Emission of neutrinos by neutron stars

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The importance of taking into account collective effects of the nucleon medium in the problem of the luminosity of neutron stars is demonstrated. New processes are introduced: nonresonance, which can be reduced to two-nucleon reactions but with allowance for multiple scattering in intermediate states, and resonance, associated with collective vibrations of the medium such as spin sound or a pion condensate. In the low-temperature approximation $T \ll \varepsilon_F^p \sim 5(\rho/\rho_0)^{4/3}$ MeV, where ρ_0 is the density of a nucleus, the contribution to the luminosity of all the most important processes is calculated. The resulting luminosity depends strongly on the mean density ρ of the nucleon matter and at larger ρ appreciably exceeds the estimates of earlier studies that did not take into account the collective effects. Therefore, neutron stars having a higher mean density (or mass) emit neutrinos much more copiously than ones having a lower density (or mass). This opens up a possibility of explaining the low experimental limits on the surface temperatures of some neutron stars assumed to exist in the remnants of young supernovas without recourse to the pion condensate hypothesis. The mean free paths of neutrinos in different direct processes are calculated. The minimal neutrino mean free path ~ 10 km is comparable to the radius of the star at a temperature $T \approx T_{op} \lesssim \varepsilon_F^p$. Lowering T_{op} compared with earlier estimates leads to the retention of neutrinos in a neutron star during the first minutes of its life. Experimental consequences of the obtained results are discussed.

I. INTRODUCTION

According to current ideas, neutron stars are formed when very massive stars collapse after supernova explosions.[1,2] The characteristic mass of a neutron star is $M \sim M_\odot$, where $M_\odot$ is the mass of the Sun, the radius being $R \sim 10$ km. In the course of the collapse, the neutron star is heated to a temperature $T \sim 10\ \mathrm{MeV} \approx 1.16 \cdot 10^{11}$ °K, after which it cools, emitting neutrinos and photons.

The first calculations of various processes that contribute to the cooling of neutron stars were made in 1965.[3,4] At that time, they were mainly of academic interest. The situation was radically changed in connection with a unique possibility that appeared recently for carrying out experiments in space.

The Einstein space observatory detected soft and nonpulsating radiation from sources at the centers of four supernova remnants. These observations were used to estimate the surface temperatures of the associated neutron stars under the assumption that all the radiation is thermal. In addition, dozens of upper limits $T_s^{\max}$ on the surface temperatures of neutron stars were obtained under the assumption that they were formed in explosions of young supernovas.[5] A new and rapidly progressing experimental field developed in which one could expect fascinating results relating to the detection of central sources and the lowering of the upper limits of the surface temperatures of neutron stars assumed to exist in supernova remnants.

Analysis of the theoretical calculations of different reactions that lead to cooling of neutron stars, a comparison of them with experimental data, and references to the literature can be found, for example, in the review of Ref. 6, and also in later publications.[7,8]

Study of various reactions led to the development of a certain scenario of neutron star cooling, according to which the main contribution to the cooling at a temperature $T_{in} \gtrsim 10^9$ °K of the internal region of the star is made by neutrino emission from this internal region, which has a density $\rho \gtrsim 10^{14}$ g/cm^3, determined basically by the modified URCA process $nn \to npe\bar{\nu}$ ($T_s \sim (10^{-2}–10^{-3}) T_{in}$; Ref. 9). The ordinary β decay of the neutron, $n \to pe\bar{\nu}$, is strongly suppressed due to the difficulty of satisfying the momentum conservation law (the Fermi momenta of the proton and electron are $p_{Fp} \approx p_{Fe} \approx 85(\rho/\rho_0)^{2/3}$ MeV/c, where $\rho_0 = 2.8 \cdot 10^{14}$ g/cm^3 is the density of a nucleus, while the neutron Fermi momentum is $p_{Fn} \approx 340(\rho/\rho_0)^{1/3}$ MeV/c, $cq \sim T \lesssim 10$ MeV). Therefore, a second particle is required in order to satisfy the momentum conservation law. As the temperature is lowered, crust neutrinos[10] and then photons begin to make a contribution.

Calculations of the cooling of neutron stars have been made for different scenarios. Different equations of state have been introduced, assumptions have been made about the presence or absence of neutron and proton superfluidity and about the magnetic field, the values of poorly known physical parameters have been varied, and so forth. It has been found that the resulting luminosities depend strongly on the assumptions made about the internal structure of the neutron stars and the choice of the physical parameters. Nevertheless, it has been shown that for all the adopted assumptions about the equation of state, the presence or absence of superfluidity, the strength and radial dependence of the magnetic field, etc., the low experimental upper limits on the surface temperatures of some neutron stars cannot be explained by means of the processes usually considered.

If in the interior regions of neutron stars there were a

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sufficient number ($\rho_\pi \sim \rho_0$) of free pions, then through the reaction $n\pi^- \to ne\bar{\nu}$ neutron stars could be cooled much more rapidly than follows from the modified URCA process.[3] However, there were no grounds for supposing free pions present in the matter of neutron stars. Indeed, at the characteristic density of neutron matter, $\rho \sim \rho_0$, the neutron chemical potential is $\mu_n \sim 60$ MeV, and the production of pions with energy $\gtrsim m_\pi \approx 140$ MeV is suppressed by the exponential factor $\sim e^{-80/T}$ (here T is given in MeV). Interest in such processes was rekindled by the hypothesis of pion condensation in dense neutron matter.[11,12] Then a condensate pion could play the part of the free pion in the reaction $n\pi^- \to ne\bar{\nu}$. The corresponding specific luminosity (the luminosity of unit volume) is found to be of the same order as if the pion were free.[13] Nevertheless, doubts were expressed—first, a pion condensate need not develop in the matter of a neutron star if the central density is below the critical value for pion condensation, this value varying in the interval $\rho_c \sim \rho_0 - 4\rho_0$ depending on the chosen parameters; second, in some models with strong nucleon–nucleon correlations, which suppress both the πNN and the πNN^*_{33} interaction vertex, a pion condensate does not arise at all.[14]

As candidates for processes leading to rapid cooling, consideration has also been given to the processes $d \to ue\bar{\nu}$, $ue \to d\bar{\nu}$, which take place on up and down quarks.[15] However, the more detailed study of Ref. 8 showed that the corresponding reactions make a much smaller contribution than the π-condensate process. Thus, it remained to conclude that either there is a pion condensate in neutron stars or that neutron stars are not present in the supernova remnants in which the values of $T_s^{\max}$ are sufficiently low. However, the latter conclusion is difficult to reconcile with the experimental data on the frequency of supernova explosions and of the formation of pulsars in the Galaxy[16] and also with the successful description of the data on the fluctuations of pulsar periods, which require objects to have very low temperatures.[17]

All the studies mentioned above were based on the assumption that the neutrinos mean free path is much greater than the radius of the system. It is only in this case that the effects of neutrino absorption can be ignored. In Refs. 18 and 19 it was shown that in the early stage of cooling, i.e., at comparatively high temperatures, the neutrino mean free path is appreciably less than the radius of the neutron star, which radiates as a black body. However, this occurs during the first minutes (in the absence of a pion condensate) or hours (if there is a pion condensate) from the time of formation of the neutron star. Then, as the cooling proceeds, the neutrino mean free path becomes greater than the radius of the system, and during the entire subsequent evolution the radiation is nonequilibrium. For comparison with the experimental values of the surface temperatures of neutron stars formed after supernova explosions that occurred many years ago these modifications are unimportant. Nevertheless, they could be extremely important, for example, for studying the possible pulsations of neutron stars immediately after their formation,[1)] and also for the description of the ejection of supernova shells.

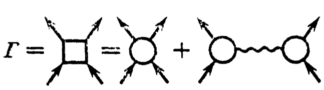
In view of how important learning whether neutron stars are present in supernova explosions is for obtaining new information about the structure of neutron stars, for testing the hypothesis of pion condensation in neutron stars, for describing the pulsations of a neutron star during the initial stage of its cooling, and also for explaining the part played by neutrinos in the ejection of the supernova shell, one should consider carefully whether all possibilities have been exhausted for increasing the matrix elements of the processes usually considered, and also whether there might not be other, as yet unstudied, processes capable of leading to rapid cooling of neutron stars. These are the matters to which the present paper is devoted.

Some of the results have been briefly presented in Ref. 21. In the following section, we introduce the formalism needed in the present work. The description of the NN interaction is constructed in accordance with Migdal's theory of finite Fermi systems, while Weinberg–Salam theory is used to describe the weak interaction. In Sec. III, we discuss new processes, forbidden in the vacuum but possible in a nucleon medium, which, as will be shown, significantly increase the resulting neutrino luminosity. We describe the approximation in which the luminosity of the neutron star will be calculated. In Sec. IV, we find the contribution of the nonresonant processes to the luminosity, and in Sec. V we estimate the luminosity of the resonant processes. The luminosities of the reactions that take place on the pion condensate are found under the assumption that it exists. It is shown that the pion condensation hypothesis is not required to explain the low experimental upper limits on the surface temperatures of some neutron stars. In Sec. VI, we find the mean free paths for the processes studied in the paper. Characteristic cooling curves are given. In Sec. VII, we formulate some unresolved problems.

II. GENERAL FORMALISM

1. Description of the strong interaction

The equation for the NN scattering amplitude has the form[12]

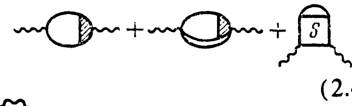
$\Gamma =$  $= \Gamma_1 + \mathcal{T}_1 D_\pi \mathcal{T}_1$, (2.1)

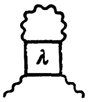
where Γ_1 is the NN scattering amplitude having no pion excitation in the channel considered, $\mathcal{T}_1$ is the vertex of the πN interaction that does not contain a pion pole, and nucleon–nucleon correlations are included in Γ_1 and $\mathcal{T}_1$; D_π is the pion Green's function:

$$D_\pi^{-1} = (\omega_\pi + \mu_\pi)^2 - k^2 - m_\pi^2 - \Pi(k, \omega_\pi, \mu_\pi, T), \quad \hbar = c = 1, \quad (2.2)$$

$D_\pi = (D_{\pi^0}, D_{\pi^+}, D_{\pi^-})$ for the π^0, π^+, π^- mesons, $D_{\pi^-}(\omega_\pi, \mathbf{k}) = D_{\pi^+}(-\omega_\pi, -\mathbf{k})$, m_π is the mass, μ_π is the chemical potential, and ω_π and $\mathbf{k}$ the frequency and momentum of the pion; $\Pi(k, \omega_\pi, \mu_\pi, T)$ is the temperature-dependent polarization operator of the pion. It is basically determined by the following graphs[12,22–24]:

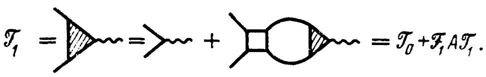
$$\Pi = \Pi^{MF} + \Pi^F, \quad (2.3)$$

$\Pi^{MF}(k,\ \omega_\pi,\ \mu_\pi,\ T\ll m_\pi)=$  , (2.4)

$\Pi^F =$  . (2.5)

In (2.4), the first graph corresponds to pion decay into a nucleon particle and hole, the second into an N^*-isobar and nucleon hole, and the third corresponds to S-wave scattering; the graph (2.5) corresponds to allowance for the pion fluctuations. The hatched vertices denote allowance for the nucleon correlations.

In the present paper, we do not require the explicit form of the pion polarization operator, and we therefore restrict ourselves to the comments already made. For the details, we refer to Refs. 12, 23, and 24. For $\mathscr{T}_1$, we have the symbolic equation[12]

 $=\mathscr{T}_0+\mathscr{F}_1 A\mathscr{T}_1.$ (2.6)

Here, $\mathscr{T}_{0\alpha}=f\,\boldsymbol{\sigma}\mathbf{k}\tau_\alpha$ is the bare vertex of the πNN interaction, $f\approx m_\pi^{-1}$, and $\mathscr{F}_1$ is a local interaction of quasiparticles in nuclear matter, this being introduced through the constants of the nuclear matter as follows[12]:

$$dn/d\varepsilon_F|_{p_F=p_0}\mathscr{F}_1=\{f+f'\boldsymbol{\tau}_1\boldsymbol{\tau}_2+(g+g'\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\}\delta(\mathbf{r}_1-\mathbf{r}_2),\quad (2.7)$$

where $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are the isotopic and $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ the spin matrices of the two nucleons, and p_0 is the Fermi momentum at the nuclear density. The constants f, f', g, g' are found by comparing the theoretical and experimental data, and

$$A=-i\int G(p+k)G(p)d^4p/(2\pi)^4 \quad (2.8)$$

is a loop of fermions (nucleons) and N^*_{33} resonances. There is an exactly similar loop correction of the local interaction Γ_1. Because of the dependence on the different constants, and also the different values of the momentum transfers and frequencies in the different vertices, the factors of the nucleon–nucleon correlations are, in general, different for the $\pi^\pm np$, $\pi^0 nn$, and $NNl\bar{\nu}$ vertices.

Since the numerical values of the parameters of the theory of finite Fermi systems for a neutron system are unknown and the expression for the results is extremely cumbersome in the general case, the nucleon correlations will be taken into account schematically in what follows, all vertices being suppressed by one and the same factor $\gamma(\omega, k)$. Because of the uncertainties associated with the different treatment of the nucleon correlations, this parameter may vary over fairly wide ranges $\gamma\sim0.3$–1.[12,25] For general understanding, we nevertheless indicate the structure of the nucleon correlation parameter:

$$\gamma_{np}(k,\omega)\sim[1+g^- p_F p_0^{-1}\Phi_1(k,\omega_\pi,\mu_\pi)]^{-1},$$
$$\gamma_{nn}(k,\omega)\sim[1+g^{nn}p_F p_0^{-1}\Phi(k,\omega_\pi)]^{-1},$$

where

$$\Phi=\Phi_1(k,\omega_\pi)+\Phi_1(-k,-\omega_\pi)$$

is the Lindhard function, and g^- and g^{nn} are constants of the theory of finite Fermi systems. We note that in a realistic situation the introduction of the two constants g^- and g^{nn} is insufficient on account of the different suppressions of the πNN and πNN^* interaction vertices,[12] and also on account of the dependence of the vertices of the $NNl\bar{\nu}$ interaction on the constants f, f', g, g'.

For simplicity, we shall in what follows ignore the local block Γ_1 compared with the contribution of the medium-softened one-pion exchange. At these neutron matter densities ($\rho\sim\rho_0-5\rho_0$) and parameter values, allowance for Γ_1 leads to a comparatively moderate change in the specific luminosities. (At characteristics values $|D^{-1}(\omega_\pi=0, k\sim p_F^n)|\lesssim1$ and $\gamma\sim1$, this change is negligible and increases to a factor ~1 as γ decreases to 0.3–0.4.) The generalization to the case $\Gamma_1\neq0$ is laborious but does not present any fundamental difficulties.

The degree of softening of the pion propagator (2.2) in the dense nucleon medium is determined by how much it differs from the corresponding vacuum value $D^{-1}_{\pi\,\mathrm{vac}}=\omega_\pi^2-k^2-m_\pi^2$. Since the characteristic quantity in (2.2) when $\rho\sim\rho_0\sim m_\pi^3$ is m_π, and the characteristic frequencies are $\omega_\pi\sim T\ll m_\pi$, in what follows we shall use in place of Re D_π^{-1} the expression $D_\pi^{-1}(\omega_\pi=0)$. At sufficiently high densities of the nuclear matter, $\rho\gtrsim\rho_0$, $\mathrm{Re}|D^{-1}(\omega_\pi=0)|$, there is a minimum at $k=k_0\sim p_F^n$. Therefore, we shall sometimes use the simpler expression[2)]

$$\mathrm{Re}|D_\pi^{-1}(\omega_\pi=0)|\approx\tilde{\omega}^2+\gamma_1(k^2-k_0^2)^2/4k_0^2. \quad (2.9)$$

The gap $\tilde{\omega}$ is found from the self-consistent equation

$$\tilde{\omega}^2=m_\pi^2+k^2+\Pi(k,\ \omega_\pi=0,\ T,\ \tilde{\omega}). \quad (2.10)$$

Under the assumption that the graph (2.5) corresponding to the pion fluctuations is small, we have

$$\tilde{\omega}^2\approx\omega_0^2=m_\pi^2+k^2+\Pi^{MF}(k,\ \omega_\pi=0,\ T). \quad (2.11)$$

Then the transition to the pion condensate occurs at the density $\rho=\rho_c$ at which $\omega_{L0}^2(k_0)=0$ and is a phase transition of the second kind. In reality, $\Pi^F\neq0$ and the π-condensate phase transition is of the first kind,[26,22–24] though in the limit $T\to0$ this difference may not be too significant for numerical reasons.[26]

2. Description of the weak interaction

For the description of the weak processes in the Weinberg-Salam model, we use the nonrelativistic expression for the interaction Lagrangian.[27]

With participation of the charged current

$$\mathscr{L}_c=\frac{G}{\sqrt{2}}\chi_p^+(\delta_{\mu0}-g_A\delta_{\mu i}\sigma_i)\chi_n l_\mu, \quad (2.12)$$

where χ_n and χ_p^+ are the spinors of the incoming neutron and the outgoing proton, $G\approx1.7\cdot10^{-5}$ GeV^{-2} is the Fermi constant of the weak interaction, $g_A\approx1.26$ is the constant of the axial-vector interaction,

$$l_\mu=\bar{u}(q_1)\gamma_\mu(1-\gamma_5)u(q_2), \quad (2.13)$$

is the leptonic current, $q_1=(\omega_1, \mathbf{q}_1)$ and $q_2=(\omega_2, \mathbf{q}_2)$ are the lepton 4-momenta, and γ_μ are the Dirac matrices.

With participation of the neutral currents,

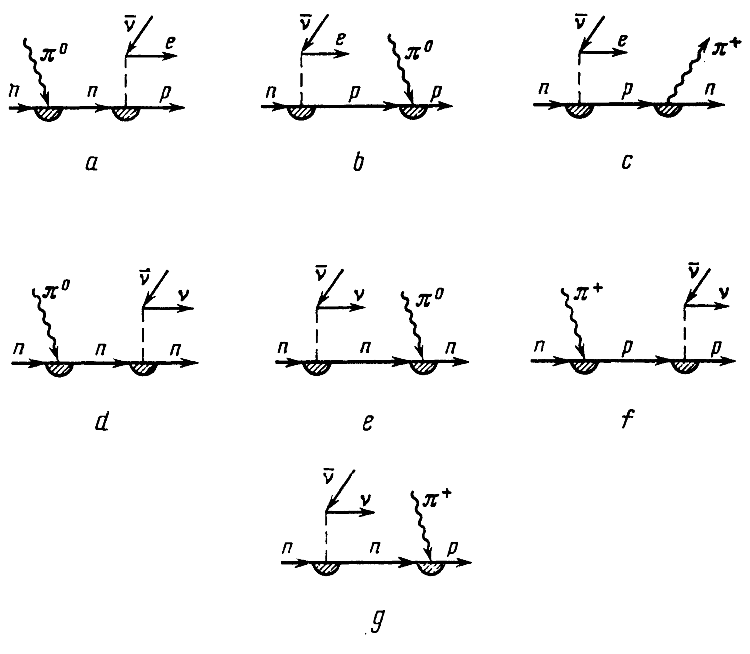


FIG. 1. Diagrams of one-nucleon processes. The continuous lines represent nucleons and leptons, as indicated in the figure. The broken lines represent the weak interaction. The wavy line corresponds to a medium pion (indicated in the figure) or to a quantum of the local NN interaction. In the presence of a pion condensate, the wavy line must be replaced by an external π-condensate field. The hatched block takes into account the nucleon-nucleon correlations.

$$\mathscr{L}_n = -\frac{G}{2^{3/2}}\chi_1^+(\delta_{\mu 0} - g_A\delta_{\mu i}\sigma_i)\chi_2 l_\mu, \tag{2.14}$$

for the neutrons and for the protons

$$\mathscr{L}_p = \frac{G}{2^{3/2}}\chi_1^+(c_V\delta_{\mu 0} - g_A\delta_{\mu i}\sigma_i)\chi_2 l_\mu,$$

$$c_V = 1 - 4\sin^2\theta_W,\ \sin^2\theta_W \approx 0{,}23. \tag{2.15}$$

The nucleon Green's function is

$$G_N = (E - p^2/2m_N^* + \mu_N)^{-1}, \tag{2.16}$$

where E and p are the energy and the momentum, μ_N is the nucleon chemical potential, and m_N^* is the effective mass. To small corrections associated with the large nucleon mass,

$$G(E+\omega) = \pm(\omega + \Delta\mu)^{-1}. \tag{2.17}$$

Here, ω is the total energy carried away by the leptons, and $\Delta\mu = 0$ for processes on the neutral currents (2.14), (2.15); $\Delta\mu = \mu_n - \mu_p$, where the indices n and p correspond to neutrons and protons, respectively, for processes on the charged currents (2.12). The $+$ and $-$ signs correspond to the cases when the weak process takes place on the outgoing and ingoing nucleon, respectively.

III. RESONANT AND NONRESONANT REACTIONS

As already mentioned in the Introduction, the ordinary neutron β decay $n \to pe\bar{\nu}$ is suppressed at the characteristic neutrino energies and momenta, $\omega_\nu \sim \gamma_\nu \sim T$, because of the difficulty of satisfying the momentum conservation law. An additional particle capable of carrying the extra momentum $\sim p_F^n$ is therefore required. Even in the most favorable case when free pions are taken as these additional particles (represented in Fig. 1 by the wavy lines), these processes are still suppressed because of the absence of free pions in the matter of a neutron star. Therefore, in earlier studies devoted to the problem of the luminosity of neutron stars the one-nucleon processes of Fig. 1 were not taken seriously. But in a nucleon medium, such processes may take place quite differently; for in a medium there is not only the comparatively high-lying pion excitation branch, which with decreasing density ρ goes over into the vacuum $\omega_\pi^{\text{vac}} = (m_\pi^2 + k^2)^{1/2}$, but also low-lying excitation branches. Spin-isospin excitations are of this type. For the π^0 mesons, the low-lying spin-isospin branch has the asymptotic behavior $\omega \approx s_0 k v_F^N$, $k \to 0$, i.e., corresponds to spin-isospin sound. For charged π^+ mesons in a neutron medium, a low-lying excitation branch occurs only when the medium has a sufficiently high density: $\rho > \rho_c^+ \sim \rho_0$. Then $\omega_{\pi^+} < -\mu_n$ and a small admixture of protons in the matter of the neutron star goes over into n and π^+, i.e., the presence of such a π_s^+ branch means the production of a π_s^+ condensate.

There are also acoustic collective vibrations associated with the local NN interaction. At $T \neq 0$, the low-lying excitation branches can be populated, and the quasiparticles corresponding to them can participate in neutrino reactions. The possible processes are shown in Figs. 1 and 2. The wavy lines

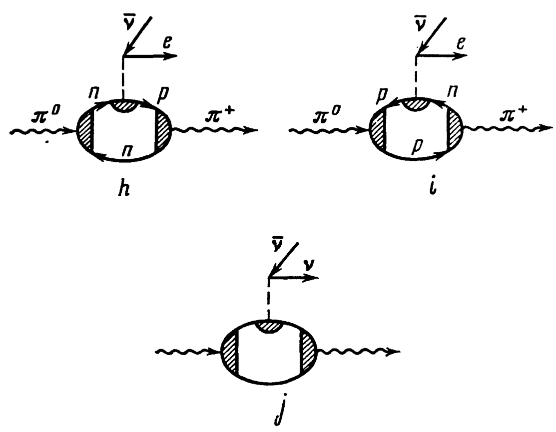
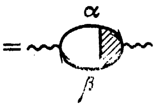


FIG. 2. Diagrams of one-pion reactions. The wavy lines correspond to medium pions (indicated in the figure) or quanta of the local interaction. The other notation is the same as in Fig. 1.

in these figures represent soft excitations, for example, spin-isospin acoustic excitations or spin-spin acoustic excitations. In the presence of a pion condensate, the wavy line in Fig. 1 must be replaced by the external π-condensate field. As Fig. 2 shows, in the initial and final states one can actually have processes without participation of a nucleon at all. These are one-pion processes if the wavy line corresponds to a pion from the spin-isospin branch, or processes corresponding to acoustic vibrations of local type. All these processes are decidedly collective and are possible only in the presence of a medium. They can be called resonant processes, since the imaginary part of the propagator of the corresponding quasi-particle is proportional to $\delta(\omega - \omega(k))$, where $\omega(k)$ is the pole of the propagator that determines the dispersion law of the excitations.

On the other hand, because of the possibility of pion decay into a particle and hole, the pole term of the polarization operator

$\Pi_p =$  (3.1)

has in a certain region of frequencies ω_π and momentum transfers $k(\omega_\pi < kv_F^N)$ an imaginary part different from a δ-function form:

$$\mathrm{Im}\,\Pi_{\alpha\beta}{}^A = -2\zeta f^2 k^2 \pi\gamma^2 \times \int (n_p{}^\beta - n^\alpha_{p+k})\,\delta(\omega_\pi + E_p{}^\alpha - E^\beta_{p+k} + \mu_\alpha - \mu_\beta)\,\frac{d^3p}{(2\pi)^3},$$

$$\mathrm{Im}\,\Pi_{\alpha\beta}{}^A = -\,\mathrm{Im}\,(D^A_{\pi\alpha\beta})^{-1}, \qquad (3.2)$$

where D_π^A is the advanced pion Green's function. Here, the indices α and β relate to the nucleon species (n,p), $\zeta = 1$ for π^0 mesons and 2 for $\pi^\pm$ mesons. For charged pions in the intermediate states (3.2) different nucleons n and p participate.

The presence of the non-δ-function imaginary part in the propagator means physically that pions are created and annihilated with the passage of time. Once created, they can participate in neutrino reactions. We shall refer to such reactions as nonresonant.

The square of the matrix element of the one-nucleon reaction in Fig. 1, summed over the spins and final states of the pion field, is

$$|M|^2 \sim \langle i|\varphi^+\varphi|i\rangle = 2\,\mathrm{Im}\,D_\pi{}^A/[\exp(\omega_\pi/T) - 1]. \qquad (3.3)$$

Using (3.3) and (3.2), and also the general expressions for the luminosities of the one- and two-nucleon processes (see (4.1) below), one can show that the calculation of the luminosity of any one-nucleon nonresonance process of Fig. 1 or one-pion nonresonance process of Fig. 2 reduces to the calculation of the luminosity of the corresponding two-nucleon process (denoted in Figs. 3–5 by the same Latin letters $a, b, \ldots$, as in Figs. 1 and 2). This is simply a consequence of the optical theorem. Generally speaking, the luminosities of all these processes could be found directly using the formalism of the optical theorem. However, the analogy with the calculations made in the literature would then be less transparent. Therefore, we here use a different, more physical language. Bearing in mind what has been said, we shall in what follows, when calculating the nonresonant processes, consider only the corresponding two-nucleon processes instead of the one-nucleon and one-pion processes.

The two-nucleon reactions that contribute to the resulting neutrino luminosity are shown in Figs. 3–5. The group of diagrams shown in Fig. 3 corresponds to the process $nn \to npe\bar{\nu}$ with charged weak currents and can be called the generalized URCA process by analogy with the name used

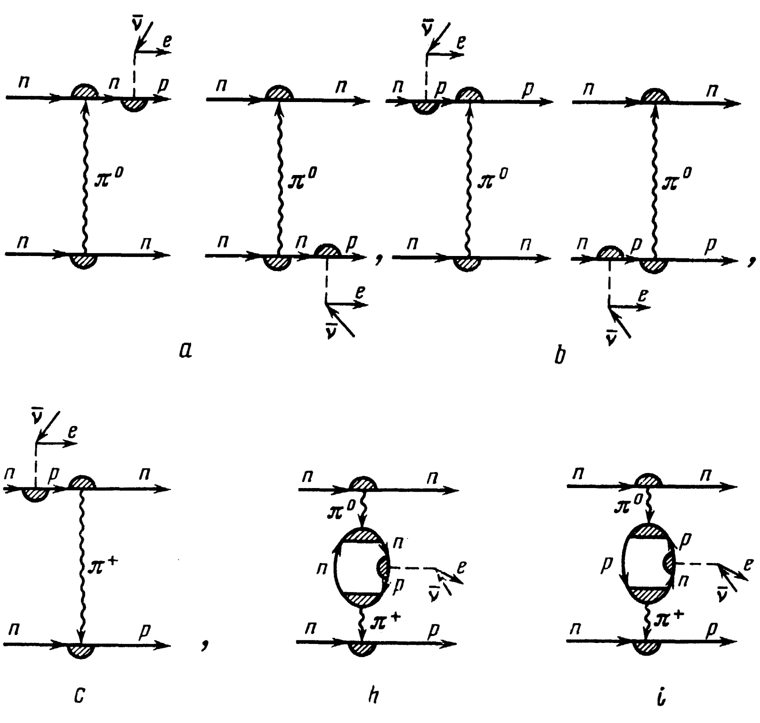


FIG. 3. Diagrams of the process $nn \to npe\bar{\nu}$ (generalized URCA process). The wavy line corresponds to the nucleon-nucleon interaction. The remaining notation is the same as in Fig. 1.

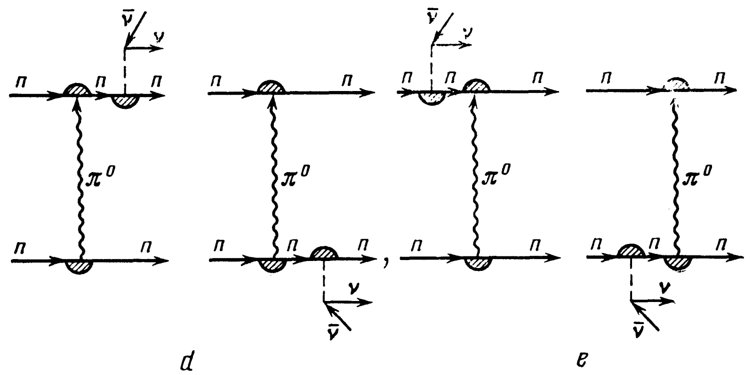


FIG. 4. Diagrams of the process $nn \to nn\nu\bar{\nu}$. The notation is the same as in Fig. 3.

earlier. Figures 4 and 5 show the diagrams of the processes $nn \to nn\nu\bar{\nu}$ and $np \to np\nu\bar{\nu}$ with neutral currents.

Two-nucleon reactions have been investigated earlier in many studies, the most detailed of which is that of Ref. 27. It was shown there that the main contribution to the specific luminosity is made by the modified URCA process $nn \to npe\bar{\nu}$. A certain contribution is also made by the production of neutrino pairs $\nu\bar{\nu}$ accompanying nucleon scattering $nn \to nn\nu\bar{\nu}$ and $np \to np\nu\bar{\nu}$ on neutral currents. The nucleon-nucleon interaction was approximated by one-pion exchange with the free pion propagator. With regard to the local interaction, it was not given a loop correction. Even in this case, allowance for the local interaction only slightly changes the luminosities of the other processes (by a factor 0.5–1). The cutoff of the one-pion exchange potential at short distances leads to even smaller modifications.

As is obvious from the description of the nucleon-nucleon interaction formulated above, in a dense nucleon medium it differs radically from the vacuum interaction, and this significantly changes the results of the earlier studies from both the qualitative and quantitative points of view.

Thus, the principal changes that we have introduced into the description of the two-nucleon reactions consist of the following: 1) the pion Green's function is significantly modified in the medium, 2) the πNN vertices acquire loop corrections, 3) weak processes also take place in intermediate particle-hole states.

It should be pointed out that the introduction of medium effects is necessary from a fundamental point of view. At the first glance, it might appear that the use of the vacuum pion propagator in the two-nucleon processes symbolizes only the belief of the authors in the hardness of the πN interaction, suppressed, for example, for some reason by strong nucleon correlations. Until reliable experimental data is obtained, such a point of view has a valid right to existence. In reality, the approximation under examination is inconsis-

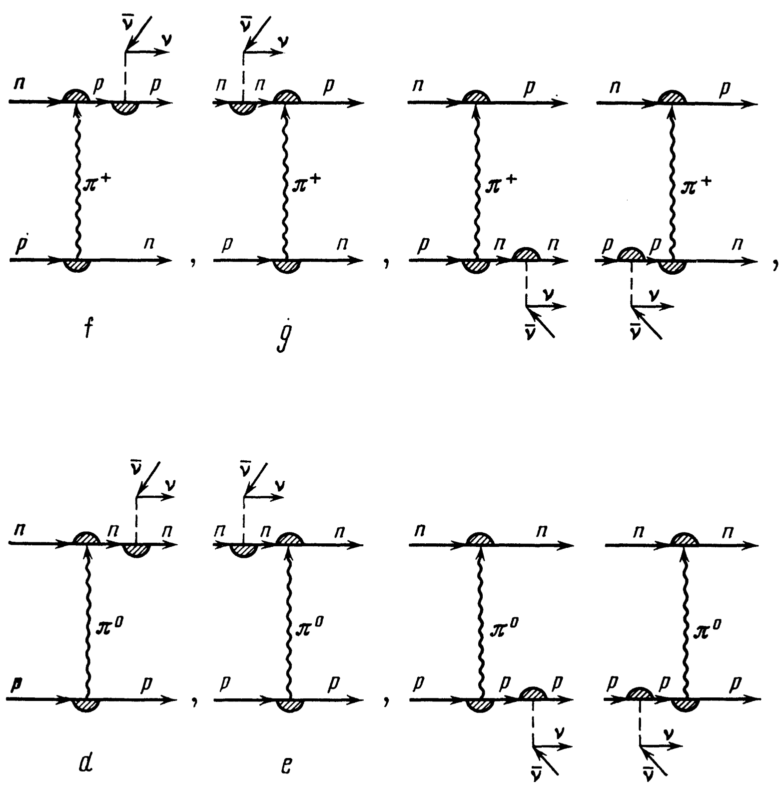


FIG. 5. Diagram of the process $nn \to np\nu\bar{\nu}$. The notation is the same as in Fig. 3.

tent. This can be seen particularly clearly if one starts from the one-nucleon processes. Complete neglect of Π in the two-nucleon process corresponds to allowing for it in first order in the one-nucleon process (i.e., in the imaginary part of D_π it is necessary to take $D_\pi = D^\pi_{\rm vac} + D^\pi_{\rm vac}\Pi D^\pi_{\rm vac}$ in the calculation of the one-nucleon process). Indeed, it is possible to have multiple scattering of pions by particle-hole pairs of the medium, corresponding to the expression of the Dyson equation in the form $D_\pi = D^\pi_{\rm vac} + D^\pi_{\rm vac}\Pi D_\pi$. It reduces to the previous equation when $|\Pi|/(m_\pi^2 + k^2) \ll 1$, $\omega \approx 0$. This can be achieved only by strongly suppressing the πNN vertex. On the other hand, significant suppression of the πNN and $NNl\bar{\nu}$ vertices leads to the appearance of a strong factor suppressing the luminosity of the process, $\sim\gamma^6 \ll 1$, in contrast to the factor ~ 0.5–1 used in Ref. 27. Thus, the approximation in which the nuclear interaction is approximated by vacuum one-pion exchange in a certain sense contradicts the optical theorem. The graphs of Fig. 2 are lost altogether. Such an approximation is justified only when $\rho \ll \rho_0$, when the medium effects are unimportant.

It is inconsistent *a fortiori* to ignore the change in the interaction in the medium but then directly take into account pion condensation when $\rho > \rho_c \sim \rho_0$. In a consistent scheme of nuclear forces, pion condensation arises precisely as a result of the softening of the pion mode with increasing density of the nuclear matter.[12] In the scheme used in our paper, all processes are described from a common point of view.

Of course, one can also have more complicated diagrams not shown in Figs. 1–5 that may also make some contribution to the resulting neutrino luminosity. We shall select the most important graphs with respect to the parameter $T/\varepsilon_F^N \ll 1$, restricting ourselves to allowance for the contributions to the luminosity that are $\sim (T/\varepsilon_F^N)^8$ inclusively. In this approximation, only the diagrams shown in Figs. 1–5 are important. The remaining graphs either make a smaller contribution to the luminosity at the parameters $\gamma < 1$, $\tilde{\omega}_\pi^2 \sim 1$ or contain powers of T/ε_F^N higher than $(T/\varepsilon_F^N)^8$.

As will be shown in Sec. VI, when $T \gtrsim T_{op}$ ($T_{op} \lesssim \varepsilon_F^p$) the effects of the absorption of neutrinos (and antineutrinos) produced in direct processes become important (T_{op} is determined from the condition that the neutrino mean free path λ_ν be equal to the radius of the neutron star). When $T \gg T_{op}$, there is equilibrium emission, and calculation of the contributions to the luminosities from the different direct processes is meaningless. When $T \sim T_{op}$, only numerical solution to the problem of cooling of the neutron star is possible. Thus, the restriction to the case of low temperatures $T \ll \varepsilon_F^p$ in the calculation of the contributions to the luminosity of the direct processes is entirely justified.

IV. TWO-NUCLEON PROCESSES

The specific luminosity of the two-nucleon processes is determined by the general expression

$$\varepsilon_\nu = 2\pi \int \left[\prod_{i=1}^{4} \frac{d^3p_i}{(2\pi)^3}\right] \frac{d^3q_1}{2\omega_1(2\pi)^3} \frac{d^3q_2}{2\omega_2(2\pi)^3}$$

$$\times \delta(E_f - E_{in})(2\pi)^3 \delta^{(3)}(\mathbf{P}_f - \mathbf{P}_{in}) s^{-1} \left(\sum |M|^2\right)$$

$$\times \omega_\nu n_1 n_2 (1-n_3)(1-n_4), \qquad (4.1)$$

where n_i are the population numbers of the particles participating in the reaction, p_i are the nucleon momenta, q_i and ω_i are the momenta and energies of the leptons, ω_ν is the total energy of the neutrinos, the δ functions take into account the energy-momentum conservation laws, $\Sigma|M|^2$ is the square of the reaction matrix element summed over the spins, and s is a symmetry factor. As in Ref. 27, we make the following simplifications:

$$d^3p_i \approx d^3p_i \delta(p_i - p_F^i)(m_i^*/p_F^i) \int dE_i, \quad i = N,$$
$$d^3p_i \approx d^3p_i \delta(p_i - p_F^i) \int dE_i, \quad i = e, \qquad (4.2)$$

these being possible on account of the strong inequality $T \ll \varepsilon_F^N$, and we also use the equation

$$1 = \int d^3k \, \delta^{(3)}(\mathbf{k} - \mathbf{p}_1 + \mathbf{p}_3), \qquad (4.2a)$$

substituting this integral in the expression (4.1), which makes it possible to reduce the angular part of the integral over the phase space to an integral over $\mathbf{k}$. The technique of the calculations is analogous to that described in Ref. 27. Therefore, we give the final results directly.

a) $nn \to npe\bar{\nu}$. The generalized URCA process, Fig. 3. For the matrix element of the URCA process $nn \to npe\bar{\nu}$ on the charged current, using the corresponding expression for the weak interaction Lagrangian (see (2.12)) and taking into account the collective effects of the nucleon medium in the pion propagator (2.2), we obtain without allowance for the exchange interaction

$$M_{\rm URCA} = \frac{2^{1/2} f^2 G}{\tilde{\omega}_{\pi^0}^2(k)(\omega + \Delta\mu)} \{l_0 \chi_3^+ \mathbf{k}\boldsymbol{\sigma}\chi_1 \chi_4^+ \mathbf{k}\boldsymbol{\sigma}\chi_2 (1 - \beta(k) + \varkappa(\omega_\pi, k))$$
$$- g_A (\mathbf{l}\mathbf{k}) \chi_3^+ \boldsymbol{\sigma}\mathbf{k}\chi_1 \chi_4^+ \chi_2 + g_A \beta(k) \chi_4^+ \mathbf{k}\boldsymbol{\sigma}\chi_2 \chi_3^+ (\mathbf{k}\boldsymbol{\sigma})(\mathbf{l}\boldsymbol{\sigma})\chi_1\}, \qquad (4.3)$$

where we have introduced the notation

$$\beta(k) = \tilde{\omega}_{\pi^0}^2(k)/\tilde{\omega}_{\pi^\pm}^2(k), \quad \varkappa(\omega_\pi, k) = \gamma[\Pi_P^{\pi^0}(\omega_\pi, k)$$
$$- {}^1/{}_2 \Pi_P^{\pi^+}(\omega_\pi, \mu_{\pi^+}, k)]/\tilde{\omega}_{\pi^\pm}^2(k), \qquad (4.3a)$$

in which Π_p is the pole term of the pion polarization operator (the first graph is (2.4)).

Using the relation

$$\mathrm{Sp}\, l_i^+ l_k = 8(q_{1i}q_{2k} + q_{1k}q_{2i} - (q_1q_2)g_{ki} + 4i\varepsilon_{jink}q_{1j}q_{2n}) \qquad (4.4)$$

and the rules for commuting the Pauli matrices, we obtain for the square of the matrix element (4.3), summed over the spins,

$$\sum |M|^2 = 128 \frac{f^4 G^2 \gamma^6}{(\omega + \Delta\mu)^2} \omega_1 \omega_2 A(k), \qquad (4.5)$$

$$A(k) = \frac{k^4}{\tilde{\omega}_{\pi^0}^4(k)} \{[1 + 3\beta^2(k)]g_A^2 + [1 - \beta(k) + \varkappa(\omega_\pi, k)]^2\}. \qquad (4.5a)$$

In deriving (4.5), we have, as in Ref. 27, ignored the terms $\sim (\mathbf{q}_1 \cdot \mathbf{k})(\mathbf{q}_2 \cdot \mathbf{k})$. In (4.5) they are, generally speaking, of the same order as the terms retained but make only a small correction to the specific luminosity. Indeed, since

$q_1,\ q_2\sim\omega_1,\ \omega_2\sim T\ll p_n,\ p_p\sim p_F{}^n,\ p_F{}^p,$
the momenta of the leptons can be ignored compared with the nucleon momenta. In particular, this will be done in the δ-function of the momentum conservation law. Then the terms of the type $(\mathbf{q}_1\cdot\mathbf{k})(\mathbf{q}_2\cdot\mathbf{k})$ in (4.5) disappear when the square of the matrix element is integrated over the phase space. Therefore, in what follows too such terms will be omitted in writing down the matrix elements in order to shorten the expressions and because they are not needed.

Note that (4.5) contains a vector current contribution absent when the vacuum pion propagator is used. The term with $\varkappa\neq 0$ corresponds to lepton production in intermediate particle-hole states.

Thus far we have ignored the requirement of symmetry of the nucleon wave functions. To satisfy this requirement, we must add to the diagrams of Fig. 3 corresponding diagrams with the ingoing and outgoing nucleons interchanged. These diagrams are added coherently to the matrix elements of the diagrams previously taken into account. The direct and exchange contributions to the resulting matrix element differ only in the position of the spinors, the value of the momentum transfer, and the sign.

For the square of the matrix element of the process in Fig. 3, summed over the spins with allowance for the exchange interaction, we have

$$\begin{gathered}|M^{exch}|^2=\frac{64f^4G^2\gamma^8\omega_1\omega_2}{(\omega+\Delta\mu)^2}\{A(k_1)+A(k_2)\\ -[2(\mathbf{k}_1\mathbf{k}_2)^2-k_1{}^2k_2{}^2]B(k_1)B(k_2)-g_A{}^2C(k_1,k_2)\},\\ B(k)=\tilde\omega_{\pi^0}{}^{-2}(k)[1-\beta(k)+\varkappa(\omega_\pi,k)],\\ C(k_1,k_2)=\tilde\omega_{\pi^0}{}^{-2}(k_1)\tilde\omega_{\pi^0}{}^{-2}(k_2)\{(\mathbf{k}_1\mathbf{k}_2)^2-[2\mathbf{k}_1\mathbf{k}_2]^2\\ -k_1{}^2k_2{}^2][\beta(k_1)+\beta(k_2)]+\beta(k_1)\beta(k_2)[2(\mathbf{k}_1\mathbf{k}_2)^2+k_1{}^2k_2{}^2]\}.\end{gathered}\qquad(4.6)$$

The luminosity of the processes shown in Fig. 3 can be found from (4.1) with allowance for (4.2), (4.2a), and (4.5) or (4.6) after direct calculations completely analogous to those made in Ref. 27. Therefore, we give the final result:

$$\varepsilon_\nu=c_0f^4G^2m_n{}^{*3}m_p{}^{*}\gamma^6p_F{}^e(p_F{}^n)^4T^8\tilde\omega_{\pi^0}{}^{-4}(p_F{}^n)\,I_{\text{URCA}},\quad c_0=11\,513/24\,1920\pi.\qquad(4.7)$$

The value of I_{URCA} is different for the cases without and with allowance for the exchange interaction:

$$I^{\text{no exch}}_{\text{URCA}}=[1-\beta(p_F{}^n)+\varkappa(0,p_F{}^n)]^2+[1+3\beta^2(p_F{}^n)]g_A{}^2,\qquad(4.8)$$

$$\begin{gathered}I^{\text{exch}}_{\text{URCA}}={}^{25}/_{84}(1+{}^{16}/_{5}\beta(p_F{}^n))^2g_A{}^2+{}^{5}/_{2}(1-\beta(p_F{}^n)\\ +\varkappa(0,p_F{}^n))^2+{}^{61}/_{42}g_A{}^2(1-\beta(p_F{}^n))^2.\end{gathered}\qquad(4.9)$$

On the transition from (4.6) to (4.7) and (4.9), we used the relations $k_1^2=k_2^2=(p_F^n)^2$ and $\mathbf{k}_1\cdot\mathbf{k}_2=\frac{1}{2}(p_F^n)^2$, which follow from the momentum conservation law. Since $I^{\text{exch}}_{\text{URCA}}>I^{\text{no exch}}_{\text{URCA}}$, allowance for the exchange interaction leads to an additional increase in the luminosity.

In the limit of low neutron matter density, $\rho\to 0$, the expression (4.7) goes over into the corresponding expression of Ref. 27.

Substituting in (4.7) the numerical values of the constants and taking into account the doubling of (4.7) by the inverse reactions not shown in Fig. 3, we have

$$\varepsilon_{\text{URCA}}\approx 2.7\cdot 10^{22}\left(\frac{m_n^*}{m_n}\right)^3\left(\frac{m_p^*}{m_p}\right)\times\left(\frac{\rho}{\rho_0}\right)^2\left(\frac{m_\pi}{\tilde\omega_{\pi^0}(p_F{}^n)}\right)^4\gamma^6 I^{\text{exch}}_{\text{URCA}}T_9{}^8.\qquad(4.10)$$

In the numerical estimates, the luminosity is measured in erg·cm^{-3}·s^{-1}, and $T_9=T/10^9$ K.

In the limit of strong softening of the pion mode, the main contribution to (4.10) is made by the neutrino processes in the intermediate particle-hole states. Taking into account only this contribution to $I^{\text{no exch}}_{\text{URCA}}\sim 5\varkappa^2(0,p_F^n)/2$ and $\varkappa(0,p_F^n)\sim\gamma(p_F^n)^2/2$ for $\tilde\omega^2\sim m_\pi^2$, we obtain from (4.10)

$$\varepsilon_{\text{URCA}}\approx 6\cdot 10^{23}\left(\frac{m_n^*}{m_n}\right)^3\left(\frac{m_p^*}{m_p}\right)\left(\frac{\rho}{\rho_0}\right)^{10/3}\left[\frac{m_\pi}{\tilde\omega_{\pi^0}(p_F{}^n)}\right]^8\gamma^8T_9{}^8.\qquad(4.11)$$

As can be seen from the comparison of (4.10) and (4.11) with the corresponding expression of Ref. 27 for the luminosity of the modified URCA process, we obtain not only a different numerical estimate for the luminosity of the process but also quite different qualitative dependences on the density ρ and the correlation parameter γ.

Taking as a rough estimate in (4.10) $\tilde\omega^2_{\pi^0}\sim\tilde\omega^2_{\pi^\pm}\sim 0.7m_\pi^2$ [3)] and choosing the factor γ in the same way as in Ref. 27 ($\gamma^6\sim 0.5$–1), we obtain $\varepsilon_{\text{URCA}}$ (present paper) $\sim 10^3\varepsilon_{\text{UCRA}}$ (Ref. 27) for $\rho\sim\rho_0$ and $\varepsilon_{\text{URCA}}$ (present paper) $\sim 10^4\varepsilon_{\text{URCA}}$ (Ref. 27) for $\rho\sim 2\rho_0$. If it is assumed that in neutron matter and in matter with $N\approx Z$ the constants characterizing the nucleon correlations are of the same order, then a more natural estimate of the parameter γ will be $\gamma\sim 0.5$. For $\gamma\sim 0.5$, $\tilde\omega^2(p_F^n)\sim m_\pi^2$ we obtain the estimate $\varepsilon_{\text{URCA}}$ (present paper) $\sim\varepsilon_{\text{URCA}}$ (Ref. 27) for $\rho\sim\rho_0$ and $\varepsilon_{\text{URCA}}$ (present paper) $\sim 10\varepsilon_{\text{URCA}}$ (Ref. 27) for $\rho\sim 2\rho_0$.

Thus, our numerical estimate shows that in a fairly wide range of variation of the parameters our luminosity for the process $nn\to npe\bar\nu$ is substantially greater than the one found previously in Ref. 27. Moreover, the result increases appreciably with increasing density.

b) $nn\to nn\nu\bar\nu$. The process is shown graphically in Fig. 4. Its matrix element without allowance for the exchange interaction can be written in the form

$$M=\frac{Gf^2g_Al_i\varepsilon_{imn}}{2^{1/2}\omega\tilde\omega_{\pi^0}{}^2(k)}\{\chi_4{}^+\mathbf{k}\boldsymbol{\sigma}\chi_2\chi_3{}^+\sigma_n\chi_1k_m+\chi_3{}^+\mathbf{k}\boldsymbol{\sigma}\chi_1\chi_4{}^+\sigma_n\chi_2k_m\}.\qquad(4.12)$$

For the square of the matrix element of the reactions shown in Fig. 4, summed over the spins, we obtain

$$\sum|M|^2=64f^4G^2g_A{}^2\gamma^6\tilde\omega_{\pi^0}{}^{-2}(k)k^4\omega^{-2}\omega_1\omega_2.\qquad(4.13)$$

The exchange interaction is taken into account in the same way as for the URCA process. As a result, we obtain

$$\sum|M^{exch}|^2=64f^4G^2g_A{}^2\gamma^6\omega^{-2}\omega_1\omega_2\times\left\{\frac{k_1{}^4}{\tilde\omega_{\pi^0}{}^4(k_1)}+\frac{k_2{}^4}{\tilde\omega_{\pi^0}{}^4(k_2)}+\frac{[\mathbf{k}_1\mathbf{k}_2]^2-2(\mathbf{k}_1\mathbf{k}_2)^2}{\tilde\omega_{\pi^0}{}^4(k_1)\tilde\omega_{\pi^0}{}^4(k_2)}\right\}.\qquad(4.14)$$

Using (4.13) or (4.14), we obtain from (4.1), (4.2), and (4.2a) the luminosity of the processes shown in Fig. 4,[4)]

$$\varepsilon_\nu \approx c_1 f^4 G^2 m_n^{*4} \gamma^6 g_A^2 (p_F^n)^5 T^8 \tilde\omega_{\pi^0}^{-4}(p_F^n) I_{nn}, \quad c_1 = 328/14\,175\pi, \tag{4.15}$$

where I_{nn} is different without and with allowance for exchange:

$$I_{nn}^{no\ exch} = \tilde\omega_{\pi^0}^4(p_F^n) \int_0^1 dx\, x^4/\tilde\omega_{\pi^0}^4(2p_F^n x). \tag{4.16}$$

Under the assumption of strong softening of the pion mode, $(\tilde\omega_{\pi^0}(k) \ll \gamma_1 k_0^2$ [see (2.9)], for $2p_F^n > k_0 \sim p_F^n$, the integral I_{nn} can be calculated analytically:

$$I_{nn}^{no\ exch} \simeq \frac{\pi}{64} \frac{\tilde\omega_{\pi^0}^{-3}(k_0)}{\gamma_1^{1/2} k_0} \left(\frac{k_0}{p_F^n}\right)^5 \tilde\omega_{\pi^0}^4(p_F^n). \tag{4.17}$$

With allowance for the exchange interaction,

$$I_{nn}^{exch} \approx \frac{1}{2\pi} \tilde\omega_{\pi^0}^4(p_F^n) \int_{x^2+y^2<1,\,x,y>0} \frac{dx\,dy}{(1-x^2-y^2)^{1/2}} \left\{ \frac{x^4}{\tilde\omega_{\pi^0}^4(2p_F^n x)} + \frac{y^4}{\tilde\omega_{\pi^0}^4(2p_F^n y)} + \frac{x^2 y^2}{\tilde\omega_{\pi^0}^2(2p_F^n x)\,\tilde\omega_{\pi^0}^2(2p_F^n y)} \right\}. \tag{4.18}$$

The luminosity (4.15) found for the process $nn \to nn\nu\bar\nu$ must also be multiplied by N_ν, where N_ν is the number of light neutrino species. Setting $N_\nu = 2$ for the two light neutrino species ν_e and ν_μ, we give a numerical estimate for the luminosity of the $nn \to nn\nu\bar\nu$ process:

$$\varepsilon_\nu(nn \to nn\nu\bar\nu) \approx 8.4 \cdot 10^{22} \left(\frac{m_n^*}{m_n}\right)^4 \left(\frac{\rho}{\rho_0}\right)^{5/3} \left[\frac{m_\pi}{\tilde\omega_{\pi^0}(p_F^n)}\right]^4 \gamma^6 T_9^8 I_{nn}^{exch}. \tag{4.19}$$

c) $np \to np\nu\bar\nu$. The process is shown graphically in Fig. 5. The square of the matrix element shown by the graphs in Fig. 5 can, summed over the spin, be reduced to the following forms:

without allowance for the exchange interaction

$$\sum |M^{no\ exch}|^2 = 64 f^4 G^2 g_A^2 \gamma^6 k^4 \omega^{-2} \tilde\omega_{\pi^0}^{-4}(k)\, \omega_1 \omega_2 (1+2\beta^2(k)), \tag{4.20}$$

with allowance for the exchange interaction

$$\sum |M^{exch}|^2 = 64 f^4 G^2 g_A^2 \gamma^6 \omega^{-2} \omega_1 \omega_2 \times \left\{ \frac{k_1^2(1+2\beta^2(k_1))}{\tilde\omega_{\pi^0}^4(k_1)} + \frac{k_2^2(1+2\beta^2(k_2))}{\tilde\omega_{\pi^0}^4(k_2)} - \frac{[2\mathbf{k}_1\mathbf{k}_2]^2}{\tilde\omega_{\pi^0}^2(k_1)\,\tilde\omega_{\pi^0}^2(k_2)} \right\}, \tag{4.21}$$

from which we obtain for the luminosity of the process in Fig. 5

$$\varepsilon_{np} \approx c_1 f^4 G^2 (m_n^*)^2 (m_p^*)^2 \gamma^6 g_A^2 p_F^e (p_F^n)^4 T^8 \tilde\omega_{\pi^0}^{-4}(p_F^n) \times (I_{np1} + I_{np2}), \tag{4.22}$$

where

$$I_{np\,1}^{no\ exch} = \beta^2(p_F^n)/8, \tag{4.23}$$

$$I_{np\,2}^{no\ exch} = 2\tilde\omega_{\pi^0}^4(p_F^n) z^{-1} \int_0^z x^4\, dx/\tilde\omega_{\pi^0}^4(2p_F^n x), \qquad z = p_F^e/p_F^n, \tag{4.24}$$

$$I_{np\,1}^{exch} = \tilde\omega_{\pi^0}^4(p_F^n) \frac{1}{z} \int_0^z dx \left\{ \frac{1}{8\tilde\omega_{\pi^0}^4(p_F^n)} - \frac{x^2}{\tilde\omega_{\pi^\pm}^2(2p_F^n x)\,\tilde\omega_{\pi^0}^2(p_F^n)} + \frac{4x^4}{\tilde\omega_{\pi^\pm}^4(2p_F^n x)} \right\}, \tag{4.25}$$

$$I_{np\,2}^{exch} = 2\tilde\omega_{\pi^0}^4(p_F^n) \frac{1}{z} \int_0^z dx \left\{ \frac{1}{8\tilde\omega_{\pi^\pm}^4(p_F^n)} - \frac{x^2}{2\tilde\omega_{\pi^0}^2(2p_F^n x)\,\tilde\omega_{\pi^\pm}^2(p_F^n)} + \frac{x^4}{\tilde\omega_{\pi^0}^4(2p_F^n x)} \right\}. \tag{4.26}$$

The numerical estimate (4.22) with allowance for its doubling when $N_\nu = 2$ gives

$$\varepsilon_\nu(np \to np\nu\bar\nu) \approx 2{,}6 \cdot 10^{21} \left(\frac{m_n^*}{m_n}\right)^2 \left(\frac{m_p^*}{m_p}\right)^2 \left(\frac{\rho}{\rho_0}\right)^2 \times \left[\frac{m_\pi^4}{\tilde\omega_{\pi^0}^4(p_F^n)} + \frac{2m_\pi^4}{\tilde\omega_{\pi^\pm}^4(p_F^n)}\right] \gamma^6 T_9^8. \tag{4.27}$$

The contribution to the luminosity of the processes $np \to np\nu\bar\nu$ and $nn \to nn\nu\bar\nu$ in a wide range of variation of the employed parameters is less than that of the process $nn \to npe\bar\nu$. Nevertheless, $\tilde\omega_\pi^2(k_0(\rho))$ can decrease more rapidly with increasing ρ than $\tilde\omega_\pi^2(p_F^n(\rho))$ or even than $\tilde\omega_\pi^4(p_F^n(\rho))/m_\pi^2$. In addition, $\tilde\omega_\pi^2(k_0) < \tilde\omega_\pi^2(p_F^n)$ always. Therefore, the relative contribution to the luminosity of the processes $nn \to nn\nu\bar\nu$ and $np \to np\nu\bar\nu$ compared with $nn \to npe\bar\nu$ can increase when the transition to high nucleon matter densities takes place.

To conclude this section, we note that we have ignored the processes on neutral currents that take place through intermediate particle-hole states since they contribute to the luminosity in a higher order in the parameter $T/\varepsilon_F^N \ll 1$ than $(T/\varepsilon_F^N)^8$.

V. LUMINOSITY OF RESONANT REACTIONS

1. Processes associated with pion condensate

When nuclear matter becomes denser, a pion condensate may arise.[11,12] In a neutron medium with density $\rho > \rho_c^+, \rho_c^\pm, \rho_c^0 \gtrsim \rho_0$, condensates of π_s^+, $\pi^\pm$, and π^0 can appear.[12] The first possibility is associated with the reaction $p \to n + \pi_s^+$, which is allowed because when $\rho > \rho_c^+$ an additional branch appears in the spectrum of the π^+ meson on which $\mu_{\pi^+} < -\mu_n$. The second is due to the fact that at $\rho = \rho_c^\pm$ the total energy of the π^+ and π^- mesons vanishes. The third possibility corresponds to the occurrence when $\rho > \rho_c^0$ of a branch with $\omega^2 < 0$ in the π^0 spectrum. The specific luminosities of the reactions associated with the pion condensate can be calculated under the simplifying condition $|\varphi|^2 \ll m_\pi^2$ by the same method as was used to calculate the luminosities of the one-pion processes. For this, the wavy line in Figs. 1 and 2, which corresponded previously to a pion from the spin-isospin branch, must be replaced by the external field. This corresponds to the replacement in the expressions for the squares of the matrix elements (3.3) of $\langle i|\varphi^+\varphi|i\rangle$ by

$$\langle i|\varphi^+\varphi|i\rangle = (2\pi)^4 a^2 \delta(\omega_\pi - \mu_c)\,\delta(\mathbf{k} - \mathbf{k}_0), \tag{5.1}$$

where $a = (a_{\pi_s^+}, a_{\pi^\pm}, a_{\pi^0})$, $\mu_c = (\mu_c^+, \mu_c^\pm, 0)$, $\mathbf{k}_0$

$= (\mathbf{k}_0^+, \mathbf{k}_0^\pm, \mathbf{k}_0^0)$ for the condensates of the π_s^+, $\pi^\pm$, or π^0 mesons.

From the expression for the luminosity of the one-nucleon processes, which differs from (4.1) by the presence of one nucleon in the final and initial states, we obtain for the process $n\pi^0_{\text{cond}} \to pe\bar{\nu}$, using (3.3) and (5.1), and also the approximation (4.2) and (4.2a),

$$\varepsilon_{\bar{\nu}e}^{\pi^0} \approx c_2 f^2 G^2 (1+g_A^2) m_n^* m_p^* \gamma^4 k_0^0 a_0^2 T^6, \quad c_2 = 457\pi/5040. \tag{5.2}$$

In deriving (5.2), we have also used the inequalities

$$p_F^n - p_F^e < k_0^0 < p_F^n + p_F^e.$$

For the luminosity of the $\pi^\pm$-condensate process $n\pi^-_{\text{cond}} \to ne\bar{\nu}$ we have

$$\varepsilon_{\bar{\nu}e}^{\pi^\pm} = \varepsilon_{\bar{\nu}e}^{\pi^0} (a_0 \to a_\pm, k_0^0 \to k_0^\pm)(1+3g_A^2)/4(1+g_A^2), \tag{5.3}$$

$$\varepsilon_{\bar{\nu}e}^{\pi_s^+} = \varepsilon_{\bar{\nu}e}^{\pi^\pm} (a_\pm \to a_+, k_0^\pm \to k_0^+, \mu_c^\pm \to \mu_c^+). \tag{5.4}$$

The estimate of the luminosity of the process $n\pi^0_{\text{cond}} \to n\nu\bar{\nu}$ gives

$$\varepsilon_{\nu\bar{\nu}}^{\pi^0} = c_3 \varepsilon_{\bar{\nu}e}^{\pi^0} g_A^2/(1+g_A^2), \quad c_3 = 224/9597, \tag{5.5}$$

and for the processes $n\pi^+_{\text{cond}} \to p\nu\bar{\nu}$ and $p\pi^-_{\text{cond}} \to n\nu\bar{\nu}$ we have

$$\varepsilon_{\nu\bar{\nu}}^{\pi^\pm} \approx c_4 \frac{[(c_V+1)^2 + 4g_A^2]}{1+g_A^2} \varepsilon_{\bar{\nu}e}^{\pi^0}, \quad c_4 = 56/9597. \tag{5.6}$$

To include in the treatment the inverse reactions, the expressions (5.2)–(5.4) must also be multiplied by 2.

The numerical estimate of the luminosity of the processes $n\pi^0_{\text{cond}} \to pe\bar{\nu}$ and $ne \to p\pi^0_{\text{cond}}\nu$ is

$$\varepsilon_{\bar{\nu}e}^{\pi^0_{cond}} \approx 6.2 \cdot 10^{27} \frac{k_0^0}{m_\pi} \left(\frac{m_n^* m_p^*}{m_n m_p} \right) \gamma^4 \frac{\theta_{\pi^0}^2}{4} T_9^6, \tag{5.7}$$

where $\theta_{\pi^0} = 2f a_{\pi^0}/g_A$ is the chiral angle.

We note that the reaction $n\pi^-_{\text{cond}} \to pe\bar{\nu}$ (Fig. 1c) has already been considered in Ref. 13 in the framework of the σ model with neglect of the nucleon correlations at the vertices of the πNN and $NNl\bar{\nu}$ interactions. Other π-condensate processes have not yet been investigated. As we have already argued, the inclusion of nucleon correlations, although done trivially, significantly changes the numerical estimates of the star's luminosity. Indeed, if the factor γ were ~ 0.3, the resulting luminosity would be suppressed by $\gamma^4 \sim 10^{-2}$ times.

It can be seen from the expressions (4.11) and (5.7) that in the early stage of cooling of the neutron star, i.e., at sufficiently high temperature, $10^2 T_0 \gtrsim T \gtrsim T_9$, processes without π condensation can actually predominate over the π-condensate processes. Thus, the conclusion of earlier studies (which did not take into account collective effects) that the low experimental upper limits on the surface temperatures of some neutron stars can be explained only by recourse to pion condensation is without support.

We note that even processes associated with excitations of a new π-condensate vacuum make a contribution to the cooling. These processes differ from the ones we considered in the earlier sections only through the difference of the pion propagator for pion excitations measured from the new and old vacuums. The corresponding dispersion laws of the new pion excitations in cold neutron matter were obtained in Ref. 28. Taking them into account, one should repeat the calculations of the processes analogous to those shown in Figs. 1–5. Here, we shall not make these calculations, which are laborious but straightforward from the conceptual point of view. We merely mention that such calculations lead to an appreciable anisotropy of the luminosity along and at right angles to the direction of the condensate wave vector.

1. Non-π-condensate resonance processes

In the absence of a pion condensate, resonance reactions are associated with spin-spin or spin-isospin sound. It is easy to see that other processes are forbidden by symmetry requirements. The main contribution to the luminosity arises from the neutral-current reactions shown in Fig. 2j, in which the wavy line now corresponds to the quantum of the spin sound or spin-isospin sound. A simple estimate gives

$$\varepsilon_\nu \sim \gamma^2 \exp\{-(s_0^2 - 1)\varepsilon_F^n/T\}. \tag{5.8}$$

To obtain (5.8), we used the dispersion law $\omega \approx s_0 k v_F^n$ for the excitations. As follows from (5.8), ε_ν decreases exponentially with decreasing temperature, and therefore such a process must be rejected in the approximation $T \ll \varepsilon_F^n$ that we use. Nevertheless, we still draw attention to the absence of the suppression factor $\sim \gamma^6$–γ^8. The resonance processes associated with charged currents make an even smaller contribution to the luminosity. Processes with charged pions are possible only in the presence of a pion condensate.

VI. NEUTRINO TRANSPORT PROBLEM

The initial stage of the cooling of the neutron star is described as follows. During the collapse to the formation of the neutron star and immediately after it the process of neutronization, accompanied by neutrino production, takes place. The neutrinos are strongly degenerate and rapidly leave the star (characteristic time ~ 1 s). The chemical potential of the neutrinos becomes less than T, but the mean free path λ_ν is still less than the radius of the star (~ 10 km). The luminosity is determined by the well-known formula $L = 4\pi(7/8)\sigma T^4$ (σ is the Stefan-Boltzmann constant). At $T = T_{op}$, the neutrino mean free path λ_ν becomes equal to the radius $R \sim 10$ km of the neutron star. For $T \ll T_{op}$, $\lambda_\nu \gg R$ and the emission is determined by the direct processes. The total luminosity is $\int \Sigma \varepsilon_\nu$, integrated over all direct processes.

The characteristic cooling time of the star, during which the neutrinos still trapped in the star are nondegenerate and there is black-body emission, can be estimated on the basis of the diffusion equation[19]:

$$\frac{17}{2} \sigma r^{-2} \frac{\partial}{\partial r} \left(r^2 T^3 \lambda_{\nu R}(T) \frac{\partial T}{\partial r} \right) = C_V \frac{\partial T}{\partial t}, \tag{6.1}$$

$$\lambda_{\nu R}(T) = \int d\omega_\nu\, \omega_\nu^3 \lambda_\nu(\omega_\nu, T)$$

$$\times \frac{d}{dT} \left[1 + \exp\left(\frac{\omega_\nu}{T}\right) \right]^{-1} \Big/ \int d\omega_\nu\, \omega_\nu^3 \frac{d}{dT} \left[1 + \exp\left(\frac{\omega_\nu}{T}\right) \right]^{-1},$$

where $T(r, t)$ is the radial distribution of the temperature of the star at the time t, and C_V is the specific heat. A dimensional estimate of the characteristic time during which the neutrinos carry heat to the surface layer of width $\sim\lambda_\nu$, from which they are radiated outside the star, is

$$t_0 \sim \lambda_{\nu R}^{-1} R^2 C_V \sigma^{-1} T^{-3}. \quad (6.2)$$

The neutrino mean free path λ_ν can be determined from the simple relation[19]

$$[1+\exp(\omega_\nu/T)]\, dL/d\omega_\nu = 2^{-1}\pi^{-2}\lambda_\nu^{-1}(\omega_\nu)\,\omega_\nu^3 T^4, \quad (6.3)$$

where $dL/d\omega_\nu$ is the derivative of the luminosity per unit volume with respect to the neutrino energy ω_ν.

Making the calculations, we obtain from (6.3) and (4.7) for the neutrino mean free path in the URCA process

$$\lambda_{\nu\,\mathrm{URCA}}^{-1} = \frac{G^2 f^4}{24\pi^7}(m_n^*)^3(m_p^*)\gamma^6(p_F^n)^4 p_F^e T^4 f(y)\, I_{\mathrm{URCA}}^{\mathrm{exch}} / \tilde{\omega}_{\pi^0}^4(p_F^n),$$

$$f(y) = y^4 + 10\pi^2 y^2 + 9\pi^4, \quad y = \omega_\nu/T. \quad (6.4)$$

Since $\lambda_\nu^{-1} \sim \varepsilon_\nu$, we can find from (6.4) and the relations obtained above for the luminosities of the various processes the corresponding values of λ_ν^{-1}. The neutrino mean free path is a minimum for the generalized URCA process. The numerical estimate (6.4) gives

$$\frac{\lambda_\nu}{R} \approx 1.8\cdot 10^8 T_9^{-4} f^{-1}(y)\gamma^{-6}\left[\frac{m_\pi}{\tilde{\omega}_{\pi^0}(p_F^n)}\right]^{-4}(I_{\mathrm{URCA}}^{\mathrm{exch}})^{-1}(\rho/\rho_0)^{-2}, \quad (6.5)$$

where the radius R of the neutron star is taken to be ~ 10 km. Equation (6.5) shows that neutrinos of mean energy $y = \bar{y} = \bar{\omega}_\nu/T \approx 4.7$, where

$$\bar{\omega}_\nu = \int f(y)\,(e^y+1)^{-1} y\, dy \Big/ \int f(y)\,(e^y+1)^{-1}\, dy,$$

produced in the process $nn \to npe\bar{\nu}$, will have a mean free path less than the radius of the star if T_9 exceeds $T_9^{op} \approx 6\tilde{\omega}^2(p_F^n)(\rho_0/\rho)^{5/6}\gamma^{-2}$. (Here, we have assumed that the main contribution to the luminosity of the $nn \to npe\bar{\nu}$ process is made by reactions with the emission of neutrinos in intermediate particle-hole states.) The analogous quantity T_9^{op} obtained in Ref. 27 for the modified URCA process was ~ 22, and in Ref. 19 it was $T_9^{op} \sim 50$. Thus, the minimal value of λ_ν can be less than was expected in Refs. 27 and 19. According to the estimate of (6.2), this can lead to additional confinement of the neutrinos during the first minutes in the life of the neutron star. Therefore, there may be basic changes in the description of the initial stage of cooling of the neutron star, when the neutrino radiation is trapped. In the first place, this will be manifested in the description of interesting physical phenomena such as the ejection of the supernova shell, vibrations of the neutron star about the equilibrium position, and associated neutrino pulsations, which, in principle, could be detected on the earth if a supernova explodes in our Galaxy.

Leaving aside these important questions, which relate to the description of the initial stage in the evolution of the neutron star (at times $t \lesssim t_0$, on the order of minutes or an hour), we turn to the time evolution of the neutron star luminosity during times $t \gg t_0$, when neutrino absorption is no longer important. It is determined by the equation

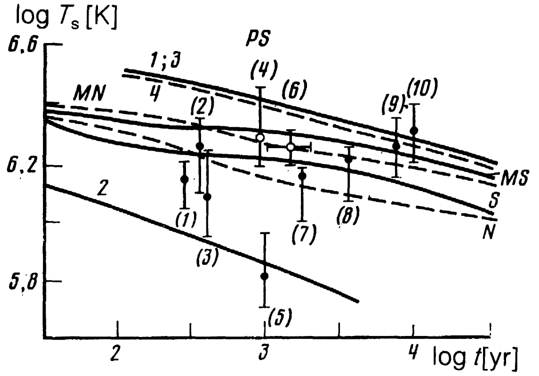


FIG. 6. Comparison of theoretical calculations with data of the Einstein observatory. The abscissa is the logarithm of the time, the ordinate the logarithm of the surface temperature. The equation of state is that of Pandharipande and Smith ($M \approx 1.3\, M_\odot$, $R \approx 8$ km, $R^\infty = 11$ km). The experimental data (see Ref. 7): 1) Cas A, 2) Kepler, 3) Tycho, 4) Crab,[5] 5) SN1006, 6) RCW103, 7) RCW86, 8) W28, 9) G350, 0-18, 10) G22, 7-02. The open circles represent observed sources, the black circles are upper limits. The curves are as follows: S for superfluid neutron stars, N for normal stars, MS for magnetic superfluid stars, and MN for magnetic normal stars.[7] Curves 1 and 2 represent our calculations with mean density $\bar{\rho} = \rho_0$, $\tilde{\omega}_\pi = m_\pi$, $\gamma = 0.47$ and $\bar{\rho} = 2\rho_0$, $\tilde{\omega}_\pi = 0.5 m_\pi$, $\gamma = 0.47$, respectively. Curves 3 and 4 are our calculations for the same values of $\bar{\rho}$ but with luminosity from Ref. 27.

$$C_V \dot{T} = -L, \quad L = \sum \int \varepsilon_\nu\, d\mathbf{r}. \quad (6.6)$$

Here, $C_V \approx C_V^n + C_V^\pi$, C_V^n is the specific heat of the baryon subsystem, and $C_V^\pi \sim C_V^n$ is the contribution to the specific heat of the pion fluctuations.[23,24] As can be seen from Eq. (6.6), the cooling time of the star is shortened by as many times as the luminosity is increased.

In Fig. 6, which is taken from Ref. 7, we give cooling curves calculated in different scenarios (with and without allowance for superfluidity and a magnetic field) with the equation of state of Pandharipande and Smith in conjunction with the experimental data of the Einstein observatory. All curves of Ref. 7 lie appreciably above some of the experimental data.

In Fig. 6, to demonstrate qualitatively the influence of the collective effects of the nucleon medium on the time evolution of the cooling of a neutron star, we give $L(t)$ curves calculated for different mean values of the nuclear matter density. The following values were taken: $\bar{\rho} = \rho_0$ (curve 1) and $\bar{\rho} = 2\rho_0$ (curve 2). The dependence $T_{in}(T_s)$ was taken from Ref. 9. We assumed

$$\tilde{\omega}_\pi(p_F^n(\rho_0)) = m_\pi, \quad \gamma(\rho_0) = 0.47;$$

$$\tilde{\omega}_\pi(p_F^n(2\rho_0)) = 0.5 m_\pi, \quad \gamma(2\rho_0) = 0.47.$$

The specific heat C_V was taken equal to the specific heat for an ideal neutron gas. Curves 3 and 4 were also calculated for $\bar{\rho} = \rho_0$ and $\bar{\rho} = 2\rho_0$ but with the luminosity of Ref. 27 and $C_V \approx C_V^{n\ id}$. As can be seen from Fig. 6, our curves 1 and 2 can be reconciled with all the experimental data under the assumption that different densitites are attained in the neutron stars (with different masses). The results of other studies cannot be reconciled with the experimental data without recourse to the hypothesis of pion condensation (under the assumption that neutron stars exist in the objects studied).

Of course, to obtain quantitative information, one should make detailed numerical calculations for specific stars, using their parameters and the currently calculated realistic dependences $\tilde{\omega}_{\pi}(n)$, $\gamma(n)$, etc., for the given equation of state. Such an analysis would probably make it possible to determine definite information about the internal structure of specific neutron stars as well as some information about the nuclear interactions in neutron matter.

VII. CONCLUDING REMARKS

If in the future central sources are discovered in supernova remnants with low values of T_s (see Fig. 6), then they could be associated with neutron stars having a denser internal region than other neutron stars with higher T_s (the other parameters being assumed more or less equal). On the other hand, if neutron stars with low surface temperatures are not found at all, it will be necessary either to conclude that in neutron stars there is no dense internal region, or to reconsider our ideas about nuclear forces in dense neutron matter.

We mention here that with increasing absolute value of the specific luminosity of the internal layers of a star there is simultaneously an increase in the relative contribution of the internal luminosity compared with the luminosity determined by the processes taking place in the crust, including the photon luminosity. Therefore, neutrino cooling of the internal layers of a neutron star is the dominant process down to lower temperatures than hitherto assumed.

Several important questions have remained outside our treatment. We have ignored the possibility of neutron superfluidity and proton superconductivity (see Ref. 29), which could significantly change the results obtained in the region of temperatures $T \lesssim \Delta_n$, $\Delta_p \sim 10^9$ °K, where Δ_n and Δ_p are the pairing gaps for the neutrons and protons. It would be interesting to study the influence of the proposed effects on the evolution stage $t \lesssim t_0$ of the neutron star. Solution of the transport problem would make it possible to study the damped vibrations of the neutron star, the pulsations of the neutrino radiation, the dependence $T(r)$ through the star, and the part played by the neutrinos in the ejection of the supernova shell. It would be important to take into account the influence of the pionic degrees of freedom on the equation of state of the neutron matter. In particular, because the neutron-proton system interacting through the soft pions is highly collisional, the isotopic composition of this system could be significantly changed. As was demonstrated in Ref. 24, even at $T = 0$ some of the nucleons may go over into N^*_{33} resonances on account of the strong NN^*_{33} interaction. There then appear additional neutrino reaction channels and the resulting luminosity can be increased still further.

In Ref. 30, to explain the low values of T_s, axions—hypothetical practically massless pseudoscalar particles associated with Peccei-Quinn symmetry,[31] were introduced. As is well known, these particles are introduced to solve the problem of CP invariance in strong interactions. The same two-nucleon processes were calculated as in Ref. 27 but with the axion instead of the neutrino. The nuclear part of the interaction was constructed in accordance with the scheme used in Ref. 27, i.e., it was effectively reduced to vacuum one-pion exchange. It is obvious that the calculations of Ref. 30 can be readily generalized with allowance for collective effects of the nucleon medium. They will be unchanged only in the limit $\rho \ll \rho_0$, when the polarization of the nuclear medium becomes unimportant.

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[1)] Neutrino pulsations could be experimentally observed if a supernova exploded in our Galaxy.[20]

[2)] This assumption is by no means necessary for us, but we shall sometimes make it to demonstrate the limiting case of strong softening. It is not the explicit form of Re D_{π}^{-1} that is important but the degree of softening.

[3)] The assumption $\tilde{\omega}^2_{\pi^0}(k) = \tilde{\omega}^2_{\pi\pm}(k)$ is unjustified even in the limit of vanishing πN interaction ($\Pi \to 0$). According to (2.2), in this case $\tilde{\omega}^2_{\pi\pm} \to m^2_{\pi} + k^2 - \mu^2_{\pi\pm}$, $\mu_{\pi^+} = -\mu_{\pi^-} = \mu_p - \mu_n$, and $\tilde{\omega}^2_{\pi^0}(k) \to m^2_{\pi} + k^2$.

[4)] In calculating the luminosity of the process $nn \to nn\nu\bar{\nu}$ and then the process $np \to np\nu\bar{\nu}$ with allowance for exchange effects, we use the orthogonality of $\mathbf{k}_1$ and $\mathbf{k}_2$, which follows from the momentum conservation laws.

[5)] If some of the radiation has a nonthermal nature, then this is not the surface temperature but an upper limit on T_s. From the recent successful description of the data on fluctuations of the periods of the Crab and Vela pulsars in a model using neutron superfluidity there follow values of T_{in} and, therefore, T_s (log $T_{s\,\mathrm{Vela}}$ [deg] ≈ 5.48, log t[yr] ≈ 4, log $T_{s\,\mathrm{Crab}}$ [deg] ≈ 6.2.[17] Such a low value of $T_{s\,\mathrm{Vela}}$ cannot be explained without recourse to the mechanisms considered in the present paper.

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