

Collimation and decollimation of atomic beams by laser radiation pressure

V. I. Balykin, V. S. Letokhov, V. G. Minogin, Yu. V. Rozhdestvenskiĭ, and A. I. Sidorov

Institute of Spectroscopy, Academy of Sciences of the USSR

(Submitted 11 July 1985)

Zh. Eksp. Teor. Fiz. **90**, 871–880 (March 1986)

A study is made of the transverse cooling of a beam of sodium atoms in an axisymmetric light field formed by a reflection axicon. It is shown that the transverse cooling decreases the angular divergence (collimates) the atomic beam. During the cooling the transverse temperature decreases from 190 to 17 mK. As the laser frequency is varied by an amount comparable to the natural width of the atom transition ($\Delta\nu_{\text{las}} = 26$ MHz), the density of atoms at the center of the beam changes by a factor of 800.

1. INTRODUCTION

Beams of charged or neutral particles are widely used in many areas of physical research. The accuracy of the experiments and the efficiency with which the particle beams can be utilized depends largely on the degree to which the particle beams are compressed in momentum space, i.e., on the degree of the collimation of the beams. The problem of collimating beams of charged particles in experimental physics has been solved.^{1,2} The motion of charged-particles beams is controlled by magnetic or electric fields, by which the particle beams can be deviated, accelerated or slowed, focused or defocused. The forces controlling the motion of the particles are derivable from potentials. Because Liouville's theorem holds for such forces, the phase volume of the ensemble of particles is independent of time and is a constant of the motion. In other words, the density of particles in phase space is a constant which is determined by the initial conditions. Therefore, potential forces can only serve to alter the shape of the phase volume occupied by the particles but cannot be used to change the size of the phase volume or to increase the phase density of the particles.

To increase the phase density one must introduce dissipative forces, i.e., forces causing a transfer of energy from the particles to some external system. There are several techniques that have been devised for increasing the phase density of charged particles. For example, radiation friction is used for light particles, electron cooling is used for protons and antiprotons, and the ionization loss method is used for heavy particles. Radiation braking leads to a loss of part of the kinetic energy of the particle beam and consequently to an increase in the degree of collimation of the beam. This method is used for high-energy particles of small mass. In the method of electron cooling the beam is collimated through the transfer of energy from the beam of protons (antiprotons) to a cold electron gas in Coulomb collisions of the protons with electrons. In the ionization loss method the density dissipation mechanism is the same as in electron cooling.

By virtue of Liouville's theorem, it is necessary to use dissipative forces to collimate any kind of particle beam. For neutral atoms the most efficient dissipative force is that of the radiation pressure arising in the resonant interaction of an atom with radiation. It is therefore reasonable to suppose that the pressure of resonance radiation can be used to solve

the problem of collimating beams of neutral particles. The possibility of using the pressure of resonance radiation to decrease the transverse velocities of atomic beams was first noted in Ref. 3. This idea was recently analyzed in Ref. 4. It was shown that a possible method of collimating an atomic beam is to irradiate it with an axisymmetric light field formed by reflecting a laser beam from the inner surface of an axicon. The proposed scheme was implemented experimentally in Ref. 5.

In the present paper we present detailed results of research on the collimation and decollimation of a thermal beam of sodium atoms by means of the transverse cooling of the beam by the pressure of resonance laser radiation.

2. QUALITATIVE ANALYSIS

The atomic-beam collimation (decollimation) scheme which we investigated is shown in Fig. 1a. In this scheme an atomic beam 2 emerging from source 1 is irradiated from all sides by an axisymmetric light field whose frequency ω is shifted toward the red from the atomic transition frequency ω_0 . The axisymmetric field was produced by reflecting the laser radiation 4 from the inner mirror surface of a conical reflection axicon 3.

In the axisymmetric light field formed by the reflection axicon, an atom having a transverse velocity $\mathbf{v}_\rho = \mathbf{v}_x + \mathbf{v}_y$ is acted upon by a force of radiation pressure which is directed counter to the radial velocity vector \mathbf{v}_ρ if $\omega < \omega_0$ and along \mathbf{v}_ρ if $\omega > \omega_0$. This direction of the force of radiation pressure is due to the radial direction of the wave vectors \mathbf{k} of the field inside the axicon. An atom moving at an angle to the axis of the cone interacts in the xy plane with two counterpropagating light waves whose intensities are equal at any point in space. In the rest frame of the atom one of the waves has a frequency $\omega - kv_\rho$, while the other has a frequency $\omega + kv_\rho$. When the laser frequency is chosen lower than the frequency of the atomic transition, $\omega < \omega_0$, the atom absorbs more efficiently the photons from the wave that is propagating counter to the radial velocity vector \mathbf{v}_ρ . This means that the force of radiation pressure acting on the atom for $\omega < \omega_0$ is directed counter to the radial velocity \mathbf{v}_ρ . The force of radiation pressure in the inner region of the axicon for $\omega < \omega_0$ causes a rapid narrowing of the transverse velocity distribution of the atomic beam, leading to a decrease in the angular divergence of the beam and an increase in the density of

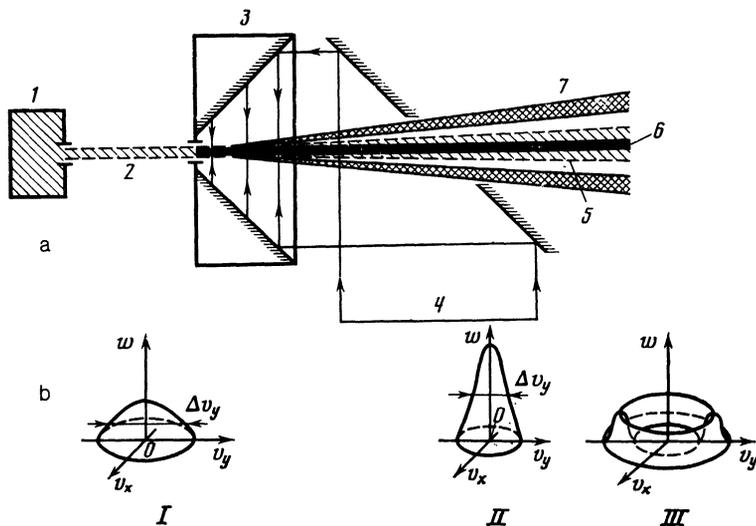


FIG. 1. Scheme for the collimation and decollimation of atomic beam by laser radiation pressure. a) Geometry of the atomic and laser beams: 1) source of atomic beam, 2) atomic beam before axicon, 3) axicon, 4) laser radiation, 5) atomic beam before interaction with the laser radiation, 6) collimated atomic beam, 7) decollimated atomic beam. b) Profile of the transverse velocity distribution of the atomic beam: I) before interaction with the radiation, II) after collimation, III) after decollimation. The xy plane is perpendicular to the axis of the atomic beam.

atoms, i.e., to an increase in the degree of collimation of the atomic beam. The characteristic time over which the collimation of the atomic beam occurs can be estimated from the equation of motion of an atom under the influence of the force of radiation pressure. For a two-level atom at low velocities $v_p \ll |\Omega|/k$, the force of radiation pressure reduces to the frictional force⁶

$$F = -\beta M v_t, \quad (1)$$

where

$$\beta = \frac{4\hbar^2 k^2}{M} \left| \frac{\Omega}{\gamma} \right| G \left(1 + \frac{\Omega^2}{\gamma^2} \right)^{-1} \left(1 + \frac{\Omega^2}{\gamma^2} + 2G \right)^{-1} \quad (2)$$

is the coefficient of dynamic friction. In expressions (1) and (2)

$$G = 1/2 z \rho^{-1} (dE_0/\hbar\gamma)^2$$

is the saturation parameter for the atomic transition, d is the matrix element for the dipole moment of the atom, 2γ is the natural width of the resonant transition, $\Omega = \omega - \omega_0$ is the detuning of the frequency ω of the resonance radiation from the frequency of the atomic transition, $\rho = (x^2 + y^2)^{1/2}$, M is the mass of the atom, and E_0 is the strength of the electric field.

The equation of motion of an atom under the influence of the frictional force (1) implies that the characteristic time for the narrowing of the velocity distribution is determined by the value of $\tau_c = \beta^{-1}$. For the typical parameters $|\Omega|/\gamma = 2$ and $G = 1$, the characteristic collimation time is $\tau_c \approx 2 \cdot 10^{-5}$ sec. If the time required for the atoms to traverse the axicon is greater than the time τ_c , then the evolution of the atomic ensemble in the field of the axicon is determined not only by the radiation pressure but also by the diffusion of the atomic momenta. The combined effect of radiation pressure and momentum diffusion leads to the establishment of a steady velocity distribution of the atoms, with an effective temperature⁶

$$T_{min} = \frac{\hbar\gamma}{2k_B} \left(\frac{\gamma}{|\Omega|} + \frac{|\Omega|}{\gamma} \right), \quad (3)$$

where k_B is Boltzman's constant. Using this value of the

temperature, one can find the collimation angle of the atomic beam at the exit from the axicon:

$$\Delta\varphi_{min} = \frac{1}{\bar{v}_z M^{1/2}} (2k_B T_{min})^{1/2} = \frac{1}{\bar{v}_z} \left(\frac{\hbar\gamma}{M} \right)^{1/2}. \quad (4)$$

Here \bar{v}_z is the average velocity of the atoms along the axis of the axicon. For a thermal atomic beam the collimation angle is of the order of 10^{-3} – 10^{-4} rad.

The given value of the collimation angle characterizes the divergences of the beam at the exit from the axicon. In the region inside the axicon the divergence is smaller, since it is determined not by the free expansion of the atoms but by the slow diffusive broadening of the beam. The corresponding limiting collimation angle at a distance l from the point of entrance of the atoms into the axicon is⁴

$$\Delta\varphi'_{min} = \lambda (\gamma/l\bar{v}_z)^{1/2}. \quad (5)$$

For an atomic beam irradiated by a light field over a length $l = 10$ cm, the collimation angle is of the order of $\Delta\varphi_{min} \sim 10^{-4}$ – 10^{-5} rad.

These estimates thus show that the use of an axicon can give rise to appreciable collimation of an atomic beam. An increase in the density of atoms $(\Delta\varphi_{in}/\Delta\varphi_{fn})^2 \approx 10^6$ due to the collimation of the beam can be achieved from beams with an initial divergence $\Delta\varphi_{in} \sim 1$ rad. Then the increase in the phase density can reach values of $(\Delta\varphi_{in}/\Delta\varphi_{fn})^4 \approx 10^{12}$.

3. OPTICAL SCHEME FOR EXCITATION OF THE ATOMS

The experiment was done with sodium atoms. A four-level scheme was used for the interaction of the sodium atoms with the radiation (Fig. 2). Two-mode laser radiation was tuned to the D_2 line of the sodium atom. A value of 1712 MHz was chosen for the frequency difference of the laser radiation. One mode of the laser radiation, with frequency ω_1 , excited atoms from the level $F = 1$ of the ground state $3S_{1/2}$ to the level $F' = 2$ of the excited state $3P_{3/2}$, while the other mode of radiation, with frequency ω_2 , excited atoms from the $F = 2$ level of the $3S_{1/2}$ state to the level $F' = 3$ of the $3P_{3/2}$ state. Excitation of the atoms from two sublevels of the ground state ruled out optical pumping and thus ensured

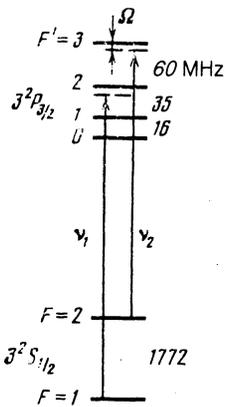


FIG. 2. Scheme for optical excitation of sodium atoms for collimation (decollimation) of an atomic beam.

a prolonged cyclical interaction of the atoms with the radiation. Excitation of the atom to two different sublevels permitted avoidance of the coherent trapping of the atomic populations in the sublevels of the ground state.⁷⁻⁹ To ascertain the efficiency at which the atoms are excited by the two-mode radiation, we measured the fluorescence signal of sodium atoms excited by the scheme of Fig. 2 as a function of the power of the laser radiation. This power dependence turned out to agree well with the dependence of the fluorescence of the two-level atom on the intensity of the single-frequency laser radiation exciting the fluorescence.

4. EXPERIMENTAL ARRANGEMENT

Figure 3 shows the arrangement of the experimental apparatus, which consisted of the following basic elements: an atomic beam source 3, a conical reflection axicon 4, two dye laser 1 and 2, a mechanical chopper *M* for modulating the laser radiation, photomultipliers 7, and total-internal-reflection prism 6. A beam of sodium atoms was shaped by means of two limiters 1 mm in diameter, one of which was placed near the aperture of the atomic oven and the other at a distance of 6 cm from the atomic oven. A conical axicon was

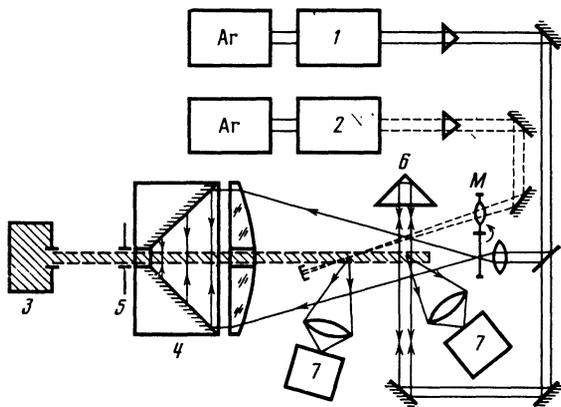


FIG. 3. Schematic of experimental apparatus. Ar) Argon laser, 1) two-mode dye laser for collimation (decollimation) of the atomic beam, 2) single-mode dye laser for measuring the density profile of the beam, 3) atomic beam source, 4) axicon, 5) limiter, 6) prism, 7) photomultiplier, M) mechanical radiation chopper.

located 2 cm from the second limiter. The length of the atom-radiation interaction region within the axicon was 35 mm. The distance from the atomic oven to the detection zone was 52 cm. The atomic beam was collimated by the radiation from two-frequency laser 1, the scheme of which was described in Ref. 10. A value of 1712 ± 2 MHz was chosen for the frequency difference between the axial modes, and the atoms were excited by the scheme illustrated in Fig. 2. The radiation was directed inside the axicon by means of a telescope formed by two lenses. The lens placed near the axicon had an aperture for passage of the atomic beam. Part of the radiation of the two-frequency laser intersected the atomic beam at an angle of 90° and was used for absolute calibration of the frequency scale.⁵

The density profile of the atomic beam was measured by detecting the fluorescence excited by single-mode dye laser 2. For this purpose, radiation focused by a long-focus lens intersected the atomic beam in the plane of Fig. 3 at an angle of 10° . The laser beam was shifted parallel to itself over several diameters of the atomic beam in the plane perpendicular to the plane of Fig. 3 by rotating a lens mounted on the mechanical chopper. The length of the interaction region of the probe field with the atomic beam in the field of view of the cathode of the photomultiplier was 1 mm. The frequency of the probe laser was tuned to the frequency of the atomic transition $3S_{1/2} - 3P_{3/2}$. The dependence of the density of atoms along the *y* axis (see Fig. 1) was measured during a spatial scanning of the laser beam. The recording time for the density profile in the transverse cross section of the beam was $\tau_r = 150 \mu\text{s}$.

It should be noted that the method of laser probing of the spatial distribution of an atomic beam has certain advantages over the conventional hot-wire method. First, it permits measurements over very short times; second, it can be used to measure the degree of collimation of an atomic beam as a function of the longitudinal velocity of the atoms; third it permits direct measurement of the intensity of atoms in the beam as a function of the beam radius. A shortcoming of the method is that it does not measure the spatial distribution of all the atoms of the beam but only that of atoms found at certain sublevels of the ground state ($F = 1$ or $F = 2$ in our case), the populations of which can change during the interaction with the strong field. To determine the true profile of the beam density in an experiment we measured the atomic density at the two hyperfine sublevels $F = 1$ and $F = 2$.

5. EXPERIMENTAL RESULTS

Figure 4 shows profiles of the atomic beam before and after interaction with the laser radiation. The power of the two-frequency radiation was set at 60 mW. The laser frequency was detuned to the red side of the atomic transition frequency ($\omega - \omega_0 \approx -3\gamma$). The curves in Fig. 4 are for atoms having a longitudinal velocity of $7.3 \cdot 10^4$ cm/sec. Comparison of the beam profiles before and after the interaction reveals, first, a substantial (fivefold) increase in the intensity at the center of the beam, and second, the presence of an appreciable narrowing (collimation) of the atomic beam. By measuring the beam diameters before and after the

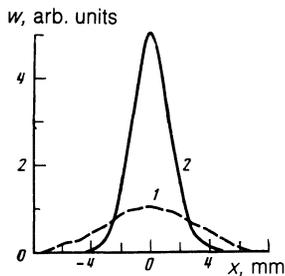


FIG. 4. Collimation of an atomic beam by laser radiation. 1) Profile of the beam prior to interaction with laser radiation, 2) after interaction.

interaction with the radiation (and with allowance for the finite diameter of the probe beam), we were able to calculate the change in the maximum transverse velocity of the atoms in the course of the collimation. For the case illustrated in Fig. 4, the maximum transverse velocity of the atoms decreased from $11.7 \cdot 10^2$ to $3.5 \cdot 10^2$ cm/s. Correspondingly, the temperature of transverse motion decreased from 190 to 17 mK. The angular divergence of the atomic beam decreased from $3.2 \cdot 10^{-2}$ to $0.9 \cdot 10^{-2}$ rad. It should be noted that the degree of collimation depends to a large extent on how well the axis of the atomic beam is matched up with the axis of the axicon and on the uniformity of the illumination of the axicon by the laser radiation.

In a somewhat different configuration, with the atomic beam source 9.5 cm away from the limiter ($\Delta\varphi_0 = 2.2 \cdot 10^{-2}$ rad), we investigated the change in the profile of the atomic beam as the magnitude and sign of the detuning of the laser frequency from the transition frequency change (Fig. 5). The laser radiation power in these measurements was 60 mW. As we see from Fig. 5, there is a certain optimum negative detuning at which one observes the maximum collimation of the atomic beam. As the amount of the detuning is decreased, the degree of collimation falls off appreciably. When the detuning is close to zero one observes a broadening of the beam and a decrease in its intensity. This broadening of the beam is probably due to momentum diffusion of the atoms.

As the detuning went positive there was a substantial broadening (decollimation) of the atomic beam. The stron-

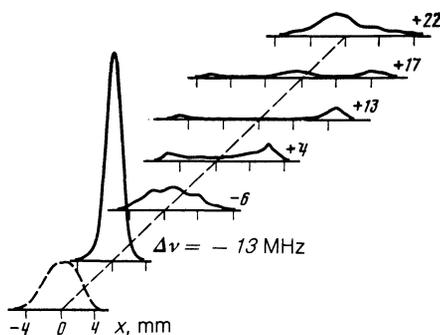


FIG. 5. Dependence of the profile of the atomic beam on the position of the laser frequency ν relative to the atomic transition frequency ν_0 ($P_{\text{las}} = 60$ mW, $\Delta\nu = \nu - \nu_0$). The dashed profile is the atomic beam prior to interaction with the laser radiation.

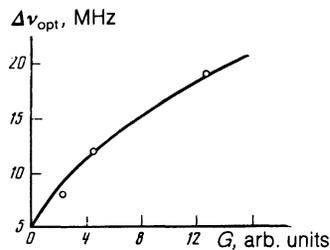


FIG. 6. Optimum detuning of the laser radiation frequency $\Delta\nu_{\text{opt}} = \nu - \nu_0$ versus the saturation parameter $G = I_{\text{las}}/I_{\text{sat}}$ (I_{sat} is the saturation intensity of the transition).

gest decollimation was observed at a detuning of +13 MHz, a value equal in magnitude and opposite in sign to the detuning at which the maximum collimation of the beam was observed. The density profile of the atomic beam at a positive detuning of 13 MHz was ring-shaped. The atomic intensity at the center of the beam at this detuning was 0.6% of the original intensity. The divergence of the atomic beam increased to $\Delta\varphi = 6 \cdot 10^{-2}$ rad. It should be noted that at this detuning the atomic beam fell completely outside its original dimensions in the transverse cross section.

Collimation of the beam was observed at detunings from -30 to -8 MHz, and decollimation at detunings from 0 to +25 MHz. The laser radiation thus effectively influenced the density profile of the atomic beam over a frequency interval of ~ 55 MHz. Over this interval the atomic intensity at the center of the beam changed by a factor of 800. The total intensity of the atomic beam, given by

$$I_x = \int I(\rho) \rho d\rho,$$

remains unchanged during the collimation (decollimation) to within the experimental accuracy. It should be noted that for each value of the laser radiation intensity there is an optimum value of the frequency of the collimating field. According to Ref. 6, the optimum detuning of the field frequency for atomic cooling depends on the intensity of the field as $(1 + G)^{1/2}$. Figure 6 shows the experimental dependence of the optimum detuning on the intensity of the laser radiation. This curve is described rather well by the function $(1 + G)^{1/2}$.

The degree of collimations of the atomic beam also depended on the interaction time of the atoms with the laser radiation, i.e., on the longitudinal velocity v_z of the atoms. In the experiment we measured the dependence of the beam collimation efficiency on the longitudinal velocity of the atoms. Figure 7 shows the dependence of the relative intensi-

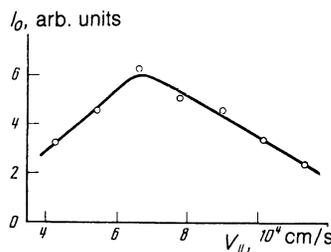


FIG. 7. Density of atoms at the center of the collimated atomic beam versus the longitudinal velocity of the atoms.

ty of atoms at the center of the beam as a function of their longitudinal velocity at a given initial divergence of the atomic beam. We see from the figure that the collimating effect of the laser radiation gets weaker at low and high atomic velocities. The optimum longitudinal velocity for collimation of the beam coincided with the most probable velocity. The decrease in the collimation efficiency at low velocities can be explained as follows. When the transverse velocities of the atoms are so low that the corresponding Doppler shift of the absorption frequency of the atom is smaller than the homogeneous linewidth of the atomic transition, the optimum detuning of the frequency of the light field for collimation is equal to $\gamma(1+G)^{1/2}$. As the detuning is decreasing further, the force of radiation pressure goes to zero.⁶ Therefore, the most efficient collimation occurs for a beam in which the transverse velocity is equal to

$$v_\rho \approx \gamma(1+G)^{1/2}/k.$$

this value of the transverse velocity corresponds to an optimum longitudinal velocity of

$$v_z \approx 2\gamma(1+G)^{1/2}\lambda/\Delta\varphi.$$

At a saturation parameter $G \approx 1$ and an initial divergence $\Delta\varphi_0 = 1.4 \cdot 10^{-2}$ rad, this velocity is $v_x = 6 \cdot 10^4$ cm/s, in rather good agreement with the experimental value.

6. NUMERICAL CALCULATION OF THE COLLIMATION OF AN ATOMIC BEAM

In the experiment described above it was impossible to detect directly the transverse velocity distribution of the atoms in the beam, and so we could not directly determine the transverse temperature. For this reason we determined the temperature of the atomic beam by numerically calculating the velocity distribution and spatial profile of the collimated atomic beam.

To describe the evolution of the atomic beam in the axicon field we used Liouville's equation for the atomic distribution function $w(r, \rho, t)$:

$$\frac{\partial}{\partial t} w + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} w = - \frac{\partial}{\partial \rho} (Fw), \quad (6)$$

where F is the force of radiation pressure.

Since the transit time of an atom through the axicon, $t_{tr} = l/\bar{v}_x \approx 5 \cdot 10^{-5}$ s, was comparable to the characteristic time β^{-1} for the narrowing of the transverse velocity distribution under the influence of frictional force (1), the change in the distribution function of the beam occurred solely on account of the force of radiation pressure (1). For this reason we have neglected in the theoretical analysis the contribution of momentum diffusion, which changes the velocity of the atom by an amount $\Delta v_{dif} \sim (Dt_{tr})^{1/2} \approx 30$ cm/s, where D is the diffusion coefficient.⁶

In this experiment the atom source could not be treated as a point source, since it was comparable in size to the limiter that was supposed to shape a beam of given angular divergence. This meant that we had to make allowance for atoms emerging from different points on the surface of the source. As a model of an extended source we chose a discrete set of point sources of atoms at different distances from the symmetry axis of the axicon.

Under these assumptions we solved Liouville's equation (6) for each point source by the method of characteristics. The characteristic equations were written in a cylindrical coordinate system whose z axis coincided with the axis of the atomic beam:

$$\dot{v}_\rho = \frac{1}{M} F(\rho, v_\rho) + v_z^2/\rho, \quad \dot{v}_t = -v_\rho v_t/\rho, \quad \dot{v}_z = 0, \quad (7)$$

$$v_\rho = \dot{\rho}, \quad v_t = \dot{\varphi}\rho, \quad v_z = \dot{z},$$

where ρ and φ are polar coordinates in the xy plane (Fig. 1). We note that the characteristic equations have a different form for a point source on the symmetry axis of the axicon than for a point source at some distance from the axis. In the first case the motion of the atom occurs only along the radius ρ , while in the second case there is an additional motion in the xy plane (Fig. 1), with an angular velocity $\dot{\varphi}$. The presence of angular dependence in (7), however, is not important, since the force depends only on ρ on v_ρ , and we therefore have an overall cylindrical symmetry.

By taking into account the contributions from all the point sources, we found the spatial and velocity distributions of the atoms in the detection region at the experimental power level of the laser radiation. The value of the frequency detuning here was chosen so that the calculated spatial distribution agreed with the experimental. This made it possible to determine the temperature of the atomic beam from the theoretical velocity distribution.

Figure 8 shows the calculated and experimental distributions over the coordinate ρ (curves 2 and 3) for an original beam divergence $\Delta\varphi_0 = 1.4 \cdot 10^{-2}$ rad. For laser radiation with $P_{las} = 45$ mW and $\Delta\nu = 3.8\gamma$ the theoretical curve turned out to be quite close to the experimental curve. The discrepancy in the curves is due mainly to the fact that we neglected the multilevel structure of the atomic transitions

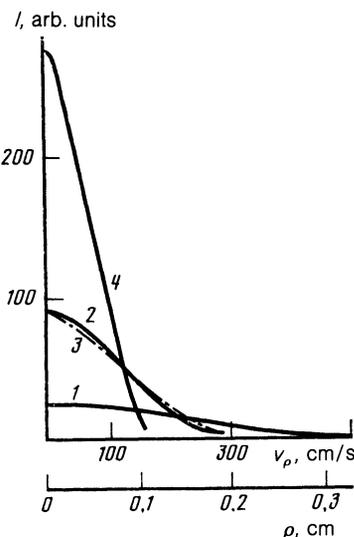


FIG. 8. Spatial and velocity distributions of the beam atoms for $P_{las} = 45$ mW and $\Delta\nu = 3.8\gamma$. 1) Theoretical spatial and velocity distributions of the atoms in the absence of the collimating radiation; 2,3) theoretical and experimental spatial distributions of the beam atoms before collimation; 4) theoretical velocity distribution of the beam atoms after collimation.

and the contribution of momentum diffusion in the calculation and replaced the real atom source by a discrete set of point sources. Starting from the velocity distribution (curve 4), we obtained a value of $T = 3.3$ mK for the transverse temperature of the collimated beam. This value agrees in order of magnitude with an earlier estimate.⁵ To determine the temperature we approximated our velocity distribution by a Gaussian curve using the least-squares method.

7. CONCLUSIONS

Several effective methods of controlling the motion of atomic beams by means of the pressure of laser resonance radiation have been discovered in recent years. The proven experimental methods include the deviation, focusing (defocusing), velocity compression, and longitudinal cooling of atomic beams.¹¹ The beam collimation (transverse cooling) method studied in the present paper opens up additional possibilities for purposeful control of the velocities and coordinates of neutral atoms. These new possibilities, we believe, include the following. First, radiative collimation of beams can eliminate the main obstacle to obtaining intense beams of cold atoms, viz., the rapid expansion of the beam atoms at right angles to the beam axis. When the beam to be cooled is collimated, this limitation is alleviated on account of both the transverse beam cooling itself and also the replacement

of the free expansion of the atoms by a much slower diffusive broadening of the beam at right angles to its axis. Second, the collimation of atomic beams, owing to its frequency selectivity, might be of interest for selectivity decreasing the angular divergence of beams of certain isotopes in schemes for the separation or detection of rare isotopes.

¹A. Septier, *Focusing of Charged Particles*, Vols. I and II, Academic Press, New York (1967).

²G. I. Budker and A. I. Skrinskii, *Usp. Fiz. Nauk* **124**, 561 (1978) [*Sov. Phys. Usp.* **21**, 277 (1978)].

³T. W. Hänsh and A. W. Schawlow, *Opt. Commun.* **13**, 68 (1975).

⁴V. I. Balykin, V. S. Letokhov, V. G. Minogin, and T. V. Zueva, *Appl. Phys. B* **35**, 145 (1984).

⁵V. I. Balykin, V. S. Letokhov, and A. I. Sidorov, *Pis'ma Zh. Eksp. Teor. Fiz.* **40**, 251 (1984) [*JETP Lett.* **40**, 1026 (1984)].

⁶V. S. Letokhov, V. G. Minogin, and B. D. Pavlik, *Zh. Eksp. Teor. Fiz.* **72**, 1328 (1977) [*Sov. Phys. JETP* **45**, 698 (1977)].

⁷E. Arimondo and G. Orriols, *Lett. Nuovo Cimento* **17**, 333 (1976).

⁸M. L. Citron, H. R. Gray, C. W. Gabel, and C. R. Stroud Jr., *Phys. Rev. A* **16**, 1507 (1977).

⁹V. G. Minogin and Yu. V. Rozhdestvenskii, *Zh. Eksp. Teor. Fiz.* **88**, 1950 (1985) [*Sov. Phys. JETP* **61**, 1156 (1985)].

¹⁰V. I. Balykin and A. I. Sidorov, *Kvantovaya Elektron. (Moscow)* **11**, 2001 (1984) [*Sov. J. Quantum Electron* **14**, 1342 (1984)].

¹¹V. I. Balykin, V. S. Letokhov, and V. G. Minogin, *Usp. Fiz. Nauk* **147**, 117 (1985) [*Sov. Phys. Usp.* **28**, 803 (1985)].

Translated by Steve Torstveit