Spin wave excitation by an ac magnetic field in a biaxial ferromagnet with a moving domain wall

L. G. Potemina

Low-Temperature Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR
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The generation and properties of spin waves are considered for waves excited by a parallel pumping field in a ferromagnet with a moving domain wall. The spin waves form near the wall and propagate in both directions from it; only one wave leaves in a given direction, and its parameters depend on the frequency ω of the ac field and on the average wall velocity. The radiation has a threshold velocity which leads to resonance peaks in the wall mobility.

INTRODUCTION

Parallel pumping is one of the best-understood methods for generating spin waves at microwave frequencies in ferromagnetic materials. In this case, an ac magnetic field polarized parallel to the magnetization interacts parametrically with the material (see the review in Ref. 1). In order for the spin waves to interact parametrically, the field amplitude must exceed a certain threshold, and the ac field frequency ω must be such that $\omega/2$ lies in the continuous spectrum of the linear oscillations. The principal resonance then occurs at twice the natural frequency of the system.

The phenomenological Landau-Lifshitz equation takes the form

$$-\mathbf{S}_t=[\mathbf{S}\Delta\mathbf{S}]+[\mathbf{S}J\mathbf{S}]+\mathbf{R}(\mathbf{S},t), \quad (1)$$

$$\mathbf{R}(\mathbf{S},t)=[\mathbf{S}\mathbf{n}](h_0+h\sin\omega t)-\lambda[\mathbf{S}\mathbf{S}_t] \quad (2)$$

in terms of dimensionless variables; it describes parametric instability of the time-dependent magnetization $S(r,t)=(S_1,S_2,S_3), S^2=1$, in a nearly uniformly magnetized biaxial ferromagnet. Here the diagonal matrix $J=\mathrm{diag}\,(J_1,J_2,J_3), J_1\leqslant J_2\leqslant J_3$, describes the anisotropy of the magnetic interactions. The external perturbation operator $\mathbf{R}(\mathbf{S},t)$ contains the dc and ac magnetic fields h_0 and $h\sin\omega t$, which point along the anisotropy axis $\mathbf{n}=(0,0,1)$ and are spatially uniform; the second term contains the dimensionless Gilbert damping parameter $\lambda\ll 1$ (Ref. 2) and describes the dissipative processes.

In this paper we propose a novel spin wave generation mechanism in which variations in the directions of the magnetic moment in the domain walls play a key role. It is found that under appropriate conditions, spin-wave excitation by an ac magnetic field in biaxial ferromagnets with moving domain walls admits a complete theoretical analysis based on Eqs. (1), (2), which are the same as in the theory of paramagnetic resonance. We note that our specific results were obtained for the one-dimensional case, whereas in ordinary experiments on domain wall dynamics in epitaxial iron-garnet films, for example, one-dimensional domain structures occur only for high quality factors $Q>1$ (Ref. 2); in this case, one can neglect the demagnetizing influence of the surface, which (among other things) gives rise to Bloch lines in the domain walls. We therefore assume a strongly magnetic material in the limit $Q\gg 1$. In addition, we require that the external magnetic field be weak compared to the field $4\pi QM_0$, where M_0 is the spontaneous magnetization per unit volume. If we pass to dimensionless variables by dividing the magnetic field strength by $4\pi QM_0$, the above conditions take the form $h,h_0\ll 1$ and $J_3-J_2=1$; the latter condition can be relaxed to $J_3-J_2\sim 1$ without significantly altering the results.

The main result of this article is that intense spin-wave radiation in a given direction can be generated by using a parallel magnetic field to pump magnetic materials in which the average domain wall motion is spatially uniform. More precisely, at frequencies $\omega>\omega_c(V)$ a moving domain wall radiates spin waves both along the opposite to its direction of motion; these spin waves have different properties which depend on the field frequency ω and the dc field strength h_0, i.e., on the mean wall velocity $V=V(h_0)$. Spin-wave excitation of this type is possible in principle for arbitrarily small amplitudes h of the ac field component. The threshold frequency $\omega_c(V)$ above which spin waves are excited depends on the wall velocity; it is less than the homogeneous magnetic resonance frequency Ω_0 and may compete with the fundamental parametric resonance frequency in experiments measuring the energy absorption rate in two-domain specimens. The effective magnetic field h_{eff} produced by the spin waves also peaks resonantly at the frequency $\omega_c(V)$; this field retards the domain walls and tends to take them out of resonance. The resonant frequency-dependence of h_{eff} should show up as a "hollow" in the monotonic dependence $V=V(h_0)$ whose position depends on the frequency ω. In this respect the behavior differs from that studied previously in Refs. 3–6, where the retardation of the domain walls was due to quasiparticles in the crystal (acoustic or optical photons, optical or surface magnons). Indeed, the horizontal sections observed there occurred at velocities V equal to the phase velocity of the quasiparticles and may disappear completely in strongly magnetic materials.

The perturbation theory developed in this paper is based on the inverse scattering method[7–13] and yields explicit formulas for the propagation of elliptically polarized spin waves whose wavelength is long compared to the characteristic width ξ^{-1} of the domain walls. The results show that the mean deviation ΔS_3 of the projection S_3 of the magnetic moment from the equilibrium value is given by

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$$\Delta S_3 \approx \pi^2 h^2 V^2 (2J_3 - J_1 - J_2)/16\xi^2\Omega_0^2 |V - v_g^\pm|^2$$

far from the domain wall (but within a spin-wave damping length). This result is valid for frequencies $\omega \lesssim \Omega_0$ and fields $h_0 < \lambda (J_2 - J_1)/2$, and the group velocities for the forward- and backward- directed radiation are given by $v_g^+ \lesssim 2V$ and $v_g^- \gtrsim 0$, respectively. We get the estimate $\Delta S_3 \sim h^2$ if $J_3 - J_2 \sim J_2 - J_1 \sim 1$ (ξ, $\Omega_0 \sim 1$). The deviation ΔS_3 is thus bounded from above only by the requirement that the perturbation theory be valid (in our case, this requires that $h \ll \omega$) and may be considerably greater than the thermal fluctuations $\Delta S_3(T)$, which are generally $\sim 10^{-7}$–10^{-5} at room temperatures. Dissipative processes attenuate the spin waves; for $\omega \lesssim \Omega_0$ and velocities $V \sim \gamma V_0$, the forward radiation has wave number $\sim \gamma \xi$ and mean free path $\sim \xi^{-1}\gamma/\lambda$, where $0 < \gamma \ll 1$ and V_0 is the maximum wall velocity. The condition $\lambda \ll \gamma$ is clearly necessary if the spin-wave radiation is to be observed leaving the sample. The damping factor λ is typically between 10^{-4} and 10^{-2} in iron-garnet films, which suggests that long-wave radiation should be observable for a wide range $\lambda V_0 \ll V \ll V_0$ of wall velocities. In crystals with large magnetic losses it should be possible to use magnetooptical methods to measure the radiation from the domain walls in an ac magnetic field; the radiation should show up as an increase in the effective width of the domain walls.

We note that the inverse scattering method is not essential for calculating how uniform dc and ac magnetic fields alter the parameters of an individual domain wall in a dissipative material in the adiabatic approximation—the equations can also be solved by direct methods. In Sec. I we eliminate the secular terms[14–16] to derive the most general equations in the adiabatic approximation; they are of independent interest and describe the one-dimensional dynamics of the soliton solutions of the Landau-Lifshitz equation in biaxially anisotropic materials subjected to a small perturbation.

1. EQUATIONS IN THE ADIABATIC APPROXIMATION

We have already noted that Eq. (1) describes the one-dimensional dynamics of the magnetization vector $\mathbf{S} = \mathbf{S}(x,t)$ in a biaxial ferromagnetic crystal containing a domain wall. We will be interested in the case when the deviations of $\mathbf{S}$ from the magnetization distribution in the domain wall are small enough so that the concept of domain wall remains meaningful. This requires that the parameter ε ($h_0, h, \lambda \sim \varepsilon$) characterizing the perturbation $\mathbf{R}$ be small. This problem is a special case of a more general one in which the one-dimensional solutions of the perturbed equation (1) are found by exactly solving the corresponding unperturbed, exactly integrable equation

$$\mathbf{S}_t = \{\mathbf{S}, H\}. \tag{3}$$

Here the Hamiltonian H is given by

$$H = \frac{1}{2} \int_{-\infty}^{\infty} dx [\mathbf{S}_x^2 - \mathbf{S}J\mathbf{S}], \tag{4}$$

and the Poisson brackets are defined by

$$\{F, G\} = \int_{-\infty}^{\infty} dx\, e_{ijk} \frac{\delta F}{\delta S_i(x)} \frac{\delta G}{\delta S_j(x)} S_k(x),$$

where the tensor e_{ijk} is the completely antisymmetric unit tensor.

According to perturbation theory,[16] one starts with any solution $\mathbf{S}^{(0)}$ of the unperturbed equation (3) which is defined by a set of N parameters p_i ($i = 1, 2, ..., N$), i.e., $\mathbf{S}^{(0)} = \mathbf{S}^{(0)}(p_i, x, t)$. One then seeks a solution of (1) of the form

$$\mathbf{S} = \mathbf{S}^{(0)}(p_i(T), x, t) + \mathbf{S}^{(1)}(x, t), \quad T = \varepsilon t, \quad \varepsilon \ll 1, \tag{5}$$

under the assumption that the small perturbation (in the principal, or adiabatic, approximation) causes the p_i to vary only slowly with time. In the next-higher approximation, the perturbation is assumed to give a correction $\mathbf{S}^{(1)}$ which is of the same order as $\mathbf{R}(\mathbf{S}^{(0)})$.

Linearizing Eq. (1) near $\mathbf{S}^{(0)}$, we obtain

$$\begin{gathered} -\mathbf{S}_t^{(1)} = [\mathbf{S}^{(1)}, \mathbf{S}_{xx}^{(0)} + J\mathbf{S}^{(0)}] + [\mathbf{S}^{(0)}, \mathbf{S}_{xx}^{(1)} + J\mathbf{S}^{(1)}] \\ + \frac{\partial \mathbf{S}^{(0)}}{\partial p_i} \frac{dp_i}{dt} + \mathbf{R}(\mathbf{S}^{(0)}) + \mathbf{R}'(\mathbf{S}^{(0)}, \mathbf{S}^{(1)}), \\ \mathbf{S}^{(0)}\mathbf{S}^{(1)} = 0. \end{gathered} \tag{6}$$

The term $\mathbf{R}'$ in (6) contains corrections of higher order in ε which can almost always be neglected, except when the deviation of $\mathbf{S}$ from $\mathbf{S}^{(0)}$ is no longer small; we will return to this matter a little later. In general, however, one must require that Eq. (6), with the inhomogeneous term

$$\mathbf{R}(\mathbf{S}^{(0)}) + \frac{\partial \mathbf{S}^{(0)}}{\partial p_i} \frac{dp_i}{dt},$$

possess a bounded solution. The solution of the corresponding homogeneous equation must be skew-optional to the above inhomogeneous term in terms of the symplectic structure defining the initial kinematic system. If we differentiate (3) with respect to p_i and use the constraint $(\mathbf{S}^{(0)})^2 = 1$, we can verify that the functions $\partial \mathbf{S}^{(0)}/\partial p_i$ satisfy the homogeneous equation and find that no secular terms are present in (6) if

$$\frac{dp_i}{dt} \int_{-\infty}^{\infty} dx\, \mathbf{S}^{(0)} \left[\frac{\partial \mathbf{S}^{(0)}}{\partial p_i} \frac{\partial \mathbf{S}^{(0)}}{\partial p_k} \right] = \int_{-\infty}^{\infty} dx\, \mathbf{R}(\mathbf{S}^{(0)}) \left[\mathbf{S}^{(0)} \frac{\partial \mathbf{S}^{(0)}}{\partial p_k} \right]. \tag{7}$$

We have thus found a closed system of N linear algebraic equations for the derivatives dp_i/dt which describe the time dependence of the parameters p_i in the adiabatic approximation.

To apply the above result to domain wall motion caused by a perturbation of general form, we take the lowest-order approximation $\mathbf{S}^{(0)}$ to be the one-soliton solution of the unperturbed equation (3):

$$\begin{gathered} S_1^{(0)} = \cos\varphi \operatorname{sech} z, \quad S_2^{(0)} = \sin\varphi \operatorname{sech} z, \quad S_3^{(0)} = \varkappa \operatorname{th} z, \\ z = \xi x - \varkappa\tau t - \xi x_0(0) = \xi(x - x_0), \\ \tau = -(J_2 - J_1)\sin\varphi\cos\varphi, \quad \xi = [J_3 - J_2 + (J_2 - J_1)\cos^2\varphi]^{1/2}. \end{gathered} \tag{8}$$

Here the parameter $\varkappa$ characterizes the polarity of the domain wall ($\varkappa = +1$ and -1 describe solitons and antisolitons, respectively); the angle φ gives the orientation of the vector $\mathbf{S}^{(0)}$ in the plane normal to the anisotropy axis $\mathbf{n}$; the width of the domain wall is $\sim\xi^{-1}$.

It is easy to see that the relevant parameters p_i for the domain wall are the angle φ and the coordinate x_0 of the center of the wall. A term containing $\partial \mathbf{S}^{(0)}/\partial t$ then appears in Eqs. (6) and (7), and p_i-derivatives are given by

$$\frac{\partial \mathbf{S}^{(0)}}{\partial p_i}=\left\{\frac{\partial \mathbf{S}^{(0)}}{\partial x_0}\Big|_{\varphi=\mathrm{const}},\quad \frac{\partial \mathbf{S}^{(0)}}{\partial \varphi}\Big|_{x_0=\mathrm{const}}\right\}.$$

If we recall that $\mathbf{R}(\mathbf{S}^{(0)})$ is perpendicular to $\mathbf{S}^{(0)}$, we obtain the dynamic equations

$$\frac{d\varphi}{dt}=\frac{1}{2}\int_{-\infty}^{\infty} dz\, \mathrm{sech}\, z\,(R_1\sin\varphi-R_2\cos\varphi),\qquad (9)$$

$$\frac{dx_0}{dt}=-\frac{\varkappa\tau}{\xi}+\frac{1}{2\xi}\int_{-\infty}^{\infty} dz\left[\frac{\tau}{\xi^2}z\,\mathrm{sech}\, z\,(R_1\sin\varphi-R_2\cos\varphi)+\varkappa R_3\right]\qquad (10)$$

for the domain wall in the adiabatic approximation after some straightforward calculations. The right-hand sides of (9) and (10) should actually contain the derivatives dx_0/dt and $d\varphi/dt$ to allow for dissipation; however, these corrections are negligible to lowest order.

Substituting (2) into Eqs. (9) and (10) and writing $h_c=\lambda(J_2-J_1)/2$, we get

$$d\varphi/dt=h_0+h\sin\omega t+h_c\sin 2\varphi,\qquad (11)$$

$$dx_0/dt=V(\varphi)=-\varkappa\tau/\xi.\qquad (12)$$

Situations when the domain wall is either stationary or moves uniformly are of interest in the study of spin-wave excitation in ferromagnets with domain walls. This behavior can only be approximated experimentally, and even here the ac field amplitude must be small and the dissipation in the material large.

We will first describe the domain wall dynamics in the adiabatic approximation when no ac magnetic field is present. Equation (11) can be solved trivially when $h=0$, and we find that the behavior of $\varphi(t)$ depends on the ratio h_0/h_c. In strong magnetic fields ($h_0>h_c$), the domain wall moves in a complicated way with periodic changes in the sign of the velocity, and the mean displacement of the center of the wall does not vanish. It is very difficult to analyze radiation associated with this type of wall motion. The weak-field case ($h_0<h_c$) is of greater experimental interest. In this case the time dependence of the wall velocity $V^{(0)}(t)$ is given by

$$V^{(0)}(t)=\varkappa(J_2-J_1)s(t)/[s^2(t)+1]^{1/2}[(J_3-J_2)s^2(t)+J_3-J_1]^{1/2},\qquad (13)$$

where

$$s(t)=\mathrm{tg}\,\varphi^{(0)}(t)=\frac{2\omega_0\,\mathrm{tg}\,\varphi_0+(h_c\,\mathrm{tg}\,\varphi_0+h)\,\mathrm{th}\,2\omega_0 t}{2\omega_0-(h\,\mathrm{tg}\,\varphi_0+h_c)\,\mathrm{th}\,2\omega_0 t}$$
$$\omega_0={}^1/_2(h_c^2-h_0^2)^{1/2}.\qquad (14)$$

The velocity approaches the limiting Walker value

$$V_W=-\frac{\varkappa}{2}(J_2-J_1)\frac{h_0}{h_c}\left[J_3-\frac{1}{2}(J_1+J_2)-\frac{1}{2}(J_2-J_1)\left(1-\frac{h_0^2}{h_c^2}\right)^{1/2}\right]^{-1/2}\qquad (15)$$

during times $\sim\omega_0^{-1}$.

The situation when the domain wall moves at the constant Walker velocity is of the greatest interest for studying the linear response of magnetic systems to external ac magnetic fields of the form $h\sin\omega t$ directed along the axis of anisotropy $\mathbf{n}$. As was noted above, in order to get steady-state behavior in which the magnetization vector in the domain wall is perturbed only slightly ($|\Delta\varphi|\ll 1$) from the average value

$$\varphi_W={}^1/_2\arcsin(h_0/h_c)-\pi/2$$

corresponding to V_W, the amplitude h must be small and the dissipation sufficiently great, i.e.,

$$h^2\ll\omega^2+4h_c^2,\quad h_0<h_c.\qquad (16)$$

Inequalities (16) permit us to seek a solution of (11) of the form $\varphi=\varphi^{(0)}+\varphi^{(1)}$, where (14) gives the time dependence of $\varphi^{(0)}$ and the correction

$$\varphi^{(1)}(t)\approx-\frac{h}{\omega^2+4h_c^2\cos^2 2\varphi^{(0)}(t)}\times[2h_c\cos 2\varphi^{(0)}(t)\sin\omega t+\omega\cos\omega t]$$

describes forced oscillations of the magnetization vector in the domain wall about the averaging value $\varphi^{(0)}(t)$; these oscillations occur at frequency ω and have a small amplitude $|\varphi^{(1)}|\ll 1$. In the limit $t\to\infty$, $\varphi^{(0)}(t)$ tends asymptotically to φ_W and the wall velocity oscillates slightly about the Walker value V_W. For $h_0=0$, $\varphi^{(0)}$ tends to the limits $\varphi_W=\pm\pi/2$. In this case the mean velocity V_W is equal to zero, i.e., the dissipation combined with the ac magnetic field $h\sin\omega t$ causes the domain wall to oscillate near the stable (Bloch) equilibrium position.

Now that $\varphi(t)$ and $x_0(t)$ are known, we can analyze the equation for the first correction $\mathbf{S}'^{(1)}$ by recasting (6) in the form

$$-\mathbf{S}_t^{(1)}=[\mathbf{S}^{(1)},\mathbf{S}_{xx}^{(0)}+J\mathbf{S}^{(0)}-\lambda\mathbf{S}_t^{(0)}]+[\mathbf{S}^{(0)},\mathbf{S}_{xx}^{(1)}+J\mathbf{S}^{(1)}-\lambda\mathbf{S}_t^{(1)}]+\mathbf{Z}\frac{d\varphi}{dt},$$
$$\mathbf{Z}=z\,\mathrm{sech}^2 z\frac{d\ln\xi}{d\varphi}(-\cos\varphi\,\mathrm{sh}\,z,-\sin\varphi\,\mathrm{sh}\,z,\varkappa)\qquad (17)$$

for a domain wall. We note that the dissipative term $\lambda[\mathbf{S}'^{(0)}\mathbf{S}_t^{(1)}]$ once again appears in the equation. Our results will show that this term is responsible for cutting off the divergences and for damping the spin waves; the other terms in $\mathbf{R}'(\mathbf{S}^{(0)},\mathbf{S}'^{(1)})$ give only small corrections to $\mathbf{S}^{(1)}$.

If we assume (16), we can set $\varphi=\varphi_W$ in all the terms in (17) containing $\mathbf{S}^{(0)}$ and its derivatives; the correction $\varphi^{(1)}(t)$ needs to be considered only in the derivative $d\varphi/dt$ in the last term. For large times ($\omega_0 t\gg 1,\omega t\gg 1$) we have

$$\frac{d\varphi}{dt}=-h\omega(\omega^2+\omega_0^2)^{-1/2}\cos\left(\omega t+\mathrm{arctg}\frac{\omega}{\omega_0}\right).\qquad (18)$$

We now show how the above problem of calculating the correction $\mathbf{S}'^{(1)}$ to the soliton solution $\mathbf{S}^{(0)}$ in an ac magnetic field can be reduced in the long-wave approximation to a system of linear equations. We will average (17) under the assumption that $\mathbf{S}'^{(1)}$ changes over a characteristic length k^{-1} much greater than the width of the domain wall, i.e., $k \ll \xi$. To do this we pass to an intrinsic coordinate system moving at $V = V_W$ (the comoving frame for the domain wall) and integrate (17) with respect to z from $-\eta$ to η, where $1 \ll \eta \ll \xi/k$. This yields the linear equation

$$-\mathbf{S}_t^{(1)} = [\varkappa \mathbf{n}\,\mathrm{sign}\, z,\ \mathbf{S}_{xx}^{(1)} + (J - J_3)\mathbf{S}^{(1)} - \lambda \mathbf{S}_t^{(1)}], \qquad (19)$$

which has a nontrivial "source" term; the boundary conditions are

$$\left[\left(1 - \frac{\lambda V}{\xi}\right)\mathbf{S}^{(1)}(z+0) - \left(1 + \frac{\lambda V}{\xi}\right)\mathbf{S}^{(1)}(z-0), \mathbf{n}\right] = V\frac{\pi}{\xi^3}\frac{d\varphi}{dt}(\cos\varphi, \sin\varphi, 0). \qquad (20)$$

Equation (19) is satisfied by elliptically polarized spin waves

$$S_j = S_0\,\mathrm{Re}\{\beta_j \exp[i(kx - \Omega(k)t)]\},$$
$$\beta_1 = [k^2 + J_3 - J_2 - i\lambda\Omega(k)]^{1/2}, \qquad (21)$$
$$\beta_2 = i\varkappa\,\mathrm{sign}\, z\,[k^2 + J_3 - J_1 - i\lambda\Omega(k)]^{1/2},$$

which are familiar in the theory of magnetism; the dispersion law ($\lambda \ll 1$) is

$$\Omega(k) \approx (\Omega_0^2(k) - h_c^2)^{1/2} - i\lambda[k^2 + J_3 - (J_1 + J_2)/2],$$
$$\Omega_0(k) = (k^2 + J_3 - J_1)^{1/2}(k^2 + J_3 - J_2)^{1/2}.$$

The boundary conditions (20) lead to two interesting conclusions in the limit $\lambda \to 0$. First, there are only two spin waves, and their frequencies are equal to ω in the comoving frame. Their wave vectors k can be found from the equation $\omega + kV = \Omega_0(k)$, which has two solutions for each domain wall velocity V for frequencies $\omega > \omega_c(V)$ (Fig. 1). The second conclusion is that the spin-wave amplitude vanishes at $V = 0$, i.e., for $\varphi = 0$, which corresponds to an unstable Néel domain wall, and for $\varphi = \pm\pi/2$, which correspond to a stable Bloch wall. This is hardly surprising, since the wall energy is equal to 2ξ and has a maximum at $\varphi = 0$ and a minimum at $\varphi = \pm\pi/2$. For a specified small amplitude $\varphi^{(1)} \sim h$, the second-order effects cause the wall energy to oscillate by an amount $\propto h$ well away from the extrema but by an amount $\propto h^2$ close to them. According to (20) the correction $\mathbf{S}'^{(1)}$, which is linear in h, should thus vanish linearly as $V \to 0$.

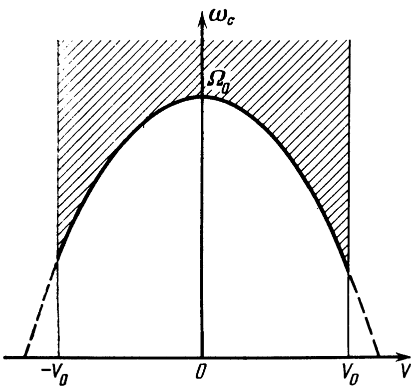

FIG. 1. Critical pump frequency vs average wall velocity; spin waves are excited in the hatched region. The maximum wall velocity is $V_0 = (J_3 - J_1)^{1/2} - (J_3 - J_2)^{1/2}$.

Further analysis of spin waves using the reduced equation (19) is unwarranted, because the spin waves are in fact generated nonlocally throughout the entire domain wall; more complicated mathematical machinery is therefore needed to find how the energy is distributed between the two harmonics. In this paper we will use perturbation theory based on the inverse scattering method.

2. APPLICATION OF THE INVERSE SCATTERING METHOD TO THE LANDAU-LIFSHITZ EQUATION

We first discuss a general procedure for solving Eq. (3) by the inverse scattering method in which one solves the Riemann matrix problem on a torus. The reader is referred elsewhere for details.[7–10]

The Landau-Lifshitz equation (3) is exactly solvable and is usually expressed as the condition for the system of linear equations

$$\Psi_x = L\Psi, \quad L = -i\sum_\alpha w_\alpha(u) S_\alpha \sigma_\alpha, \qquad (22)$$

$$\Psi_t = M\Psi,$$
$$M = i\sum_{\alpha,\beta,\gamma} w_\alpha(u) S_\beta S_{\gamma x} \sigma_\alpha e_{\alpha\beta\gamma} - i\sum_{\alpha,\beta,\gamma} w_\beta(u) w_\gamma(u) S_\alpha \sigma_\alpha |e_{\alpha\beta\gamma}|, \qquad (23)$$

to be consistent. Here σ_α ($\alpha = 1,2,3$) are the Pauli matrices, the elliptic functions $w_\alpha(u)$ are defined in the period parallelogram

$$R = \{u : |\mathrm{Re}\, u| \leqslant 2K, \quad |\mathrm{Im}\, u| \leqslant 2K'\},$$
$$w_1(u) = \rho\,\mathrm{ns}(u, m), \quad w_2(u) = \rho\,\mathrm{ds}(u, m),$$
$$w_3(u) = \rho\,\mathrm{cs}(u, m),$$

where

$$\rho = \tfrac{1}{2}(J_3 - J_1)^{1/2}, \quad m = [(J_2 - J_1)/(J_3 - J_1)]^{1/2}, \quad 0 \leqslant m \leqslant 1,$$

and $ns(u,m)$, $ds(u,m)$, and $cs(u,m)$ are the Jacobi elliptic functions.

We are interested in boundary conditions of the domain wall type with polarity $\varkappa = \pm 1$,

$$\mathbf{S}(x, t) \to (0, 0, \pm\varkappa), \quad x \to \pm\infty, \qquad (24)$$

and therefore introduce the Jost functions for Eq. (22), i.e., fundamental solutions with the asymptotic behavior

$$\Psi_+(x, u) \to \exp[-i\sigma_3 \varkappa w_3(u) x], \quad x \to +\infty,$$
$$\Psi_-(x, u) \to i\exp[i\sigma_3 \varkappa w_3(u) x]\sigma_2, \quad x \to -\infty \qquad (25)$$

for $\mathrm{Im}\, u = 0$ and $2K'$.

The scattering matrix $T(u)$ relates the Jost functions: $\Psi_+(x,u) = \Psi_-(x,u)T(u)$ and is given by

$$T(u)=\begin{pmatrix} a(u) & -b(u) \\ \overline{b(\bar u)} & \overline{a(\bar u)} \end{pmatrix}, \quad \det T(u)=1, \tag{26}$$

$$\begin{aligned} a(u+2K)&=-a(u), \quad \overline{a(\bar u+2iK')}=-a(u),\\ b(u+2K)&=b(u), \quad \overline{b(\bar u+2iK')}=b(u). \end{aligned} \tag{27}$$

We define the functions

$$\begin{aligned} f_+(x,\ u)&=\Psi_-(x,\ u)T_+(u)\exp[i\varkappa w_3(u)x\sigma_3],\\ f_-(x,\ u)&=\exp[-i\varkappa w_3(u)x\sigma_3]T_-(u)\Psi_-^{-1}(x,\ u), \end{aligned} \tag{28}$$

$$T_+(u)=\begin{pmatrix} a(u) & 0 \\ \overline{b(\bar u)} & 1 \end{pmatrix}, \quad T_-(u)=\begin{pmatrix} \overline{a(\bar u)} & b(u) \\ 0 & 1 \end{pmatrix}$$

on the curves

$$\Gamma_1=\{u: |\mathrm{Re}\ u|\leqslant 2K,\ \mathrm{Im}\ u=0\},$$

$$\Gamma_2=\{u: |\mathrm{Re}\ u|\leqslant 2K,\ \mathrm{Im}\ u=2\varkappa K'\}.$$

For $\varkappa=1$ we can continue $f_\pm$ analytically into the regions $R_\pm$ (the upper and lower halves of the parallelogram R), while for $\varkappa=-1$ the continuation is into $R_\mp$. In either case, the extended functions are doubly periodic with the periods $4K$ and $4K'$. After analytic continuation into the half of R in which f_+ is analytic, the coefficient $a(u)$ satisfies $|a(u)|\to 1$ and $u\to 0$ and can have zeros at symmetrically located points

$$u_{0k}\,(0\leqslant \mathrm{Re}\ u_{0k}\leqslant 2K, \quad 0\leqslant\varkappa\ \mathrm{Im}\ u_{0k}\leqslant 2K'),$$

$$u_{1k}=u_{0k}-2K, \quad u_{2k}=\bar u_{0k}+2i\varkappa K', \quad u_{3k}=u_{0k}-2K+2i\varkappa K'.$$

The matrix f_+ becomes singular at the zeros of $a(u)$, so that its columns are proportional:

$$f_+^{(1)}(x,u_{0k})=\bar b_k f_+^{(2)}(x,u_{0k})\exp[2i\varkappa w_3(u_{0k})x].$$

The coefficient $b(u)$ and the characteristics of the discrete spectrum (the zeros u_k of $a(u)$ and the normalization coefficients b_k, $k=1,2,\ldots,N$) constitute a complete and independent set of scatting data for the potential $\mathbf{S}(x,t)$. Equation (23) governs the time evolution of the scattering data; we obtain

$$\begin{aligned} u_{0k}(t)&=u_{0k}(0),\\ b(u,\ t)&=b(u,\ 0)\exp[-4i\varkappa w_1(u)w_2(u)t],\\ b_k(t)&=b_k(0)\exp[-4i\varkappa w_1(u_{0k})w_2(u_{0k})t]. \end{aligned}$$

We see easily that the Cauchy problem for Eq. (3) with the boundary conditions (24) has the solution

$$\begin{aligned} \boldsymbol{\sigma}\mathbf{S}&=f_+(x,\ 0)\sigma_3 f_+^{-1}(x,\ 0), \quad \varkappa=+1,\\ \boldsymbol{\sigma}\mathbf{S}&=-f_-^{-1}(x,\ 0)\sigma_3 f_-(x,\ 0), \quad \varkappa=-1 \end{aligned} \tag{29}$$

so that the problem reduces to calculating $f_\pm$ from the scattering data.

We note that by construction, the classical Riemann matrix problem on the torus can be formulated for the functions $f_\pm$. Indeed, the definition (28) yields the boundary conditions for the Riemann problem, i.e., conjugacy conditions for the functions $f_\pm$ on the contour contour $\Gamma=\Gamma_1\cup\Gamma_2$ (Im $u=0$):

$$\begin{aligned} f_-(x,\ u)f_+(x,\ u)&=G(x,\ t;\ u),\\ f_-(x,\ u-2iK'\varkappa)f_+(x,\ u+2iK'\varkappa)&=G(x,\ t;\ u+2iK'\varkappa), \end{aligned} \tag{30}$$

$$G(x,t;u)=\begin{pmatrix} 1 & b(u,t)\exp[-2i\varkappa w_3(u)x] \\ \overline{b(\bar u,t)}\exp[2i\varkappa w_3(u)x] & 1 \end{pmatrix}.$$

In our case, $f_\pm$ must satisfy the constraints

$$f_+(u+2K)=\sigma_3 f_+(u), \quad \overline{f_+(\bar u+2iK')}=\sigma_3 f_+(u), \tag{31}$$

$$f_-(u+2K)=f_-(u)\sigma_3, \quad \overline{f_-(\bar u+2iK')}=f_-(u)\sigma_3, \tag{32}$$

$$f_\pm^+(u)=f_\mp(\bar u).$$

As usual, we factor the solution of the Riemann problem as $f_+=f^r_+f^R_+$, $f_-=f^R_-f^r_-$, where $f^r_\pm$ is the solution of the regular Riemann problem ($\det f^r_\pm\neq 0$); the functions $f^R_\pm$ have prescribed zeros and can be continued analytically to meromorphic functions on the torus if we set $f^R_-(u)f^R_+(u)=I$.

If the $f^R_\pm$ are known (their calculation lies beyond the scope of this paper), we can formulate a regular Riemann problem for the $f^r_\pm$:

$$\begin{aligned} f_-^r(u)f_+^{(r)}(u)&=\tilde G(x,\ t;\ u),\\ \tilde G(x,\ t;\ u)&=f_+^R(u)G(x,\ t;\ u)(f_+^R(u))^{-1}. \end{aligned} \tag{33}$$

Unlike the functions $f^R_\pm$, which preserve the symmetry of the complete solution $f_\pm$ of (30), the regular component $f^r_\pm$ has the properties

$$f_\pm^r(u+2K)=\sigma_3 f_\pm^r(u)\sigma_3, \quad f_\pm^r(\bar u+2iK')=\sigma_3 f_\pm^r(u)\sigma_3, \tag{34}$$

$$(f_\pm^r(u))^+=f_\mp^r(\bar u), \tag{35}$$

which are exactly the same as the ones derived in Refs. 9 and 10 for boundary conditions of another type.

The last step in reconstructing the potential $\mathbf{S}(x,t)$ is to solve the Fredholm integral equation that arises in the problem (33). This equation involves the boundary values of f^r_+ (f^r_-) on the contour Γ and can be expressed in the following form, which is invariant under the transformations (34) ($u\in\Gamma$, $\varkappa=1$):

$$\begin{aligned} {}^1/_2 f_+^r(u)(1+\tilde G^{-1}(u)) &= f_0+\frac{1}{2\pi i}\oint_\Gamma d\mu[N_1(\mu-u)f_+^r(\mu)(1-\tilde G^{-1}(\mu))\\ &+N_2(\mu-u)\sigma_3 f_+^r(\mu)(1-\tilde G^{-1}(\mu))\sigma_3], \end{aligned} \tag{36}$$

where

$$N_1(\mu)={}^1/_8(\mathrm{ns}\ \mu+\mathrm{ds}\ \mu)(1+\mathrm{cn}\ \mu),$$

$$N_2(\mu)={}^1/_8(\mathrm{ns}\ \mu+\mathrm{ds}\ \mu)(\mathrm{cn}\ \mu-1).$$

According to (34), the matrix f_0 in (36) contains only two unknown parameters; their ratio is determined by the condition that $f_+(x,0)$ be unitary.

We next consider the magnetization distribution for the case when $\mathbf{S}$ initially differs slightly from the magnetization $\mathbf{S}^{(0)}(x,t)$ for a pure soliton pulse; that is, $|b(u,t)|\ll 1$ and Eq. (36) can be solved by successive approximation. The term $\mathbf{S}^{(0)}(x,t)$ corresponds to $b^{(0)}=0$ in the scattering problem, and we have

$$S^{(0)}=\sigma \mathbf{S}^{(0)}=\varkappa f_+^R(x,0)\sigma_3(f_+^R(x,0))^{-1}. \quad (37)$$

According to (29), the first-order correction $\mathbf{S}^{(1)}$ is given by

$$\mathbf{S}^{(1)}\sigma=\varkappa[f_1(x,0),S^{(0)}], \quad (38)$$

where

$$f_1(0)=-\frac{1}{2\pi i}\int_\Gamma d\mu[N_1(\mu)G_1(\mu)+N_2(\mu)\sigma_3G_1(\mu)\sigma_3].$$

Here we have used the fact that $G_1(u)=\widetilde{G}^{-1}(u)-I$ and $f_1(u)=f^r_\pm(u)-I$ are small to the same order.

We next find the contribution from $\mathbf{S}^{(1)}$ to the energy H and momentum P of the system from the equations

$$P=-2i\varkappa\ln a(0),\quad H=-4i\rho\varkappa a'(0)/a(0),$$

which express H and P in terms of the coefficient $a(u)$. The representation

$$\ln a(u)=\ln a_s(u)+\frac{\varkappa}{2\pi i}\int_0^{2K}d\mu\,\mathrm{cs}(\mu-u-\varkappa i0)\ln|a(\mu)|^2 \quad (39)$$

holds for $a(u)$, which is analytic in the region $R_\pm$ ($\varkappa=\pm 1$). The second term in (39) is nonzero only if the continuous spectrum is excited, i.e., if $|a(u)|<1$. Denoting the corresponding contribution to H and P by E_{rad} and P_{rad}, respectively, and retaining only the first nonvanishing term in the expansion of the logarithm in powers of $|b(u,t)|$, we find that

$$E_{rad}=\frac{2}{\pi\rho}\int_0^{2K}du\,w_1(u)w_2(u)|b(u,t)|^2, \quad (40)$$

$$P_{rad}=\frac{1}{\pi\rho}\int_0^{2K}du\,w_3(u)|b(u,t)|^2. \quad (41)$$

Finally, we analyze an almost-integrable one-dimensional modification of Eq. (1) which coincides with the exactly soluble equation (3), except for the presence of the small term $\mathbf{R}$. To do this we develop a perturbation theory which is based on the inverse scattering method[11-13] and yields evolution equations for the scattering data. The latter are determined from the scattering problem (22), (25), but the small perturbation $\mathbf{R}$ makes them time-dependent:

$$\frac{\partial a(u,t)}{\partial t}=i\int_{-\infty}^{\infty}dx\det\{\psi(x,u),\hat{R}(u)\varphi(x,u)\}, \quad (42)$$

$$\frac{\partial b(u,t)}{\partial t}=-4i\varkappa w_1(u)w_2(u)b(u,t)+i\int_{-\infty}^{\infty}dx\det\{\hat{R}(u)\varphi(x,u),\bar{\psi}(x,u)\}, \quad (43)$$

$$\frac{du_k}{dt}=-\frac{i}{a'(u_k)}\int_{-\infty}^{\infty}dx\det\{\psi(x,u_k),\hat{R}(u_k)\varphi(x,u_k)\}, \quad (44)$$

$$\frac{db_k}{dt}=-4i\varkappa w_1(u_k)w_2(u_k)b_k-\frac{i}{a'(u_k)}\int_{-\infty}^{\infty}dx\det\left\{\left[\frac{\partial\varphi(x,u)}{\partial u}\right]_{u=u_k}-b_k\left[\frac{\partial\psi(x,u)}{\partial u}\right]_{u=u_k},\hat{R}(u_k)\varphi(x,u_k)\right\}, \quad (45)$$

$$\hat{R}(u)=\sum_{\alpha=1}^{3}w_\alpha(u)R_\alpha(\mathbf{S})\sigma_\alpha.$$

Here we have denoted the columns of the matrix-values functions by $\Psi_\pm$, where

$$\Psi_+=(\psi,\bar{\psi}),\quad \Psi_-=(-\bar{\varphi},\varphi).$$

A system of equations which describes the evolution of $\mathbf{S}^{(0)}$ in the adiabatic approximation[1)] and is completely equivalent to the system (7) derived in the previous section can be obtained for the case of a pure-soliton initial pulse $\mathbf{S}^{(0)}$, defined by the parameters $p_i=\{u_i,b_i\}$, by using the values for the soliton case to approximate the true scattering parameters in the integrands in (44) and (45). Of course, the form of these solutions differs from the pure soliton case, and effects due to spin-wave radiation are also present; these are described by the first-order correction $\mathbf{S}^{(1)}$, which is related to the modified scattering data by the same formula (38) that holds in the exactly soluble model.

The Jost functions

$$\psi_s(x,u)=\frac{\exp\{-i\varkappa w_3(u)x\}}{(2\,\mathrm{ch}\,z)^{1/2}}\begin{pmatrix}\exp\frac{z}{2}\\ -a_s(u)\exp\frac{-z}{2}\end{pmatrix},$$
$$\varphi_s(x,u)=\frac{\exp\{i\varkappa w_3(u)x\}}{(2\,\mathrm{ch}\,z)^{1/2}}\begin{pmatrix}\exp\frac{-z}{2}\\ a_s(u)\exp\frac{z}{2}\end{pmatrix}, \quad (46)$$

where

$$a_s(u)=\frac{\varkappa w_3(u)+i\xi/2}{-w_1(u)\cos\varphi+iw_2(u)\sin\varphi} \quad (47)$$

correspond to the particular magnetization distribution (8) in the domain wall. If we substitute expressions (46) evaluated at the points u_k in the discrete spectrum into the right-hand sides of (44) and (45), we obtain Eqs. (9) and (10), which describe the response of an isolated soliton to a general perturbation. The first-order correction $\mathbf{S}^{(1)}$ to the adiabatic approximation $\mathbf{S}^{(0)}$ is given by (38) with

$$G_1(u)=-\mathrm{sech}\,z\sum_{\alpha=1}^{3}g_\alpha(u)\sigma_\alpha,$$

$$g_1(u)=\mathrm{Re}\,[a_s(u)B(x,t;u)],\quad g_2(u)=\mathrm{Im}\,[a_s(u)B(x,t;u)], \quad (48)$$

$$2B(x,t;u)=\overline{b(u,t)}\exp(z+2i\varkappa w_3(u)x)-b(u,t)\exp(-z-2i\varkappa w_3(u)x),$$

$$g_3(u)=\mathrm{Re}[b(u,t)\exp(-2i\varkappa w_3(u)x)].$$

Here the coefficient $b(u,t)$ satisfies the equation

$$-\partial b(u,t)/\partial t = 4i\varkappa w_1(u) w_2(u) b(u,t) + U(u,\varphi)\exp[2i\varkappa w_3(u) x_0(t)] \quad (49)$$

up to terms of second order in the small perturbation $\mathbf{R} \approx \mathbf{R}[\mathbf{S}^{(0)}(z),t]$. Here U is given by

$$U(u,\varphi) = \frac{1}{2\xi}\int_{-\infty}^{\infty} dz\, \mathrm{sech}\, z[\overline{a_s(u)} e^{-z}(iw_1(u)R_1 - w_2(u)R_2)$$

$$-a_s(u)e^{z}(iw_1(u)R_1 + w_2(u)R_2)$$

$$-2iw_3(u)R_3]\exp(2i\varkappa w_3(u)z/\xi).$$

3. SPIN WAVE EXCITATION IN A MAGNET WITH A MOVING DOMAIN WALL

In this section we employ perturbation theory based on the inverse scattering method to study how spin waves are excited by an ac magnetic field $h \sin \omega t$ in a ferromagnet with a moving domain wall. We have already observed that for suitable restrictions (16) on the amplitude h, the mean wall velocity tends to the Walker value V_W, which is determined by the net balance between the dissipation and the dc component of the magnetic field. For a perturbation of the form (2), Eq. (49) governing the time dependence of the coefficient $b(u,t)$ takes the simple form

$$\frac{\partial b(u,t)}{\partial t} - i\Omega_0(u) b(u,t) = -\frac{\pi\tau}{2\xi^2}\frac{d\varphi}{dt}\,\mathrm{sech}\frac{\pi\zeta(u)}{2}\exp[2i\varkappa w_3(u)x_0(t)],$$

$$\zeta(u) = 2\varkappa w_3(u)/\xi, \quad \Omega_0(u) = -4\varkappa w_1(u) w_2(u). \quad (50)$$

We analyze the case when the domain wall moves at the constant velocity $V = V_W$ immediately before the ac field is turned on. The right-hand side of (50) then vanishes, and the modulus $|b(u,t)|$ is independent of t. This situation corresponds to a steady-state distribution in the system of radiated spin waves; that is, the system saturates and stops absorbing energy It is therefore reasonable to set $b(u,t) = 0$, where we recall that the angle φ is completely determined (it is equal to the Walker angle φ_W).

We assume further that the ac magnetic field $h \sin \omega t$ causes φ to vary slightly relative to φ_W, i.e., $|\Delta\varphi| \ll 1$. We can then set $\varphi = \mathrm{const}$ in (50) everywhere except in the derivative $d\varphi/dt$. After the steady state has been reached (this occurs during a time $t \sim \omega^{-1}$), the solution of (50) is given by

$$\Delta b(u,t) = b_\omega(u,t) - b_{-\omega}(u,t), \quad (51)$$

where

$$b_\omega(u,t) = \frac{i\varkappa D_\omega}{f_\omega(u)}\{\exp[-itf_\omega(u)] - 1\} \times \frac{\exp[i(\omega t + \zeta(u)\xi x_0(t) + \delta_\omega)]}{\mathrm{ch}[\pi\zeta(u)/2]}$$

$$f_\omega(u) = \omega - \Omega_0(u) + \zeta(u)\xi V,$$

$$D_\omega = -\frac{\pi h \omega V}{4\xi(\omega^2 + \omega_0^2)^{1/2}}, \quad \delta_\omega = \mathrm{arctg}\frac{\omega}{\omega_0}.$$

Here we have omitted the term containing the initial value $b(u,0)$, because this term is "forgotten" by the system as $t \to \infty$; we have also used expression (18) for $d\varphi/dt$ and the approximation $x_0(t) \approx x_0(0) + Vt$ for the time-dependence of the coordinate x_0 of the center of the domain wall.

If we compare this result with the assertions made in Sec. 1, we find that the computational procedure correctly treates the damping of the "source" (moving domain wall) but neglects the dissipation of the spin waves. Before proceeding with the analysis, we therefore observe that the desired correction $\mathbf{S}^{(1)}(x,t)$ is a superposition of excitations whose amplitude is linear in the ac field amplitude h and which therefore do not interfere with one another. In this case, the dissipation causes each radiated harmonic to relax independently. We will assume that the spin waves are radiated faster than the characteristic damping times for $\mathbf{S}^{(1)}$. In other words, we can neglect the delay and the change in the amplitude of $\mathbf{S}^{(1)}$ caused by dissipation over distances comparable to the domain wall width. This approximation is justified at least in the long-wave limit due to the smallness of the parameter $|\lambda V/\xi| \ll 1$—indeed, up to terms of order $\lambda V/\xi$ the matching condition (20) for $\mathbf{S}^{(1)}(z,t)$ contains the relaxation constant λ only through the source parameters. Moreover, according to the linear theory, which is valid outside the domain wall, the only effect of dissipation in the long-wave limit is to rescale the spectral characteristics of the radiation [the frequency $\Omega_0(k)$ and the degree of polarization β_1/β_2, see Eq. (21)]. In the inverse-scattering spectral representation, the frequency $\Omega_0(k)$ corresponds to the quantity $\Omega_0(u) = -4\varkappa w_1(u) w_2(u)$, and the amplitudes β_1 and β_2 correspond to the function $w_2(u)$ and $w_1(u)$. Since we want to allow for the dissipative term $\lambda[\mathbf{S}^{(0)}\mathbf{S}_t^{(1)}]$ in Eq. (17), we must make a suitable change of variables. In so doing, we must of course ensure that the dissipative term leaves the adiabatic approximation $\mathbf{S}^{(0)}$ unchanged; in addition, the properties of the scattering characteristics ensuring that $\mathbf{S}^{(1)}$ is real must be preserved. It turns out that these requirements can be satisfied only by renormalizing the frequency function $\Omega_0(u)$ in expression (51) for the coefficient $b(u,t)$:

$$\Omega_0(u) \to \Omega(u) = \Omega_0(u) + i\Gamma(u), \quad \Gamma(u) = \lambda[w_1^2(u) + w_2^2(u)]. \quad (52)$$

We assume in (52) that the frequency $\Omega_0 = (J_3 - J_1)^{1/2}(J_3 - J_2)^{1/2}$ for homogeneous ferromagnetic resonance is large compared to the relaxation frequency h_c.

Noting that the imaginary correction to $\Omega(u)$ causes the corresponding exponential in $b_\omega(u,t)$ to decay for large times, we get the following formulas for $\mathbf{S}^{(1)}(z,t)$:

$$S_+^{(1)} = S_1^{(1)} + iS_2^{(1)} = 2[f_1^{21}(0)\,\mathrm{th}\, z - \varkappa e^{i\varphi} f_1^{11}(0)\,\mathrm{sech}\, z],$$

$$S_3^{(1)} = -2\varkappa\,\mathrm{sech}\, z\,\mathrm{Re}[f_1^{21}(0)e^{-i\varphi}], \quad (53)$$

where

$$f_1^{11}(0)=\frac{D_\omega}{4\pi\rho i\,\mathrm{ch}\,z}\oint_\Gamma du\frac{w_3(u)}{\mathrm{ch}[\pi\zeta(u)/2]}\times\mathrm{Im}\frac{\exp[i(\omega t+\delta_\omega-\zeta(u)z)]}{f_\omega(u)-i\Gamma(u)}$$

$$f_1^{21}(0)=\frac{D_\omega}{4\pi\rho\,\mathrm{ch}\,z}\times\oint_\Gamma du\frac{w_2(u)\,\mathrm{Re}\,m(z,u)+iw_1(u)\,\mathrm{Im}\,m(z,u)}{\mathrm{ch}[\pi\zeta(u)/2][f_\omega^2(u)+\Gamma^2(u)]^{1/2}},$$

$$m(z,u)=a_\nu(u)\,\mathrm{ch}\,[z+i\zeta(u)z-i\omega t-i\delta_\omega-i\arg(f_\omega(u)+i\Gamma(u))].$$

As $|z|$ increases, the function $f_1^{11}(0)$ decays exponentially over distances comparable to the width of the domain wall, while $f_1^{21}(0)$ determined the asymptotic behavior $S_+^{(1)}$ ($|z|\to\infty$) of interest:

$$S_+^{(1)}=\pm\frac{D_\omega}{|X(k_\pm)|}(|(k_\pm^2+J_3-J_1)^{1/2}-(k_\pm^2+J_3-J_2)^{1/2}|e^{i\Phi_\pm}-|(k_\pm^2+J_3-J_1)^{1/2}+(k_\pm^2+J_3-J_2)^{1/2}|e^{-i\Phi_\pm})\times\exp\left(\mathrm{Im}\,k_\pm\frac{|z|}{\xi}-i\alpha_\pm\right),$$

$$\Phi_\pm=\pm\left(\omega t+\delta_\omega-\frac{z}{\xi}\,\mathrm{Re}\,k_\pm\right)-\arg X(k_\pm), \tag{54}$$

$$\alpha_\pm=\arg[(k_\pm^2+J_3-J_1)^{1/2}+(k_\pm^2+J_3-J_2)^{1/2}],$$
$$X(k_\pm)=i\Omega_0(k_\pm)(k_\pm+i\xi)(V-\partial\Omega_0(k_\pm)/\partial k_\pm\mp 2i\lambda k_\pm)\times\{\mathrm{sech}\,(\pi k_\pm/2\xi)[-\cos\varphi(k_\pm^2+J_3-J_1)^{1/2}+i\sin\varphi(k_\pm^2+J_3-J_2)^{1/2}]\}^{-1}.$$

Here the upper and lower signs are for $z\gg 1$ and $z\ll -1$, respectively, and the solutions $k_\pm$ of the equation

$$k_\pm V+\omega-\Omega_0(k_\pm)\mp i\lambda(k_\pm^2+J_3-(J_1+J_2)/2)=0, \tag{55}$$

satisfy $|k_\pm|\ll\xi$ and $\mathrm{Im}\,k_\pm<0$.

Expression (54) shows that the correction $\mathbf{S}^{(1)}$ is anomalously large when the pump frequency coincides with the resonance frequency $\omega_c(V)$; the latter is given by the consistency condition for the two equations

$$\Omega_0(k_c)=\omega+k_cV,\quad \partial\Omega_0(k_c)/\partial k_c=V. \tag{56}$$

Figure 1 shows a plot of $\omega=\omega_c(V)$ (see also Ref. 17, p. 71).

We seek a solution of the dispersion equation (55) in the form $k_\pm=k_c+\delta k_\pm$ for near-resonant wave numbers $k_c=k_c(\omega_c(V),V)$ by expanding $\Omega_0(k_\pm)$ in the small parameter $|\delta k_\pm|/k_c\ll 1$. The result is

$$\delta k_\pm=(\pm 1-i)\lambda^{1/2}[V\Omega_0(k_c)/2k_c\Omega_0''(k_c)]^{1/2} \tag{57}$$

and substitution into (54) gives

$$\begin{pmatrix}S_1^{(1)}\\ S_2^{(1)}\end{pmatrix}\approx\frac{\pm 2D_\omega\exp\{\mathrm{Im}\,\delta k_\pm(|z|/\xi)\}}{\Omega_0(k_c)\Omega_0''(k_c)\,\mathrm{ch}\,(\pi k_c/2\xi)\,|\delta k_\pm|}\times\begin{pmatrix}-(k_c^2+J_3-J_2)^{1/2}\cos\Phi_\pm\\ (k_c^2+J_3-J_1)^{1/2}\sin\Phi_\pm\end{pmatrix} \tag{58}$$

as $z\to\pm\infty$. This shows that $\mathbf{S}^{(1)}$ grows as $\lambda^{-1/2}$ if we formally let $\lambda\to 0$ (recall that the relaxation constant λ is bounded from below by the requirement that $h_c>h_0$). The above estimate is therefore valid only to the extent that the perturbation theory is applicable.

For $\omega>\omega_c(V)$ away from resonance, $\mathbf{S}^{(1)}$ has the form of a traveling wave which propagates along the sample and is damped over a distance of $\sim\lambda^{-1}$. In the nondissipative limit ($\lambda=0$, $\mathrm{Im}\,k_\pm=0$), $\mathbf{S}^{(1)}$ is an ordinary elliptically polarized spin wave. For $z\to\pm\infty$ we have

$$\begin{pmatrix}S_1^{(1)}\\ S_2^{(1)}\end{pmatrix}\approx\pm\frac{2D_\omega\,\mathrm{sech}\,(\pi k_\pm/2\xi)}{\Omega_0(k_\pm)\,|V-v_g^\pm|}\begin{pmatrix}-(k_\pm^2+J_3-J_2)^{1/2}\cos\Phi_\pm\\ (k_\pm^2+J_3-J_1)^{1/2}\sin\Phi_\pm\end{pmatrix} \tag{59}$$

with phase

$$\Phi_\pm=\pm[\Omega_0(k_\pm)t-k_\pm x+\delta_\omega]-\arg(ik_\pm-\xi)+\arg[-\cos\varphi(k_\pm^2+J_3-J_1)^{1/2}+i\sin\varphi(k_\pm^2+J_3-J_2)],$$

frequency $\Omega_0(k_\pm)=\omega+k_\pm V$, and group velocity $v_g^\pm=\partial\Omega_0(k_\pm)/\partial k_\pm$.

It is important to note that the forward ($v_g^+>V$) and backward ($v_g^-<V$) radiation observed far from the domain wall have unequal frequencies which in general differ from the frequency ω of the external magnetic field. A magnetic material with two domains and a moving domain wall thus acts as a frequency converter whose conversion factor can be regulated by means of an external dc magnetic field h_0 in accordance with Eqs. (15) and (55).

For pump frequencies $\omega<\Omega_0$, it is clear that the emission (59) can begin only for domain wall velocities that exceed a critical value $V_c=V_c(\omega)$, and in this case the radiation in the backward direction is absorbed by the wall. However, if $\omega>\Omega_0$ then emission occurs for all velocities $V\neq 0$, and the group velocities v_g^+ and v_g^- are oppositely directed. The resonance factor $|V-v_g^\pm|^{-1}$ in (59) is also noteworthy, as it highlights the similarity between linear Landau damping and spin-wave generation—resonance occurs when $V=v_g^+=v_g^-$.

Finally, no spin waves are generated when $\omega<\omega_c(V)$. In this case, $k_+(\omega,V)\approx k_-(\omega,V)\approx -i|k_\pm|$ and the correction $\mathbf{S}^{(1)}(z,t)$ is localized near the domain wall in a region of width $\sim|k_\pm|^{-1}\gg\xi^{-1}$.

We close this section by discussing how the above features of spin-wave excitation might be observed experimentally. First of all, we can use (40) and (51) to calculate the absorbed power $W(\omega)=dE_{\rm rad}/dt$. In an unbounded material, damping causes $W(\omega)$ to vanish as $t\to\infty$; in this case, the energy distribution in the spin-wave system is stationary and there is no energy flow as $|x|\to\infty$. In practive, however, the dimensions l are finite and some energy flows across the edge of the sample; this situation can be described in terms of scattering data with the boundary conditions (24), provided that when we calculate $W(\omega)$ we restrict ourselves in the coefficients $b_\omega(u,t)$ to times t which are short compared to the relaxation times. Physically, this is equivalent to calculating the amount of energy absorbed in a region of width l containing the moving domain wall, where l is much greater than the wavelength of the spin waves but less than their damping lengths. This requirement is particularly important for estimating $W(\omega)$ near resonance because the absorbed power behaves as $t_c^{1/2}$, where $t_c \sim k_c/\lambda V\Omega_0(k_c)$ is the relaxation time. Recalling the above discussion for $W(\omega)$, for $\omega > \omega_c(V)$ we find

$$W(\omega) \approx 2D_\omega^2 \sum_{+(-)} |V - v_g^\pm|^{-1} \operatorname{sech}^2 \frac{\pi k_\pm}{2\xi} \tag{60}$$

in the nondissipative limit if

$$\xi \gg \operatorname{Re} k_\pm \gg l^{-1} \gg \operatorname{Im} k_\pm, \tag{61}$$

and at resonance $\omega = \omega_c(V)$ we have

$$W(\omega) = 4D_\omega^2 \frac{\operatorname{sech}^2(\pi k_c/2\xi)}{\Omega_0''(k_c)\,|\delta k_\pm|}, \tag{62}$$

if

$$\xi \gg k_c \gg l^{-1} \gg |\delta k_\pm|. \tag{63}$$

By using (58) and (59) to calculate the mean energy flux-$\langle \mathbf{S}_x \mathbf{S}_t \rangle_\omega$ across the boundary of the sample (averaged over the period $2\pi/\omega$ of the ac magnetic field), one can show that (60) and (62) contain only part of the energy which is carried off by waves whose amplitude is linear in the ac field amplitude h. The remaining energy comes from the correction which is quadratic in h. However, for sufficiently long wavelengths $l^{-1} \ll |k_\pm| \ll \xi$ and $\omega > \omega_c(V)$, both approaches yield the same resonant dependence on the ac field frequency ω; when inequalities (61) and (63) are satisfied, the above estimates (60) and (62) should therefore give a qualitatively correct description of the absorption in long samples of finite dimensions when a moving domain wall is present.

We close with another observation regarding the effects on the wall mobility of a uniform ac magnetic field $h \sin \omega t$ parallel to the anisotropy axis $\mathbf{n}$. Recall that for $h = 0$, the wall velocity depends on the dc field strength h_0 as described by Eq. (15). The above procedure can be used to calculate the contribution from spin-wave radiation to dP/dt, the change in the momentum of the system per unit time. Applying Eq. (41), we conclude that the radiation generates an effective dc magnetic field h_{eff} which must be allowed for in the adiabatic equations. This field

$$h_{eff} = D_\omega^2 \sum_{+(-)} \frac{k_\pm \operatorname{sech}^2(\pi k_\pm/2\xi)}{\Omega_0(k_\pm)\,|V - v_g^\pm|} \tag{64}$$

is small (of order h^2) when $\omega > \Omega_0$ for all velocities $0 < V < V_0$ (cf. Fig. 1) and vanishes at $V = 0$. Equation (64) also describes h_{eff} when $\omega_c(V) < \omega < \Omega_0$ well away from resonance ($V \neq V(\omega)$). However, as the wall velocity V approaches the threshold $V_c(\omega)$, h_{eff} increases rapidly to a maximum

$$h_{eff} \approx \frac{2D_\omega^2 k_c \operatorname{sech}^2(\pi k_c/2\xi)}{\Omega_0(k_c)\,\Omega_0''(k_c)\,|\delta k_\pm|} \tag{65}$$

at $V = V_c(\omega)$. We see that when $V = V_c(\omega)$, the additional field h_{eff} behaves as $\sim \lambda^{-1/2}$ and diverges in a nondissipative magnetic material. In practice, this rise in h_{eff} can destabilize the uniform average motion of the domain wall unless an additional compensating dc magnetic field is used. It is also noteworthy that after the dc field h_0 is turned off, the average wall velocity drops nearly to zero during a time of order h_c^{-1}. If the deceleration of the wall begins when $V > V_c(\omega)$, there will be a peak in the energy absorption as V decreases through the resonance region.

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[1)] This result was obtained by the author in collaboration with A. M. Kosevich and Yu. C. Kishvar' and was reported at the All-Union Seminar on the Landau-Lifshitz equation at Kiev in April 1984.

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