Oscillations of cross section for photoproduction of an electron-positron pair in an external field

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The production of an electron-positron pair by two circularly polarized photons in an external crossed field is investigated. It is shown that in a weak field the total cross section is determined not only by the perturbation-theory series but also by the oscillating terms. The dependence of the cross section on the photon energy is found. The conditions under which the effect of an external field can be observed are indicated.

Two-photon formation of an electron-positron pair in a constant uniform crossed field was considered by many workers.[1–4] They have established that the differerential cross section of the reaction oscillates when the external-field strength and the interacting-phonon energies are varied. It was shown in a recent paper[5] that the presence of a constant crossed field leads also to oscillations of the total cross section. The nonrelativistic approximations used in Ref. 5, however, prevented an investigation of the dependence of the oscillation amplitudes on the photon energies. This question is the subject of the present paper.

In contrast to Refs. 1–4, in which one of the photons is assumed to be linearly polarized, we consider pair production by two circularly polarized photons, for it is in just this case that the dependence on the polarizations is most strongly pronounced. We assume that photons with momenta k^μ and k'^μ propagate counter to one another with the photon k^μ moving along the Poynting vector of the crossed field. Using the standard technique of quantum electrodynamics with an external field,[6] we can obtain in the lowest order of perturbation theory in the quantized field, the following integral representation of the cross section for pair production:

$$\sigma^\pm=-\frac{ir_0^2t^3}{2}\int_{-1}^{1}\frac{d\beta}{(1-\beta^2)^2}\int_{-\infty}^{\infty}\frac{d\rho}{\rho-i0}\times\exp\left\{i\rho\left[1-\frac{t}{1-\beta^2}\left(1+\frac{\xi^2\rho^2}{12}\right)\right]\right\}f^\pm, \quad (1)$$

$$f^\pm=f_1^\pm-i\rho f_2^\pm-\rho^2f_3^\pm,$$

$$f_1^\pm=\frac{2}{t}-1+\xi^2\left(1+\frac{1+\beta^2}{t}\right)+\frac{1\mp1}{2}\frac{1+\beta^2}{t}\left(-2+\frac{1-\beta^2}{t}-2\xi^2\right),$$

$$f_2^\pm=\xi^2\left(\frac{3+\beta^2}{1-\beta^2}-\frac{1\mp1}{t}(1+\beta^2)\right),\qquad f_3^\pm=\frac{\xi^2}{2}\frac{2+\xi^2(1+\beta^2)}{1-\beta^2}$$

In these equations m and e are the mass and charge of the electron, $r_0=e^2/4\pi m$ its classical radius, ω and ω' the photon frequencies, H the crossed-field strength, $t=m^2/\omega\omega'$, $\xi=eH/m\omega$. Note that the probability of the process depends only on the product of the helicities of the initial photons, so that a distinction need be made only between the cases of coinciding (plus) and opposing (minus) polarizations of the photons.[1)]

The integrand in (1) is the representation in the spectral variable $\beta=(p-q,k)/(p+q,k)$, where p^μ and q^μ are the 4-momenta of the electron and positron. On integrating with respect to ρ, the result (1) can be represented in the form

$$\sigma^\pm=r_0^2t^3\int_{-1}^{1}\frac{d\beta}{(1-\beta^2)^2}\left\{\int_x^\infty\Phi(y)\,dy\,f_1^\pm-\Phi(x)f_2^\pm z-\Phi'(x)f_3^\pm z^2\right\}, \quad (2)$$

where we have introduced the Airy function

$$\Phi(x)=\frac{1}{2}\int_{-\infty}^{\infty}ds\exp\left\{i\left(xs+\frac{s^3}{3}\right)\right\},$$

$$z=\left(\frac{4(1-\beta^2)}{t\xi^2}\right)^{1/3},\quad x=z\left(\frac{t}{1-\beta^2}-1\right).$$

This expression, when averaged over the photon polarizations, agrees with that given in Ref. 2.

We consider the case of relatively weak external fields, i.e., we assume that the conditions $\xi^{2/3}\ll1-t\,(t<1)$ are satisfied. Note that the order of integration cannot be changed in (1), since the integrand has a singularity at the point $\beta=\pm1$. It is known, however, that the Airy function and its derivatives decrease exponentially as $x\to+\infty$. The contribution from the vicinities of the points $\beta=\pm1$ is therefore exponentially small and can be disregarded when the asymptotic expansion is calculated. This permits the use of the two-dimensional standard-phase method.[7]

The change of variable $y=\rho\xi$ allows us to rewrite the argument of the exponential in (1) in the form

$$\frac{i}{\xi}S(y,\beta)=\frac{i}{\xi}y\left[1-\frac{t}{1-\beta^2}\left(1+\frac{y^2}{12}\right)\right].$$

It can be shown that, apart from exponentially small terms, the integral in (1) is equal to the sum of the contributions of the stationary-phase points and of the contribution of the region where the factor in front of the exponential has a singularity in the variable y. In the integration region we have four stationary points:

$$A(y=0,\ \beta=v),\ A'(y=0,\ \beta=-v),$$
$$B(y=-2v/t^{1/2},\ \beta=0),\ C(y=2v/t^{1/2},\ \beta=0)$$

(see Fig. 1), where $v=(1-t)^{1/2}$ can be intepreted as the initial velocity of the produced fermions in the photon c.m.s.

Let us clarify the introduced constraint on the field in-

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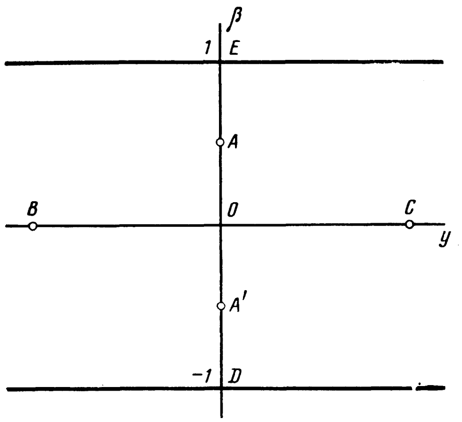

FIG. 1.

tensity. We make for this purpose the substitutions $y = (v/t^{1/2})y'$, $\beta = v\beta'$. As a result, the argument of the exponential takes the form

$$\frac{i}{\xi} v^3 \frac{y'}{t^{1/2}} \frac{1-\beta'^2-y'^2/12}{1-v^2\beta'^2}.$$

The location of the stationary points becomes now independent of the parameter t and it can be seen that the stationary-phase method can be used only if $v^3/\xi \gg 1$.

We obtain now the contribution made to the integral in (1) by the region DE in which the pre-exponential factor has a singularity, and which contains in addition two stationary points A and A' (see Fig. 1). We integrate in (2) by parts and return to a notation similar to that of (1). The singularity in y is thus eliminated. Using the fundamental theorem of the stationary-phase method (see [7]) we get

$$\sigma_{DE}^{\pm} = \pi r_0^2 t^4 \frac{1}{v} \sum_{j=0}^{k-1} \frac{\xi^j}{j!} L^j \left[\frac{\varphi(y,\beta)}{(1-\beta^2)^2} \exp\left\{ \frac{i}{\xi}\left(S(y,\beta) + \frac{2v}{t} y(\beta - v) \right)\right\}\right]_{y=0,\,\beta=v} + R_k, \tag{3}$$

$$\varphi(y,\beta) = \frac{2\beta}{3}(2t+1-\beta^2) \int d\beta \frac{f_1^{\pm}}{(1-\beta^2)^2} - f_2^{\pm} + \frac{iy}{\xi} f_3^{\pm},$$

where

$$L = -\frac{it}{2v} \frac{\partial^2}{\partial y\, \partial \beta},$$

and the remainder term is

$$R_k \infty \xi^{1+k-[2/3k]}.$$

It can be easily shown that this expressions, with the terms rearranged, conicides with the perturbation-theory series that can be obtained by formally expanding the integrand of (1) in terms of the parameter ξ.

It remains to consider the contributions of the stationary points C and B. Clearly, these contributions are complex-conjugates, so that it suffices to consider the point C. The function $S(y,\beta)$ has at this point a maximum and can be expanded in its vicinity in the series

$$S(y,\beta) = \frac{4}{3}\frac{v^3}{t^{1/2}} - \frac{vt^{1/2}}{2}\left(y - \frac{2v}{t^{1/2}}\right)^2 - \frac{2v}{3t^{1/2}}(1+2t)\beta^2 + W(y,\beta).$$

Here $W(y,\beta)$ is the remainder of the series. Applying again the fundamental theorem of the stationary-phase method, we get

$$\sigma_C^{\pm} = -\pi r_0^2 t^3 \frac{\xi\sqrt{3}}{2v(1+2t)^{1/2}} \sum_{j=0}^{k-1} \frac{\xi^j}{j!} \tilde{L}^j \left[\frac{f^{\pm}(y,\beta)}{y(1-\beta^2)^2} \times \exp\left\{ \frac{i}{\xi}\left(W(y,\beta) + \frac{4}{3}\frac{v^3}{t^{1/2}} \right)\right\}\right]_{y=2v/t^{1/2},\,\beta=0} + \tilde{R}_k, \tag{4}$$

where

$$\tilde{L} = -\frac{i}{8vt^{1/2}}\left(4\frac{d^2}{dy^2} + \frac{3t}{1+2t}\frac{d^2}{d\beta^2} \right),$$

and the remainder term is

$$\tilde{R}_k \infty \xi^{1+k-[2/3k]}.$$

We can thus represent the cross section for electron-positron pair photoproduction as a sum of a monotonic part and an oscillating one:

$$\sigma^{\pm} = \sigma^{\pm}_{\rm mon} + \sigma^{\pm}_{\rm osc},$$

$$\sigma^{\pm}_{\rm mon} = \sigma_{DE}^{\pm}, \qquad \sigma^{\pm}_{\rm osc} = \sigma_C^{\pm} + \sigma_B^{\pm}.$$

It is important that to calculate the oscillating part it sufficies to know the differential cross section of the process at $(pk) \approx (qk)$, i.e., $\beta \approx 0$ (in the absence of a field this corresponds to production of electrons and positrons emitted perpendicular to the propagation direction of the initial photons in the c.m.s.). To calculate the monotonic part, on the other hand, it is not enough to know the differential cross section at $(pk)/(qk) \approx (1+v)(1-v)$, i.e., $\beta \approx v$. In fact, let us calculate the monotonic part of the cross section, after excluding the vicinities of the points A and A'. We introduce for this purpose a function $\chi(y)$ with the following properties: $\chi(y)$ is infinitely differentiable, is identically equal to unity in the vicinity of the point O, and is equal to zero near the points B and C. The contribution sought for is then

$$\tilde{\sigma}_{DE}^{\pm} = -\frac{ir_0^2 t^3}{2} \int_{-1}^{1} d\beta \int_{-\infty}^{\infty} dy \frac{f^{\pm}(y,\beta)\chi(y)\theta(|\,|\beta| - v| - \varepsilon)}{(y-i0)(1-\beta^2)^2} \times \exp\left\{ \frac{i}{\xi} S(y,\beta) \right\},$$

where $\varepsilon > 0$ is some small quantity. Regarding the integral as iterated and recognizing that $|\partial S/\partial y| \geqslant \delta(\varepsilon) > 0$, in the integration region, it can be easily calculated by using the known relation

$$\int_{-\infty}^{\infty} \exp\left\{ \frac{i}{\xi} S(y) \right\} \frac{f(y)}{y-i0} dy = 2\pi i f(0) \exp\left\{ i\frac{S(0)}{\xi} \right\} \theta(S'(0)) + O(\xi^{\infty}).$$

Taking next the limit as $\varepsilon \to 0$ we get

$$\tilde{\sigma}_{DE}^{\pm} = \pi r_0^2 t \left\{ \left(1 \mp 1 + t - \frac{t^2}{2} \right) \ln\frac{1+v}{1-v} - (t+1\mp 3)v + \xi^2\left(\frac{t^2}{2}\ln\frac{1+v}{1-v} + (t\pm 2)v \right) + O(\xi^{\infty}) \right\}. \tag{5}$$

It can be seen from (5) that the contribution of the region with the singularity differ from the known Breit-Wheeler cross section (see [8] in that it contains terms that depend on the external field, and to calculate σ_{mon} it is necessary to know the differential cross section at all possible momentum values. A similar situation is observed in solid-state physics. For example, the monotonic part of the magnetization depends on all the electrons in the metal, whereas the oscillating part is determined only by the conduction electrons in the vicinity of the Fermi surface.

We present the final result of the calculations. We transform for convenience from ξ to the parameter $\varkappa = \xi/t$. As already mentioned, the expansion of the monotonic part in powers of $\varkappa^2$ coincides with the perturbation theory series. Its first two terms are

$$\bar{\sigma}_{\text{mon}}^{-}=\sigma_0^{-}+\pi r_0^2 t^3\varkappa^2\left[\frac{t^2}{2}\ln\frac{1+v}{1-v}-\frac{6-15t+17t^2-6t^3}{6v^3}\right],$$

$$\overset{+}{\sigma}_{\text{mon}}=\sigma_0^{+}+\pi r_0^2 t^3\varkappa^2\left[\frac{t^2}{2}\ln\frac{1+v}{1-v}-\frac{16-24t+22t^2-31t^3+16t^4}{16v^5}\right] \qquad (6)$$

($\sigma_0^{\pm}$ denotes the Breit-Wheeler cross section). When the photons have different polarizations, the coefficient of $\varkappa^2$ is always negative; otherwise it becomes positive at $t \geqslant 0.9$. With decreasing photon energy the correction increases, and for $v \ll 1$ we have

$$\bar{\sigma}_{\text{mon}}^{-}\approx\frac{8}{3}\pi r_0^2 v^3\left(1-\frac{1}{8}\frac{\varkappa^2}{v^6}\right),\qquad \overset{+}{\sigma}_{\text{mon}}\approx 2\pi r_0^2 v\left(1+\frac{1}{32}\frac{\varkappa^2}{v^6}\right). \qquad (7)$$

The separation parameter at $v \ll 1$ is thus $(\varkappa/v^3)^2$.

We proceed now to the oscillating part of the cross section. It is expanded in powers of $\varkappa$. The principal part of the series is

$$\bar{\sigma}_{\text{osc}}^{-}=\pi r_0^2 t\varkappa\left(\frac{3t}{1+2t}\right)^{1/2}\frac{t(3t-1)}{2}\cos\left[\frac{4}{3\varkappa}\left(\frac{1}{t}-1\right)^{3/2}\right],$$

$$\overset{+}{\sigma}_{\text{osc}}=\pi r_0^2 t\varkappa\left(\frac{3t}{1+2t}\right)^{1/2}\frac{t^2(2-3t)}{2(1-t)}\cos\left[\frac{4}{3\varkappa}\left(\frac{1}{t}-1\right)^{3/2}\right]. \qquad (8)$$

It can be seen from (8) that the amplitude of the oscillations vanishes at $t = 1/3$ for σ^- and $t = 2/3$ for σ^+, leading to a change of the oscillations by π.

At $v \ll 1$, the cross section averaged over the photon polarizations is

$$\sigma_{\text{av}}\approx\pi r_0^2 v\left[1+\frac{1}{32}\left(\frac{\varkappa}{v^3}\right)^2-\frac{1}{4}\frac{\varkappa}{v^3}\cos\frac{4v^3}{3\varkappa}\right]. \qquad (9)$$

Equation (9) coincides with the result of [5].

It should be noted that the oscillating part of the cross section is odd in $\varkappa$. This typical of all previously investigated processes in a crossed field, viz., charged-pion decay,[9] π^0 and K^0 decays,[10] and allowed nuclear β decay.[11] A similar behavior was observed also in the investigation of the analytic properties of the photon scattering amplitude in a crossed field.[12]

In view of the imposed constraint $1 - t \gg \xi^{2/3}$, Eqs. (6) and (8) are not valid near the value $t = 1$ that corresponds to the threshold of pair production in the absence of an external field. It is easy to obtain for the behavior of the cross section near the threshold ($v \ll \varkappa^{1/3}$)

$$\sigma^{-}\approx\pi r_0^2\varkappa\frac{5}{3\sqrt{3}},\qquad \sigma^{+}\approx\pi r_0^2\varkappa^{1/3}\frac{\Gamma(5/6)}{(12)^{1/6}\pi^{1/2}}. \qquad (10)$$

The asymptotic form of the cross section in the below-threshold region, i.e., at $t > 1$, can be calculated by the saddle-point method:

$$\sigma^{-}\approx\pi r_0^2 t\varkappa\left(\frac{3t}{1+2t}\right)^{1/2}\frac{t(3t-1)}{4}\exp\left\{-\frac{4}{3\varkappa}\left(1-\frac{1}{t}\right)^{3/2}\right\},$$

$$\sigma^{+}\approx\pi r_0^2 t\varkappa\left(\frac{3t}{1+2t}\right)^{1/2}\frac{t^2(3t-2)}{4(t-1)}\exp\left\{-\frac{4}{3\varkappa}\left(1-\frac{1}{t}\right)^{3/2}\right\}. \qquad (11)$$

Naturally, it is exponentially small (see Ref. 2). A plot of the cross section vs the parameter t at $\xi = 0.1$, obtained by numerical calculations, is shown in Fig. 2.

Note that in the region $v \lesssim \alpha$, where α is the fine-structure constant, our analysis is generally speaking only qualitative, since no account has been taken of the Coulomb corrections.

Let us determine more accurately the region where the foregoing equations are valid. The results (6)–(11) are exact for a reaction in a crossed field but, as shown in Ref. 1, they can be used to describe the pair production process in any constant field, if one considers photons in whose c.m.s. the external field differs little from the crossed field.

In conclusion, we examine the experimental conditions needed to observe the influence of the external field. We introduce for this purpose the parameter

$$a=\frac{4}{3\varkappa}\left(\frac{1}{t}-1\right)^{3/2}$$

and, following (13), assume that it is not specified exactly, but has a distribution characterized by a probability density function $\rho(a_0, D)$ where a_0 and D are respectively the mean value and the variance of a. Assume that the phonon frequencies, the angle θ between their propagation directions, and the parameter $\varkappa$ are all known together with their mean squared deviations $\Delta\omega$, $\Delta\omega'$, $\Delta\theta$, $\Delta\varkappa$. The variance of the

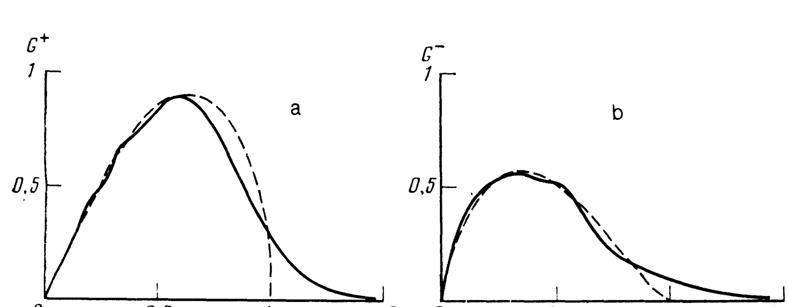

FIG. 2. Pair-production cross section vs the parameter t for like (a) and unlike (b) photon polarizations, $G^{\pm} = \sigma^{\pm}/\pi r_0^2$. The solid and dashed curves are for $\xi = 0.1$ and $\xi = 0$, respectively.

parameter a is then given by

$$D=\frac{4v^2}{\varkappa^2t^3}\left\{\left(\frac{\Delta\Omega}{4\pi}\right)^2+\left(\frac{\Delta\omega}{\omega}\right)^2+\left(\frac{\Delta\omega'}{\omega'}\right)^2+\frac{4}{9}v^4\left(\frac{\Delta\varkappa}{\varkappa}\right)^2\right\},$$

where $\Delta\Omega=\pi(\Delta\theta)^2$. Let, for the sake of argument, a have a Gaussian distribution and let $D\ll a_0^2$. Averaging the oscillating part, it can be easily shown that if $D\gg 1$ the oscillation amplitude becomes exponentially small. It follows hence that cross-section oscillations can be observed only when $D\lesssim 1$. At the same time, an upper bound on D is necessary also for measurement of the monotonic correction to the cross section. This bound can be easily obtained from the condition $\Delta\sigma<|\sigma_{\rm mon}-\sigma_0|$, where $\Delta\sigma$ is the cross-section measurement error; if $D\gg 1$, we have

$$\Delta\sigma\approx\left[\sigma_{\rm osc}^2+D\left(\frac{d\sigma_{\rm mon}}{da}\right)^2_{a=a_0}\right]^{1/2}.$$

The influence of the external field increases as the pair-production cross section threshold $t=1$ is approached. It is therefore natural to attempt to observe this influence just there. The resolution of the cross section into monotonic and oscillating components is meaningless in this region. It can be shown that to observe the influence of the field near the threshold, it is necessary to have

$$D'=\frac{4\varkappa^{4/3}}{\varkappa^2t^3}\left\{\left(\frac{\Delta\Omega}{4\pi}\right)^2+\left(\frac{\Delta\omega}{\omega}\right)^2+\left(\frac{\Delta\omega'}{\omega'}\right)^2+\frac{4}{9}\varkappa^{4/3}\left(\frac{\Delta\varkappa}{\varkappa}\right)^2\right\}\leqslant 1.$$

The cross section at $t\approx 1$ becomes comparable already at $\varkappa\sim 10^{-2}$ with the maximum Breit-Wheeler cross section. If the external-field strength is $H\approx 3\times 10^5$ G, the required photon energies are $\omega'\approx 730$ GeV and $\omega\approx 0.4$ eV. From the condition $D'\lesssim 1$ we obtain then $\Delta\omega'/\omega'\lesssim 10^{-2}$.

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[1)]We use here a system of units with $c=\hbar=1$.

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