Relation between localized and mobile states of electrons in quantization of Hall resistance in silicon MIS structures

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The dependences of the plateau width in the quantum Hall effect on the magnetic field and on the mobility of two-dimensional electrons are investigated, and it is shown that at a given temperature the dimensionless width of the plateau depends only on the product $\mu H \equiv \omega\tau$. This dependence is universal and describes practically all the experimental results obtained on several dozen different silicon MIS structures, and has a maximum at $\mu H = 10$ to 12. The absolute value of the ratio of the localized states to the total number of states on the Landau level and its dependence on the parameter μH are determined from the temperature dependences of the diagonal and Hall magnetoresistance components measured in the regions of the minima and of the plateau.

§I. INTRODUCTION

The energy spectrum of two-dimensional ($2D$) electrons placed in a strong perpendicular magnetic field is known to be discrete and to constitute a set of Landau levels. The imperfections or potential fluctuations present in real structures broaden the Landau levels to a definite value, and lead to the appearance of localized states at the gaps of the energy spectrum. It is precisely the localized states that cause respectively broad minima and a plateau on the plots of the diagonal (ρ_{xx}) and Hall (ρ_{xy}) components of the magnetoresistance tensor vs the $2D$-electron concentration (n) or vs the magnetic field (H) at low temperatures and in strong magnetic fields.[1] This phenomenon, named the "quantum Hall effect" (QHE) is observed when the next Landau sublevel is completely occupied, i.e., when the condition $n = \nu eH/h$ is met,[1)] where ν is the filling factor and is an integer, e is the electron charge, and h is Planck's constant.

At very low temperatures ($T \rightarrow 0$), when Fermi energy of the $2D$-electrons energy lands, as their concentration changes, in the region of localized states in an energy-spectrum gap (i.e., near an integer ν), ρ_{xx} vanishes and ρ_{xy} remains constant[2]:

$$\rho_{xx}=0, \quad \rho_{xy}=h/\nu e^2. \tag{1}$$

With rising temperature, the electrons and holes are thermally activated from the Fermi level to the corresponding Landau sublevels, and Eqs. (1) no longer hold. A necessary condition for the observation of the QHE is also the presence of mobile states below the Fermi level, for otherwise we have $\sigma_{xy} = 0$ (Ref. 2), contrary to experiment. Localization in a system of $2D$ electrons can be due not only to fluctuations of the potential[3–6] but also, for example, to electron-electron interaction.[7,8] In this case, the relation between the localized and mobile states should differ substantially and have a different dependence, say, on H.

Information on the ratio of the number n_l of the localized states to the total number $n_H = eH/h$ of the states on a Landau level can be obtained by investigating the width of the plateau. Indeed, at $T = 0$, so long as a plateau is observed on the $\rho_{xy}(n)$ or $\rho_{xy}(\nu)$ plot, only localized states are occupied, so that the plateau width Δn measured in units of the $2D$-electron concentration determines the concentration of the localized states, while the dimensionless plateau width in the scale of the filling factors determines the ratio of the number of localized states to the total number of states on the Landau level. If the $2D$-electron localization is due to fluctuations of the potential on the interface, the ratio n_l/n_H and the plateau width should be sensitive to the parameter that characterizes the value of these fluctuations. Such a parameter can be the $2D$-electron mobility (μ) measured at $H = 0$.[9,10,11] It can be concluded from general considerations that if the disorder on the interface is decreased ($\mu \rightarrow \infty$) at $H =$ const, the initial cause of the QHE vanishes and the plateau width should tend to zero.[9,11] If, on the other hand, the disorder is increased ($\mu \rightarrow 0$), then $\mu H \equiv \omega\tau \rightarrow 0$, which is equivalent to vanishing of the magnetic field, and $\Delta\nu$ should naturally tend to zero. Optimal conditions (in terms of μ and H) in the sense that the dimensionless width of the plateau is a maximum, exist thus for observing the QHE.

Our aim was to find the optimum conditions for QHE observation in silicon metal-insulator-semiconductor (MIS) structures, to study the quantum singularities in the Hall effect as functions of various parameters, and to determine the absolute value of the ratio of number of localized electronic states to the total number of states on a nondegenerate Landau level. To this end we investigated the dependences of the plateau of ρ_{xy} and of the minima of ρ_{xx} on the magnetic field, on the $2D$-electron mobility, and on the temperature.

§2. MIS STRUCTURES AND EXPERIMENTAL TECHNIQUE

We investigated about forty Si(100) MIS structures from four different p-silicon plates. The structures had rectangular geometry, were equipped with several pairs of potential and Hall contacts, and differed greatly in quality: the maximum $2D$-electron mobility varied from structure to structure in the range 0.5–5.2 m^2/V·s at $T = 1.5$ K. Table I

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TABLE I.

Plate No.	No. of structures	d_{SiO_2}, Å	ρ_0, Ω·cm	d, mm	l, mm	No. of pot. cont.	Barrier	μ_m, m²/V·cm
1	19	2000	20	0.28	2.5	5	Al+Mo	0.6–3.5
2	14	2000	20	0.28	2.5	5	Al+Mo	0.5–2.5
3	4	1300	10	0.4	1.2	4	Al	2.0–5.2
4	4	2000	20	0.09	1.0	5	Mo	0.5–1.5

lists the main characteristics of these structures: their number, the thickness d_{SiO_2} of the oxide layer, the substrate resistivity ρ_0, the channel width d, the channel length l, the number of potential contacts, the barrier material, and the range of variation of the mobility μ_m at the maximum.

The samples were cut from KDB-10 and KDB-20 silicon plates (respective resistivities 10 and 20 Ω·cm) 150 mm in diameter. To improve the surface quality of the mechanically ground plates, they underwent a number of cycles of oxidation followed by etching of the oxide. HCl and H_2O were used as additives when the final thin oxide coating was deposited, and the oxidation rate did not exceed 6 Å/min.

We measured both the Hall mobility μ_H and the mobility μ_σ as determined from the conductivity at $H=0$. These quantities were always very close: $\mu \approx \mu_H \approx \mu_\sigma$. Typical plots of $\mu_H(n)$ and $\mu_\sigma(n)$ were published earlier (e.g., in Ref. 10). All the measurements were made with ac current of density $\leqslant 3\cdot 10^{-6}$ A/cm at frequencies 20–30 Hz, $T=1.5$ K, and $H \leqslant 20$ T.

In some cases, $\mu(n)$ was varied by applying a voltage V_{SB} between the 2D-electron layer and the substrate. To set the structure in equilibrium with a steady depletion layer, the structures were illuminated with white light of wavelength λ ($0.4\ \mu\text{m} < \lambda < 100\ \mu\text{m}$) through the semitransparent barrier. Only in this case were reproducible results obtained, and the dependence of the threshold voltage on $(E_g/e - V_{SB})^{1/2}$ was strictly linear[12] (E_g is the Si band gap).

§3. PLATEAU WIDTH VARIATION WITH MAGNETIC FIELD AND 2D-ELECTRON MOBILITY

The plateau width Δn (or $\Delta\nu$) is usually defined as the interval, in the scale of n (or ν), in which the following inequality holds at a given constant temperature T (see Fig. 1):

$$\left| \frac{\rho_{xy}}{h/\nu e^2} - 1 \right| < \delta. \qquad (2)$$

The choice of δ is arbitrary, usually 10^{-4}–10^{-2} (Ref. 9). When so defined, the absolute value of the plateau width has no particular meaning, since it depends strongly on δ (and T). It will be shown below, however, that the functions $\Delta\nu(\mu)$ and $\Delta\nu(H)$ as well as the optimal values of μ and H needed for best observation of the QHE do not depend on δ (at $10^{-4} < \delta < 10^{-2}$). Yet the definition (2), in view of its simplicity, makes it easy to reduce a large number of experimental data and summarize them on a single graph. We shall therefore use definition (2) in the present section, which is devoted to the dependences on μ and H.

Figure 1 shows two characteristic $\rho_{xy}(n)$ plots obtained at $T=1.5$ K and $H=15$ T for two different MIS structures, in which the 2D-electron mobilities at $n=7.3\cdot 10^{11}$ cm^{-2} ($\nu=2$) were 0.8 and 1.8 m²/V·s. It can be seen that an increase of μ modifies substantially the $\rho_{xy}(\nu)$ plot in the region $\nu \approx 2$, viz., it decreases the width of the plateau in accordance with the definition (2), shown in Fig. 1 for $\delta = 10^{-3}$. A decrease of the plateau width with increasing μ is clearly observed in this case for $10^{-4} \leqslant \delta \leqslant 10^{-2}$. With further increase of the accuracy to $\delta = 10^{-5}$, the error in the determination of the plateau width increases, and at $\delta \lesssim 10^{-5}$ $\Delta\nu$, and hence the slopes of both curves shown in Fig. 1, are equal in our experiment to within the measurement accuracy. We shall therefore use hereafter the data obtained for $10^{-4} \leqslant \delta \leqslant 10^{-2}$.

Figure 2a shows plots of the dimensionless plateau width ($\Delta\nu$) vs the 2D-electron mobility, obtained for different MIS structures at constant $T=1.5$ K, $\delta=10^{-3}$, $H=6.3$ T, and $H=15$ T for $\nu=1$ (the dark symbols in Fig. 2a show also the results obtained at $V_{SB} \neq 0$). The largest experimental error occurs in the change of the 2D-electron mobility and is connected not with the accuracy with which μ is measured but with the fact that the plateau, measured in the n scale, has a length segment in which μ can vary by virtue of the $\mu(n)$ dependence. It can be seen from Fig. 2a that: 1) the plots of $\Delta\nu(\mu)$ for different H have a maximum $(\Delta\nu)_{max}$ at μ^*; 2) the values of $(\Delta\nu)_{max}$ for different H are practically equal; 3) the larger H the lower μ^* at which the plateau width is a maximum; 4) the experimental results for $V_{SB} \neq 0$ under conditions when the equilibrium state in the depletion layer was reached by additional illumination agrees well with the data obtained at $V_{SB}=0$.

A similar dependence of the plateau width on the magnetic field is observed at constant $T=1.5$ K, $\delta=10^{-3}$ and

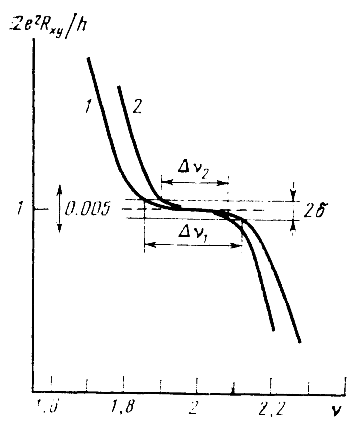

FIG. 1. Hall resistivity ρ_{xy} vs 2D-electron concentration n plotted at $T=1.5$ K and $H=15$ T for two different MIS structures, in which the mobilities at $n=7.3\cdot 10^{11}$ cm^{-2} ($\nu=2$) were 0.8(1) and 1.8 m²/V·s (2). The plateau width is shown for $\delta=10^{-3}$ in accordance with definition (2), $V_{SB}=0$ V.

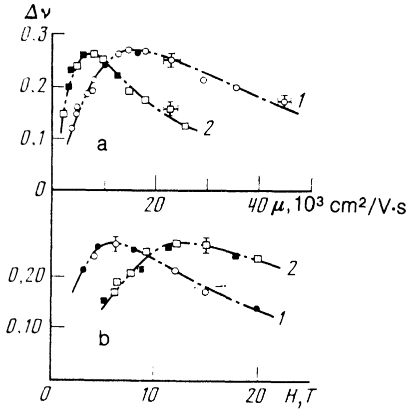

FIG. 2. (a) Plateau width $\Delta\nu$ vs mobility μ of 2D electrons, obtained for $\nu=2$, $\delta=10^{-3}$, and $T=1.5$ K for $H=6.3$ T (1) and $H=15$ T (2). Light symbols—$V_{SB}=0$ V, each light symbol corresponds to a different MIS structure. Dark symbols—$V_{SB}\neq 0$. (b) $\Delta\nu(H)$ plots obtained for $\nu=2$, $\delta=10^{-3}$, and $T=1.5$ K for $\mu=(1.7\pm 0.1)$ m²/V·s (1) and $\mu=(0.8\pm 0.2)$ m²/V·s (2). Light symbols—$V_{SB}=0$, V, dark—$V_{SB}\neq 0$ V.

for $\mu=0.8$ and 1.7 m²/V·s (Fig. 2b). To determine the dependences at constant μ we used, first, the region of the $\mu(n)$ dependence near the maximum, where the condition $\mu=\mu_m=$ const is satisfied in a sufficiently wide range of n (and hence also of $H=nh/ev$). Second, when the specified value $\mu=\mu_x$ did not land in the region of the $\mu(n)$ maximum, we determined two values of n_x for which $\mu(n_x)=\mu_x$ and chose H such that $H=n_x h/ev$. Third, for $\mu_x<\mu_m$, it was always possible to choose a value V_{SB} at which the maximum mobility was comparable with μ_x. The data obtained by the third method are shown in Fig. 2b by dark symbols and agree well with the remaining ones. The $\Delta\nu(H)$ dependences at constant μ lead likewise to three conclusions: 1) the dependence of $\Delta\nu$ on H at constant μ has a maximum $(\Delta\nu)_{\max}$ at H^*; 2) the values of $(\Delta\nu)_{\max}$ are the same for different μ; 3) the larger μ the smaller H^* at which the maximum $\Delta\nu$ is realized.

Plots of $\Delta\nu(\mu)|_{H=H_x}$ and $\Delta\nu(H)|_{\mu=\mu_x}$, similar to those shown in Fig. 2, were obtained also for other values of H,μ, and δ and for $\nu=1,3,4,6$. The observed behavior of the $\Delta\nu(\mu,H)$ plots suggests that the dimensionless width of the plateau is determined only by the product $\mu H\equiv\omega\tau$. This leads to a possibility of plotting the plateau width vs the magnetic field and vs the 2D-electron mobility in dimensionless coordinates and verifying the universality of the function $\Delta\nu(\mu H)$.

To this end, we reduced practically all the data obtained at $T=1.5$ K for different MIS structures under greatly differing values of the parameters μ and H. The results of this reduction are shown in Fig. 3 for $\delta=10^{-2}$ and 10^{-3} and for the different $\nu=2$, 3, and 4. What is surprising is that, for a given ν, practically all the experimental points fit fairly well a single $\Delta\nu(\mu H)$ plot that can be regarded as universal, and that the function $\Delta\nu(\mu H)$ has a clearly pronounced maximum at $\mu H=10$ to 12. Both properties (the universality and the location of the $\Delta\nu$ maximum at $\mu H\sim 10$) are obtained for all investigated values $\nu=2$, 3, and 4, even though these three values of ν correspond to different energy-spectrum splittings—spin, intervalley, and cyclotron, respectively.

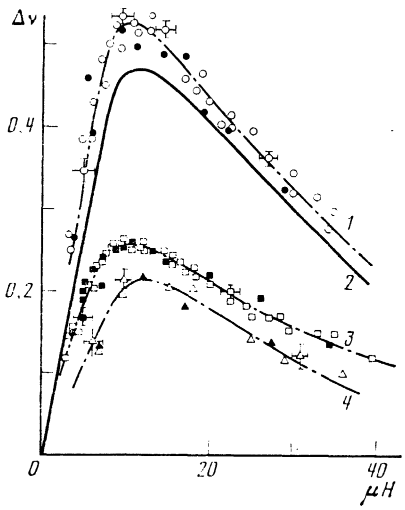

FIG. 3. Dimensionless plateau width $\Delta\nu$ vs the parameter μH, obtained at $T=1.5$ K: 1) $V=4$, $\delta=10^{-2}$; 2) curve drawn by least squares through the points corresponding to $\nu=2$ and $\delta=10^{-2}$; 3) $\nu=2$, $\delta=10^{-3}$; 4) $\nu=3$, $\delta=10^{-2}$. Light and dark symbols—$V_{SB}=0$ V and $V_{SB}\neq 0$ V, respectively. The results were obtained for all structures from the four plates. The mobility varied by more than 50 times.

It is important that the two basic properties of the function $\Delta\nu(\mu H)$ do not depend on the choice of δ (at $10^{-4}\leqslant\delta\leqslant 10^{-2}$). Figure 3 shows also a curve drawn by least squares through the experimental points obtained at $\delta=10^{-2}$ for $\nu=2$; this curve has likewise a maximum at $\mu H=10$–12.

§4. RELATION BETWEEN LOCALIZED AND MOBILE STATES

The conductivity of a weakly disordered 2D system is known to decrease logarithmically at $H=0$ and $T=0$ with increasing size of the system. This logarithmic dependence of the conductivity on the sample dimension reduces to a logarithmic dependence on the temperature, under the condition that the characteristic sample dimension d exceeds the diffusion length $l_d=v_F(\tau_p\tau_\varepsilon)^{1/2}$, and the mean free path is $l_p=v_F\tau_p$, where v_F is the Fermi velocity, and τ_p and τ_ε are the momentum and energy relaxation times.[14] At $T=0$ all the states in the 2D system are localized, but at $T\neq 0$ there exist electronic states in which the localization length L exceeds l_d, l_p, or d, and these states can be regarded as mobile. Similarly, in the presence of a strong magnetic field, at $T=0$ all the states on the wings of the Landau level in an unbounded 2D system at $T=0$ are localized, with the possible exception of one state situated at the center of the Landau level, for which the localization length becomes infinite. At $T\neq 0$ all the states for which the localization length exceeds l_d, l_p, or d will be mobile, and those with $L<l_d$, l_p, or d will be localized. One of the most important problems in the QHE is the determination of the absolute value of the ratio of the localized states to the total number of states on a Landau level. The usually employed plateau-width definition (2) cannot extract from experiment the absolute values of n_l/n_H with any degree of reliability, inasmuch as within

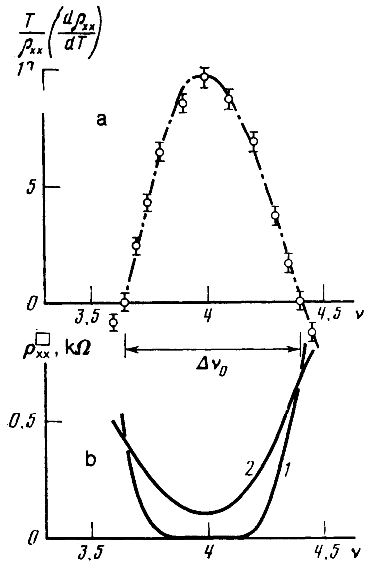

FIG. 4. Plots of $(T/\rho_{xx})(d\rho_{xx}/dT$ vs ν at $T = 1.6$ K (a) and of $\rho_{xx}(\nu)$ at $T = 1.5$ K and 3.7 K (curves 1 and 3 of Fig. b, respectively), obtained in the vicinity of $\nu = 4$ at $H = 5.7$ T and $\mu = (1.9 \pm 0.1)$ m²/V·s; $\Delta\nu_0$ is the absolute value of the dimensionless width of the plateau.

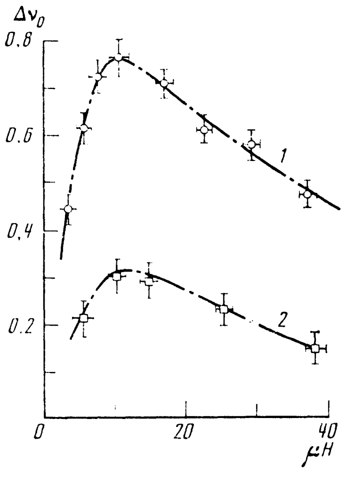

FIG. 5. Plots of $\Delta\nu_0(\mu H)$ obtained at $\nu = 4$ (curve 1) and $\nu = 3$ (curve 2) at $T = 1.6$ K.

the framework of this definition the plateau width depends strongly on the artificially introduced parameter δ. One can get around this deficiency by investigating the temperature dependence of ρ_{xx} or of $\partial\rho_{xy}/\partial n$ at different values of the occupation factor in the region of the plateau, i.e., in the vicinity of integer ν values.

It can be seen from Figs. 4a and 4b that, in the ν scale, the different points have qualitatively different temperature dependences at $1.5 < T < 4.2$ K. Whereas near the integer $\nu = 4$ the function $\rho_{xx}(T)$ is well described by the expression $\rho_{xx} \sim \exp(-W/kT)$, where W is the activation energy, on deviation from $\nu = 4$ in either direction the $\rho_{xx}(T)|_{\nu=\text{const}}$ dependences become weaker and cannot be described by a single exponential.[15] It is important that in the ν scale there exists an interval $\Delta\nu_0$ (see Figs. 4a and 4b) within which the values of ρ_{xx} (and of $|\partial\rho_{xx}/\partial n|$) decrease with temperature for all ν. This interval corresponds to location of the Fermi surface in the region of localized states for regardless of whether the activation or hopping conduction is realized, the $\sigma_{xx}(T)$ dependence has the property that σ_{xx} (and ρ_{xx}) decreases with decreasing temperature. If, however, the Fermi energy lands in the region of mobile states, an increase of ρ_{xx} with decreasing T is observed (see Fig. 4a).

The interval $\Delta\nu_0$ determined in this manner is no longer dependent on one artificial parameter (such as δ), and in our opinion its absolute value can serve as a measure of the ratio of the localized states to the number of states on a Landau level. An important test of the introduced definition was an investigation of the dependence of $\Delta\nu_0$ on the parameter μH. Figure 5 shows this dependence obtained for $\nu = 3$ and $\nu = 4$ for different MIS structures and for different μ and H. It can be seen that within the framework of this definition the form of the $\Delta\nu_0(\mu H)$ dependence agrees qualitatively with that determined above and has likewise a maximum at $\mu H = 10$–12. The absolute value of the ratio n_l/n_H at 1.5 K $< T < 4.2$ K reaches a maximum $(n_l/n_H)_{\max} = 0.76 \pm 0.05$ at $\mu H = 10$–12 for $\nu = 4$, to which the largest gap in the electron energy spectrum corresponds. A substantially lower value $(n_l/n_H)_{\max} = 0.32 \pm 0.05$ is obtained for the smaller gap $\nu = 3$, but likewise at $\mu H = 10$–12.

It is important that the values of n_l/n_H and the dependence on μH were insensitive to the sizes of the investigated samples, and that the positions of the outermost points of the $\Delta\nu_0$ interval were practically independent of temperature in the investigated range ($1.5\ K < T < 4.2$ K). Nonetheless, with further lowering of the temperature ($T \to 0$ K) the character of the $\rho_{xx}(T)$ dependences can change and can turn out to depend on the temperature and the size of the sample. We do not known to this day whether all the states of the $2D$ electrons are localized at $T = 0$ K in the presence of a magnetic field. In the most perfect structures, when the temperature is lowered way down to 10 mK, a QHE is observed with nonzero fraction of mobile electronic states in which ρ_{xx} increases as $T \to 0$ (Ref. 16).

§5. DISCUSSION OF RESULTS AND CONCLUSION

Investigation of the QHE parameters and of the ratio n_l/n_H as functions of the $2D$-electron mobility is particularly meaningful in connection with the discovery of the fractional QHE—observation of a plateau in $\rho_{xy}(\nu)$ and of minima in $\rho_{xx}(\nu)$, corresponding not to integer but to fractional values $\nu = 1/3$, 2/3, 2/5, 3/5, 4/5 etc. (Refs. 17–21,10). The point is that the fractional QHE is observed only in samples of very high quality, with anomalously high $2D$-electron mobility. In first-order approximation, the observation of the fractional QHE can be interpreted as a result of predominance of the electron-electron interaction over effects due to disorder and to fluctuations of the potential.[22,23] One of the essential results of the present paper is that the $2D$-electron mobility measured at $H = 0$ turns out to be a good parameter that characterizes the disorder and describes the universal dependences of the integer QHE at $H \neq 0$. This conclusion agrees with the results of Refs. 10 and 24, in which the parameter μ could likewise be used to describe disorder in an investigation of fractional QHE.

Another important result of this paper is that there exist parameters that describe the integer QHE and are determined by the dimensionless product $\mu H \equiv \omega\tau$, whereas in the fractional QHE the dependences of μ and H are separated.[24] This fact agrees with the present-day premise that the fractional and integer QHE have different physical causes. The fractional QHE is a consequence of electron-electron interaction and of condensation of the $2D$-electron gas into an incompressible Fermi liquid,[22,23] whereas the integer QHE is due to localization of noninteracting electrons in Landau-level wings.

We have determined here, in addition, the following numerical quantities: 1) the value of μH at which the effects of integer quantization of the Hall resistance are maximal ($\mu H = 10$ to 12), 2) the ratio of the localized states to the total number of states on a Landau level and its dependence on μH (maximum value $n_l/n_H = 0.76 \pm 0.05$ for $\nu = 4$ and $1.5 < T < 4.2$ K). The values obtained are dimensionless and can therefore be compared with the values determined in other $2D$-electron systems, e.g., in heterojunctions. There are at present no published data indicating whether the $\Delta\nu(\mu H)$ dependence in heterojunctions is universal, nor are the values of μH at which $\Delta\nu$ is a maximum known. There are, however, data indicating that in GaAs–$Al_xGa_{1-x}As$ heterojunctions the ratio n_l/n_H can reach 0.97 (Refs. 25 and 16). We deem it therefore of interest to continue this investigation in other $2D$- electron systems.

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[1)] We use SI units throughout.

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