Hysteresis in a two-level system and frictional force in a standing light wave

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(Submitted 12 December 1985)
Zh. Eksp. Teor. Fiz. **90**, 1619–1634 (May 1986)

Equations for the density matrix of a two-level atom in the field of a standing light wave are solved quasiclassically. The adiabatic state representation is used and an allowance is made for the mixing of these states due to spontaneous emission and Landau-Zener transitions. The frictional force acting on an atom is found from this solution for a wide range of atomic and field parameters. In a strong field the friction is due to hysteretic effects in stimulated transitions. At low velocities this force exhibits a narrow peak which is not broadened by the field (adiabatic resonance). Coherent "shakeup" as a result of the Landau-Zener transitions causes the frictional force to have an oscillatory structure.

1. INTRODUCTION

We shall consider two types of problem. In the first we shall employ the quasiclassical method to solve the equation for the density matrix of a two-level atom in a field described by $V_0 \sin\omega t$. Such equations are frequently encountered in nonlinear resonance optics when describing the behavior of an atom in the field of a standing light wave or in a bichromatic (two-frequency) field. In the second type of problem the radiation pressure force acting on an atom in the field of a standing wave is obtained from the solutions over a wide range of atomic and field parameters. The second problem is very important[1–3] since resonant electromagnetic fields can influence significantly the translational motion of heavy particles.

The radiation pressure force in a strong field of a standing wave was first calculated in Ref. 4 in a quasisteady approximation. It was shown that, in addition to a gradient force, a particle experiences also a frictional force which depends on the sign of the detuning. The same problem was considered for weak fields in Refs. 5–9. The case of slow atoms was studied in Refs. 10–12. Moreover, a numerical solution of this problem was obtained in Ref. 13 for a certain range of parameters.

In a strong field the main role is played by stimulated transitions which form two quasienergy atomic states. Spontaneous relaxation mixes the states and gives rise to hysteresis in the response of the system. The other source of mixing is represented by Landau-Zener transitions in the vicinity of field nodes. These transitions and the consequent interference effects were studied in Refs. 14 and 15 without allowance for relaxation.

In the present paper (Sec. 4) we obtain a solution of equations for the density matrix allowing for both mechanisms for mixing of quasienergy states: noncoherent (spontaneous relaxation) and coherent (Landau-Zener transitions).

We shall show that in a strong standing-wave field a frictional force appears as a result of hysteresis of the gradient force ("delayed" gradient force) and it may exceed considerably the spontaneous radiation pressure. Considered as a function of the velocity, it has a large-amplitude resonance at small velocities such that $kv \sim \gamma$, where γ is the line width. A strong field does not broaden this resonance, but simply alters its profile. This resonance appears in the case of a sufficiently large detuning; we shall refer to it as adiabatic.

In Sec. 3 we shall use perturbation theory to show that an adiabatic resonance appears in the sixth order with respect to the field and that it is associated with the appearance of shifted components in the resonant fluorescence spectrum of an atom.

In the range of high velocities and small values of the detuning the frictional force exhibits an oscillatory structure because of interference effects when an atom travels under the simultaneous influence of two potentials. The visibility of the interference pattern is determined by the competition between the spontaneous relaxation and the "shakeup" due to the Landau-Zener transitions (Sec. 5).

2. PRINCIPAL EQUATIONS

In a resonant inhomogeneous field $E(\mathbf{r})\exp(-i\Delta t)$ (Δ is a small detuning from a resonance) an atom experiences a force

$$F = \mathrm{Tr}(\hat{d}\rho)\nabla E^* + \text{c.c.}, \tag{1}$$

where $\hat{d}$ is the dipole moment operator and ρ is the density matrix of the atom. For simplicity, the atom is assumed to have two levels and the level degeneracy as well as the field polarization are ignored. The magnitude of this force depends strongly on the spatial structure of the field. In a traveling wave

$$E(\mathbf{r}) = E_0 \exp(ikx)$$

we obtain the familiar expression for the force[2]

$$F \equiv F_x = \hbar k\gamma w(\Delta - kv_x), \quad w(\Delta) = V_0^2/(\Delta^2+\gamma^2/4+2V_0^2), \quad V_0 = dE_0/\hbar, \tag{2}$$

where γ is the rate of decay of the upper level and w is its population. The quantity F is limited by the rate of spontaneous transitions, and under saturation conditions ($w \approx 1/2$) it reaches its maximum value $F_{sp} = \hbar k\gamma/2$.

We shall be interested in the case of a standing wave $E(\mathbf{r}) = E_0 \sin kx$, when the field intensity gradient has its highest possible value. In this case it is the stimulated transitions and delay effects caused by the spontaneous relaxation that play the main role in a strong field.

 0038-5646/86/050951-09$04.00

It is convenient to consider these effects in the time domain using a system of coordinates moving at the velocity of the atom, $v \equiv v_x$. Such a description is justified in many cases of practical importance when the velocity of a particle varies slowly in the interaction process. This condition is realized, in particular, when the kinetic energy $Mv^2/2$ exceeds the potential energy of the interaction $\sim dE_0$.

In the time representation ($x = vt$) the induced dipole moment is found by solving the equation for the density matrix of an atom $\rho(t)$:

$$d\rho/dt = -i[\hat{H}\rho] - \hat{\gamma}\rho, \quad \rho = \begin{pmatrix} \rho_{bb} & \rho_{ba} \\ \rho_{ab} & \rho_{aa} \end{pmatrix}, \quad \mathrm{Tr}\,\rho = 1, \tag{3}$$

$$\hat{H} = -\begin{pmatrix} \frac{\Delta}{2} & V(t) \\ V(t) & -\frac{\Delta}{2} \end{pmatrix},$$

$$\hat{\gamma}\rho = \begin{pmatrix} \gamma\rho_{bb} & \frac{\gamma}{2}\rho_{ba} \\ \frac{\gamma}{2}\rho_{ab} & -\gamma\rho_{bb} \end{pmatrix}, \quad V(t) = V_0 \sin\omega t, \quad \omega = kv, \; V_0 = dE_0/\hbar.$$

The indices a and b denote the ground and excited states.

We shall consider an asymptotic (when $\gamma t \gg 1$) periodic solution of the density matrix $\rho(t + 2\pi/\omega) = \rho(t)$. In addition to the obvious invariance under the substitution $t \to t + 2\pi/\omega$, the system of equations (3) is invariant under a translation equal to half the period π/ω, accompanied by a simultaneous reversal of the sign of the off-diagonal elements of the density matrix $[\rho_{ba}(t + \pi/\omega) = -\rho_{ba}(t)]$. Therefore, the asymptotic solution satisfies the "half-period" condition

$$\rho(t+\pi/\omega) = \sigma_3 \rho(t) \sigma_3, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

The force (1) is directed along the x axis and in the coordinate system of the atom is given by

$$F(t) = \frac{2}{v}\frac{dV}{dt}\,\mathrm{Re}\,\rho_{ba}. \tag{5}$$

In the case of the steady-state periodic solution the induced dipole moment

$$p(t) = \rho_{ba}(t) = \sum_n p_{2n+1} \exp[i(2n+1)\omega t]$$

can be represented in the form of two terms $p(t) = p_s(t) + p_r(t)$, where

$$p_s(t) = i\sum_n p_{2n+1} \sin(2n+1)\omega t = V(t)\mathcal{P}_s[V^2(t)],$$

$$p_r(t) = \sum_n p_{2n+1} \cos(2n+1)\omega t = \frac{1}{\omega}\frac{dV}{dt}\mathcal{P}_r[V^2(t)].$$

Here, $\mathcal{P}_s$ and $\mathcal{P}_r$ are certain functions of time which are determined by the instantaneous value of the field and have the period π/ω. Therefore, $p_s(t)$ represents the synchronous (phase-matched) part of the dipole moment which is governed by the value of the field at a moment of time t, whereas the delayed part of the dipole moment $p_r(t)$ depends not only on $V(t)$, but also on the derivative dV/dt. We must stress that we are speaking here of the synchronous and delayed behavior of $p(t)$ relative to slow oscillations of the field at the Doppler frequency $\omega = kv$ and not relative to fast optical oscillations of the field.

This division of the dipole moment gives rise to two characteristic terms in the optical pressure force:

$$F = F_g + F_r,$$

$$F_g(t) = \frac{2}{v}\frac{dV}{dt}\,\mathrm{Re}\,p_s(t) = \frac{1}{v}\frac{d(V^2)}{dt}\,\mathrm{Re}\,\mathcal{P}_s[V^2(t)], \tag{6}$$

$$F_r(t) = \frac{2}{v}\frac{dV}{dt}\,\mathrm{Re}\,p_r(t) = \frac{2}{v\omega}\left(\frac{dV}{dt}\right)^2 \mathrm{Re}\,\mathcal{P}_r[V^2(t)].$$

The force F_g is the total derivative of a periodic function with respect to time and it obviously vanishes when averaged over the oscillation period π/ω. In the coordinate representation ($vt = x$) this force is a gradient of some effective potential, $F_s = -\partial U_{\mathrm{eff}}/\partial x$.

The delayed part of the dipole potential creates a frictional force F_r which cannot be represented as a time derivative of a periodic function and which therefore does not disappear as a result of averaging over the field period. It can be shown that the delayed (hysteretic) part of the force is an odd function of the detuning Δ and of the velocity v of the atom.

3. FEATURES OF PERTURBATION THEORY IN THE CASE OF A STANDING WAVE FIELD

The system of equations (3) rewritten in terms of the Bloch variables $p = \rho_{ba}$ and $q = \rho_{bb} - \rho_{aa}$ becomes

$$\left(\frac{d}{dt} + \gamma\right) q = 4V(t)\,\mathrm{Im}\,p - \gamma, \tag{7}$$

$$\left(\frac{d}{dt} - i\nu\right) p = -iV(t)q, \quad \nu = \Delta + i\gamma/2. \tag{8}$$

We shall consider large values of the detuning from a resonance:

$$\Delta \gg V_0, \omega, \gamma \tag{9}$$

and fairly general relationships among the parameters V_0, ω, and γ. Then, Eqs. (7) and (8) can be solved by expanding them in powers of $1/\Delta$. In the first order in $1/\Delta$ we obtain the synchronous part of the dipole moment $p_s(t) \approx -V(t)/\Delta$, which gives the familiar expression for the gradient force in the case of weak saturation

$$F_g = -\partial U_{\mathrm{eff}}/\partial x, \quad U_{\mathrm{eff}} = \hbar V^2(x)/\Delta. \tag{10}$$

The effective potential of Eq. (10) is governed by stimulated transitions and represents the Stark shift of the atomic levels in a resonant inhomogeneous external field.

In higher order in $1/\Delta$, we not only have small corrections to the effective potential $\sim (V_0/\Delta)^2$, $(\gamma/\Delta)^2$, and $(\omega/\Delta)^2$, which we shall ignore, but also a frictional force. This force appears because of the delay effects associated with spontaneous emission.

The delay effect in a traveling wave field gives rise to the spontaneous radiation pressure of Eq. (2), which is directed along the wave vector and is less than F_{sp}. In a standing wave there is a contribution equal to the difference between the forces of the spontaneous radiation pressure (2) for each of two waves traveling in opposite directions (counterpropagating waves).[6-9] Moreover, there is a contribution from the delay effect in stimulated transitions associated with the repeated scattering of photons between two opposed light waves. This quantity can be called the delayed gradient force. The effects in question can easily be identified by the following simple procedure for solving the Bloch equations.

In solving Eq. (8) it is sufficient to allow for the time derivative using perturbation theory

$$p \simeq \frac{V}{v} q + \frac{1}{iv^2}\frac{d}{dt}(Vq),$$
$$\operatorname{Re} p \approx \frac{Vq}{\Delta} + \frac{\gamma}{\Delta^3}\frac{dV}{dt}, \quad \operatorname{Im} p \approx -\frac{\gamma V}{2\Delta^2} q - \frac{1}{\Delta^2}\frac{d}{dt}(Vq). \tag{11}$$

The second term in Rep, where the substitution $q \approx -1$ is made, represents the contribution of the difference between the spontaneous radiation pressure forces to the frictional force F_r. The first term in Rep represents the gradient force [in particular, if $q \approx -1$, it is described by Eq. (10)] and also the delayed gradient force. The latter can be found by calculating the delay effect in the population difference.

Substituting Imp in Eq. (7), we obtain the following equation for the function $A(t) = -\varepsilon(t)q(t)$ (Ref. 4):

$$\frac{dA}{dt} = -\gamma\frac{\varepsilon^2+1}{2\varepsilon^2}A + \frac{\gamma}{\varepsilon}, \quad \varepsilon(t) = [1+(2V(t)/\Delta)^2]^{1/2}. \tag{12}$$

Integrating Eq. (12) subject to the initial condition $A(-\infty) = 1$, we obtain

$$A(t) = \gamma\int_0^\infty \frac{d\tau}{\varepsilon(t-\tau)}\exp\left[-\frac{\gamma}{2}\int_0^\tau d\tau'(1+\varepsilon^{-2}(t-\tau'))\right]. \tag{13}$$

Equation (13) is valid for arbitrary values of the saturation parameter $(V_0/\Delta)^2$ and will be used in Sec. 4. However, in the case of a weak field ($V_0 \ll \Delta$), it simplifies greatly so that omitting small local corrections of the form $[V(t)/\Delta]^2$, we find that $q(t)$ is described by

$$q(t) \approx -1 - \frac{2}{\Delta^4}\frac{d}{dt}\int_0^\infty d\tau \exp(-\gamma\tau) V^4(t-\tau). \tag{14}$$

It should be noted that the delay effect appears in q only in terms of fourth order and higher with respect to the field.

The physical interpretation of the hysteresis is clearest in the adiabatic state representation and is discussed in Sec. 5. Using Eqs. (14) and (11), we can find the optical pressure force averaged over the field period (the averaging procedure is represented by angular brackets):

$$\langle F(t)\rangle = \langle F_r\rangle = \hbar k\gamma\frac{\omega V_0^2}{\Delta^3}\left\{1 - \frac{V_0^4}{\Delta^2(\gamma^2+4\omega^2)}\right\}. \tag{15}$$

The first term is associated with the spontaneous radiation pressure. If $\Delta > 0$, it is positive and results in acceleration, whereas if $\Delta < 0$, it results in deceleration of particles.

In moderately weak fields, when the spatially dependent Stark shift exceeds the resonance width

$$V_0 > V_c = (\gamma\Delta)^{1/2}, \tag{16}$$

the second term associated with the delayed gradient force becomes more important. Then, $\langle F\rangle$ vanishes not only for $v = 0$, but also for

$$|v| = v_c = \frac{\gamma}{2k}\left[\left(\frac{V_0}{V_c}\right)^4 - 1\right]^{1/2}$$

and if $\Delta > 0$ and $|v| < v_c$, it becomes the deceleration force. For sufficiently large field amplitude $\Delta > V_0 > \Delta(\gamma/\Delta)^{1/6}$, the maximum value of the force $|\langle F\rangle|$ exceeds F_{sp}. The delayed gradient force considered as a function of the velocity is dispersive with a characteristic width $\omega \sim \gamma$. At high velocities $\omega \gg \gamma$, the contribution of the delayed gradient force becomes small. This makes it possible to write down the interpolation formula for $\langle F_r\rangle$ valid for arbitrary velocities, using only the assumption that $\Delta \gg V_0, \gamma$:

$$\langle F_r\rangle = \hbar k\gamma\left\{w(\Delta-\omega) - w(\Delta+\omega) - \left(\frac{V_0}{\Delta}\right)^6\frac{\omega\Delta}{\gamma^2+4\omega^2}\right\}. \tag{17}$$

The force (17) considered as a function of the velocity exhibits resonances associated with the independent action of two opposed waves. These resonances are located in the range of high velocities $\omega = \pm\Delta$, and if $V_0 \gg \gamma$ they have the field width. Secondly, $\langle F_r\rangle$ has a resonance in the range of low velocities $\omega \sim \gamma$ associated with the simultaneous action of two waves. A special feature of this resonance, which we shall call adiabatic, is the absence of field broadening. Figure 1 shows schematically the force (17).

4. QUASICLASSICAL SOLUTION OF EQUATIONS FOR THE DENSITY MATRIX

As shown above, in the limit of slow atoms and strong forces the friction is governed mainly by the delay effect in stimulated transitions. We can generalize this result to the case of arbitrary saturation by obtaining a quasiclassical solution of the system of equations (3) for the density matrix of an atom subject to the condition

$$V_0, \ \Delta \gg \omega, \ \gamma, \tag{18}$$

but for arbitrary values of the parameters V_0/Δ and ω/γ. This situation was analyzed in Refs. 14 and 15 ignoring

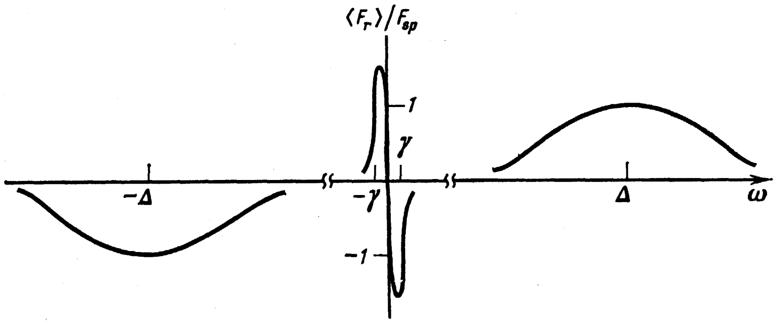

FIG. 1. Dependence of the average frictional force of Eq. (17) on the velocity in the case when $\Delta > V_0 > (\gamma\Delta)^{1/2}$.

spontaneous emission, and it was found that a convenient basis for the description of the state of an atom is provided by adiabatic solutions of the Schrödinger equation with the Hamiltonian (3):

$$\psi_1=\begin{pmatrix} v \\ u \end{pmatrix},\quad \psi_2=\begin{pmatrix} u^* \\ -v^* \end{pmatrix},$$

$$u(t)=\left(\frac{1}{2}+\frac{1}{2\varepsilon(t)}\right)^{1/2}\exp\left(-i\frac{\Delta}{2}\int_0^t \varepsilon\, dt'\right),$$

$$v(t)=-\operatorname{sign}\left(\frac{V(t)}{\Delta}\right)\left(\frac{1}{2}-\frac{1}{2\varepsilon(t)}\right)^{1/2}\exp\left(-i\frac{\Delta}{2}\int_0^t \varepsilon\, dt'\right), \tag{19}$$

$$\varepsilon(t)=\left(1+\left(\frac{2V(t)}{\Delta}\right)^2\right)^{1/2}.$$

The states ψ_1 and ψ_2 correspond to the motion of an atom experiencing potentials $\pm U(x) = \pm \hbar\Delta\varepsilon\cdot(x)/2$; when the field is switched off, they reduce to the ground and excited states, respectively. If the detuning from a resonance is sufficiently large

$$\Delta\gg\Delta_0,\quad \Delta_0=(V_0\omega)^{1/2}\ll V_0, \tag{20}$$

the adiabatic solutions (19) are valid for all values of t. In the case of a small detuning $\Delta\lesssim\Delta_0$, the adiabatic approximation is no longer obeyed in narrow regions of width $\delta t\sim 1/\Delta_0\ll 1/\omega$ near the nodes of the field. In such regions of the closest approach of the adiabatic terms $\pm U$ the mixing of the states ψ_1 and ψ_2 is strong, and this is described by a unitary Landau-Zener transition matrix

$$S_L=\begin{pmatrix} (1-R^2)^{1/2}e^{i\chi} & R \\ -R & (1-R^2)^{1/2}e^{-i\chi} \end{pmatrix}, \tag{21}$$

$$R=\exp(-\pi\xi),\quad \xi=\Delta^2/8\Delta_0^2,$$

$$\chi=\frac{\pi}{4}+\arg\Gamma(1-i\xi)-\xi\ln(e/\xi),$$

where $\Gamma(x)$ is the gamma function.

The problem can be solved allowing for spontaneous relaxation by the following procedure. Far from the field nodes the density matrix can easily be found in the adiabatic state basis (19). In the vicinity of a field node the system of equations (3) is solved by the Landau-Zener method. Matching these results, we obtain a general solution for $\rho(t)$ over half the field period π/ω and we can then find the arbitrary constants from the condition (4).

Region far from a field node

We shall consider specifically a time interval π/ω with a field node at $t=0$. For the values of t not too close to a field node ($|t|\gg 1/\Delta_0$) we shall go over to the density matrix in the adiabatic state representation (19) by a unitary transformation $S_A(t)$:

$$\rho(t)=S_A(t)\rho_A(t)S_A^+(t),\quad S_A=\begin{pmatrix} u^* & v \\ -v^* & u \end{pmatrix},$$

$$\frac{d}{dt}\rho_A=-S_A^+(\hat{\gamma}\rho)S_A,\quad \rho_A=\begin{pmatrix} \rho_{22} & \rho_{21} \\ \rho_{12} & \rho_{11} \end{pmatrix}. \tag{22}$$

In the equation for $\rho_A(t)$ we can ignore the terms which are proportional to uv and $(uv)^*$, since they oscillate rapidly if $|t|\gg 1/\Delta_0$ and give rise to corrections of order $\gamma/\varepsilon\Delta\ll 1$. Then, the population difference $A=\rho_{11}-\rho_{22}$ and the coherence $B=\rho_{21}$ of adiabatic states are described by independent equations

$$dA/dt=-{}^1/_2\gamma(1+1/\varepsilon^2)A+\gamma/\varepsilon,$$
$$dB/dt=-{}^1/_4\gamma(3-1/\varepsilon^2)B. \tag{23}$$

A general solution of these equations for the interval $0<t<\pi/\omega$ is

$$A(t)=A_0e^{-\mu_1(t)}+G(t),\quad \mu_1(t)=\frac{\gamma}{2}\int_0^t\left(1+\frac{1}{\varepsilon^2}\right)dt',$$

$$G(t)=\gamma\int_0^t\frac{dt'}{\varepsilon(t')}\exp[\mu_1(t')-\mu_1(t)], \tag{24}$$

$$B(t)=B_0e^{-\mu_2(t)},\quad \mu_2(t)=\frac{\gamma}{4}\int_0^t\left(3-\frac{1}{\varepsilon^2}\right)dt'.$$

The solution for the case when $-\pi/\omega<t<0$ differs only in the integration constants.

Region near a field node

We shall now consider a small region near a field node: $|t|\sim 1/\Delta_0\ll 1/\omega, 1/\gamma$. We can linearize here the interaction operator $V(t)\approx V_0\omega t$ and simplify considerably the relaxation operator

$$\hat{\gamma}\rho\approx\gamma\sigma_3/2,$$

where σ_3 is the Pauli matrix. The above relationship means that we are ignoring the terms $\gamma p/2$ and γq in the Bloch equations (7) and (8); these terms contribute small corrections of order $\gamma t\ll 1$ to a homogeneous solution. The inhomogeneous term γ in Eq. (7) should generally be retained because the difference q between the populations under strong saturation conditions is small and can be of the order of the contribution γt made by the inhomogeneous source.

Then, in the vicinity of a field node the system (3) becomes

$$\frac{d}{dt}\rho=-i[\hat{H}_L\rho]-\gamma\sigma_3/2,\quad \hat{H}_L=-\Delta\sigma_3/2-\Delta_0^2t\sigma_1. \tag{25}$$

The solution of the system is obtained using the evolution operator $S(t)$ of the Schrödinger equation with the Hamiltonian $\hat{H}_L$ from the theory of the Landau-Zener transitions:

$$\rho(t)=S(t)\tilde{\rho}(t)S^+(t),\quad \tilde{\rho}(t)=\tilde{\rho}(0)-\frac{\gamma}{2}\int_0^t dt'\, S^+(t')\sigma_3S(t'),$$

$$i\frac{dS}{dt}=\hat{H}_LS,\quad S=\frac{1}{2^{1/2}}\begin{pmatrix} \Phi_1^*+\Phi_2^* & \Phi_1-\Phi_2 \\ -\Phi_1^*+\Phi_2^* & \Phi_1+\Phi_2 \end{pmatrix}, \tag{26}$$

$$\Phi_1=\exp(-i\tau^2/2)\,\Phi({}^1/_2+i\xi/2,\ {}^1/_2;\ i\tau^2),\quad \tau=\Delta_0t,$$

$$\Phi_2=-i(\Delta/2\Delta_0)\tau\exp(-i\tau^2/2)\,\Phi({}^1/_2+i\xi/2,\ {}^3/_2;\ i\tau^2),$$

where $\Phi(\alpha,\gamma;x)$ is the confluent hypergeometric function.

Matching of solutions

The solutions (22) and (26) for $\rho(t)$ should be identical in the regions of their overlap $1/\Delta_0\ll|t|\ll 1/\omega,\ 1/\gamma$. If

$|t| \gg 1/\Delta_0$, the evolution operator $S(t)$ differs from $S_A(t)$ by the constant matrices

$$S(t) \approx S_A(t) \begin{cases} S_+, & t>0 \\ S_-, & t<0 \end{cases}, \quad S_+ = \begin{pmatrix} b^* & a \\ -a^* & b \end{pmatrix}, \quad S_- = S_+(a \leftrightarrow b),$$

$$a = \{{}^1/_2(1+R)\}^{1/2} \exp(i\chi_1),$$
$$b = \{{}^1/_2(1-R)\}^{1/2} \exp[-i(\chi-\chi_1)], \tag{27}$$
$$\chi_1 = -{}^1/_2\xi \ln(2e/\xi) + \arg\Gamma({}^1/_2 - i\xi/2).$$

The matrices $S_\pm$ are related to the Landau-Zener transition matrix, Eq. (21), by

$$S_+ S_-^+ = S_L.$$

Therefore, in the overlap regions the relationship between the solutions is

$$\rho_A(t) = S_\pm \tilde{\rho}(t) S_\pm^+. \tag{28}$$

The lower $\pm$ signs refer to $t \gtrless 0$.

Equation (22) for ρ_A in an overlap region can be simplified retaining only the source

$$\frac{d}{dt}\rho_A \approx -\frac{\gamma}{2} S_A^+(t)\sigma_3 S_A(t) \approx -\frac{\gamma}{2\varepsilon}\sigma_3,$$

$$\rho_A(t) = \rho_A(+0) - \frac{\gamma}{2}\sigma_3 \int_0^t \frac{dt'}{\varepsilon(t')},$$

$$\rho_A(+0) = \begin{pmatrix} (1-A_0)/2 & B_0 \\ B_0^* & (1+A_0)/2 \end{pmatrix}. \tag{29}$$

Substituting Eqs. (26) and (29) into Eq. (28), we obtain the relationship between the constants of integration:

$$\rho_A(+0) = S_+ \tilde{\rho}(0) S_+^+ - \frac{\gamma}{2} \int_0^\infty dt \left\{ S_+ S^+(t) \sigma_3 S(t) S_+^+ - \frac{\sigma_3}{\varepsilon(t)} \right\}. \tag{30}$$

The integral on the right-hand side of this relation converges for $t \sim 1/\Delta_0$ and, therefore, the upper limit of this integral can be replaced with infinity. We then naturally have $\varepsilon(t) \approx \{1 + (2\Delta_0^2 t/\Delta)^2\}^{1/2}$.

Similarly, matching of the solutions at $t<0$ gives the relationship between $\rho_A(-0)$ and $\tilde{\rho}(0)$. We thus obtain the following relation between the constants of the general solution (24) to the left and right of a field node:

$$\rho_A(+0) = S_L \rho_A(-0) S_L^+ - (S_+ \sigma^+ S_+^+ C + \text{H.c.}), \quad \sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{31}$$

$$C = \gamma \int_0^\infty dt (\Phi_1^2 - \Phi_2^2 - 2ab/\varepsilon) = (\gamma\Delta/4\Delta_0^2) ab [2\ln(\xi/2) + i/\xi - \psi({}^1/_2 + i\xi/2) - \psi(1 - i\xi/2)],$$

where $\psi(x) = d\ln\Gamma(x)/dx$.

It therefore follows that the "shakeup" experienced by an atom near a field node establishes an effective boundary condition (31) for the adiabatic solution. The first term on the right-hand side in the first equation of the system (31) represents a coherent perturbation due to the Landau-Zener transitions, whereas the second term is due to incoherent transitions caused by the spontaneous relaxation.

Using Eqs. (31), we continue the solution by half the field period π/ω and use the "half-period" condition (4) to find the constants of the adiabatic solution (24)

$$A_0 = \frac{(1-R^2)(\operatorname{ch}\mu_2 + \cos 2\varphi)(G + \gamma|\Delta| g/2\Delta_0^2) - R^2(G \operatorname{sh}\mu_2 - (\pi\gamma|\Delta|/2\Delta_0^2)\sin 2\varphi)}{(1-R^2)(1-e^{-\mu_1})(\operatorname{ch}\mu_2 + \cos 2\varphi) + R^2(1+e^{-\mu_1})\operatorname{sh}\mu_2}$$

$$\mu_{1,2} = \mu_{1,2}(\pi/\omega), \qquad G = G(\pi/\omega), \tag{32}$$

$$\varphi = \chi + \frac{\Delta}{2}\int_0^{\pi/\omega} \varepsilon\, dt, \qquad g = \ln\xi - \operatorname{Re}\psi(1 + i\xi).$$

The function $B(t)$ will not be needed later, which is why the fairly complicated expression for B_0 is not given.

Equations (22), (24), and (32) represent the solution of the problem formulated above and they describe the density matrix of an atom under the quasiclassical conditions of Eq. (18), allowing for spontaneous emission and for the Landau-Zener transitions.

5. FRICTIONAL FORCE IN A STRONG FIELD

It follows from Eq. (22) that $\rho(t)$ contains three terms. One of them, proportional to $A(t)$, exhibits no fast time oscillations. The other two, proportional to $B(t)$ and $B^*(t)$, oscillate at a high (Rabi) frequency.

In calculating the radiation pressure force we can ignore these fast oscillations. Then, F is determined only by the function A and is of the form $(x = vt)$

$$F(x) = -\frac{\partial U}{\partial x} A(x), \quad U(x) = \frac{\hbar\Delta}{2}\varepsilon(x). \tag{33}$$

The function A allows both for the delay due to spontaneous emission and for coherent "shakeup" which an inner state of an atom experiences on passing through a field node.

Quasisteady conditions

If $\Delta \gg \Delta_0$, the probability R^2 of a Landau-Zener transition is exponentially small, the function $g \simeq -(1/12\xi^2)$ is also small, and we have $A_0 \simeq G/(1-e^{-\mu_1})$, so that Eq. (24) for $A(t)$ can be represented in the form

$$A(t) = \frac{\gamma}{1-e^{-\mu_1}} \int_0^{\pi/\omega} \frac{d\tau}{\varepsilon(t-\tau)} \exp\left\{ -\frac{\gamma}{2}\int_0^\tau d\tau' \left[1 + \frac{1}{\varepsilon^2(t-\tau')} \right] \right\}. \tag{34}$$

This is identical with Eq. (13) if we allow for periodicity of the function $\varepsilon(t)$. In the case of slow atoms $\omega \ll \gamma$ the delay time is short $(\tau \sim 1/\gamma \ll 1/\omega)$ and $\varepsilon(t-\tau)$ in Eq. (34) can be expanded in powers of τ. This expansion is valid for all values of t if the condition $\omega/\gamma \ll 1$ is satisfied in the case of moderate saturation, whereas in the case of strong saturation we need a more stringent condition $\omega/\gamma \ll \Delta/V_0 \ll 1$.

If we confine ourselves to terms of the zeroth and first order in ω/γ, we obtain

$$F_g = -\frac{\partial U_{\text{eff}}}{\partial x}, \qquad U_{\text{eff}} = \frac{\hbar\Delta}{2}\ln[1+\varepsilon^2(x)],$$
$$F_r = -\frac{2\hbar v\Delta}{\gamma}\frac{\varepsilon^2(\varepsilon^2-1)}{(1+\varepsilon^2)^3}\left(\frac{d\varepsilon}{dx}\right)^2. \tag{35}$$

These expressions are in agreement with the results of Refs. 10–12. The gradient force is determined by the logarithmic potential: if $V_0 \gtrsim \Delta$, the depth of modulation is $\delta U_{\text{eff}} \sim \hbar\Delta$ and it rises slowly (logarithmically) as a function of the field. The frictional force F_r represents the delayed gradient force. In the case of weak saturation $(\gamma\Delta \ll V_0^2 \ll \Delta^2)$ it is identical with the results of perturbation theory (15). When saturation is close to unity $(V_0 \sim \Delta)$, the value of F_r has a characteristic scale $F_r \sim \hbar k\Delta\omega/\gamma$. In the case of strong saturation $(V_0 \gg \Delta)$ the frictional force is strongly inhomogeneous in space and its local maximum located near a field node exceeds $\langle F_r \rangle$ by a factor V_0/Δ. The expressions in Eq. (35) are valid in a wide range of values of the detuning Δ_0, $\gamma \ll \Delta \ll V_0^2/\gamma$. Outside this range we generally have to allow for the contribution of the spontaneous radiation pressure force.[10–12]

In the case of fast atoms $\gamma \ll \omega \ll \Delta^2/V_0$, retaining in Eq. (34) the terms of the zeroth and first order in γ/ω, we obtain

$$F_g = -\frac{\partial U_{\text{eff}}}{\partial x}, \qquad U_{\text{eff}} = C_g U(x),$$
$$C_g = \frac{4\cos\alpha K(\sin\alpha)}{\pi(1+\cos\alpha)}, \qquad \sin\alpha = \left[1+\left(\frac{\Delta}{2V_0}\right)^2\right]^{-1/2}, \tag{36}$$
$$\langle F_r \rangle = -\frac{C_r\hbar\gamma\Delta}{2v},$$
$$C_r = \frac{4}{\pi^2}\frac{K(\sin\alpha)[E(\sin\alpha)+\cos^2\alpha K(\sin\alpha)]}{1+\cos\alpha} - 1.$$

Here, K and E are complete elliptic integrals. The effective potential differs from the adiabatic potential U only by a constant factor C_g, which is proportional to the difference between the populations of the adiabatic states. The parameter $\gamma C_r/\omega$ represents the magnitude of the hysteresis. At low values of V_0/Δ the probability of spontaneous transitions is low, so that $C_g \approx 1$ and $C_r \approx 1/2(V_0/\Delta)^6$ in agreement with the formulas (10) and (15) obtained from perturbation theory. In the strong saturation case $(V_0 \gg \Delta)$ the populations of the adiabatic states equilibrate:

$$C_g = \frac{4}{\pi}\frac{\Delta}{V_0}\ln(8V_0/\Delta), \qquad C_r = \frac{4}{\pi^2}\ln(8V_0/\Delta) - 1.$$

Therefore, the amplitude of modulation of the potential and the average friction force saturate to within logarithmic terms in a strong field.

When the field is strong $(V_0/\Delta \gg 1)$, we can obtain simple expressions for the average force in two overlapping velocity intervals: 1) $\omega/\gamma \gg \Delta/V_0$; 2) $\omega/\gamma \ll 1$. In the first interval the values of τ close to t are important in the integral of Eq. (34), so that to within logarithmic terms the average frictional force is given by

$$\langle F_r \rangle = -\frac{2}{\pi}\frac{\hbar k\Delta\gamma^2}{\gamma^2+4\omega^2}\operatorname{cth}\left(\frac{\pi\gamma}{4\omega}\right)\ln\Lambda,$$
$$\Lambda = \begin{cases} 8V_0/\Delta, & \omega/\gamma \gg 1, \\ 8e^{-c}V_0\omega/\gamma\Delta, & \Delta/V_0 \ll \omega/\gamma \ll 1, \quad c=0.577. \end{cases} \tag{37}$$

If $\omega \gg \gamma$, this agrees with Eq. (36). In the velocity interval $\Delta/V_0 \ll \omega/\gamma \ll 1$ the frictional force depends weakly (logarithmically) on the velocity:

$$\langle F_r \rangle = -\frac{2\hbar k\Delta}{\pi}\operatorname{sign} v \ln\left(4.5\frac{V_0\omega}{\gamma\Delta}\right). \tag{38}$$

It is clear from Eq. (37) that the width of an adiabatic resonance on the ω scale is governed by the value of γ even under strong saturation conditions. In other words, there is no field broadening of a resonance and only the profile is altered compared with the case of a weak field described by Eq. (15).

In the second velocity interval $(\omega/\gamma \ll 1)$ the average frictional force depends on the parameter $\gamma\Delta/V_0\omega$, which governs the competition between the processes of spontaneous relaxation and growth of the field near a node. Calculations yield

$$\langle F_r \rangle = -\frac{2\hbar k\Delta}{\pi}Q\left(\frac{\gamma\Delta}{4V_0\omega}\right)\operatorname{sign} v,$$

$$Q(\beta) = \frac{1}{2}\int_0^\infty dx\, e^{-\beta x} \times \int_{-\infty}^{\infty} dy \frac{1+\beta y}{(1+y^2)^{3/2}}\frac{\exp[\beta\operatorname{arctg} y - \beta\operatorname{arctg}(x+y)]}{[1+(x+y)^2]^{1/2}}. \tag{39}$$

The asymptotic form of the function $Q(\beta)$ is

$$Q(\beta) \approx \begin{cases} 3\pi/16\sqrt{2\beta}, & \beta \gg 1, \\ \ln(2e^{-c}/\beta), & \beta \ll 1. \end{cases} \tag{40}$$

We can easily see that Eqs. (37)–(39) are identical in the velocity range $\Delta/V_0 \ll \omega/\gamma \ll 1$. In the case of very slow atoms when $\omega/\gamma \ll \Delta/V_0$, Eq. (39) is identical with the result of averaging of Eq. (35) over a field period.

Therefore, in a strong saturating field the frictional force is described always by Eqs. (37) and (39) irrespective of the atomic velocities. The force then obeys $\langle F_r \rangle \propto v$ in the range $\omega/\gamma < \Delta/4V_0$, rises logarithmically with the velocity when $\Delta/4V_0 < \omega/\gamma < 1$, reaches a maximum on the order of $\hbar k\Delta \ln(V_0/\Delta)$, and falls as $1/v$ when $\omega/\gamma > 1$.

Hysteresis

A qualitative understanding of the hysteresis may be gained by considering the weak saturation case $(V_0 \ll \Delta)$. The population of the adiabatic state ψ_1 is then close to unity $\rho_{11} = (1+A)/2 \approx 1$, and the population $\rho_{22} = (1-A)/2$ of the state ψ_2 is small and satisfies, in accordance with Eq. (22), the following equation

$$\frac{d}{dt}\rho_{22} + \gamma\rho_{22} \approx \gamma\left[\frac{V(t)}{\Delta}\right]^4. \tag{41}$$

An atom in the state ψ_1 has a potential energy $U > 0$ when

$\Delta > 0$ and is subject to the gradient force of Eq. (10). In the case of motion in the positive direction to the left of a field antinode the atom is decelerated by this force, whereas to the right it is accelerated. The probability of a transition to the state ψ_2 as a result of spontaneous emission is proportional to V^4 and peaks in the region of a field antinode. After such a transition an atom moves in a potential $-U$, which now decelerates its motion to the right of a field antinode. The decay of the population ρ_{22} occurs at a constant rate γ. If an atom returns to the initial state ψ_1 in the region of a field node, then after a period π/ω the deceleration is a maximum. Therefore, forward and reverse spontaneous transitions between adiabatic states have different spatial (temporal) localizations and a situation typical of hysteretic phenomena is observed.

The sign of the potential U is reversed for negative values of the detuning and there is a corresponding change in the sign of $\langle F_r \rangle$.

It should be noted that spontaneous transitions between the adiabatic states ψ_1 and ψ_2 govern the shifted components in the resonance fluorescence spectrum of atoms in an external field. In the case of an atom at rest the intensities of these components are the same; this is a consequence of the law of conservation of energy in the scattering processes. The appearance of the frictional force during the motion of an atom relative to a standing field implies breakdown of the symmetry between the shifted components. Obviously, if $\Delta > 0$, when a particle is decelerated by the delayed gradient force, the intensity of the anti-Stokes component is greater than that of the Stokes component.

Landau-Zener transitions

If $\Delta \lesssim \Delta_0$, then the expression for the force contains new functions of the parameters of the problem which include both a smooth dependence on the velocity of a particle, which enters via the parameters ω/γ and Δ/Δ_0, and a sharp dependence, which is due to the quasiclassical phase $\varphi \sim V_0/\omega \gg 1$. The latter circumstance has the effect that a change in the velocity gives rise to oscillations of the optical pressure force and their period is $\delta v/v \sim \omega/V_0 \ll 1$. The dependence of the force on the phase φ is due to an interference effect which occurs when an atom is traveling under the simultaneous influence of two adiabatic potentials $\pm U(x)$. The interference effects are considered in Refs. 14 and 15 under the conditions of pure coherent interaction with the field (i.e., ignoring spontaneous relaxation).

In the case of noncoherent interaction the "visibility" of the interference structure is governed by the competition between the processes of spontaneous relaxation and "shakeup" in the Landau-Zener transitions.

Since the saturation is strong for $\Delta \sim \Delta_0$, the main contribution to $\langle F_r \rangle$ comes from vicinities of field nodes and can simplify greatly the general expression for the frictional force. In a wide range of velocities $\omega/\gamma \gg \Delta/V_0$, we have

$$\langle F_r \rangle = -\frac{\hbar\gamma\Delta}{2v}\frac{1}{1+(\gamma/2\omega)^2}\left\{-1+\frac{4\mu}{\pi^2}\frac{(1-R^2)(\ln\Lambda+g/2)(1+\cos 2\varphi/\mathrm{ch}\,3\mu)+(\pi R^2/2)\sin 2\varphi/\mathrm{ch}\,3\mu}{(1-R^2)\,\mathrm{th}\,\mu\,(1+\cos 2\varphi/\mathrm{ch}\,3\mu)+R^2\,\mathrm{th}\,3\mu}\right\}, \tag{42}$$

$\mu = \pi\gamma/4\omega$.

The quantity Λ which occurs in the above expression has the same meaning as in Eq. (37). If $R = 0$, Eq. (42) becomes identical with Eq. (37).

We shall now consider the typical behavior of $\langle F_r \rangle$ for finite values of R.

In the case of slow atoms ($\Delta/V_0 \ll \omega/\gamma \ll 1$) the parameter μ is large so that the dependence on the phase φ disappears from Eq. (42) and there are no interference effects. Consequently, the average force is described by the expression

$$\langle F_r \rangle = -\frac{2\hbar k\Delta}{\pi}\,\mathrm{sign}\,v\{(1-R^2)(\ln\Lambda+g/2)-\pi\omega/\gamma\}. \tag{43}$$

If $1 - R^2$ is very small, then the main role is played by the logarithmic term which differs from the quasisteady-state expression (38) by the factor $1 - R^2$. In the case of small values of the detuning ($\Delta \ll \Delta_0$) we have $g \approx c + \ln\xi$ and the frictional force

$$\langle F_r \rangle = \frac{2\hbar k^2 v\Delta}{\gamma}\left(1-\frac{\Delta^2}{\Delta_1^2}\right), \quad \Delta_1 = 2\omega\left[\frac{V_0}{\gamma\ln(2{,}12\Delta_0/\gamma)}\right]^{1/2} \tag{44}$$

regarded as a function of Δ changes its sign at $\Delta = \Delta_1$.

The interference effects begin to appear when $3\mu \lesssim 1$. We shall consider the case of fast atoms when $\gamma/\omega \ll R$, so that the "shakeup" as a result of the Landau-Zener transitions is not small and is not balanced by relaxation. Then, we obtain the following formula from Eq. (42):

$$\langle F_r \rangle = -\frac{\hbar\gamma\Delta}{2v}\left\{-1+\frac{4}{\pi^2}\frac{[\ln(8V_0/\Delta)+g/2]\cos^2\varphi+\frac{1}{2}\pi\delta^2\sin\varphi\cos\varphi}{\cos^2\varphi+\delta^2}\right\}, \quad \delta^2 = 3R^2/2(1-R^2). \tag{45}$$

We can see that $\langle F_r \rangle$ is a rapidly oscillating function of the velocity which changes sign. The nature of the oscillations depends on the value of the parameter Δ/Δ_0.

If $\Delta/\Delta_0 \gtrsim 1$, then with the logarithmic precision we obtain

$$\langle F_r \rangle = -\frac{\hbar\gamma\Delta}{2v}\left\{-1+\frac{4}{\pi^2}\ln\left(\frac{8V_0}{\Delta}\right)\frac{\cos^2\varphi}{\cos^2\varphi+\delta^2}\right\}. \tag{46}$$

If $\delta^2 \approx \frac{3}{2}R^2 \ll 1$, the value of $\langle F_r \rangle$ as a function of the velocity has narrow peaks ($\Delta > 0$) or dips ($\Delta < 0$) near the points where $\varphi = \pi(n+\frac{1}{2})$. This is the condition for a multiphoton resonance with respect to ω in a strong field ($V_0 \gg \Delta \gg \omega$) subject to an allowance for the adiabatic behavior at field nodes (Landau-Zener resonance[14,15]).[1)] The width of the peaks or dips is $\delta v/v \sim \omega R/V_0 \ll 1$ and their relative amplitude is of the order of $\ln(V_0/\Delta)$. At a resonance point the force changes its sign and is equal to $\hbar\gamma\Delta/2v$. The frictional force of Eq. (46) averaged over rapid oscillations is identical with the quasisteady-state expression (36) containing the renormalized coefficient C_r. At low values of R there is naturally no renormalization.

In the range of small values of the detuning ($\Delta \ll \Delta_0$, i.e., $\delta^2 \gg 1$) we can ignore the logarithmic term in Eq. (45), so that

$$\langle F_r \rangle = \frac{\hbar\gamma\Delta}{2v}\left[1 - \frac{2}{3\pi}\cos\left(\frac{4V_0}{\omega}\right)\right]. \tag{47}$$

Near a resonance ($\Delta \ll \Delta_0$) the general expression for the force (45) simplifies considerably for arbitrary velocities:

$$\langle F_r \rangle = \frac{\hbar\gamma\Delta}{2v}\frac{1}{1+(\gamma/2\omega)^2}\left\{1 - \frac{2}{\pi}\frac{\mu}{\operatorname{sh} 3\mu}\cos\frac{4V_0}{\omega}\right\}. \tag{48}$$

If $\omega \gg \gamma$, then Eqs. (47) and (48) naturally become identical.

We can see that if $\Delta \ll \Delta_0$, then the nature of the force changes qualitatively and its sign is opposite to the quasisteady-state result of Eq. (36).

Figure 2 shows the dependence of the frictional force $\langle F_r \rangle$ on Δ and ω calculated using Eqs. (24), (32), and (33), and also by numerical integration of the Bloch equations.

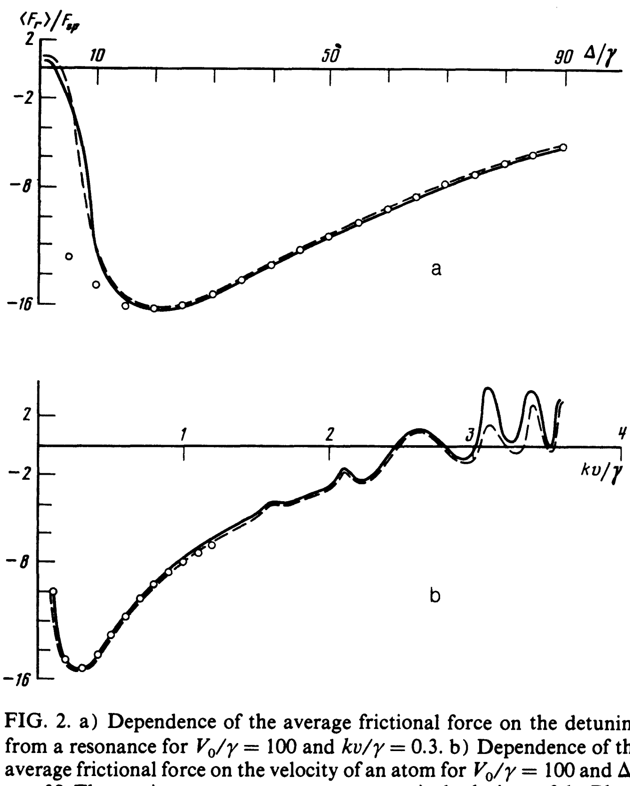

FIG. 2. a) Dependence of the average frictional force on the detuning from a resonance for $V_0/\gamma = 100$ and $kv/\gamma = 0.3$. b) Dependence of the average frictional force on the velocity of an atom for $V_0/\gamma = 100$ and $\Delta/\gamma = 30$. The continuous curves represent numerical solutions of the Bloch equations, and the dashed curves are calculations based on Eq. (32); open circles give the quasisteady-state solution of Eq. (34).

6. CASE OF FAST ATOMS

We shall now consider the case of a small detuning and fairly fast atom:

$$\gamma,\ \Delta \ll \omega \tag{49}$$

without any restriction on the relationship between V_0 and ω.

The solution of the equations for the density matrix can be found using perturbation theory with respect to γ and Δ in the basis of diabatic states in which a rigorous allowance is made for the interaction with the field when $\gamma = \Delta = 0$ (Refs. 17 and 18). Let us assume that $\rho(t) = S(t)\tilde{\rho}(t)S^+(t)$, where

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\varphi} & e^{-i\varphi} \\ e^{i\varphi} & -e^{-i\varphi} \end{pmatrix}, \quad \varphi(t) = \int V\,dt = -\frac{V_0}{\omega}\cos\omega t.$$

Then, the equations for $A(t) = \rho_{11} - \rho_{22}$ and $B(t) = \rho_{21}$, where ρ_{ij} $(i,j = 1,2)$ are elements of the matrix $\tilde{\rho}$, become

$$\begin{aligned} \frac{dA}{dt} &= -\frac{\gamma}{2}A + i\Delta(e^{2i\varphi}B - \text{c.c.}), \\ \frac{dB}{dt} &= -\frac{3}{4}\gamma B - \frac{\gamma}{4}e^{-4i\varphi}B^* - i\frac{\Delta}{2}e^{-2i\varphi}A - \frac{\gamma}{2}e^{-2i\varphi} \end{aligned} \tag{50}$$

and they can be solved subject to the "half-period" condition $A(t+\pi/\omega) = -A(t)$, $B(t+\pi/\omega) = B^*(t)$. In the lowest order in Δ and γ, we have[18]

$$A(t) \sim \Delta, \quad B(t) \approx B_0 = -\frac{2J_0(z)}{3+J_0(2z)}, \quad z = \frac{2V_0}{\omega}, \tag{51}$$

where J_0 is a Bessel function.

The expression for the average force obtained after integration by parts using the system (50) can be represented in the form

$$\begin{aligned} \langle F_r \rangle &= -\frac{1}{v}\left\langle \frac{dV}{dt}A(t) \right\rangle \\ &= \frac{\hbar\gamma\Delta}{2v}\left\{1 + \langle (1+i\varphi)e^{2i\varphi}B + \text{c.c.}\rangle - \frac{\gamma}{2\Delta}\langle \varphi A \rangle\right\}. \end{aligned} \tag{52}$$

In the lower order in γ and Δ we can drop the last term, and use Eq. (51) for B. As a result, we obtain

$$\langle F_r \rangle = \frac{\hbar\gamma\Delta}{2v}\left\{1 - \frac{4J_0(z)[J_0(z) - {}^1/_2 zJ_1(z)]}{3+J_0(2z)}\right\}. \tag{53}$$

In a strong field ($V_0 \gg \omega$), this expression is identical with Eq. (47).

7. CONCLUSIONS

Therefore, the adiabatic state representation and allowance for mixing of states by spontaneous relaxation and the Landau-Zener transitions makes it possible to obtain a very informative solution of the Bloch equations in a wide range of the parameters of the problem. An analysis given above shows that there are three characteristic regions of the parameters in which the nature of these solutions changes qualitatively. This is shown schematically in Fig. 3. In region I the frictional force is governed primarily by the difference between the spontaneous radiation pressure forces in the case of two opposed waves. In region II the delayed gradient force predominates and it is described by a quasi-

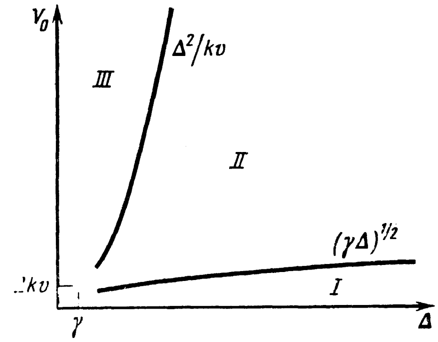

FIG. 3. Characteristic regions of solutions of the Bloch equations: I) weak stimulated transitions $V_0 < (\gamma\Delta)^{1/2}$; II) quasisteady-state region $\Delta > \Delta_0$, where the solution is described by Eq. (13); III) Landau-Zener transitions with $\Delta \lesssim \Delta_0$, where the solution is described by Eqs. (24) and (32).

steady-state solution and when the detuning is sufficiently large ($\Delta > (V_0\gamma)^{1/2}$) there is a narrow peak (adiabatic resonance) in the velocity interval $kv \lesssim \gamma$. If $\Delta > 0$, this force decelerates particles. The force reaches its maximum value, as can be seen from Fig. 2, for $\Delta \sim 0.2 V_0$ and $\omega \sim 0.3\gamma$ and it is then of the order of $|\langle F_r \rangle|_{\max} \sim 0.1 kdE_0$. Recently the basis of adiabatic ("dressed") states has been used also to analyze the frictional force in the quasisteady-state case.[19] The results obtained agree with those given above.

Since the slow-atom limit ($\omega \ll \gamma$) for which results are obtained in Refs. 10–12 is of special interest, we note that in the case of strong saturation ($V_0 \gg \Delta$) these results are valid in reality subject to a more stringent condition $\omega/\gamma \ll \Delta/V_0$. Finally, in region III the Landau-Zener transitions give rise to coherence between adiabatic states and the density matrix ρ exhibits oscillations with the Rabi frequency. Consequently, the frictional force considered as a function of the velocity is oscillatory and is due to the Landau-Zener resonances. These features can be seen most easily in the case of fast atoms $\omega \gtrsim \gamma$.

We shall conclude by noting that the above solution of the Bloch equations can be used also to study other problems in nonlinear resonance optics.

The authors are grateful to L. N. Shchur for his advice and to A. Tsvetnov for his help in the numerical calculations.

[1)]In a weak field $\varphi = \pi\Delta/2\omega$ these resonances also coincide with the Doppler structure.[16] In a strong field their positions and widths change considerably, and this why we shall call them the Landau-Zener resonances.

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Translated by A. Tybulewicz