Reflection of weak perturbations by a shock-wave front. Nonlinear analysis

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(Submitted 15 August 1985)
Zh. Eksp. Teor. Fiz. **90**, 744–753 (February 1986)

The interaction between a weak perturbation of the flow behind the front of a shock wave and the front itself is investigated in the vicinity of resonances corresponding in the linear approximation to an infinite reflection coefficient. The conditions under which regular reflection is impossible are determined. Under these conditions a weak perturbation in the form of a three-wave configuration can propagate in advance along the shock-wave front from the intersection line of the fronts of the incident wave and of the unperturbed shock wave. This perturbation modifies the shock-wave front in a way that makes regular reflection again possible. According to the results, infinitely small perturbations of the flow behind the shock-wave front do not lead to finite perturbations of the front.

INTRODUCTION

Sound reflection by a shock-wave front was theoretically investigated by Kontorovich[1,2] and by D'yakov.[3] This question was reconsidered much later by Fowles.[4] According to Refs. 2 and 3, the ratio p_r/p_f of the pressure amplitudes of the reflected (p_r) and incident (p_f) waves becomes infinite under certain conditions. This reflection singularity was named resonance in Refs. 2 and 3. Resonances exist[2] when the following inequalities obtain:

$$L_0 \equiv \frac{1-\theta M^2-M^2}{1+\theta M^2-M^2} < L < 1+2M, \tag{1}$$

where $L = J^2(\partial v/\partial p)_H$, $J = [(p-p_0)/(v_0-v)]^{1/2}$ is the flux of matter through the shock-wave front, $(\partial v/\partial p)_H$ is the derivative of the specific volume v with respect to pressure along the shock adiabat, M is the shock-wave Mach number relative to the flow behind it, and $\theta \equiv v_0/v$ is the degree of compression in the shock wave.

In the theory of stability of shock waves to small perturbations (linearization of the hydrodynamic equations with respect to perturbations and solution of the characteristic equation for the complex frequency[5–7]; this theory will be called hereafter linear for short), the inequalities (1) determine the conditions under which there exist perturbations that remain undamped and do not increase in time (corrugations). Corrugation perturbations of the front surface lead in case (1) to downstream propagation of sound waves from the shock front.[5] Inequalities (1), following Ref. 5, are therefore also called conditions for spontaneous emission of sound by the shock-wave front. It is noted in Ref. 2 that the conditions for the existence of resonances and for the emission of sound waves coincide because the resonance equation ($p_r/p_f = \infty$) correspond to the characteristic equation of the stability problem.[5]

The evolution of shock-front-surface perturbations was investigated in Ref. 3 by the method of the theory of shock-wave intersection. The perturbations were specified in the form of three-wave configurations. It was shown that the conditions for stationary existence of such surface perturbations are determined by inequalities (1). This coincidence is not accidental, since the corrugation perturbation of the surface, considered in Refs. 5–7, can be represented as a superposition of three-wave configurations.[8] In this sense, analysis of the evolution of three-wave configurations led to a result previously known from the linear theory. The shock-wave intersection theory, however is relatively simpler and more lucid. This advantage of the method becomes particularly important for the solution of a much more complicated problem, that of shock-wave stability in the nonlinear approximation.

The possibility of shock-wave instability in the region (1) was not excluded in Refs. 2 and 3, but it was emphasized there that this question calls for a special investigation. In Ref. 4, however, the presence of resonances is treated already more categorically, albeit without proof, as a direct manifestation of shock-wave instability. The subdomain of the values of L that satisfy Eq. (1) is accordingly included in Ref. 4 in the shock-wave instability domain.

The present paper explores the true character of the enhancement of weak perturbations by reflection from a shock-wave front under conditions for which $p_r/p_f \to \infty$ in the linear approximation, and checks whether reflection can produce an infinitely weak wave a finite perturbation and thereby lead to instability of the shock-wave front. These questions can be answered only by forgoing the linear approximation. As in Ref. 8, the investigation is based on the theory of shock-wave intersection in a quadratic approximation.

§1. LINEAR APPROXIMATION

It is convenient to start from the results of the investigation[8] of three-wave configurations in an approximation linear in the weak-field amplitude. We have in mind configurations made up of the fronts of an unperturbed shock wave 1, a perturbed one 2, and a weak wave 3 (Fig. 1). The condition for the existence of a three-wave configuration with a specified angle γ between fronts 1 and 3 is expressed as

$$\psi(\gamma) = 0, \tag{2}$$

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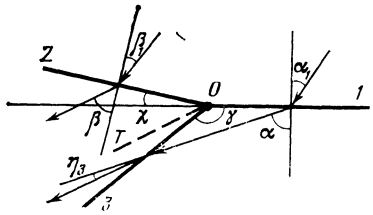

FIG. 1. Configuration of three waves with a weak compression wave 3: 1—unperturbed shock wave, 2—perturbed shock wave, T—tangential discontinuity. The arrows indicate the current-line directions in a coordinate frame with immobile point O.

where the function $\psi(\gamma)$ multiplied by the jump of pressure p_3 in wave 3 is the difference between the flux rotation angles η_i in wave 1 and waves 2 and 3, in a coordinate frame with an immobile point 0 (Fig. 1);

$$\eta_1=\alpha-\alpha_1,\quad \eta_2=\beta-\beta_1,\quad p_3\psi(\gamma)=\eta_2-(\eta_1+\eta_3).$$

Using the relations of Ref. 8 between the current-line rotation angles, amplitudes, and wave orientation, we can show that in the linear approximation

$$\psi(\gamma)=\frac{\theta-1}{2(p-p_0)(1+A^2)}\left[A-\frac{\theta}{A}+2S-L\left(A+\frac{\theta}{A}\right)\right],\quad (3)$$

$$A=(1+\Gamma^2)^{1/2}M^{-1}-\Gamma,\quad S=M(1+\Gamma^2)^{1/2}-\Gamma,\quad \Gamma=\mathrm{ctg}\,\gamma.\quad (4)$$

The incident and reflected waves $3f$ and $3r$, together with the initial shock wave 1 and the perturbed one 2, make up a four-wave configuration (Fig. 2). In the linear approximation, the perturbations of the shock wave by waves $3f$ and $3r$ are additive. Therefore the condition for existence of a four-wave configuration, which stipulates that all the current lines in sector IV be parallel, is represented in the linear approximation, in analogy with (2), in the form

$$p_r\psi(\gamma_r)+p_f\psi(\gamma_f)=0.\quad (5)$$

The relations between the angles γ for the incident (γ_f) and reflected (γ_r) waves follow from the requirement that the propagation velocities V_t of the waves $3f$ and $3r$ along the front of wave 1 be equal. This requirement is determined in the linear approximation by the equality[2,4,8]

$$\frac{1-M\cos\gamma_r}{\sin\gamma_r}=\frac{1-M\cos\gamma_f}{\sin\gamma_f}=\frac{V_t}{c},\quad (6)$$

where c is the speed of sound behind the shock-wave front.

The angle γ_f, can specified in the range $0\leqslant\gamma_f\leqslant\gamma_0 \equiv\arccos M$. Accordingly, γ_r varies in the range $\pi\leqslant\gamma_r\leqslant\gamma_0$. It follows from (5) that

$$p_r/p_f=-\psi(\gamma_f)/\psi(\gamma_r).\quad (7)$$

This result, in terms of other variables, was obtained in Ref. 2 (see also Ref. 4).

A complete quantitative quadratic-approximation calculation of the reflection of a weak perturbation from a shock wave entails very laborious calculations. The results of such calculations are given in §4. The quantitative character of the solution, however, is easier to investigate by using a phenomenological approach.

§2. PHENOMENOLOGICAL NONLINEAR ANALYSIS OF WEAK-WAVE REFLECTION

The condition for the existence of a four-wave configuration, given the angle γ_f, can be represented, with the terms quadratic in p_f and p_r taken into account, in the form

$$p_f\psi(\gamma_f)+p_r\psi(\gamma_r)+ap_r^2+a_1p_rp_f+a_2p_f^2=0.\quad (8)$$

We emphasize that here, just as in (5), the function $\psi(\gamma)$ is defined by Eqs. (3) and (4). The coefficients a, a_1, and a_2 depend on γ_f, on parameters determined by the thermodynamic properties of the material at the given point on the shock adiabat, and on the pressure and density of the material ahead of the shock wave.

Being interested in a solution near resonance, i.e., in the vicinity of a point at which $\psi(\gamma_r)=0$, $p_r\gg p_f$, we retain in the quadaratic form contained in (8) only the maximum term (it is assumed that $a\neq 0$ at the resonance point; the possibility of violation of this inequality is considered in §4). We have then in lieu of (8)[1)]

$$p_f\psi(\gamma_f)+p_r\psi(\gamma_r)+ap_r^2=0.\quad (9)$$

The following properties of the function $\psi(\gamma)$ will be found important if the parameter L satisfies conditions (1): a) $\psi(\gamma_f)<0$; b) $\psi(\gamma_r)>0$ at $\gamma_r>\gamma_r^L$, where γ_r^L is the root of Eq. (2), i.e., the resonant value of the angle γ_r at a given L.

Let us prove inequalities a) and b). Let a certain value of γ_f be given. We denote by L_f the value of the parameter L that causes the function (3) to vanish at the point $\gamma=\gamma_f$. L_f is bounded in the range $-1<L_f<L_0$ (see Ref. 8). Consequently, the inequality $L>L_f$ is valid under conditions (1). Taking this inequality into account and substituting $\psi(\gamma_f)$, in accordance with (3), in the form

$$\psi(\gamma_f)={}^1/_2(L_f-L)(\theta-1)(A+\theta/A)[(p-p_0)(1+A^2)]^{-1},$$

we arrive at inequality a).

The angle γ_r at which the function (3) with a specified value of the parameter L from the region (1) vanishes increases monotonically[8] with increasing L (if θ and M are fixed). This and (3) lead directly to the inequality b).

The quadratic equation (9), which goes over far from resonance into (7), has a root

$$X=\frac{p_r}{p_f}=\frac{-\psi(\gamma_r)+(\mathrm{Det})^{1/2}}{2ap_f},\qquad \mathrm{Det}=\psi^2(\gamma_r)-4\psi(\gamma_f)ap_f.\quad (10)$$

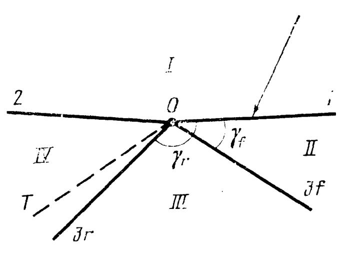

FIG. 2. Configuration of four waves: 1—unperturbed shock wave, 2—perturbed shock wave, 3—weak shock wave or weak rarefaction wave ($3f$—incident, $3r$—reflected), T—tangential discontinuity. The arrow indicates the direction of the current lines ahead of the shock wave in a coordinate system with immobile point O.

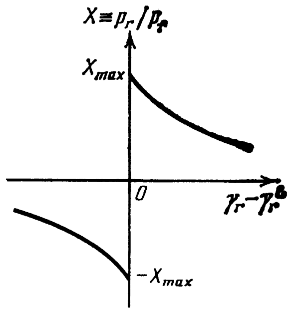

FIG. 3. The ratio p_r/p_f vs the reflection angle γ_r in the vicinity of the resonance point $\gamma_r = \gamma_r^L$ for the case $ap_f > 0$.

The second root of (9) fails to satisfy, even far from resonance, the condition $p_r \to 0$ as $p_f \to 0$, and is similar to the solution of the strong family in the problem of regular reflection from a rigid wall (Refs. 9 and 10).

The root (10) is positive at $\psi(\gamma_r) > 0$ (i.e., at $\gamma_r > \gamma_r^L$) regardless of the sign of ap_f. If $ap_f > 0$ the maximum of X is reached at the point $\psi(\gamma_r) = 0$ and is equal to $(-\psi(\gamma_f)/ap_f)^{1/2}$. In this case p_r is a maximum:

$$p_r = p_{r,\max} = (-\psi(\gamma_f)p_f/a)^{1/2}.$$

If $ap_f < 0$ the maximum value of X is reached at the point $\psi(\gamma_r) = 2(\psi(\gamma_f)ap_f)^{1/2}$ and amounts to $(\psi(\gamma_f)/ap_f)^{1/2}$. Both results can be written in the same form:

$$X_{\max} = \left(-\frac{\psi(\gamma_f)}{|ap_f|}\right)^{1/2}, \quad |p_{r,\max}| = \left(-\frac{\psi(\gamma_f)\,|p_f|}{|a|}\right)^{1/2}. \quad (11)$$

At $ap_f > 0$ there is a region of values of γ_f in which Det < 0 and Eq. (9) has no real solution. This case is considered in §3.

It can be seen from (11) that $X_{\max} \to \infty$ as $p_f \to 0$. In this case, however, $p_{r,\max} \to 0$. This means that infinitely small perturbations that are reflected from the shock-wave front remain infinitely small, and only the order of their smallness changes. As for finite perturbations, according to (11) they remain finite on reflection.

On going through the resonance point in the case $ap_f > 0$ by increasing γ_f, the sign of $\psi(\gamma_r)$ is reversed and the solution that passes far from the resonance in (7) is the second root of Eq. (9):

$$X = -[\psi(\gamma_r) + (\mathrm{Det})^{1/2}]/2ap_f. \quad (12)$$

This root is negative: a compression wave is reflected in the form of a rarefaction wave and vice versa. At the resonance itself the solution is not ambiguous (see Fig. 3). We emphasize that although the picture of the flow is ambiguous, the qualitative character of wave enhancement on reflection in the vicinity of the resonance point is unchanged: if $p_f \to 0$ then $p_r \to 0$ at a rate not slower than $p_f^{1/2}$.

§3. THE CASE $ap_f < 0$, Det < 0

In this case Eq. (9) has no real solution. Accordingly, wave reflection does not reduce to formation of a configuration with only four waves. In this case one can expect the asymptotic evolution of the reflection to be accompanied by a larger change of pressure in the perturbed shock wave compared with $p_{r,\max}$ [Eq. (11)]. However, increasing the pressure by a certain value not exceeding $2p_{r,\max}$ suffices already to make the velocity V_t of the autonomous three-wave configuration[8] larger than or equal to the velocity V_t of the incident wave. This causes a three-wave configuration to separate and proceed forward (to the right in Fig. 4) from the line of intersection of the fronts of the incident wave and of the unperturbed shock wave. This can be seen, for example, from the following: the angle γ of the three-wave configuration increases in the case $ap_f < 0$ with increasing amplitude of the pressure of the wave 3, and becomes equal to the angle $\gamma_{r,\min}$ that corresponds to the boundary of the existence of the four-wave configuration (defined by the condition Det $= 0$) at a value of p_3 that satisfies the equation $\psi(\gamma_{r,\min}) + ap_3 = 0$. This value of p_3, as seen from a comparison with (10), is double the value of $p_{r,\max}$. At equal γ, however, V_t increases with increasing amplitude of the weak wave. For this purpose it is necessary only to satisfy the usual condition $\tilde{\beta} \equiv (\partial^2 v/\partial p^2)_s > 0$, where s is the entropy.

In the angle range $\pi > \gamma_r > \pi/2$, the flow in the vicinity of the point O' is supersonic (see Fig. 4). There are, however no other incoming waves, other than the incident wave and the initial shock wave, to cause the point O' in the laboratory frame with supersonic velocity. Under these conditions the velocity V_t for the point O' can be not larger but only equal to the velocity V_t for the point O. The point O' is localized in this case on the forward boundary of the viscous structural layer of the front of the incident wave. (In view of the inequality $p_{3'} \gg p_f$, the width of the front of wave 3′ is relatively small and it is reasonable to assume localization of this front in the strongly spread-out structural layer of the incident-wave front.) The pressure of the incident wave on this boundary is vanishingly small. The pressure $p_{3'}$ is determined by the condition that the velocities V_t be equal for the points O and O'.

After the three-wave configuration passes forward (see Fig. 4), the reflection conditions change: a) In the case $ap_f > 0$ the resonant value of the angle γ_r increases with in-

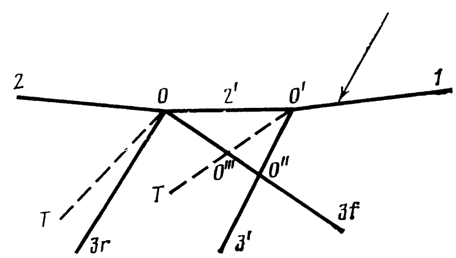

FIG. 4. Complex reflection in the vicinity of the resonant incidence angle in the case $ap_f < 0$: 1—unperturbed shock wave, 2′—shock wave perturbed beforehand by an emerging weak shock wave (or by a weak rarefaction wave) 3′; 3f and 3r—incident and reflected waves, 2—perturbed shock wave. The three- and four-wave configurations are made up respectively by the waves 1–3′–2′ and 2′–3f–3r–2; T—tangential discontinuities. The arrow indicates the direction of the current lines ahead of the shock-wave front in a coordinate system with immobile point O. The small changes of the directions of the wave fronts and of the tangential discontinuity at the points O' and O'' are not marked.

creasing pressure of the three-wave configuration that passes forward. This can be verified by taking into account the already mentioned increase of the three-wave-configuration angle γ with increasing $p_{3'}$, by representing this increase as a result of interaction of two three-wave configurations propagating in the same direction and having weak emerging waves with positive pressure amplitudes. The second configuration overtakes the first. The signs of the pressure increment resulting from the reflection of the second configuration by the first, of the angle γ, of the velocity of the material, and of other parameters (it is important that both configurations have weak waves 3, Fig. 1) coincide with the signs of the corresponding quantities (p_3, $\gamma_2 - \gamma_1$, and others) in the second three-wave configuration. Here γ_1 and γ_2 denote respectively the angles γ for the first and second configurations. b) Given the orientation of the incident wave, meaning hence given the velocity V_t of this wave, the angle γ_r decreases with increase of pressure $p_{3'}$, of the forward-passing three-wave configuration, provided that $ap_f < 0$. This can be seen from the following. Owing to the increase of the sound velocity with pressure (the condition $\beta > 0$) and to the additional drag by the flow behind the three-wave configuration, the velocity V_t of the reflected wave increases at a fixed angle γ_r. The value of V_t for the reflected wave, however, cannot increase, since it must be equal to the value V_t specified for the incident wave irrespectively of the pressure $p_{3'}$, of the three-wave configuration. The required constancy of V_t is ensured by decreasing the angle γ_r. Recall that $\gamma_r > \gamma_0$ and according to (6) we have $(dV_t/d\gamma)_{\gamma=\gamma_r} > 0$.

Since the changes of the angles γ_r^L are of opposite sign [as indicated in Subsecs. a) and b)], the reflection of a wave with specified incidence angle such that Det < 0 can be transformed, by increasing the pressure $p_{3'}$, of the forward-emitted three-wave configuration, into the regime corresponding to the region where Eq. (9) has a real solution with negative X (see Fig. 3 and the solution (12) at $\psi(\gamma_r) < 0$, $ap_f < 0$). The minimum three-wave-configuration pressure $p_{3',\min}$, sufficient for this purpose is implicitly defined by the condition

$$\mathrm{Det}_P = 0. \qquad (13)$$

The meaning of this equation is that for reflection from the front of the perturbed shock wave 2′ (Fig. 4) the root X should correspond to the boundary of the domain of negative solutions of Eq. (9). The subscript P in in (13) means that the solution pertains to the case of reflection from a perturbed shock wave.

The inequality Det < 0 for reflection in an unperturbed shock wave at low pressure of the incident wave is satisfied in a narrow range of angles γ_f on both sides of the resonance points. The lower and upper limits of this interval are determined respectively by the conditions Det $= 0$, $\psi(\gamma_r) > 0$ and Det $= 0$, $\psi(\gamma_r) < 0$. The pressure $p_{3',\min}$ decreases monotonically from a value of order $p_{r,\max}$, on the lower boundary to zero on the upper. An estimate shows that $p_{3',\min}/p_{r,\max} < 4$ on the lower boundary. More definite data on this pressure ratio can be obtained by quantitative analysis of configurations similar to those shown in Fig. 4, as functions of the parameters in the statement of the problem, such as the coefficient a and others.

The pressure $p_{3'}$ at the point O' (we designate it by $p_{3',O}$) can, however, not be varied arbitrarily. It is determined by the indicated condition that the velocities V_t be equal for the points O and O', and is bounded by the inequality $p_{3',O} < 2p_{r,\max}$. If $p_{3',\min} > p_{3',O}$, the needed pressure increase from $p_{3',O}$ to $p_{3',\min}$ can apparently be produced by bending of the lines O–O' and of the front 3′ (see Fig. 4). This question calls for additional numerical investigation.

The qualitative restructuring of the reflection picture with forward emission of a three-wave configuration recalls, in some respects, the transition to an irregular (Mach) reflection of a wave from a rigid wall. Thus, regular reflection from a shock-wave front becomes impossible, as does reflection from a rigid wall,[10] starting with the incidence angle at which the reflected waves of the "weak" and "strong" families coincide. For the solution (10), (12) this corresponds to the case Det $= 0$. A regular reflection from a rigid wall is possible, however, at all incidence angle, if the incident wave is weak enough. In addition, the character of the considered "irregular" reflection from the shock-wave front is entirely different: a wave system that is more complicated than in Mach reflection from a rigid wall is produced, with three intersection points O, O', and O'' (see Fig. 4). In contrast to the nonstationary Mach reflection from a rigid wall, four rather than three points waves intersect at the point O''.

The considered pictures of the reflection are qualitatively different in different ranges of the incidence angle and at different signs of ap_f. An important common feature, however is that as the amplitude of the incident wave p_f tends to zero the amplitudes p_i of the pressure in the reflected wave ($p_i = p_r$) and in the three-wave configuration that moves forward ($p_i = p_{3'}$) (if such a configuration is produced) do not tend to zero more slowly than $p_f^{1/2}$, i.e., $|\lim(p_i/p_f^{1/2})_{p_f\to 0}| \leqslant |\mathrm{const}| \neq \infty$.

§4. CALCULATION OF THE COEFFICIENT *a*

In the linear approximation, the parameters that determine the four-wave configuration (unperturbed and perturbed shock waves, incident and reflected weak shock wave) are the pressure p_f and the orientation of the incident-wave front, the point corresponding to the unperturbed shock wave on the shock adiababat (which passes through the specified point of the initial state of the substance), the slopes of the shock adiabat, and the isentropes at this point. Specification of the point on the shock adiabat defines the quantities θ, M, and J^2. If J^2 is known the slope of the shock adiabat is characterized by the parameter L. The slope of the isentrope is determined by specifying the speed of sound c. In the quadratic approximation, this set of parameters is supplemented by quantities that characterize the changes of the shock-adiabat and isentrope slopes. These quantities can be expressed in terms of the derivatives dL/dp and $\tilde{\beta} \equiv (\partial^2 v/\partial p^2)_s$.

To express in explicit form the condition that all the current line in the sector IV (Fig. 2) be parallel it is necessary, in the quadratic approximation (9), to take into account the following: a) The change of the shock-wave velocity D and of the degree of compression θ in second order in the pressure p_r, expressed in terms of the second derivatives d^2D/dp^2 and $d^2\theta/dp^2$. b) The changes of the velocity D_r of the weak shock and of the angle γ_r in first order in p_r. The dependence of D_r on p_r is determined[2)] by the equation $D_r = c + \tilde{\beta}c^3p_r/4v^2$. The changes of D_r and γ_r lead to a corresponding change of the quantity S defined in Eq. (4). The number M in (4) must then be replaced by the ratic Mc/D_r. We note that the quantity A [Eq. (4)], unlike S, remains unchanged. This can be easily verified if it is recognized that A can be expressed (see Ref. 8) in the form $A = \theta V_t/D$ and that the velocities V_t and D do not depend on p_r. c) The second derivatives of the trigonometric functions of the angles that characterize the rotations of the flow as it crosses the fronts of the waves 1, 2, and 3 (see Fig. 2).

After these calculation stages are completed, the coefficient a in (9) can be ultimately expressed in terms of the aforementioned parameters γ_f, θ, M, v, $p-p_0$, L, dL/dp, c and $\tilde{\beta}$. Omitting the rather laborious algebra, we present the final result and describe the nature of its terms:

$$a=a_{H\chi}+a_{H\rho}+a_{Ht}+a_{s\gamma}+a_{s\rho}+a_{st},$$

$$a_{H\chi}=-\frac{(\theta-1)(\theta-A^2)}{4(p-p_0)^2A(1+A^2)}\left[\frac{dL}{dp}(p-p_0)+\frac{1}{2}(L^2-1)\right],$$

$$a_{H\rho}=-\frac{(\theta-1)A}{2(p-p_0)^2(1+A^2)}\left[\frac{dL}{dp}(p-p_0)+L(L-1)-2\theta L^2\right],$$

$$a_{Ht}=-\frac{(\theta-1)(\theta^2+A^2)}{4(p-p_0)^2A(1+A^2)^2}\left[(\theta+1)(1+L^2)+2(A^2-1)L(1+L)+\frac{4(\theta-1)A^4L^2}{\theta^2+A^2}\right],$$

$$a_{s\gamma}=\frac{(\theta-1)^2M^2(S^2-1)\tilde{\alpha}}{2(p-p_0)^2S(1+A^2)},\qquad \tilde{\alpha}=\frac{c^4}{2v^3}\tilde{\beta},$$

$$a_{s\rho}=\frac{bS}{1+S^2},\qquad b=\frac{1}{2\rho}\left(\frac{\partial^2\rho}{\partial p^2}\right)_s=\frac{\tilde{\beta}}{2v}-\frac{v^2}{c^2},$$

$$a_{st}=-\frac{(\theta-1)^2M^4S^3}{(p-p_0)^2(1+S^2)^2}. \qquad (14)$$

Here A and S are functions of M and γ_r, defined by Eqs. (4) at $\gamma=\gamma_r$. The dependence of γ_r on γ_f is given by Eq. (6). From Eq. (4) for A and from the first equation of (6) follows invariance of (4) to replacement of γ_r by γ_f. Recall that at the resonance point, given the angle γ_r, the parameter L is not independent and is determined by Eqs. (2)–(4). For an ideal gas with a Poisson adiabatic exponent the coefficients $\tilde{\alpha}$, $\tilde{\beta}$, and b are given by

$$\tilde{\alpha}=\frac{\varkappa+1}{2},\qquad \tilde{\beta}=\frac{\varkappa+1}{\varkappa^2}\frac{v}{p^2},\qquad b=-\frac{\varkappa-1}{2p^2\varkappa^2}.$$

After reducing similar terms, the sum $a_{H\chi}+a_{H\rho}$ takes the somewhat simpler form

$$a_{H\chi}+a_{H\rho}=-\frac{\theta-1}{4(p-p_0)^2A(1+A^2)}\left[(A^2+\theta)\frac{dL}{dp}(p-p_0)+4A^2L\left(\frac{L-1}{2}-\theta L\right)+\frac{1}{2}(\theta-A^2)(L^2-1)\right].$$

The terms of the sum (14) are connected with the following corrections of the approximation quadratic in p_r: $a_{H\chi}$ corrected for the angle χ is determined by the second derivative d^2D/dp^2; $a_{H\rho}$ corrected for the density in the perturbed shock wave is determined by the second derivative $(\partial^2\rho/\partial p^2)_H$; a_{Ht} corrected for the difference between the flux rotation by the fronts of the perturbed and unperturbed shock waves is expressed in terms of the second derivatives of the trigonometric functions; $a_{s\gamma}$ with the correction to the angle γ determined by derivative dD_r/dp; $a_{s\rho}$ corrected for the density in the weak wave 3 is determined by the second isentropic derivative $(\partial^2\rho/\partial p^2)_s$; a_{st} corrected for the flux rotation angle in the weak wave 3 at fixed values of the angle γ and of the density ρ in the wave is expressed in terms of the second derivatives of the trigonometric functions.

The cause of the singularity (pole) of the coefficient as γ at the point $S=0$ is the minimum, at this point, of the velocity V_f as a function of the angle γ [see (6)]. This point, however, is of no particular interest, since $\gamma_r=\gamma_f$ there, and both the denominator in (7) vanishes along with the numerator. Resolution of this indeterminacy shows that the resonance is replaced here by the equality $p_r=p_f$.

Simultaneous satisfaction of the resonance condition and of the equality $a=0$ imposes stringent and difficultly implemented requirements on the equation of state. A random vanishing of a at the resonance point is not very likely.

We denote by a^- that part of the sum (14) which does not contain $(p-p_0)dL/dp$. According to (14),

$$a^-\equiv a+\frac{(\theta-1)(A^2+\theta)}{4A(1+A^2)(p-p_0)}\frac{dL}{dp}.$$

Random calculations of $(p-p_0)^2a^-$ by means of Eqs. (14) and (2)–(4) for a number of values of θ and M at $\alpha\approx 1$, $b[p/(p-p_0)]^2\approx b\approx -0.1$ in the entire possible range of resonant values of the angle γ_r (from γ_0 to π) have shown that the sign of a^- can be arbitrary. A varient of such calculations for the case $\theta=4$, $M=0.5$, $\tilde{\alpha}=1.2$, $b=-0.1$ yields:

γ_r	60	70	80	90	120	140	160	170	179.9
L	−0.143	$4.13\cdot10^{-3}$	$9\cdot10^{-3}$	0.25	0.892	1.40	1.83	1.96	2
$(p-p_0)^2a^-$	$-\infty$	−4.02	−3.33	−3.16	−2.25	−1.06	−0.078	$-3\cdot10^{-4}$	$2.77\cdot10^{-4}$

The sign of the derivative dL/dp obviously cannot be arbitrary and is not firmly correlated with the values of θ and M. The coefficient a can therefore be either positive or negative, depending on the parameters of the problem.

CONCLUSION

1. A nonlinear (quadratic) analysis of the reflection of weak shock waves or weak rarefaction waves from the

shock-wave front at incidence angles for which the linear theory yields infinite enhancement of the incident wave upon reflection (called a resonance) shows that when the pressure p_f of the incident wave tends to zero the pressure of the reflected wave tends to zero at a rate not slower than $p_f^{1/2}$.

2. The conditions were determined under which a simple reflection picture that can be reduced to formation of only a four-wave configuration is impossible. Under these conditions an weak perturbation in the form of a three-wave configuration can propagate in advance of the shock-wave front from the intersection line of the fronts of the incident and unperturbed shock waves. This perturbation modifies the front of the shock wave in such a way that the indicated simple reflection picture becomes again possible.

3. The pressures of the weak wave that travels ahead of the three-wave configuration and of the reflected wave in the case of complex reflection (see item 2) tends to zero as $p_f \to 0$ at a rate not slower than $p_f^{1/2}$.

4. According to the results of items 1 and 3, infinitely small perturbation of the flow behind the front of a shock-wave that satisfies the conditions (1) for the existence of resonances (generation of sound[5–7]) does not lead to finite perturbations of the front. This, together with the results of investigation of three-wave configurations[8] indicates that shock-waves that satisfy the resonance condition (1) are stable.

5. The problem of reflection of perturbations from a shock wave front, just as in the more thoroughly studied case of wave reflection from a rigid partition,[9,10] has indeterminate solutions. To determine which of the possible solutions is actually realized under some specified conditions, particularly under conditions when simple (regular, see item 2) reflection is impossible, it would be of interest to investigate experimentally the reflection of weak compression and rarefaction waves from the surfaces of shock waves that satisfy the resonance condition (1) in the vicinity of resonant incident angles.

[1)]If the reflected wave is a rarefaction wave of finite amplitude, the results that follow are valid accurate to the rarefaction-wave structure. In contrast to a shock wave, a rarefaction wave has two "fronts" that diverge at a small angle (if the amplitude p_r is small) from the common line of intersection with the shock-wave front. The space between these fronts contained the Prandtl-Meyer flow.[9,10]

[2)]This formula follows from relation (95.3) of Ref. 9.

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Translated by J. G. Adashko