Parametric excitation of spin waves under conditions of frequency drift

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Parametric excitation of spin waves is investigated under conditions of frequency drift at a constant rate. This drift may be due either to accumulation of nonequilibrium spin waves via relaxation of parametric spin waves, or to variation of an external electric field at a constant rate. The waveform of a packet of parametric spin waves in **k** space and the dependence of the number and phase of the waves on the pump power and on the drift velocity are determined. It is shown that the spin-wave behavior depends substantially on the sign of the quantity $Sd\omega_{\mathbf{k}}/dt$, where S is the matrix element of the pair-wave interaction. In particular, at $Sd\omega_{\mathbf{k}}/dt>0$ the number of waves decreases with increasing drift velocity. At $Sd\omega_{\mathbf{k}}/dt<0$ the dependence of the number of parametric spin waves on the drift velocity exhibits hysteresis. The stability of a system of parametric spin waves to small perturbations is investigated. Under conditions of intrinsic drift due to accumulation of nonequilibrium spin waves, the number of the waves has a smooth dependence on the supercriticality. This can lead to a systematic error in the determination of the spin-wave damping by measuring the parametric-spin-wave excitation threshold.

1. INTRODUCTION

A nonlinear theory of parametric excitation of waves has by now been developed in considerable detail. In first-order approximation in the wave interaction (S-theory) it is possible to determine characteristics such as the number N of the waves, phase correlations, the absorbed power, the frequencies of collective oscillations, and others (see the review[1]). In this approximation, the stationary distribution function of the parametric spin waves (PSW) is singular:

$$n_{\mathbf{k}\omega}=n_\Omega(k_\Omega^2/v_\Omega)\delta(\omega_{\mathbf{k}}-\omega_p/2)\delta(\omega-\omega_p/2). \tag{1.1}$$

Here

$$\langle a_{\mathbf{k}\omega}a^*_{\mathbf{k}'\omega'}\rangle=n_{\mathbf{k}\omega}\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')$$

is the autocorrelation function of the wave amplitudes, ω_p is the parametric-pump frequency, k_Ω and v_Ω are the wave vector and the group velocity of the waves in the direction of the solid angle Ω on the resonant surface $\omega_{\mathbf{k}}=\omega_p/2$ in **k**-space. Allowance for the scattering of the waves by one another leads to finite widths of the packet with respect to both the true frequencies, $\Delta\omega$, and the eigenfrequencies $\Delta\omega_{\mathbf{k}}$, where $\Delta\omega^{-1}$ is the correlation time of the PSW amplitudes $a_{\mathbf{k}}(t)$, while $\Delta\omega_{\mathbf{k}}$ characterizes the dimensions of the PSW packet in **k** space.

The existing theory explains most experimental facts on parametric excitation of spin waves in ferro- and antiferromagnets (see, e.g., Refs. 1–8). However, the results of a number of experiments initiated in 1977 contridicted not only the nonlinear but also the linear theory of parametric instability.[9,10]

In these experiments, performed with ferro- and antiferromagnets, an anomalously rapid damping of the PSW wave was observed compared with the relaxation frequency determined from the threshold. This character of the damping can be attributed to the presence of elastic scattering of the waves by inhomogeneities. A thorough investigation has revealed that in antiferromagnets the anomalous relaxation is most readily due to elastic scattering of PSW by the nuclear spins.[11] As a ferromagnets, elastic scattering in them is weak and cannot lead to a substantial change of the damping.[12,13] Moreover, analysis[13] of the experimental data has shown that in ferromagnets there is only one microscopic relaxation time, and that both dampings ($\Gamma_{\mathbf{k}}$ of the anomalous correlator and $\gamma_{\mathbf{k}}$ determined from the threshold) are caused by the same proper relaxation of the spin waves.

It was proposed even in the first study of relaxation of anomalous PSW in ferromagnets[10] that a PSW packet has an appreciable width in terms of the eigenfrequencies, $\Delta\omega_{\mathbf{k}}\gtrsim\gamma_{\mathbf{k}}$. In a packet having a large wave-vector width the anomalous correlator should be damped with a characteristic time $\Gamma_{\mathbf{k}}^{-1}$, where

$$\Gamma_{\mathbf{k}}=\gamma_{\mathbf{k}}+\Delta\omega_{\mathbf{k}}. \tag{1.2}$$

The presence of a considerable eigenfrequency width of the PSW packet was proved in Ref. 12 by direct experiment. A later more detailed study[13] has shown that the packet is not only broadened but also shifts in **k**-space. This shift should lead to broadening of the packet in terms of the eigenfrequencies $\omega_{\mathbf{k}}$. If the wave eigenfrequency $\omega_{\mathbf{k}}$ drifts in **k** space, new waves enter into resonance at each instant of time, and those PSW that were at resonance before are damped. One can expect that if the eigenfrequency changes after a relaxation time $\gamma_{\mathbf{k}}^{-1}$ by an amount of order $\gamma_{\mathbf{k}}$, the width $\Delta\omega_{\mathbf{k}}$ of the packet becomes of the order of $\gamma_{\mathbf{k}}$. The cause of this frequency drift is, in our opinion, the accumulation of nonequilibrium spin waves (NSW) produced as a result of PSW dissipation. The presence of a large number of NSW at small k was observed long ago.[14,15] It is natural to expect the number of NSW to increase linearly with time in a certain time interval and then reach a stationary regime. The nonlinear shift of the frequency on account of the NSW accumulation is

$$\omega_{\mathbf{k}}=\omega_{\mathbf{k}}^0+2\sum_{\mathbf{k}'}T_{\mathbf{k}\mathbf{k}'}n_{\mathbf{k}'}, \tag{1.3}$$

where $n_{\mathbf{k}}$ are the occupation numbers of the NSW and $T_{\mathbf{k}\mathbf{k}'}$ is the four-magnon interaction amplitude. If it is assumed that

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relaxation of one PSW produces one NSW (see Sec. 3), we get

$$\frac{1}{2\gamma_k^2}\frac{d\omega_k}{dt}=\sum_{k'} T_{kk'}\frac{dn_{k'}}{dt}\approx\frac{2TN_p}{\gamma}, \tag{1.4}$$

where T is the characteristic value of $T_{kk'}$ and N_p is the number of the PSW. Substituting for N_p the expression that follows from the S-theory:

$$|S|N_p=\gamma(\zeta^2-1)^{1/2}, \tag{1.5}$$

where S is the matrix element of the PSW-pair interaction and $\zeta^2=h^2/h_c^2$ is the ratio of the pump power to threshold, we get

$$\frac{1}{\gamma_k^2}\frac{d\omega_k}{dt}\approx\frac{2T}{|S|}(\zeta^2-1)^{1/2}. \tag{1.6}$$

The values of T and S are of the same order, so that the estimate (1.6) demonstrates the presence of an appreciate drift, where the dependence of the width on the supercriticality agrees qualitatively with the experimental data.[10,12,13]

In recent experiments artificial spin-wave frequency drift was produced by varying the external magnetic field at a constant rate. To interpret the experimental data for either artificial or natural drift of the eigenfrequency, the corresponding S theory must be modified. In the S-theory approximation the study of the wave-packet form consists of two states. In Sec. 2 we obtain first the form of the PSW packet and its amplitude, given the total pump P_k and the thermal noise. This is followed in Sec. 3 by a self-consistency procedure, since the self-consistent pump P_k acting on the PSW is itself dependent on the wave amplitude and phase. As a result, the behavior of the PSW system depends substantially on the sign of the derivative $Sd\omega_k/dt$. At $Sd\omega_k/dt<0$, starting with a certain drift velocity, the PSW packet can have three stationary states. To determine the solution that can be realized in experiment, we check in Sec. 4 the stability of the solutions obtained and determine the frequencies of the collective PSW oscillations. We show, in particular, that if the stationary state of the PSW in the absence of drift is stable, the stable states at $Sd\omega_k/dt<0$, are those with amplitudes close to the thermal noise as well as the highest-amplitude state that goes over as $d\omega_k/dt\to 0$ into the S-theory solution (1.1), (1.5).

In Sec. 3 we investigate also the dependence of the number of PSW on the pump power in the presence of a self-consistent natural drift with velocity proportional to the number of the PSW.

2. FORM OF PSW IN THE PRESENCE OF FREQUENCY DRIFT

1. The S-theory equations formulated in terms of correlators are

$$\begin{aligned}&\partial n_k/\partial t+2\gamma_k n_k+2\,\mathrm{Im}\,(P_k^*\sigma_k)=2\gamma_k n_k^0,\\&\partial\sigma_k/\partial t+2[\gamma_k+i(\omega_k-\omega_p/2)]\sigma_k+2iP_k n_k=0,\end{aligned} \tag{2.1}$$

where $n_k=\langle|a_k|^2\rangle$ and $\sigma_k=\langle a_k a_{-k}\rangle$ are the normal and anomalous correlators of the PSW amplitudes, and n_k^0 is the thermodynamic-equilibrium distribution function of the waves.[1] The quantity P_k is the self-consistent pump:

$$P_k=hV_k+\sum_{k'} S_{kk'}\sigma_{k'}, \tag{2.2}$$

where $S_{kk'}=T_{k,-k,k',-k'}$ is the amplitude of the four-wave interaction of a pair of waves with opposite momenta. The PSW packet is localized near the resonance surface $\omega_k=\omega_p/2$ and falls off rapidly with increasing distance from it. It is convenient to transform in Eqs. (2.1) to new variables, the frequency detuning $\varepsilon=\omega_k-\omega_p/2$ and the solid angle $\Omega=(\theta,\varphi)$ of the wave vector $\mathbf{k}$:

$$n_{\Omega,\varepsilon}=v_\Omega n_k/k_\Omega^2. \tag{2.3}$$

The quantities γ_k, n_k^0, v_k, $S_{kk'}$, are continuous functions of $|\mathbf{k}|$, and using the fact that the PSW packet is narrow we obtain therefore from (2.1)

$$\begin{aligned}&\partial n_{\varepsilon,\Omega}/\partial t+2\gamma_\Omega n_{\varepsilon,\Omega}+2\,\mathrm{Im}\,(P_\Omega^*\sigma_{\varepsilon,\Omega})=2\gamma_\Omega n^0,\\&\partial\sigma_{\varepsilon,\Omega}/\partial t+2[\gamma_\Omega+i(\varepsilon+g(t))]\sigma_{\varepsilon,\Omega}+2iP_\Omega n_{\varepsilon,\Omega}=0,\end{aligned} \tag{2.4}$$

where $g(t)$ is the drift of the eigenfrequency:

$$g(t)=\omega_k(t)-\omega_k^0. \tag{2.5}$$

In the stationary drift-free case, these equations have as solutions

$$g(t)=\text{const}=0,\quad n_{\varepsilon,\Omega}=|P_\Omega|^2 n^0/(\varepsilon^2+\gamma_\Omega^2-|P_\Omega|^2). \tag{2.6}$$

In the limit as $n^0\to 0$ this solution tends to the standard S-theory solution. In the case $|P_\Omega|>\gamma_\Omega$ the solution of the system (2.4) increases exponentially with a growth rate

$$\nu_{\varepsilon,\Omega}=-\gamma_\Omega+(|P_\Omega|^2-\varepsilon^2)^{1/2}. \tag{2.7}$$

The region of the instability of the waves in $\mathbf{k}$ space is given by the inequality $\nu_{\varepsilon,\Omega}>0$, which corresponds to

$$|\varepsilon|<(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}. \tag{2.8}$$

In the linear-drift case of interest to us we have

$$g(t)=\alpha(t-t_0). \tag{2.9}$$

Such a frequency drift can lead to the appearance of PSW packets that move in $\mathbf{k}$ space. The PSW distribution in angle in such a packet is constant, and the eigenfrequency ω_k^0 varies at a constant rate. At each instant of time in the course of the drift, some waves land on the resonance surface $\omega_k(t)=\omega_p/2$, and others depart from it. The incoming waves have thermal-fluctuation amplitudes, and in order for them to increase significantly it is necessary that the instability threshold $|P_\Omega|>\gamma_\Omega$. In this case the waves that land in the instability region increase with a growth rate

$$\nu_\Omega\approx|P_\Omega|-\gamma_\Omega \tag{2.10}$$

[see (2.7)], pass through it within a time

$$\tau=2(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}|\alpha|^{-1}\approx 2[2\gamma(|P|-\gamma)]^{1/2}|\alpha|^{-1}, \tag{2.11}$$

and their number increases to

$$n_\Omega(\tau)\approx n^0\exp(2\nu_\Omega\tau)\approx n^0\exp[4(2\gamma)^{1/2}(|P|-\gamma)^{3/2}|\alpha|^{-1}] \tag{2.12}$$

on the instability-region boundary. These wave leave next the instability region and are damped within a time $\sim\tau$. Far from the instability region, the drift can be neglected and we have in accordance with (2.6).

$$n_{\varepsilon,\Omega}-n^0\propto\varepsilon^{-2}. \qquad (2.13)$$

The central frequency $\omega_{\mathbf{k}}$ of the PSW packet is on the edge of the instability region and differs from $\omega_p/2$ by

$$\Delta\omega_{\mathbf{k}}=\omega_{\mathbf{k}}-\omega_p/2=(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}\alpha|\alpha|^{-1}. \qquad (2.14)$$

It follows from (2.12) that in the absence of thermal noise the PSW amplitude is zero.

For a qualitative analysis of the PSW amplitude and phase past the threshold it is convenient to use the condition that the pumped-in and dissipated energy fluxed be equal:

$$hV\sin\psi=\gamma, \qquad (2.15)$$

where ψ is the phase difference between the PSW anomalous operator and the pump.[1] The vector $S\sigma$ on the vector diagram (Fig. 1) makes thus an angle $\psi=\arcsin(\gamma/hV)$ with the pump hV (we assume here for the sake of argument that $S>0$). The sum $hV+S\sigma$ is the renormalized pump. We estimate now the angle between P and $S\sigma$. It follows from Eqs. (2.1) that in the stationary case

$$\sigma_{\mathbf{k}}=-iP_{\mathbf{k}}n_{\mathbf{k}}[\gamma_{\mathbf{k}}+i(\omega_{\mathbf{k}}-\omega_p/2)]^{-1}, \qquad (2.16)$$

therefore

$$\operatorname{Re}P_{\mathbf{k}}^*\sigma_{\mathbf{k}}=-(\omega_{\mathbf{k}}-\omega_p/2)|P_{\mathbf{k}}|^2n_{\mathbf{k}}[\gamma_{\mathbf{k}}^2+(\omega_{\mathbf{k}}-\omega_p/2)^2]^{-1}. \qquad (2.17)$$

At not too fast a frequency drift the packet is narrow in frequency, so that $|P_{\mathbf{k}}|^2=\gamma_{\mathbf{k}}+\Delta\omega_{\mathbf{k}}^2$, at the center of the packet, and hence

$$\operatorname{Re}P_{\mathbf{k}}^*\sigma_{\mathbf{k}}\approx-\Delta\omega_{\mathbf{k}}n_{\mathbf{k}}. \qquad (2.18)$$

Thus, at a positive drift $\Delta\omega_{\mathbf{k}}>0$ and the angle between $S\sigma$ and P is obtuse. Therefore $|S\sigma|$ is smaller than the amplitude $(|hV|^2-\gamma^2)^{1/2}$ of the PSW in the absence of drift. In the case of negative drift, $\Delta\omega_{\mathbf{k}}<0$ and the angle between $S\sigma$ and P is acute, $|S\sigma|>(|hV|^2-\gamma^2)^{1/2}$ at any drift velocity. Strictly speaking, at negative drift the condition (3.15) can be met also in the case when the angle between the vector $S\sigma$ and hV is $\pi-\arcsin(\gamma/hV)$ (line 2). For such a solution to exist, it is necessary that the drift be large enough. As shown in Sec. 4, however, this solution is unstable.

2. To investigate the PSW packets produced in the case of drift with constant velocity (2.9), it is convenient to change from the variable ε to the variable w, the running value of the frequency detuning, $w=\varepsilon+\alpha t$. The stationary drift of the packet is described by solutions that do not depend explicitly on the time. They satisfy a system of three linear equations:

$$\begin{gathered}\left(\alpha\frac{\partial}{\partial w}+2\gamma_\Omega\right)n_{w,\Omega}+2\operatorname{Im}(P_\Omega^*\sigma_{w,\Omega})=2\gamma_\Omega n^0,\\ \left(\alpha\frac{\partial}{\partial w}+2\gamma_\Omega+2iw\right)\sigma_{w,\Omega}+2iP_\Omega n_{w,\Omega}=0,\\ \left(\alpha\frac{\partial}{\partial w}+2\gamma_\Omega-2iw\right)\dot{\sigma}_{w,\Omega}-2iP_\Omega^*n_{w,\Omega}=0.\end{gathered} \qquad (2.19)$$

The solutions of this system should obviously, satisfy the boundary conditions

$$(n_{w,\Omega}-n^0),\quad \sigma_{w,\Omega},\quad \dot{\sigma}_{w,\Omega}\to0\quad \text{as}\quad w\to\pm\infty. \qquad (2.20)$$

The solutions of the homogeneous system (2.19) have as $|w|\to\infty$ the asymptotic forms

$$\operatorname{Im}(P_\Omega^*\sigma_{w,\Omega})\propto\exp[2w(-\gamma_\Omega\pm iw)/\alpha];\quad \exp(-2\gamma_\Omega w/\alpha).$$

Not one of these solutions tends to zero as $w/\alpha\to\infty$, so that no linear combination of the homogeneous-system solutions satisfies the boundary conditions (2.20). This means physically that parametric excitation of drifting waves is impossible in the absence of thermal noise. To determine the form a drifting PSW packet we must therefore find for the system (2.19) a regular solution that satisfies the boundary conditions (2.20). The requirement that the solution be regular is equivalent to the external-stability condition imposed on the stationary state of the PSW in the absence of drift.

3. Equations (2.19) that describe stationary drift of a PSW packet can be made nondimensional by introducing the new variables

$$x_\Omega=w/\gamma_\Omega,\qquad \eta_\Omega=\alpha/2\gamma_\Omega^2,\qquad \Pi_\Omega=P_\Omega/\gamma_\Omega. \qquad (2.21)$$

The solid angle Ω in Eqs. (2.19) is a parameter; we therefore omit the dependence on the solid angle when investigating the eigenfrequency makeup of the packet at fixed Ω. It is convenient to introduce in (2.19) new variables

$$U(x)=\operatorname{Re}(\Pi^*\sigma(x)),\qquad V(x)=-\operatorname{Im}(\Pi^*\sigma(x)),$$
$$M(x)=n(x)-n^0 \qquad (2.22)$$

and use a Fourier transformation to lower the order of the differential operator:

$$f(x)=\int f_\lambda e^{i\lambda x}\,d\lambda. \qquad (2.23)$$

As a result we obtain from (2.19)

$$\begin{gathered}(1+i\eta\lambda)M_\lambda-V_\lambda=0,\qquad (1+i\eta\lambda)U_\lambda-i\partial V_\lambda/\partial\lambda=0,\\ (1+i\eta\lambda)V_\lambda+i\partial U_\lambda/\partial\lambda-|\Pi|^2[M_\lambda+n^0\delta(\lambda)]=0.\end{gathered} \qquad (2.24)$$

Following the change of variable

$$z=(1+i\eta\lambda)^2/i\eta \qquad (2.25)$$

we obtain from (2.24)

$$\frac{d^2V}{dz^2}+\left(-\frac{1}{4}-\frac{i|\Pi|^2}{4\eta z}\right)V=\frac{i|\Pi|^2}{2\eta z}n^0\delta\left(z+\frac{i}{\eta}\right). \qquad (2.26)$$

Solutions of this equations[16,17] are Whittaker functions with index $m=\frac{1}{2}$, viz., $W_{\varkappa,\frac{1}{2}}(z)$ and $W_{-\varkappa,\frac{1}{2}}(-z)$, where

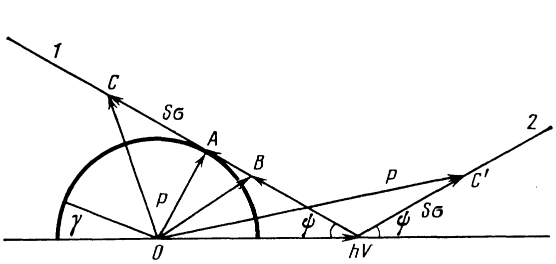

FIG. 1. Complex plate of the pump vector. hV—external pump, γ—damping, P—self-consistent pump: OA—drift-free, $OB(OC)$—positive (negative) drift. $S\sigma=\Sigma_{\mathbf{k}'}S_{\mathbf{kk}'}\sigma_{\mathbf{k}'}$—field of the pair-system reaction.

$$\varkappa=-i|\Pi|^2/4\eta, \tag{2.27}$$

$$W_{\varkappa,1/2}(z)=\frac{e^{-z/2}}{\Gamma(1-\varkappa)}\int_0^\infty\left(1+\frac{z}{t}\right)^\varkappa e^{-t}\,dt. \tag{2.28}$$

We are interested in the solution of (2.26) on a parabolic contour specified by the condition (2.25) for real λ:

$$\operatorname{Re} z=2\lambda, \qquad \operatorname{Im} z=(\lambda^2\eta^2-1)/\eta.$$

As $\lambda\to+\infty$

$$|W_{\varkappa,1/2}(z)|\to 0, \qquad |W_{-\varkappa,1/2}(-z)|\to\infty.$$

As $\lambda\to-\infty$

$$|W_{\varkappa,1/2}(z)|\to\infty, \qquad |W_{-\varkappa,1/2}(-z)|\to 0.$$

Consequently

$$V(z)=\begin{cases} AW_{\varkappa,1/2}(z), & \lambda>0 \quad (\operatorname{Re} z>0)\\ BW_{-\varkappa,1/2}(-z), & \lambda<0 \quad (\operatorname{Re} z<0)\end{cases} \tag{2.29}$$

At $\lambda=0$ ($z=-i/\eta$) the function $V(z)$ is continuous, but its derivative is not. As a result we obtain

$$AW_{\varkappa,1/2}(-i/\eta)=BW_{-\varkappa,1/2}(i/\eta),$$
$$AW'_{\varkappa,1/2}(-i/\eta)+BW_{-\varkappa,1/2}(i/\eta)=-|\Pi|^2n^0/2. \tag{2.30}$$

The integral amplitude of the PSW packet (the number of waves N_Ω) is expressed in terms of the Wronskian of the Whittaker functions $W_{\varkappa,\frac{1}{2}}(z)$ and $W_{-\varkappa,\frac{1}{2}}(z)$ and of their values at the point $z=-i/\eta$:

$$N_\Omega=\int M_\Omega(x)\frac{k_\Omega^2}{v_\Omega}\gamma_\Omega\,dx$$
$$=-\gamma_\Omega\frac{\pi k_\Omega^2}{v}|\Pi_\Omega|^2n^0\Delta^{-1}W_{\varkappa,1/2}\left(-\frac{i}{\eta}\right)W_{-\varkappa,1/2}\left(\frac{i}{\eta}\right), \tag{2.31}$$

where

$$\Delta=W'_{\varkappa,1/2}(z)W_{-\varkappa,1/2}(-z)+W_{\varkappa,1/2}(z)W'_{-\varkappa,1/2}(-z)=-e^{i\pi\varkappa}. \tag{2.32}$$

The integral characteristics of the anomalous correlator are expressed in similar form

$$V_\Omega=-\gamma_\Omega\frac{k_\Omega^2}{v_\Omega}\int\operatorname{Im}(\Pi_\Omega^*\sigma_\Omega(x))\,dx=N_\Omega, \tag{2.33}$$

$$U_\Omega=\gamma_\Omega\frac{k_\Omega^2}{v_\Omega}\int\operatorname{Re}(\Pi_\Omega^*\sigma_\Omega(x))\,dx$$
$$=i\gamma_\Omega\frac{\pi k_\Omega^2}{v_\Omega}|\Pi_\Omega|^2n^0\Delta^{-1}\frac{d}{dz}[W_{\varkappa,1/2}(z)W_{-\varkappa,1/2}(-z)]_{z=-i/\eta}. \tag{2.34}$$

Table I lists the values of the integral amplitudes N_Ω and U_Ω referred to $\pi k_\Omega^2|\Pi_\Omega|^2\gamma_\Omega n^0/v_\Omega$ in all limiting cases that arise at various values of the parameters η (the drift velocity) and renormalized pump $|\Pi|$.

Up to the threshold of the parametric instability and at low drift velocity $|\eta|\ll 1$ we have the usual below-threshold heating of the wave system. In this case

$$N_\Omega=\pi\frac{k_\Omega^2}{v_\Omega}n^0\frac{|P_\Omega|^2}{(\gamma_\Omega^2-|P_\Omega|^2)^{1/2}}, \tag{2.35}$$

just as in the absence of drift. In the case of a small excess above threshold and very small drift $|\eta|\ll(|\Pi|^2-1)^{3/2}$ we get

$$N_\Omega=\pi\frac{k_\Omega^2}{v_\Omega}n^0\frac{|P_\Omega|^2}{(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}}\exp\frac{4(|P_\Omega|^2-\gamma_\Omega^2)^{3/2}}{3\alpha\gamma_\Omega}. \tag{2.36}$$

This is just the case that we have discussed qualitatively in Subsec 1 of Sec. 2, and the present result (2.35) agrees qualitatively with the earlier estimate (2.12). It can be seen from the table that the amplitude ratio is here

$$\frac{U_\Omega}{N_\Omega}\approx-(|\Pi|^2-1)^{1/2}\operatorname{sign}\eta=-\frac{\alpha}{|\alpha|}\frac{(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}}{\gamma_\Omega}. \tag{2.37}$$

At a higher drift velocity $|\eta|\gtrsim(|\Pi|^2-1)^{3/2}$ the number of the PSW has a limit close to the thermal value:

$$N_\Omega\approx\pi\frac{k_\Omega^2}{v_\Omega}[1+(2\gamma_\Omega^2/|\alpha|)^{1/3}]\frac{|P_\Omega|^2}{\gamma_\Omega}n^0. \tag{2.38}$$

With increasing drift velocity, the integral amplitude $\operatorname{Re}(P_\Omega^*\sigma_\Omega)$ decreases in proportion to $|\eta|^{-1}$.

TABLE I.

	$-\frac{1}{\Delta}W_{\varkappa,1/2}\left(-\frac{i}{\eta}\right)W_{-\varkappa,1/2}\left(\frac{i}{\eta}\right)$	$\frac{i}{\Delta}\frac{d}{dz}\left[W_{\varkappa,1/2}(z)W_{-\varkappa,1/2}(-z)\right]_{z=-i/\eta}$
$\lvert\eta\rvert\ll\lvert\Pi\rvert$ $(\lvert\alpha\rvert\ll\gamma\lvert P\rvert)$	$\left.\begin{matrix}\lvert\Pi\rvert<1\\(\lvert P\rvert<\gamma)\end{matrix}\right\}$ $\frac{1}{(1-\lvert\Pi\rvert^2)^{1/2}}$	0
$\lvert\eta\rvert\ll(\lvert\Pi\rvert^2-1)^{3/2}$ $(\lvert\alpha\rvert\ll(\lvert P\rvert^2-\gamma^2)^{3/2}/\gamma)$	$\left.\begin{matrix}\lvert\Pi\rvert>1\\(\lvert P\rvert>\gamma)\end{matrix}\right\}$ $\frac{2\operatorname{ch}(2\xi)}{(\lvert\Pi\rvert^2-1)^{1/2}}$	$-2\operatorname{sh}(2\xi)$
$\left.\begin{matrix}(\lvert\Pi\rvert^2-1)^{3/2}\ll\lvert\eta\rvert\ll 1\\((\lvert P\rvert^2-\gamma^2)^{3/2}/\gamma\ll\lvert\alpha\rvert\ll\gamma^2)\end{matrix}\right\}$	$\lvert\eta\rvert^{-1/3}\left[\frac{2^{5/3}\pi}{3^{4/3}\Gamma^2(2/3)}+\frac{\lvert\Pi\rvert^2-1}{3^{1/2}\lvert\eta\rvert^{1/3}}\right]$	$-3^{-1/2}\operatorname{sign}\eta$
$\left.\begin{matrix}\lvert\eta\rvert\gg\lvert\Pi\rvert\\(\lvert\alpha\rvert\gg\lvert P\rvert\gamma)\end{matrix}\right\}$	$\frac{\exp(\pi\lvert\varkappa\rvert)\operatorname{sh}(i\varkappa\pi)}{i\varkappa\pi}$	$-\frac{2}{\pi}\exp(\pi\lvert\varkappa\rvert)\times$ $\times\operatorname{sh}(i\varkappa\pi)\ln\frac{4\eta^2}{\lvert\Pi\rvert^2+4\lvert\eta\rvert e^{-C}}$

**Note.* $2\xi=\eta^{-1}\{|\Pi|^2\arctan[(|\Pi|^2-1)^{1/2}]-(|\Pi|^2-1)^{1/2}\}$, $\frac{2^{5/3}\pi}{3^{4/3}\Gamma^2(2/3)}\approx 1.26$, $C=0.577\ldots$ is the Euler constant.

We determine now the position of the center of the packet $\Delta\omega_k$ and its width δ_k in terms of the eigenfrequencies. If the PSW packet amplitude is large enough, waves having frequencies far from resonance make a small contribution. Neglecting these "tails," we get

$$\bar{w}=\int wV(w)\,dw\Big/\int V(w)\,dw$$
$$=-\frac{i\gamma}{2V(0)}\left[\left(\frac{\partial V}{\partial\lambda}\right)_{\lambda=-0}+\left(\frac{\partial V}{\partial\lambda}\right)_{\lambda=+0}\right],$$
$$\overline{w^2}=\int w^2V(w)\,dw\Big/\int V(w)\,dw \qquad (2.39)$$
$$=-\frac{\gamma^2}{2V(0)}\left[\left(\frac{\partial^2 V}{\partial\lambda^2}\right)_{\lambda=-0}+\left(\frac{\partial^2 V}{\partial\lambda^2}\right)_{\lambda=+0}\right],$$
$$\delta_k^2=\overline{w^2}-\bar{w}^2.$$

In the case when $|\eta|\ll(|\Pi|^2-1)^{3/2}$,

$$\Delta\omega_k=\frac{\alpha}{|\alpha|}(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}-\frac{\alpha|P_\Omega|^2}{8\gamma_\Omega(|P_\Omega|^2-\gamma_\Omega^2)},$$
$$\delta_k^2=\frac{|\alpha|\,|P_\Omega|^2}{4\gamma_\Omega(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}}. \qquad (2.40)$$

In the other case, when the number of PSW is substantially higher than that of the thermal ones, $|P|^2\gg\alpha\gg|P|\gamma$, we have

$$\Delta\omega_k=\frac{|P_\Omega|^2}{2\alpha}\gamma_\Omega\ln\frac{\alpha^2}{\gamma_\Omega^2(|P_\Omega|^2+2|\alpha|e^{-C})},$$
$$\delta_k^2=|P_\Omega|^2-\gamma_\Omega^2+\frac{|P_\Omega|^2}{4}\ln\frac{\alpha^2}{\gamma_\Omega^2(|P_\Omega|^2+2|\alpha|e^{-C})}\gg\Delta\omega_k^2. \qquad (2.41)$$

In all other limiting cases the PSW numbers are of the same order as of the thermal and will not be considered here.

For the number of PSW to be quite large and experimentally observable, the following condition must be met:

$$|\eta|<|\Pi|^2\,\mathrm{arctg}[(|\Pi|^2-1)^{1/2}]-(|\Pi|^2-1)^{1/2}. \qquad (2.42)$$

We determine now the total number of waves in a packet and the integral amplitude of the anomalous correlator, by integrating (2.31) and (2.34) over the angles in **k** space, since the values N_Ω, V_Ω, and U_Ω were obtained for a given direction Ω:

$$N=\int N_\Omega\,d\Omega,$$
$$\Sigma=\int\sigma_k\frac{k_\Omega^2}{v_\Omega}\,d\Omega\,d\omega_k \qquad (2.43)$$
$$=\int\frac{\Pi_\Omega}{|\Pi_\Omega|^2}(U_\Omega-iV_\Omega)\,d\Omega=-i\Pi N(1-i\mu)\,|\Pi|^{-2},$$
$$\mu=-\mathrm{Re}(\Pi^*\Sigma/\gamma N).$$

The expressions for N and μ in various limiting cases take the forms

$$N=N_T(\pi|\alpha|\gamma_0/\beta)^{1/2}|P_0|^2(|P_0|^2-\gamma_0^2)^{-3/4}$$
$$\times\exp[{}^4/_3(|P_0|^2-\gamma_0^2)^{3/2}/|\alpha|\gamma_0], \qquad (2.44)$$
$$\mu=(|P_0|^2/\gamma_0^2-1)^{1/2}\,\mathrm{sign}\,\alpha-\alpha|P_0|^2/8(|P_0|^2-\gamma_0^2)$$

at $|\alpha|\ll(|P_0|^2-\gamma_0^2)^{3/2}/\gamma_0$. We have used here the fact that the surface-wave packet is narrow in terms of angle, so that we can put

$$|P(\cos\theta)|^2=|P_0|^2(1-\beta\cos^2\theta), \qquad (2.45)$$

where N_T is the number of thermal waves in a layer of width $\Delta\omega_k=\pi\gamma_k$ near the resonance surface:

$$N_T=4\pi^2 n^0 k^2\gamma/v. \qquad (2.46)$$

This is the level from which development of parametric instability begins. In the limit when $\gamma_0^2\ll|P_0|^2$, $|\alpha|\ll|P_0|^2$ we have

$$N=N_T|\alpha|(\pi\beta^{1/2}|P_0|\gamma_0^2)^{-1}\exp(\pi|P_0|^2/|\alpha|),$$
$$\mu=(4|P_0|^2/\alpha)\ln(|\alpha|/2\gamma|P_0|). \qquad (2.47)$$

At $|\alpha|\gg|P_0|^2\gg\gamma^2$

$$N=N_T\frac{|P_0|^2}{\gamma_0^2},\qquad \mu=\frac{|P_0|^2}{\alpha}\ln\frac{|\alpha|}{2\gamma_0}. \qquad (2.48)$$

In the case $\gamma_0^2\gg|\alpha|\gg(|P_0|^2-\gamma_0^2)^{3/2}/\gamma_0$

$$N=N_T|P_0|^2/\beta^{1/2}\gamma_0^2,\qquad \mu=(\alpha/2\gamma_0^2)^{1/2}. \qquad (2.49)$$

3. PSW INTERGAL AMPLITUDE—SELF-CONSISTENCY PROCEDURE

Knowing the expressions (2.44)–(2.49) for the number of waves in terms of the total pump, and the connection between P_k and the anomalous correlator (2.2), we determine now the dependence of the total number N of the PSW on the pump amplitude h.

1. For artificial drift, the self-consistency procedure reduces to finding the connection between the number of waves and the pump power, using relation (2.2) from which it follows that

$$|hV|^2=|P|^2+2\gamma\mu SN+(1+\mu)^2\gamma^2|P|^{-2}(SN)^2, \qquad (3.1)$$

where

$$\mu=-\mathrm{Re}(P^*\Sigma)/\gamma N. \qquad (3.2)$$

The solution of this equation is

$$\left.\begin{matrix}|S|N_1\\|S|N_2\end{matrix}\right\}=\frac{|P|^2S}{(1+\mu^2)|S|}\{-\mu\pm[(1+\mu^2)|hV|^2|P|^{-2}-1]^{1/2}\}. \qquad (3.3)$$

If the following condition is met

$$|\eta|\ll(|\Pi|^2-1)^{3/2},\qquad |\Pi|\qquad (|\alpha|\ll(|P|^2-\gamma^2)^{3/2}/\gamma,\ |P|\gamma), \qquad (3.4)$$

the number N of the PSW is large compared with the thermal noise, and μ is determined by Eq. (2.44). From (3.3) we obtain

$$\left.\begin{matrix}|S|N_1\\|S|N_2\end{matrix}\right\}=-\frac{\alpha S}{|\alpha S|}(|P|^2-\gamma^2)^{1/2}\pm\gamma(\zeta^2-1)^{1/2},\qquad \zeta^2=\frac{|hV|^2}{\gamma^2}. \qquad (3.5)$$

The simplest to analyze is the case $S\alpha>0$. In this case only the solution $N_1(\zeta)$ in (3.5) is meaningful. Figure 2 shows plots of $N_{1,2}$ vs $|P|$ from (3.5) and (2.44) at a fixed pump power h^2, low thermalnoise level

$$\xi=|S|N_T/\gamma\ll1 \qquad (3.6)$$

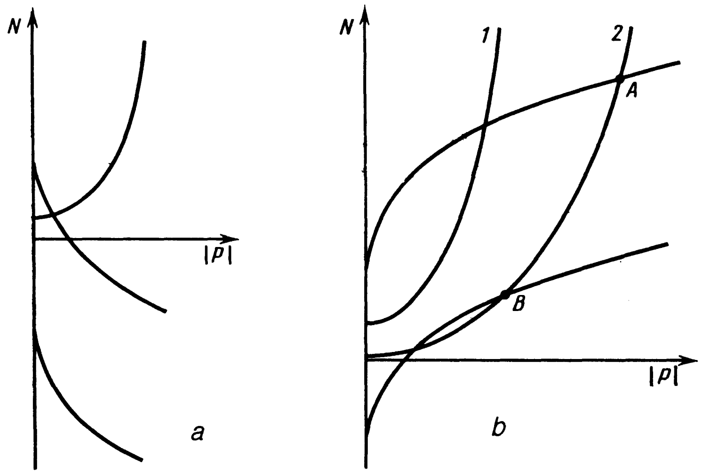

FIG. 2. PSW integral amplitude vs self-consistent pump from Eqs. (3.5) and (2.44): a—$S\partial\omega_k/\partial t>0$; b—$S\partial\omega_k/\partial t<0$; curves 1 and 3 correspond to different thermal-noise levels ($N_T^{(1)}>N_T^{(2)}$).

and various drift velocities. If the supercriticality exceeds the thermal-noise level $(\zeta^2-1)^{1/2}>\xi$, it follows from (3.5) and (2.44) that

$$|S|N_1=\gamma(\zeta^2-1)^{1/2}-\{^3/_4\alpha\gamma\ln[(\zeta^2-1)^{1/2}/\xi]\}^{1/3}. \tag{3.7}$$

The first term in (3.7) corresponds to the known S-theory result. The number of PWS decreases with increasing drift velocity. At very low supercriticalities, $(\zeta^2-1)^{1/2}<|\eta|^{1/3}$, Eqs. (3.7) are not valid, since the condition (3.4) is violated. In this case the number of waves is small compared with the thermal ones and is described by Eq. (2.49). At large supercriticalities the condition (3.4) is likewise not met, and one must use for N Eqs. (2.47) and (2.48), but the behavior of N as a function of the drift velocity α remains qualitatively the same. The calculated plots of N for $\xi=10^{-3}$ and various values of the pump power and drift velocity are given in Fig. 3.

At negative drift velocity both solutions (3.5) can be meaningful. It can be seen from Fig. 2 that at not too low noise levels and not too high drift velocities there exists a unique solution corresponding to the root N_1 in (3.5) and described by Eq. (3.7). As $S\alpha\to-0$ this expression coincides, naturally, with the S-theory result. If the noise level is low and the drift velocity high enough, we have in accordance with Fig. 2 three solutions for the number N of the waves and for the renormalized pump P.

The smallest of the solutions N corresponds to a situation in which the noise level is so low, and the drift velocity high, that the time of passage through the stability region is too short for the PSW amplitude to increase to a significant values. The two other solutions correspond to the points A and B in Fig. 2; the solution corresponding to the larger value of N is represented by the point A and is stable, while the intermediate value is represented by the point B and, as shown in Sec. 4, is unstable.

Figure 4 shows a set of plots of the number N of the waves vs the pump power ζ^2, calculated for various values of the drift velocity α and for a thermal noise amplitude $\xi=10^{-3}$.

Thus, at fixed drift velocity, the dependence of the number of PSW on the pump power exhibits hysteresis. If the radicand in (3.3) is positive ($\zeta>\zeta_2$), where

$$\zeta_2^2=\frac{|h_2V|^2}{\gamma^2}=\frac{|P|^2}{1+\mu^2}\approx1+\frac{\alpha^{2/3}}{4\gamma^{4/3}\{^3/_4\ln[(\zeta^2-1)^{1/2}/\xi]\}^{1/3}}, \tag{3.8a}$$

two stable states of the PSW can exist. With increasing pump power at $\zeta>\zeta_1$, where

$$\zeta_1^2-1=\left\{\frac{3}{4}\frac{|\alpha|}{\nu^2}\ln\left[\left(\frac{|\alpha|}{4\gamma^2}\right)^{1/3}\xi^{-1}\right]\right\}^{2/3}, \tag{3.8b}$$

there remains a single state, which turns as $\alpha\to0$ into the standard S-theory solution with an amplitude close to the known expression (1.5).

2. If the drift of the frequency ω_k is due to accumulation of the PSW relaxation products, it must be determined in a

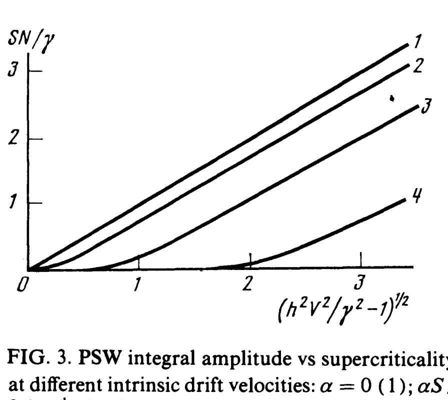

FIG. 3. PSW integral amplitude vs supercriticality at $S\partial\omega_k/\partial t>0$ and at different intrinsic drift velocities: $\alpha=0$ (1); $\alpha S/2\gamma^2|S|=10^{-2}$ (2), $3\cdot10^{-1}$ (3), 3 (4).

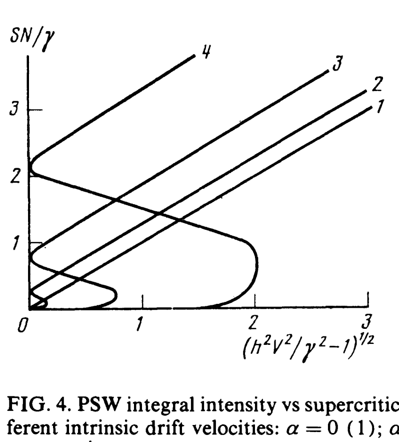

FIG. 4. PSW integral intensity vs supercriticality at $S\partial\omega_k/\partial t<0$ at different intrinsic drift velocities: $\alpha=0$ (1); $\alpha S/2\gamma^2|S|=-10^{-2}$ (2), $-3\cdot10^{-1}$ (3), -3 (4).

self-consistent manner. The PWS drift velocity is determined by the rate of the NSW accumulation.

It is easy to verify that the number $n = \Sigma n_{\mathbf{k}}$ of NWS is equal to the number of PSW dissipated within a time $(2\gamma_{\mathbf{k}})^{-1}$. Indeed, the main PSW relaxation channel is via coalescence with formation of a new NWS. The resultant spin wave (SW), however, again decays to form two SW of lower frequency. As a result, one PWS and a thermal SW with momentum $\mathbf{k}_1$ are transformed into two SW with momenta $\mathbf{k}_3$ and $\mathbf{k}_4$, i.e., relaxation of one PSW increases by unity the number of SW that are not connected with the pump (this accumulation mechanism was proposed by V. S. L'vov).

Four-magnon scattering processes to not lead to excitation of high-frequency SW. Indeed, in four-magnon processes there are preserved not only the SW energy $E = \Sigma\, \omega_{\mathbf{k}} n_{\mathbf{k}}$ but also their number n. Therefore, if the SW were initially excited in the low-frequency region, most of them will subsequently remain localized in this $\mathbf{k}$-space region. Thus, the NSW should accumulate and lead to drift of the eigenfrequencies $\omega_{\mathbf{k}}$. The velocity of the natural drift is therefore specified by Eq. (4). More detailed estimates can be found in our preprint.18 An expression for $T_{\mathbf{kk}'}$ in a ferromagnetic given in Ref. 19. In the limit $\mathbf{k}' \gg \mathbf{k}$ we have for $M \parallel [100]$

$$T_{\mathbf{kk}'} = 2\pi g^2 \left[1 + \left(\frac{\omega_M}{\omega_p}\right)^2\right]^{1/2} \times \left\{ 9\frac{\omega_a}{\omega_M} - \frac{2}{3} + \frac{1}{\sqrt{2}}\left(\left[1 + \left(\frac{\omega_M}{\omega_p}\right)^2\right]^{1/2} - 1\right)^{1/2} \right\}, \tag{3.9}$$

where $\omega_M = 4\pi g M$, g is the gyromagnetic ratio, M is the magnetization , ω_p is the pump frequency, and ω_a is the frequency connected with the crystallographic anisotropy. For an yttrium-iron garnet at room temperature we have $\omega_M = 2\pi\cdot 4.9\cdot 10^9\ \mathrm{s}^{-1}$, $\omega_a = 2\pi\cdot 0.23\cdot 10^9\ \mathrm{s}^{-1}$. As a result, for the frequency $\omega_p = 9.4\cdot 2\pi\cdot 10^9\ \mathrm{s}^{-1}$ at which the experiments of Refs. 10, 12, and 13 were performed, the drift velocity is anomalously small: $2T|S|^{-1} \approx 3.5\times 10^{-2}$.

When the pump frequency is changed by 30% the value of $2T|S|^{-1}$ for $\mathbf{k}' \gg \mathbf{k}$ is increased by an order of magnitude. If the parameter $2T|S|^{-1}$ is not too large, the number of the PSW can be found with the aid of expression (3.7)

$$|S|N = (|hV|^2 - \gamma^2)^{1/2} - \{3TN\gamma^2 \ln[(|hV|^2/\gamma^2 - 1)/\xi]\}^{1/2}. \tag{3.10}$$

At low supercriticalities $\zeta < \bar{\zeta}$ we have

$$\bar{\zeta}^2 - 1 = (3T/2|S|)\ln \xi^{-1}, \tag{3.11}$$

$$TN = {}^1/_3\gamma(\zeta^2 - 1)^{3/2} \ln[(\zeta^2 - 1)^{1/2}/\xi]. \tag{3.12}$$

At high pump powers the solution is close to (1.5) of the S-theory. A numerical calculation of the pump power for different values of the parameter $2T|S|^{-1}$ shows that even at low values of this parameter ($2T|S|^{-1} \gtrsim 10^{-3}$) the solution deviates substantially from the S theory. It should be noted, however, that at $\zeta < \bar{\zeta}$ the number of PSW increases insignificantly, so that it is impossible to observe in experiment the increase of the number of the PSW at a small excess above threshold. If the threshold power is defined as the pump

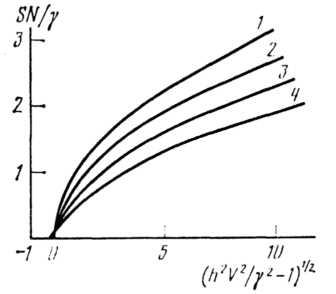

FIG. 5. Dependence of N on h^2/h_t^2 (h_t is the threshold pump amplitude) at different drift velocities: 1—$\alpha = 0$, 2—$2T/|S| = 0.3$, 3—$2T/|S| = 1$, 4—$2T/|S| = 3$.

power at which the number of PSW is 2% of the corresponding number $\zeta^2 = 2$, the "measurable" threshold power ζ_t increases. Figure 5 shows the numerically calculated dependence of the number of PSW on the pump power in units of h^2/h_t^2. The plot obtained in these coordinates is close to the S theory, but the measured threshold differs from the true one by 10–20%.

4. STABILITY OF STATIONARY STATE AND COLLECTIVE SPIN-WAVE OSCILLATIONS IN THE CASE OF FREQUENCY DRIFT

To investigate the stationary state of PSW one must consider the evolution of small perturbations against the background of this state. In this case,

$$M(x,t) = M_0(x) + \tilde{M}(x,t), \quad U(x,t) = U_0(x) + \tilde{U}(x,t), \quad V(x,t) = V_0(x) + \tilde{V}(x,t), \tag{4.1}$$

where M, U, and V are expressed in terms of the correlators by Eqs. (2.22). In the axial-symmetry case that we are considering, it is convenient to expand the solution in axial harmonics:

$$\tilde{M}_\Omega(x) = \sum_p \tilde{M}_p(x) e^{ip\varphi}, \quad \tilde{U}_\Omega(x) = \sum_p \tilde{U}_p(x) e^{ip\varphi}, \quad \tilde{V}_\Omega(x) = \sum_p \tilde{V}_\Omega(x) e^{ip\varphi}. \tag{4.2}$$

The Fourier transformation (2.23) recasts the linearized system in a form similar to (2.24):

$$\begin{aligned}
&(1 - i\Omega/2\gamma + i\eta\lambda)\tilde{M}_p(\lambda) - \tilde{V}_p(\lambda) \\
&= 2\pi(\gamma|\Pi|^2)^{-1} S_p[V(\lambda)\tilde{U}_p(0) - U(\lambda)\tilde{V}_p(0)], \\
&(1 - i\Omega/2\gamma + i\eta\lambda)\tilde{V}_p(\lambda) - i\partial\tilde{U}_p(\lambda)/\partial\lambda - |\Pi|^2\tilde{M}_p(\lambda) \\
&= 2\pi\gamma^{-1}[T_p U(\lambda)\tilde{M}_p(0) + S_p M(\lambda)\tilde{U}_p(0)], \\
&(1 - i\Omega/2\gamma + i\eta\lambda)\tilde{U}_p(\lambda) + i\partial\tilde{V}_p(\lambda)/\partial\lambda \\
&= 2\pi\gamma^{-1}[T_p V(\lambda)\tilde{M}_p(0) + S_p M(\lambda)\tilde{V}_p(0)],
\end{aligned} \tag{4.3}$$

where T_p and S_p are axial harmonics of the matrix elements $T_{\mathbf{kk}'}$ and $S_{\mathbf{kk}'}$ (Ref. 1). The operator in the left-hand side of (4.3) is of the same form as operator (2.24). The eigenfunctions of the corresponding homogeneous system are expressed in terms of the Whittaker functions $W_{\varkappa,1/2}(y)$, $W_{-\varkappa,1/2}(-y)$ and their derivatives, where $y = (1 - i\Omega/2\gamma + i\eta\lambda)^2/i\eta$.

The right-hand side of the system (4.3) contains the known functions $M(\lambda)$, $U(\lambda)$, and $V(\lambda)$ expressed in terms of the Whittaker functions $W_{\varkappa,1/2}(z)$, $W_{-\varkappa,1/2}(-z)$, where $z=(1+i\eta\lambda)^2/i\eta$, and of the values of the functions $\widetilde{M}$, $\widetilde{U}$, and $\widetilde{V}$ at $\lambda=0$. We multiply the system (4.3) by the solution of the conjugate system and integrate the result with respect to λ from ∞ to 0. In the case $|\alpha|\ll(|P|^2-\gamma^2)^{3/2}/\gamma$ of greatest interest to us the maximum contribution to the integrals is made by the region near $\lambda=0$. As a result we obtain a system of algebraic equations; equating its determinant to zero, we get the eigenvalues

$$\Omega_{1,2}=-i\gamma\pm\{4[S_pN+(|P|^2-\gamma^2)^{1/2}\operatorname{sign}\alpha](2T_p+S_p)N-\gamma^2\}^{1/2} \tag{4.4}$$

(see Ref. 18 for details). The obtained collective-oscillation frequencies coincide with the oscillation frequencies of a narrow PSW packet localized in k space on the surface $\omega_{\mathbf{k}}=\omega_p/2+\Delta\omega_{\mathbf{k}}$, i.e., where the center of the packet of drifting PSW is located.

Of greatest interest in experiment is the instability of the $p=0$ mode, and we confine ourselves for simplicity to an investigation of the stability with respect to this mode. The stablity condition for $p=0$ is

$$S_0(2T_0+S_0)(|S_0|N-S_0\Delta\omega_{\mathbf{k}}/|S_0|)>0. \tag{4.5}$$

The problem of stability of oscillations with $p=0$ reduces thus to a determination of the sign of the quantity

$$q=S_0N+(|P|^2-\gamma^2)^{1/2}\operatorname{sign}(S\alpha). \tag{4.6}$$

We assume for the sake of argument that $S_0(2T_0+S_0)>0$, i.e., that the PSW state is stable in the absence of drift. For a PSW system with drift to be stable in this case it is necessary to satisfy the inequality $q>0$. This condition is met if $S_0\alpha>0$ (positive drift). If, however, $S_0\alpha<0$ then, as follows from (3.5), the stable state corresponds to a positive root in (3.5), i.e., to the largest N at a given pump power. In the derivation of (4.4) we have actually assumed that the number of the PSW is exponentially large compared with the thermal noise, and thus we have therefore shown only that the states corresponding to the points A and B in Fig. 2 are respectively stable and unstable. As for the state represented by the point C, the PSW level in it is close to thermal, and the renormalization of the pump is small compared with unity: $|P|\approx|hV|$. Such a state is stable. Note that if the PSW packet is unstable in the absence of drift, $S_0(2T_0+S_0)<0$, we can at sufficiently high negative drift velocity ($S_0\alpha<0$) attain the stable state corresponding to the point B in Fig. 2.

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[1]V. E. Zakharov, V. S. Starobinets, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].
[2]V. S. L'vov, Zh. Eksp. Teor. Fiz. **69**, 2079 (1975) [Sov. Phys. JETP **42**, 1057 (1975)].
[3]I. V. Krutsenko, V. S. L'vov, and G. A. Melkov, *ibid.* **75**, 1114 (1978) [**48**, 561 (1978)].
[4]V. S. L'vov and V. B. Cherepanov, *ibid.* **76**, 2266 (1979) [**49**, 1145 (1979)].
[5]A. S. Bakai, I. V. Krutsenko, G. A. Melkov, and G. G. Sergeeva, *ibid.* **76**, 231 (1979) [**49**, 118 (1979)].
[6]L. A. Prozorova and A. I. Smirnov *ibid.* **67**, 1952 (1974) [**40**, 970 (1975)].
[7]V. V. Zautkin and B. I. Orel, *ibid.* **79**, 281 (1980) [**52**, 142 (1980)].
[8]V. I. Ozhogin and A. Yu. Yakubovskii, *ibid.* **67**, 287 (1974) [**40**, 144 (1975)].
[9]A. I. Smirnov, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 126 (1978) [JETP Lett. **27**, 117 (1978)].
[10]V. S. Zhitnyuk and G. A. Melkov, Zh. Eksp. Teor. Fiz. **75**, 1755 (1978) [Sov. Phys. JETP **48**, 884 (1978)].
[11]A. I. Smirnov and S. V. Petrov, *ibid.* **80**, 1628 (1981) [**53**, (1981)].
[12]V. S. Zhitnyuk and G. A. Melkov, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 149 (1980) [JETP Lett. **32**, 137 (1980)].
[13]V. S. Zhitnyuk, G. A. Melkov, and V. B. Cherepanov, Kinetics of Parametric Instability of Spin Waves in Ferrites [in Russian], Preprint No. 138, IAiE SO AN SSSR, Novosibirsk, 1980.
[14]H. Le Galle and J. P. Jamais, in: Proc. Internat. Conf. on Magnetism, MKM-73, Vol. 1 (2), Nauka, 1974, p. 20.
[15]V. N. Venitskii, V. V. Eremenko, and E. V. Matyushkin, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 239 (1978) [JETP Lett. **27**, 222 (1978)].
[16]E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, Cambrdige Univ. Press, 1928.
[17]A. Erdelyi, Ed. *Higher Transcendental Functions*, Vol. 2., McGraw, 1953.
[18]E. V. Podivilov and V. B. Chereepanov, Frequency Drift of Parametrically Excited Spin Waves [in Russian], Preprint No. 246, AIiE SO AN SSSR, Novosibirsk, 1985.
[19]V. S. L'vov, Classical Hamiltonian Formalism and Spin Waves Novosibirsk, 1981.

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