (12) on the diffusion rate D of the phase is analogous to the restrictions that arise in the cases of frequency and phase detection. Because of the phase insensitivity of the decribed measurement, the factor $(1 - \cos 2\delta\varphi)$, which is small when $|\delta\varphi| \ll 1$, is present.

It is interesting to note that there does in principle exist (if we do not concern ourselves with the physical method of preparing the necessary initial state) a further possibility for obtaining the antibunching phenomenon and suppressing the photo-observation noise in the case of interference of two waves from laser sources. This possibility arises when waves that are equal in mean amplitude and almost (but not completely) opposite in phase add up in the scattering channel, $c^2 n_1 = s^2 n_2$, $\varphi_{12}(0) = \pi \pm \delta\varphi$, $|\delta\varphi| \ll 1$, each of the waves having at the initial time less uncertainty with respect to the phase variable than a wave in a coherent state (see Fig. 1b):

$$(1/4n_i)[2-\xi_i/(1+\xi_i)] \leqslant \langle \Delta\varphi_i^2 \rangle \leqslant 1/2n_i. \quad (13)$$

The left-hand inequality in (13) follows from the uncertainty relation and determines the quantum limit for the uncertainty of the distribution of the phase φ_i in the given representation with respect to coherent states. The inequality is found in the limit $n_i \gg 1$ for a weight function Gaussian with respect to the energy and phase variables. The right-hand inequality in (13) means that the distribution with respect to the phase is narrower than in a coherent Glauber state. Obviously, this is possible only for fields with excess fluctuations of the intensity.

It can be shown that in the given case the phase contribution to the square of the fluctuation of the observable (10) is negative at short times. An analysis, which we omit, shows that for such an initial condition one does indeed obtain antibunching of the photons and suppression of the photo-observation noise, but these quantum features are rapidly (during a time $t \gtrsim C^{-1}$) destroyed by phase diffusion. In Fig. 1b, this corresponds to spreading of the regions of uncertainty with loss of the initial squeezing.

6. PHYSICAL INTERPRETATION OF QUANTUM WAVE PHENOMENA

The phenomena of photon antibunching and suppression of the natural fluctuations of photo-observation cannot, as is well known, be explained in terms of classical wave notions and have an essentially quantum nature. At the same time, a physical picture and some quantitative conclusions can quite often be obtained by representing a light beam as a stream of photons. From this point of view, antibunching is a relative ordering in the photon stream due, for example, to repulsive statistics of the emission events in the light source. If a light beam is sent through a semitransparent mirror, the fluctuations in the photocounts in one of the secondary waves can be found by assuming for each incident photon random scattering into one or other channel. The scattering probability is determined by the corresponding energy transmission coefficient. The result of such arguments agrees, as can be shown, with the conclusions of quantum theory.

A fundamental feature of the interference of quantum light fields is that the fluctuations in the scattered waves cannot be explained by energy-balance considerations for any choice of the probabilities of scattering of the photons incident on the dividing device. In the case of random and independent scattering of the photons, the fluctuations of the intensity could only increase. But in fact the situation is quite different: The secondary waves may have just as low (Fig. 1a) or even lower (Fig. 1b) fluctuations in the case of photodetection as the incident waves.

For the physical interpretation of the quantum wave phenomena, one must use wave notions (interference of amplitudes) and treat the measurements in terms of reduction of the quantum state. Suppose that at a certain instant of time the field has produced a photoelectron in the observation channel 1. The statistical description of the subsequent events is given by a reduced field density matrix ρ^R such that

$$\rho^R = b_1\rho b_1^+/\mathrm{Sp}\{b_1\rho b_1^+\}.$$

It is convenient to assume that before the measurement the weight function has a Gaussian dependence on the coordinates α_i', α_i'', $i = 1, 2$, where $\alpha_i = (\alpha_i' + i\alpha_i'')\exp(i\psi_i)$. This state of the field is not identical to that generated by a system of two laser sources, but in the case of a large number of photons and small amplitude and phase fluctuations, and also for a suitable orientation of the regions of uncertainty, there is a fairly close physical correspondence between these states.

Figure 2 shows qualitatively how the regions of uncertainty for the primary fields are changed as a result of the reduction (i.e., by the detection of the photon in the scattering channel 1).[3)] If phase-matched sub-Poissonian fields interfere, absorption of a photon by the detector weakens energetically both incident waves in such a way that their regions of uncertainty are displaced toward the center of the coordinates. The intensity of the scattered wave 1 is reduced (in these arguments, $c = s = 2^{-1/2}$), i.e., there is antibunching and, as a consequence, ordering in time of the stream of scattered photons. This ordering is the physical reason for the suppression of the photo-observation fluctuations. In the scattering channel 2, in which the intensities are subtracted, the intensity is not changed, since the shifts of the regions of uncertainty are compensated.

In the case of interference under the same conditions of

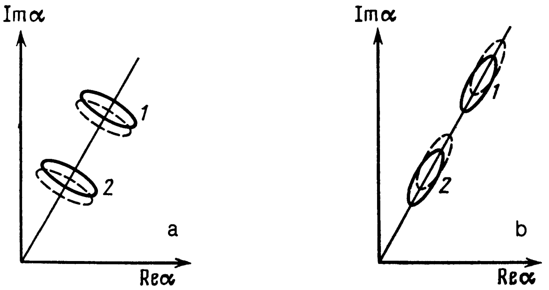

FIG. 2. Regions of quantum uncertainty for the incident waves before (continuous curves) and after (broken curves) reduction corresponding to emission of a photoelectron in the observation channel 1: a) sub-Poissonian fields, b) waves with excess fluctuations of the intensity.

waves possessing excess fluctuations of the intensity (Fig. 2b), the arguments are analogous except that absorption of a photon by the detector in channel 1 does not so much weaken energetically both incident waves as it does rather indicate a burst of their intensity. The regions of uncertainty are shifted in the direction opposite to the case of sub-Poissonian waves, and the intensity of the secondary wave is higher than the average.

APPENDIX

To investigate the quantum fluctuations of light in the interference pattern, it is necesary to know the second- and fourth-order correlation functions of the operators of the intensity of the incident waves. We give below the results of the corresponding calculations. Since the field equation of motion (1) has a fairly general form, these results can also be helpful for other physical models of a quantum field source. The correlation functions are found in the leading order and following order in the large parameter $n_i \gg 1$, the mean number of photons in the resonator, and have the form

$$\langle a_i^+(t)a_i(t)\rangle = n_i,$$

$$\langle a_i^+(t)a_j(t)\rangle = (n_i n_j)^{1/2}\exp\{i(\omega_{ij}t-\varphi_{ij}(0))-Dt-\langle\Delta\varphi_{12}^2\rangle/2\}$$
$$\times\left\{1+\left[\frac{1}{4n_i}\left(1-\frac{\xi_i}{2}\right)+(i\leftrightarrow j)\right]\right\},$$

$$\langle a_i^+(t_1)a_i(t_2)\rangle = n_i\exp\{-(i\omega_i+D_i)\Delta t\}$$
$$\times[1-(\xi_i/4n_i)(1-\exp(-\Gamma_i\Delta t))],$$

$$g_{iiii}(t_1,t_2)=n_i^2+n_i\xi_i\exp(-\Gamma_i\Delta t),\quad g_{ijji}(t_1,t_2)=n_i n_j,$$

$$g_{ijij}(t_1,t_2)=n_i n_j\exp\{-(i\omega_{ij}+D)\Delta t\}$$
$$\times\{1-[(\xi_i/4n_i)(1-\exp(-\Gamma_i\Delta t))+(i\leftrightarrow j)]\},$$

$$g_{iijj}(t_1,t_2)=n_i n_j\exp\{i\omega_{ij}(t_1+t_2)$$
$$-2i\varphi_{ij}(0)-4Dt_1-D\Delta t-2\langle\Delta\varphi_{12}^2\rangle\}$$
$$\times\left\{1+\left[\frac{1}{n_i}-\frac{\xi_i}{4n_i}(1-\exp(-\Gamma_i\Delta t))+(i\leftrightarrow j)\right]\right\},$$

$$g_{ijjj}(t_1,t_2)=n_i^{1/2}n_j^{3/2}\exp\{i\omega_{ij}t_1-i\varphi_{ij}(0)-Dt_1-\langle\Delta\varphi_{12}^2\rangle/2\}\varkappa,$$

$$g_{jijj}(t_1,t_2)=n_i^{1/2}n_j^{3/2}\exp\{i\omega_{ij}t_2-i\varphi_{ij}(0)-Dt_2-\langle\Delta\varphi_{12}^2\rangle/2\}\varkappa,$$

where

$$\varkappa=1+(\xi_j/2n_j)\exp(-\Gamma_j\Delta t)+[(1-\xi_i/2)/4n_i+(i\leftrightarrow j)].$$

In these relations $i,j=1,2$, $i\neq j$; ω_1 and ω_2 are the frequencies of the incident waves. The spectral parameters of the waves are for generality assumed to be different, $\omega_1\neq\omega_2$, $\Gamma_1\neq\Gamma_2$. By Δt we denote the time difference t_2-t_1, with $t_2\geqslant t_1$.

[1)]We do not consider here the possibilities of nonperturbing measurements,[14-16] when the investigated field does not interact directly with the photodetector.

[2)]We found $\langle\delta N_1^2\rangle$ directly; its analysis gives the same physical results.

[3)]The reduced weight function does not factorize, and therefore to find the distribution for wave 1 we found the trace with respect to the variables α_2', α_2'' and vice versa.

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Translated by Julian B. Barbour