Magnetodynamic nonlinearity in a metal-dielectric-metal system

A. A. Krokhin, I. B. Snapiro, and V. A. Yampol'skii

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

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We predict the existence of a stable decreasing portion of the current-voltage characteristic (CVC) of a system consisting of two thin metal plates separated by a dielectric layer. The nonlinearity of the CVC is related to the effect of the self-magnetic field on the conductivity of the sample. The CVC becomes multivalued (N-shaped) when the external magnetic field h_0 exceeds a critical value h_{cr}, and arises because the trajectories of electrons trapped by the magnetic field are cut off. Numerical estimates demonstrate the feasibility of experimental detection of the predicted effect.

1. A constant electric current flowing in a metal produces a magnetic field which affects the dynamics of electrons, and thus the conductivity. The nonlinear influence of this internal magnetic field on the conductivity results in deviations of the current-voltage characteristic (CVC) from Ohm's Law. This magnetodynamic nonlinearity is most clearly exhibited in thin metal samples with diffuse boundaries when the classical size effect[1–6] comes into play. When

$$d \ll l \ll L \tag{1}$$

the conductivity is basically due to a fairly small group of electrons which move almost parallel to the surface of the metal instead of colliding with it. Here L is the length of the sample, d is its characteristic thickness, and l is the electron mean free path. Reducing the number of effective electrons under linear operating conditions decreases the conductivity of a thin plate by a factor of $(l/d)/\ln(l/d) \gg 1$ as compared with the conductivity σ_0 of a bulk sample.[7]

The effect of a current-induced magnetic field is that electrons which travel along straight-line trajectories under linear conditions and collide with the boundaries of the sample are trapped by the magnetic field, and begin to move without collisions. Thus, in the nonlinear regime, the number of effective electrons is increased, and the greater the current, the more electrons there are.

In a plane-parallel plate, the current-induced magnetic field is distributed antisymmetrically, i.e., it vanishes at the center of the plate and takes the values H and $-H$ on the opposite faces, with

$$H = 2\pi I/cD. \tag{2}$$

Here I is the total current, D is the horizontal dimension of the plate in the direction perpendicular to the current, and c is the speed of light. With this alternating field distribution within the plate, there exist scalloping electron trajectories near the plane $x = x_0$ where the sign of the magnetic field changes (see Fig. 1). While moving along these trajectories, electrons do not collide with the faces of the plate, and they effectively interact with the electric field over their whole mean free path. When

$$d \ll (Rd)^{1/2} \ll l \tag{3}$$

(where R is the Larmor radius for the field H), the conductivity of the trapped electrons

$$\sigma_{tr} \sim \sigma_0 (d/R)^{1/2} \propto I^{1/2} \tag{4}$$

exceeds that of all other classes, and the voltage drop V across the sample is proportional to the square root of the current, $I^{1/2}$ (Refs. 3, 4).

The internal magnetic field $\mathbf{h}_0$, which is collinear with the current-induced magnetic field, displaces the plane $x = x_0$ where the direction of the total magnetic field changes toward one of the sample surfaces. If $h_0 > H$, the magnetic field in the plate will be constant, there will be no trapped electrons, and the sample will operate under essentially linear conditions, even when (3) is satisfied. As the current in the sample increases, the plane $x = x_0$ makes its appearance, and there is a transition from the linear to the nonlinear regime. Thus, an external magnetic field shifts the nonlinear portion of the CVC toward higher current. The change in CVC slope associated with the transition to nonlinearity is more abrupt in an external field than when $h_0 = 0$ (Ref. 4). For high currents ($H \gg h_0$), the field h_0 exerts a negligible influence, and the equation for the CVC has the same form as for $h_0 = 0$. Note that as the current increases, the plane $x = x_0$ tends asymptotically toward the middle of the plate.

We now consider a sample consisting of two identical thin plates with an insulating layer between them—a sandwich. The plates are connected in parallel. For $h_0 = 0$, the plane $x = x_0$ lies between the plates, and there are no trapped electrons in either. The sandwich CVC is therefore linear. In an external magnetic field, this situation changes drastically. When $H > h_0$, the plane where the total magnetic field changes direction can migrate to any position within either plate, depending on the current. For the sake of definiteness,

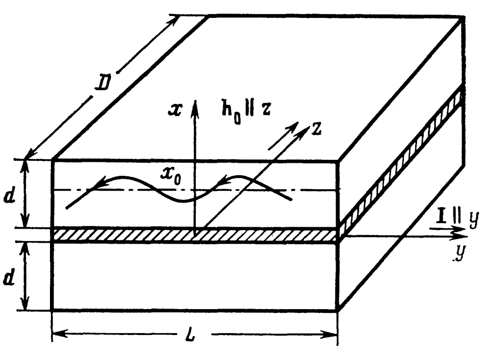

FIG. 1. Coordinate system and trajectory of a trapped electron.

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let us assume it is the upper plate ($h_0 > 0$; see Fig. 1). Until the plane $x = x_0$ crosses the middle of the upper plate, the sandwich CVC does not differ qualitatively from the nonlinear CVC of a plate of thickness $2d$. As the current increases, the number of trapped electrons grows, and the conductivity of the sandwich increases. It reaches a maximum when the plane $x = x_0$ crosses the middle of the upper plate. With a further increase in current, this plane crosses into the lower half of the plate, and the number of trapped electrons starts to drop, due to collisions with the lower face. In other words, the electron orbits are interrupted.

The cutoff current I_c delineates two regions. In the first, for $I < I_c$, the number of effective electrons increases with increasing current, and in the second, for $I > I_c$, it decreases. The change in sign of the derivative of the conductivity with respect to current should result in a discontinuity in the CVC at $I = I_c$.

The magnitude of the discontinuity, i.e., the jump in the derivative of the function $V(I)$, increases as the external magnetic field h_0 increases. This is consistent with the earlier assertion that the slope of $V(I)$ increases with increasing h_0. A rigorous calculation of the sandwich CVC below demonstrates that for some critical value $h_0 = h_{cr}$ of the field, the derivative dV/dI goes to infinity at $I = I_c$ (curve 2 in Fig. 2), and for $h_0 > h_{cr}$, a decreasing segment appears on the CVC (curve 3 in Fig. 2). We stress that the appearance of a negative differential impedance is due to the combination of two factors: the cutoff of the electron orbits, and the presence of a strong field h_0. The first produces a discontinuity in $V(I)$, and the second sets the critical value of the discontinuity.

Note that the decreasing segment of the CVC of a thin metal plate due to magnetodynamic nonlinearity had been predicted,[5] with a field h_0 oriented along the current being considered. The CVC thus obtained was S-shaped. The sandwich CVC being investigated in the present paper is N-shaped. Thus, it can be said that in pure metals at low temperatures, there is a nonthermal nonlinearity mechanism which is responsible for the same variety of CVC shapes as in semiconductors.

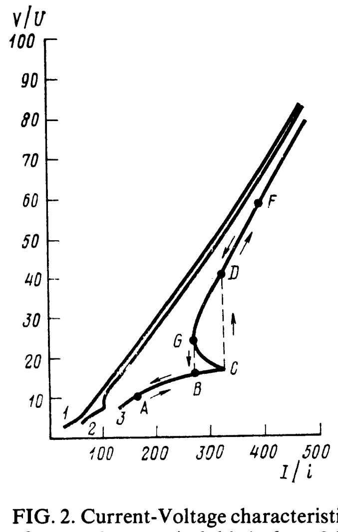

FIG. 2. Current-Voltage characteristic of the sandwich for various values of external magnetic field: 1) $h_0 = 0.75h_{cr}$; 2) $h_0 = h_{cr}$; 3) $h_0 = 1.5h_{cr}$.

Note that the sandwich critical field h_{cr} is two orders of magnitude lower than the critical field computed for a single plate.[5] Thus, the experimental detection of the decreasing segment of the CVC is a much simpler proposition for the sandwich than for the plate.

2. We now compute the CVC of a sandwich consisting of two identical plates of thickness d, connected in parallel electrically. The origin of the coordinate system is put at the center of the sandwich. The current is directed along the y-axis, with the x-axis perpendicular to the face of the sandwich. The constant external uniform field $\mathbf{h}_0$ lies along the z-axis (Fig. 1).

The equation for the CVC is obtained by simultaneously solving the Boltzmann equation and the equations of magnetostatics. However, in the present instance, we can avoid this procedure and derive the sandwich CVC by using the known[4] form of the CVC for a single thin plate in an external magnetic field.

We assume that we know the relation between the electric field E in the plate, the magnetic field H, which is proportional to the current in this plate, and the external field h_0:

$$f\left(\frac{E}{\mathcal{E}}, \frac{H}{h}, \frac{h_0}{h}\right) = 0. \qquad (5)$$

Here $\mathcal{E}$ amd h are the natural units for field measurements introduced in Ref. 4:

$$\mathcal{E} = 4chl/3\pi\sigma_0 d^2, \quad h = \gamma c p_F d/el^2, \quad \gamma \approx 0.86,$$

where p_F is the Fermi level momentum and e is the absolute value of the charge on the electron. In the geometry of Fig. 1, the plane x_0 and the group of trapped electrons can only exist in the upper plate, and it is just to this plate that we apply Eq. (5). In this equation, H pertains to the field produced by the current passing only through the upper plate, and h_0 is the absolute difference between the external magnetic field and the field produced by the current in the lower plate. The sign of the latter field must satisfy the rule that the equation for the CVC of the plate is invariant under inversion of the external magnetic field.

Thus, the sandwich CVC can be derived from Eq. (5) with the substitutions

$$\frac{H}{h} \to \frac{H}{h} - \frac{E}{\mathcal{E}} \ln \frac{R_+}{d}, \qquad \frac{h_0}{h} \to \left| \frac{h_0}{h} - \frac{E}{\mathcal{E}} \ln \frac{R_+}{d} \right|. \qquad (6)$$

We have assumed here[4] that the conductivity of the lower plate is

$$\sigma = \frac{3}{8} \sigma_0 \frac{d}{l} \ln \frac{R_+}{d}, \qquad R_+ = \frac{c p_F}{e(H + h_0)}. \qquad (7)$$

3. The actual form of the function $f(E/\mathcal{E}, H/h, h_0/h)$, which determines the CVC of a thin metal plate, was derived in Ref. 4. Carrying out the substitution (6) in Eq. (3.15) of Ref. 4, we obtain

$$\frac{H}{h} = \left(\frac{E}{\mathcal{E}}\right)^2 \left\{1 - \left|1 - \frac{h_0}{h} \frac{\mathcal{E}}{E \ln(R_+/d)}\right|\right\}^3 + \frac{E}{\mathcal{E}} \ln\left(\frac{R_+}{d}\right) \left\{1 + \left|1 - \frac{h_0}{h} \frac{\mathcal{E}}{E \ln(R_+/d)}\right|\right\}. \qquad (8)$$

Just as in the case of the CVC of the plate in Eq. (3.15) of Ref. 4, Eq. (8) can be used in the limit of large nonlinearity,

$$\frac{(Rd)^{1/2}}{l}\ln\frac{R_+}{d}\ll 1, \tag{9}$$

when the sample conductivity is due to a group of trapped electrons.

If the electric field in the sample satisfies the inequality

$$(E/\mathscr{E})\ln(R_+/d)\leqslant h_0/h, \tag{10}$$

the quantity between the vertical bars in Eq. (8) is negative. The upper plate is then immersed in a positive external magnetic field which is the sum of the field h_0 and the field produced by the lower plate. Under these circumstances, the plane $x = x_0$ lies within the upper part of the plate ($x_0 \geqslant d/2$), and the sandwich CVC is a monotonic function $E(H)$ which is essentially the same as the nonlinear CVC of a plate of thickness $2d$:

$$\frac{H-h_0}{h}=\left(\frac{E}{\mathscr{E}}\right)^2\left[2-\frac{h_0}{h}\frac{\mathscr{E}}{E\ln(R_+/d)}\right]^3. \tag{11}$$

We can easily obtain the values of the cutoff fields H_c and E_c from Eqs. (10) and (11), for which the absolute values in Eqs. (8) vanish:

$$\begin{gathered}H_c=h_0+\frac{h_0^2}{h}\ln^{-2}\left(\frac{l^2}{d^2}\frac{h}{H_c+h_0}\right),\\ E_c=\mathscr{E}\frac{h_0}{h}\ln^{-1}\left(\frac{l^2}{d^2}\frac{h}{H_c+h_0}\right).\end{gathered} \tag{12}$$

This point on the sandwich CVC corresponds to the plane $x = x_0$ being right in the middle of the upper plate. The derivative

$$\left.\frac{dE}{dH}\right|_{H=H_c-0}=\frac{\mathscr{E}}{5h_0}\ln\left(\frac{l^2}{d^2}\frac{h}{H_c+h_0}\right), \tag{13}$$

which characterizes the slope of the CVC at cutoff, is finite for all h_0.

We now consider the regime of higher electric fields,

$$h_0/h\leqslant(E/\mathscr{E})\ln(R_+/d). \tag{14}$$

In this region, Eq. (8) is quadratic in $E/\mathscr{E}$, giving

$$\frac{E}{\mathscr{E}}=\left(4\ln\frac{R_+}{d}\right)^{-1}\times\left\{\frac{H+h_0}{h}\pm\left[\left(\frac{H+h_0}{h}\right)^2-8\left(\frac{h_0}{h}\right)^3\ln^{-2}\left(\frac{R_+}{d}\right)\right]^{1/2}\right\}. \tag{15}$$

The choice of sign in (15) is determined by the "matching" condition at the point H_c given by (12). At this point, the expression inside the square root in (15) is a perfect square,

$$\left(\frac{h_0}{h}\right)^2\left[2-\frac{h_0}{h}\ln^{-2}\left(\frac{l^2}{d^2}\frac{h}{H_c+h_0}\right)\right]^2,$$

which vanishes when $h_0 = h_{cr}$. The critical field comes from the equation

$$\frac{h_{cr}}{h}=2\ln^2\left(\frac{l^2}{d^2}\frac{h}{4h_{cr}}\right). \tag{16}$$

Thus, the result of taking the square root and choosing a sign in (15) will depend on the ratio between the fields h_0 and h_{cr}.

If the external field strength is less than the critical value ($h_0 < h_{cr}$), we must take the plus sign in (15). The function $E(H)$ is then increasing. As $H\to\infty$, h_0 can be neglected in (15), and the sandwich CVC is given by the almost linear asymptote

$$H/2h=(E/\mathscr{E})\ln(R_+/d). \tag{17}$$

Figure 2 (curve 1) shows the typical form of the sandwich CVC for $h_0 < h_{cr}$. There is a kink in the CVC at H_c, since the derivative on the right,

$$\left.\frac{dE}{dH}\right|_{H=H_c+0}=\frac{\mathscr{E}}{h_{cr}-h_0}\ln\left(\frac{l^2}{d^2}\frac{h}{H_c+h_0}\right) \tag{18}$$

is not the same as the derivative (13) on the left.

At the critical value $h_0 = h_{cr}$ of the external magnetic field, the vertex of the hyperbola (15) falls at $H = H_c$, and the derivative (18) goes to infinity there. It is easy to obtain the coordinates of the vertex of the hyperbola for $h_0 = h_{cr}$ from (12) and (16):

$$H_c(h_{cr})=3h_{cr},\quad E_c=\mathscr{E}(2h_{cr}/h)^{1/2}. \tag{19}$$

The sandwich CVC is shown in Fig. 2 (curve 2) for $h_0 = h_{cr}$. Note that the equation for the critical field for the CVC of a thin plate in an external magnetic field directed along the current[5] differs from (16) only by a numerical factor on the right-hand side. However, this factor is a large one: instead of 2, the factor is 625 in Eq. (3.9) of Ref. 5. Thus, experimentally, the falling segment of the CVC can more easily be realized in a sandwich than in a plate.

There is a segment with negative differential impedance in the sandwich CVC for $h_0 > h_{cr}$. In this case, the point with coordinates H_c and E_c winds up on the decreasing branch of the hyperbola (15). The falling segment of the CVC is located between H_c and H^*, where

$$H^*=-h_0+\left(\frac{2h_0}{h}\right)^{3/2}h\ln^{-1}\left(\frac{l^2}{d^2}\frac{h}{H_c+h_0}\right) \tag{20}$$

is the coordinate of the vertex of the hyperbola. Above the point $E(H^*)$ lies the ascending branch of the hyperbola (15), which for $H\to\infty$ is described by the linear asymptote (17). When $h_0 > h_{cr}$, $E(H)$ is multivalued in the interval $H^* < H < H_c$: three values of E correspond to one value of H (curve 3 in Fig. 2).

4. Up to this point, we have been considering a sandwich containing plates of the same thickness. We now discuss the effect of a difference between plate thicknesses d_1 and d_2 on the previous results (d_1 is the thickness of the upper plate, and d_2 the lower).

First of all, we note that when there is no external magnetic field, the point x_0 lies within one of the plates, and not in between them, as in the symmetric sandwich with $d_1 = d_2$. At large currents, when the field h_0 can be neglected, there is accordingly a group of trapped electrons in the sandwich with trajectories which wind back and forth along the plane $x = x_0 = (d_1^2 - d_2^2)/2d_1$. The asymptote of the CVC is therefore nonlinear ($V\sim I^{1/2}$).

We shall not explicitly write out the sandwich CVC, the structure of which, on the falling segment, is fairly insensitive to the ratio d_1/d_2. However, the very fact of the existence of a falling segment of the CVC as well as the critical field value h_{cr}, depend on this ratio. A straightforward calculation gives the following equation for the field h_{cr}:

$$\frac{h_{cr}}{h}=2\ln^2\left(\frac{l^2}{d_1d_2}\frac{h}{4h_{cr}}\right)\left[3\left(\frac{d_1}{d_2}\right)^2-2\left(\frac{d_1}{d_2}\right)^4\right]^{-1}. \quad (21)$$

Equation (21) implies that the optimum conditions for observing the falling segment of the CVC occur when $d_1/d_2 = 3^{1/2}/2$. The coefficient in front of the logarithm in (21) is then at a minimum, and is equal to 16/9.

We now direct our attention to the fact that the dependence of the critical field h_{cr} on the thicknesses d_1 and d_2 is quite asymmetrical. A reduction in the thickness d_2 of the lower plate results in a rapid rise in the critical field, and to the disappearance of the falling segment of the CVC when $d_2 \leqslant (2/3)^{1/2} d_1$. Also, a reduction in the thickness d_1 of the upper plate results in a rise in h_{cr} as well. However, the negative differential impedance effect does not disappear, no matter how small we make d_1/d_2. Thus, to exhibit the falling segment of the CVC experimentally, the upper plate must be the thinner (for the directions of current and external field shown in Fig. 1).

5. It is well known that the falling segment of a static CVC can turn out to be unstable to circuit current or voltage fluctuations. Under these circumstances, it is not possible to observe the theoretically calculated falling segment, since the system readjusts itself as a result of the development of instability, and the decreasing segment either disappears completely, or it undergoes a significant change in shape. We therefore will discuss the problem of the stability of the decreasing segment of the sandwich CVC derived above.

For a fixed current, it is not possible to go over to the decreasing segment, since the change in voltage V occurs along one of the two stable rising segments. If the current increases from zero, the voltage will gradually increase along the line ABC (Fig. 2). At point C, there is a voltage discontinuity: it jumps to the higher value at point D. Any further change in voltage takes place along the rising segment DF. When the current is reduced along the curve FDG, the system passes through point D, and the voltage discontinuity takes place at a lower current-at point G. This behavior of $V(I)$ is related to hysteresis of the CVC.

It is possible to go over to the decreasing segment with a controlled voltage, since the measured function $I(V)$ is then single-valued. It is in this setting that the problem of stability of the decreasing segment of the CVC is of interest.

First of all, we note that on the decreasing segment of the sandwich CVC for each plate, the voltage dependence of the current in the plate is an increasing function. Thus, independent current fluctuations are quickly damped out, and instability can only arise when there is positive feedback between the fluctuating fields in the plates. However, the neutrality condition $\operatorname{div} \mathbf{j} = 0$ for metals limits the form of electromagnetic fluctuations in a plate, and as will be demonstrated below, it turns out to be impossible for coupled fluctuations to exist in a sandwich.

Let us look at the simplest nonuniform fluctuation along the current which ensures that the voltage across the sample is constant, i.e.,

$$\int E_y\, dy=0,$$

where E_y is the fluctuation field. In order for a fluctuation in one plate to affect the conductivity of the other, the fluctuation must contain the z-component of the magnetic field,

$$H_z=H(x)\exp(iky+i\omega t).$$

Maxwell's equations tell us that there are also x- and y-components of the electric field in such a fluctuation:

$$E_x=E_x(x)\exp(iky+i\omega t),\quad E_y=E_y(x)\exp(iky+i\omega t).$$

An investigation of the stability to fluctuations of the type described reduces to an analysis of the dispersion relation $\omega = \omega(k)$ obtained by simultaneously solving Maxwell's equations in the two plates. However, in the present case, the dispersion relation cannot be obtained in explicit form. It follows from this that at the interface, all fluctuating field components vanish. Therefore, fluctuation processes in the plates develop independently, and cannot result in instability. Note that the possibility of a stable falling segment of the CVC was demonstrated in a series of experiments in semiconductors [8–10].

6. To conclude, we present the necessary numerical estimates. In Fig. 2, we show the results of a calculation of the CVC of a sandwich consisting of two plates of thickness $d = 10^{-3}$ cm, length and width $L = D = 1$ cm, $p_F = 10^{-19}$g · cm/sec, and $l = 0.1$ cm. With these values, the scaling factors h and $\mathscr{E}$ for the field measurements are $h = 0.54$ Oe, $\mathscr{E} = 3.1 \cdot 10^{-5}$ V/cm. The corresponding scaling current and voltage are $i \approx 0.86$ A, $U \approx 3.1 \cdot 10^{-5}$V. The critical field strength obtained by solving Eq. (16) is $h_{cr} \approx 20$ Oe. The decreasing segment of the CVC in curve 3 is located at a current $240 < I < 280$ A. The power density dissipated in the sample on the decreasing segment of the CVC, $P = IV/LD = 0.2$ W/cm^2, can easily be carried off by liquid helium. These estimates demonstrate the possibility of observing the predicted decreasing segment of the CVC experimentally.

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[1]B. N. Aleksandrov, Zh. Eksp. Teor. Fiz. **43**, 1231 (1962) [Sov. Phys. JETP **43** (1962)].
[2]M. Yaqub and J. F. Cochran, Phys. Rev. Lett. **10**, 390 (1963).
[3]É. A. Kaner, N. M. Makarov, I. B. Snapiro, and V. A. Yampol'skii, Pis'ma Zh. Éksp. Teor. Fiz. **39**, 384 (1984) [JETP Lett. **39**, 463 (1984)].
[4]É. A. Kaner, N. M. Makarov, I. B. Snapiro, and V. A. Yampol'skii, Zh. Éksp. Teor. Fiz. **87**, 2166 (1984) [Sov. Phys. JETP **87**, 1252 (1984)].
[5]É. A. Kaner, N. M. Makarov, I. B. Snapiro, and V. A. Yampol'skii, Zh. Éksp. Teor. Fiz. **88**, 1310 (1985) [Sov. Phys. JETP **61**, 776 (1985)].
[6]É. A. Kaner, I. B. Snapiro, and V. A. Yampol'skii, Fiz. Nizk. Temp. **11**, 477 (1985) [Sov. J. Low Temp. Phys. **11**, (1985)].
[7]K. Fuchs, Proc. Cambr. Phil. Soc. **34**, 100 (1938).
[8]D. A. Kichigin and V. P. Lobachev (Lobachjov), Sol. State Commun. **9**, 619 (1971).
[9]D. A. Kichigin, V. P. Lobachev, and O. A. Mironov, Fiz. Tekh. Poluprovodn. **7**, 775 (1973) [Sov. Phys.-Semiconductors **7**, 532 (1973)].
[10]S. A. Kostylev, V. A. Shkut, and V. Ya. Krys', Pis'ma Zh. Éksp. Teor. Fiz. **21**, 232 (1975) [JETP Lett. **21**, 104 (1975)].

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