Tunneling of pairs between superconductors of different multiplicity

A. M. Gulyan

Institute of Physical Studies, Academy of Sciences of the Armenian SSR
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Tunneling between superconductors is analyzed as a test for identifying states with triplet pairing. The amplitude of the quantum oscillations in the paramagnetic susceptibility is calculated; these oscillations should accompany the Josephson effect in junctions between triplet superconductors.

1. INTRODUCTION

The discovery of anomalous behavior in superconducting lanthanide and actinide alloys has stimulated interest in models that describe Cooper pairing in states with orbital angular momentum $l \neq 0$. Such materials are now referred to as heavy-fermion superconductors (see, e.g., Ref. 1 for more details).

The higher-multiplicity states of superfluid pairs may be associated with an anisotropic gap in the one-particle excitation spectrum and thus give rise to observable effects, such as a power-law dependence of the specific heat on the temperature. Although there are some experimental indications[2] that such effects occur, the results cannot be interpreted unambiguously[3] and further work is needed.

The Josephson effect is of fundamental importance in this regard. The Josephson technique[4] is based on the absence of coherent effects between superconductors of different multiplicity (the Josephson effect itself and the proximity effect both vanish in this case[5]). The reason is plain if we recall that the Josephson effect is produced by interference among the wave functions for the condensate. The suppression of the interference is in the final analysis a consequence of the orthogonality of the wave functions (spherical harmonics) in momentum space.

The above explanation neglects the changes in the ground state that occur when a tunnel contact is present in the superconductor. In addition, because boundaries are inevitably present between the linings of the junction, the state of the lining material may be globally altered (the symmetry of the pair states may change, textures may form, etc.). Unlike the case for superfluid ^{3}He, no detailed work has been done on this question for superconductors of higher multiplicity. However, it has been found[6,7] that because of magnetic inhomogeneities, the triplet and singlet states should coexist with comparable amplitudes; for this to occur, it suffices that at least one of the coefficient V_1 in the pairing interaction

$$V(\mathbf{p}-\mathbf{p}')=\sum_l V_l P_l(\theta) \qquad (1)$$

be less than zero (here θ is the angle between the momenta $\mathbf{p}$ and $\mathbf{p}'$, and $V_1<0$ means that attraction occurs for the lth harmonic). The results in Ref. 8 on the Josephson effect in heavy-fermion superconductors thus also require further study.

In this connection we will analyze another manifestation of coherence for a junction between triplet superconductors. Specifically, we will show below that Josephson oscillations in the pair current in a "triplet" junction should be accompanied by oscillations of the paramagnetic susceptibility.

2. PAIR TUNNELING

We consider isotropic superconducting phases belonging to the $O\times R$ symmetry class in the terminology of Ref. 1, and the Gor'kov-Galitskiĭ model[9] will be used to describe them at $T=0$.[1)] As in Ref. 9, we consider the anomalous Green's functions

$$\overset{+}{F}_{m\alpha\beta}(x,x')=\langle N+2,l,m|T\psi_\alpha^+(x)\psi_\beta^+(x')|N,0\rangle, \qquad (2)$$

which depend on the projection m of the momentum l along a given direction. We assume that all $2l+1$ components in (2) are equally probable, so that

$$\langle T\psi_\alpha^+(x)\psi_\beta^+(x')\rangle=\sum_m \overset{+}{F}_{m\alpha\beta}(x,x'), \qquad (3)$$

and

$$\langle T\psi_\alpha(1)\psi_\beta^+(2)\psi_\gamma(3)\psi_\delta^+(4)\rangle=G_{\alpha\beta}(1,2)G_{\gamma\delta}(3,4)-G_{\alpha\delta}(1,4)G_{\gamma\beta}(3,2)-\sum_m F_{m\alpha\gamma}(1,3)\overset{+}{F}_{m\beta\delta}(2,4). \qquad (4)$$

A single summation is present in (4) because each of the $2l+1$ components of the Bose order parameter plays the role of a self-consistent field for that component only. The anomalous Green's functions can be expressed in terms of spherical harmonics:

$$\begin{aligned}\overset{+}{F}_{m\alpha\beta}(\mathbf{p},\varepsilon)&=F_m^*(p,\varepsilon)Y_{lm}^*(\theta,\varphi)\hat{I}_{\beta\alpha}^*,\\ F_{m\alpha\beta}(\mathbf{p},\varepsilon)&=-F_m(p,\varepsilon)Y_{lm}(\theta,\varphi)\hat{I}_{\alpha\beta},\end{aligned} \qquad (5)$$

where

$$Y_{lm}(\theta,\varphi)=\left[\frac{2l+1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}\right]^{1/2}P_l^m(\theta)e^{-im\varphi},$$

and the $P_l^m(\theta)$ are the associated Legendre polynomials. The spin matrix for pairs in the singlet state is equal to $\hat{I}^{(s)}=i\sigma_y$, where the σ_i are the Pauli matrices. The choice of $\hat{I}^{(t)}$ for triplet pairing is not unique; we have

$$\hat{I}^{(t)}=\alpha\hat{1}+\beta\hat{\sigma}_x+\gamma\hat{\sigma}_z, \qquad (6)$$

where the coefficients α,β,γ must satisfy the unitarity condition $\hat{I}\hat{I}^+=1$. If no fields act on the spins, we set $\alpha^2=\beta^2=\gamma^2=1/3$. The propagator $G_{\alpha\beta}(\mathbf{p},\varepsilon)$ in the isotropic case is given by the usual formula

$$G_{\alpha\beta}(\mathbf{p},\varepsilon)=\delta_{\alpha\beta}G(p,\varepsilon).$$

The Green's function amplitudes are given by

$$G(p,\varepsilon)=\frac{\varepsilon+\xi}{(\varepsilon-\varepsilon_p+i\delta)(\varepsilon+\varepsilon_p-i\delta)},$$

$$F_m^{*}(p,\varepsilon)=-i\frac{\Delta^{*}}{(\varepsilon-\varepsilon_p+i\delta)(\varepsilon+\varepsilon_p-i\delta)}\left(\frac{4\pi}{2l+1}\right)^{1/2}, \qquad (7)$$

where the spectrum is $\varepsilon_p=(\xi^2+|\Delta|^2)^{1/2}$.

Now assume that a static bias voltage V is applied to the junction (we set $e=\hbar=1$). If we use the Hamiltonian

$$\hat{H}_T=\sum_{\alpha}\iint d^3\mathbf{r}_1\,d^3\mathbf{r}_2\{\hat{T}(\mathbf{r}_1,\mathbf{r}_2)\psi_{1\alpha}^{+}(\mathbf{r}_1)\psi_{2\alpha}(\mathbf{r}_2)+\text{h.c.}\}, \quad (8)$$

to describe the tunneling, where the subscripts 1 and 2 correspond to the casings of the junction, we obtain the diagrams shown in Fig. 1 for $G_{\varepsilon\varepsilon-\omega}$. The symbols Δ and Δ^* correspond to the functions $F_{\varepsilon\varepsilon-\omega}$ and $F^{+}_{\varepsilon\varepsilon-\omega}$, whose pairing determines the sign of the diagram. Similar diagrams also hold for the functions $F_{\varepsilon\varepsilon-\omega}$ and $F^{+}_{\varepsilon\varepsilon-\omega}$; the thin rightward and leftward arrows in Fig. 1a then correspond to the functions $G^0_\varepsilon=(\varepsilon-\xi)^{-1}$ and $\bar{G}^0_\varepsilon=(\varepsilon-\omega+\xi)^{-1}$, respectively, while in Fig. 1b they correspond to $G^0_\varepsilon=(\varepsilon-\omega-\xi)^{-1}$ and $\bar{G}^0_\varepsilon=(\varepsilon-\omega+\xi)^{-1}$. The fat unshaded arrows (the self-energy components associated with tunneling) correspond to the Green's functions for the second lining.

Removing the thin lines from the diagrams and subtracting the expression corresponding to Fig. 1b from 1a, we find

$$\omega G_{\alpha\beta}=\sum_{m_1}\{-\hat{F}_{m_1}\hat{\Delta}_{m_1}^{*}+\hat{\Delta}_{m_1}\hat{F}^{+}+\hat{G}\hat{\sigma}^1-\hat{\sigma}^1\hat{G}\}_{\alpha\beta}$$

$$+\sum_{m_1m_2}\{-\hat{F}_{m_1}\hat{\sigma}_{m_2}^{2+}+\hat{\sigma}_{m_2}^{2}\hat{F}_{m_1}^{+}\}_{\alpha\beta} \qquad (9)$$

for the time derivative of the Green's function, where the self-energy parts $\hat{\sigma}^i$ are induced by the Hamiltonian (8). The remaining calculations simplify considerably in the semiclassical limit $V\ll|\Delta_i|$. Using the "multiplication rules" valid in this limit,[13] we easily find that for $T=0$ only the terms in the second sum in (9) give a nonvanishing contribution. We then obtain

$$J=\dot{N}=(4\pi N(0)^2Ra_0)^{-1}$$

$$\times\lim_{t\to+0}\sum_{m_1m_2}\int\frac{d^3\mathbf{p}_1\,d^3\mathbf{p}_2\,d\varepsilon}{(2\pi)^7}e^{i\varepsilon t}[-e^{2iVt}F_{m_1\alpha\beta}(\mathbf{p}_1,\varepsilon)$$

$$\times F^{+}_{m_2\beta\alpha}(\mathbf{p}_2,\varepsilon+V)+e^{-2iVt}F^{+}_{m_1\alpha\beta}(\mathbf{p}_1,\varepsilon)F_{m_2\beta\alpha}(\mathbf{p}_2,\varepsilon+V)]|T(\mathbf{p}_1,\mathbf{p}_2)|^2 \qquad (10)$$

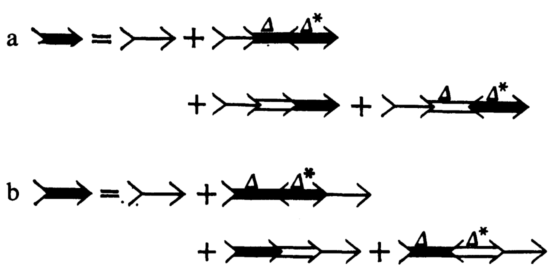

FIG. 1.

for the current J, where R is the ohmic resistance of the barrier, $N(0)$ is the density of the normal electron levels, and a_0 is defined below. Because the Green's functions peak sharply near the Fermi surface, the "transmission function" $|\hat{T}(\mathbf{p}_1,\mathbf{p}_2)|^2$ may be assumed[14] to depend only on the angle θ between the momentum vectors. Expanding it in a series of Legendre polynomials and using the addition theorem, we obtain

$$|\hat{T}(\theta)|^2=\sum_{l,m}a_l\frac{(l-|m|)!}{(l+|m|)!}P_l^m(\theta_1)P_l^m(\theta_2)\exp[im(\varphi_1-\varphi_2)]. \qquad (11)$$

If we substitute (11) into (10) and integrate first over the azimuthal angles and then over the remaining variables, we get the result

$$J=\frac{\pi|\Delta|}{2R}\cdot\frac{1}{2}\operatorname{Sp}(\hat{I}_1\hat{I}_2^{+})\left(\frac{a_l}{a_0(2l+1)}\right)\delta_{ll_1}\delta_{ll_2}\sin 2Vt, \qquad (12)$$

where for simplicity we have set $|\Delta_1|=|\Delta_2|=|\Delta|$. In deriving (12) we have used the structure of F (5) and F^+ (7), and the semiclassical approximation enabled us to set $V=0$ in the integration with respect to ε.

Equation (12) implies the following result in Ref. 4: the Josephson effect between triplet and singlet superconductors can be present only if the tunneling is accompanied by a spin reversal, because

$$\operatorname{Sp}(\hat{I}^{(s)}\hat{I}^{(t)})=0. \qquad (13)$$

Another consequence of (12) is that the pair current vanishes when $l_1\neq l_2$ (in the $O\times R$ symmetry class), even if the spin states have the same symmetry.

We stress once again that these results are valid only in our idealized model, and the actual behavior (as mentioned in the Introduction) may differ. However, the results can be used to analyze oscillations in the paramagnetic susceptibility, which persist even in the case of mixed symmetry.

3. OSCILLATIONS IN THE PARAMAGNETIC SUSCEPTIBILITY

We assume that both junction casings consist of superconductors with identical and odd values of l. Then in addition to giving rise to the Josephson effect, the oscillations in the number of pairs in the casings should also influence the paramagnetic susceptibility of the superconducting casings.

In the Gor'kov-Galitskiĭ model, the paramagnetic susceptibility of a triplet superconductor at $T=0$ is given by[15]

$$\chi_{ik}=i\mu_0^2\sum_{\alpha\beta\gamma\delta}(\sigma_i)_{\alpha\beta}(\sigma_k)_{\gamma\delta}\int\frac{d_\varepsilon\,d^3\mathbf{p}}{(2\pi)^4}\Big\{-G_{\delta\alpha}(\mathbf{p},\varepsilon)G_{\beta\gamma}(\mathbf{p},\varepsilon)$$

$$+\sum_m F_{m\delta\beta}(\mathbf{p},\varepsilon)F^{+}_{m\alpha\gamma}(\mathbf{p},\varepsilon)\Big\}. \qquad (14)$$

With Eqs. (5)–(7) this gives

$$\chi_{ik}={}^2/_3\chi\delta_{ik},\quad \chi=\mu_0^2p_F^2/\pi^2v_F=\mu_0^2N(0), \qquad (15)$$

where χ is the Pauli susceptibility of the degenerate Fermi gas and μ_0 is the Bohr magneton. The result (15) is familiar in the theory of superfluid ^{3}He.

We assume that the time dependence of the propagators is semiclassical and differentiate (14) with respect to t. The resulting derivative $\dot{G}$ is given by expression (9), and the functions $\dot{F}$ and $\dot{F}^+$ can be calculated using similar expansion diagrams.[2)] Using well-known formulas for the Legendre polynomials, we arrive at the expression

$$\dot{\chi}_{zz} = -\frac{32}{3\pi^2}\mu_0^2\frac{\nu a_l}{N(0)a_0(2l+1)}\operatorname{Im}\int\frac{p_1^2p_2^2\,dp_1\,dp_2\,d\varepsilon}{(2\pi)^5}\times G(p_1,\varepsilon)F(p_1,\varepsilon)F_2^*(p_2,\varepsilon+V) \quad (16)$$

after a straightforward calculation; here ν is the "tunnel frequency."[13] If we use Eqs. (7) for the amplitudes and again set $V=0$ in the integrand, we find by closing the path of integration in the upper half-plane that

$$\dot{\chi}_{zz} = -\frac{8}{3\pi^2}\mu_0^2\frac{\nu a_l|\Delta|^2}{N(0)a_0(2l+1)}\times\iint\frac{p_1^2p_2^2\,dp_1\,dp_2}{(2\pi)^4}\frac{(2\varepsilon_1+\varepsilon_2)\xi_1}{\varepsilon_2\varepsilon_1^3(\varepsilon_1+\varepsilon_2)^2}\sin 2Vt. \quad (17)$$

The rest of the integration is elementary, and we obtain

$$\dot{\chi}_{zz} = -\frac{4}{3\pi}\mu_0^2\frac{\nu a_l|\Delta|N(0)}{a_0(2l+1)\varepsilon_F}\sin 2Vt, \quad (18)$$

which implies that the relative magnitude of the oscillating susceptibility $\tilde{\chi}_{zz}$ is equal to

$$\tilde{\chi}_{zz}/\chi_{zz} = [a_l|\Delta|\nu/\pi(2l+1)a_0\varepsilon_F V]\cos 2Vt. \quad (19)$$

This shows that the oscillations are phase-shifted relative to the current oscillations. If the thickness of the junction lining is comparable to the correlation length in the superconductor ($\sim 10^{-4}$ cm) and the barrier resistance is $RS \sim 10^{-5}$ $\Omega\cdot$cm^2, then $\nu \sim 10^6$ s^{-1}. For small enough V, e.g., for $V \sim \nu$ (a few nanovolts), the oscillation amplitude is 10^{-3}–10^{-4} times the Pauli value. We note that the tunneling Hamiltonian (8) does not permit us to find the ratio a_1/a_0 of the coefficients appearing in (19) and (12); it can be determined from independent experiments by measuring the critical Josephson current for a triplet junction.

The oscillations in the paramagnetic susceptibility may show up near magnetic resonance—for example, as a shift in the paramagnetic resonance lines which depends on the bias voltage V. Such a shift would indicate the presence of triplet pairs.

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[1)] In spite of some reservations expressed in Refs. 10–12, we believe that the model in Ref. 9, suitably supplemented by a single gauge-invariant phase factor for all of the m-components of the anomalous Green's function, correctly describes states with the $O\times R$ symmetry for arbitrary l in the weak coupling approximation.

[2)] The latter procedure can be avoided by observing that (14) implies that $\chi_{ik}\equiv 0$ when $\sigma_i = i\sigma_y$ (singlet pairing); the contribution of the second term is therefore proportional to that of the first (in effect, we have an analog of the normalization condition for the matrix Green's function).

[1]G. E. Volovik and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **88**, 1412 (1985) [Sov. Phys. JETP **61**, 843 (1985)].

[2]H. R. Ott, H. Rudigier, T. M. Rice, *et al.*, Phys. Rev. Lett. **52**, 1915 (1984).

[3]N. E. Alekseevskiĭ, V. N. Narozhnyĭ, V. I. Nizhankovskiĭ, *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 421 (1984) [JETP Lett. **40**, 1241 (1984).

[4]J. A. Pals, W. van Haeringer, and M. H. van Maaren, Phys. Rev. B **15**, 2592 (1977).

[5]E. D. Fenton, Solid State Commun. **34**, 917 (1980).

[6]N. R. Werthamer, H. Suhl, and T. Soda, in: Proc. LT-8/R. O. Davies ed., Butterworths, London (1963), p. 140.

[7]E. W. Fenton, Solid State Commun. **53**, 501 (1985).

[8]F. Steglich *et al.*, "Heavy fermions in Kondo lattice compounds," MMM Conf., San Diego (1984).

[9]L. P. Gor'kov and V. M. Galitskiĭ, Zh. Eksp. Teor. Fiz. **40**, 1124 (1961) [Sov. Phys. JETP **13**, 792 (1961)].

[10]R. Balian, in: Lectures on the Many-Body Problem, Vol. 2, E. R. Caianiello ed., Academic Press, New York (1964), p. 147.

[11]D. Hone, Phys. Rev. Lett. **8**, 370 (1962).

[12]R. Balian, L. H. Nosanow, and N. R. Werthamer, Phys. Rev. Lett. **8**, 372 (1962).

[13]A. M. Gulyan and G. F. Zharkov, Zh. Eksp. Teor. Fiz. **89**, 156 (1985) [Sov. Phys. JETP **62**, 89 (1985)].

[14]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **51**, 1535 (1966) [Sov. Phys. JETP **24**, 1035 (1966)].

[15]I. A. Privorotskiĭ, Zh. Eksp. Teor. Fiz. **45**, 1961 (1963) [Sov. Phys. JETP **18**, 1346 (1963)].

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