Instability of parametrically excited spin waves interacting with acoustic waves

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Coupled oscillations of a packet of parametric spin waves (PSW) and of the elastic subsystem of a crystal are investigated. The frequencies and damping rates of these oscillations are determined. It is shown that the coupled oscillations can be unstable, as confirmed by experiment.

1. Parametric microwave pumping produces in a magnet a narrow-frequency-band packet of random-phase parametric spin waves (PSW) (see, e.g., the review by Zakharov *et al.*[1]). Smirnov[2] has recently observed in experiment the acoustic oscillations induced by parametric excitation of spin waves in an antiferromagnet. The oscillations were observed at the frequencies $\Omega \approx \pi c/L \approx 2\pi(2\text{–}3)10^6\,\mathrm{s}^{-1}$ of the lower acoustic modes of the sample (c is the speed of sound and L the sample dimension). The frequency variation $\Delta\Omega$ of such oscillations was below 100 Hz. The lifetime Γ^{-1} of the acoustic modes in the absence of pumping was 1.5×10^{-5} s, and the lifetime of the PSW was $\gamma^{-1}\approx 10^{-6}$ s. The excited acoustic oscillations were thus highly coherent. We report here an investigation of the mechanism by which coherent acoustic oscillations are generated, and show that they result from instability of coupled collective oscillations of PSW and sound.

2. The Hamiltonian of the magnetoelastic subsystem of a magnet can be written in the form

$$H=H_m+H_{el}+H_{mel}. \tag{1}$$

Here

$$H_{el}=\sum_\alpha\int \Omega_\varkappa^\alpha b_\varkappa^{\alpha *} b_\varkappa^\alpha\, d\varkappa \tag{2}$$

is the Hamiltonian of the free elastic oscillations, $b_\varkappa^\alpha$ is the amplitude of a sound wave with wave vector $\varkappa$ and polarization α, and $\Omega_\varkappa^\alpha=c_\alpha\varkappa$ is the phonon-dispersion law. For amplitudes which are not too far above the critical level, the spin-system Hamiltonian need only include terms that are diagonal in PSW pairs (S-theory approximation)[1]:

$$H_m=\int \omega_{\mathbf{k}} a_{\mathbf{k}}^* a_{\mathbf{k}}\, d\mathbf{k}+\frac{1}{2}\int (hV_{\mathbf{k}}e^{-i\omega_p t}a_{\mathbf{k}}^* a_{-\mathbf{k}}^*+\text{c.c.})\, d\mathbf{k}$$
$$+\int T_{\mathbf{kk}'}a_{\mathbf{k}}^* a_{\mathbf{k}} a_{\mathbf{k}'}^* a_{\mathbf{k}'}\, d\mathbf{k}\, d\mathbf{k}'+\frac{1}{2}\int S_{\mathbf{kk}'}a_{\mathbf{k}}^* a_{-\mathbf{k}}^* a_{\mathbf{k}'} a_{-\mathbf{k}}\, d\mathbf{k}\, d\mathbf{k}'. \tag{3}$$

Here $a_{\mathbf{k}}$ is the complex amplitude of the spin waves, $\omega_{\mathbf{k}}$ their spectrum, h the microwave-pump amplitude, ω_p its frequency, $V_{\mathbf{k}}$ the coefficient of the coupling between the PSW and pump, and

$$T_{\mathbf{kk}'}=T_{\mathbf{k},\mathbf{k}',\mathbf{k},\mathbf{k}'},\qquad S_{\mathbf{kk}'}=T_{\mathbf{k},-\mathbf{k},\mathbf{k}',-\mathbf{k}'}, \tag{4}$$

where $T_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}$ is the amplitude of the four-magnon interaction. At low elastic-oscillation amplitude, the magnetoelastic-interaction Hamiltonian is

$$H_{mel}=\sum_\alpha\int (V_{\mathbf{kk}'\varkappa}^\alpha a_{\mathbf{k}}^* a_{\mathbf{k}'} b_\varkappa^\alpha+\text{c.c.})\,\delta(\mathbf{k}-\mathbf{k}'-\varkappa)\, d\mathbf{k}\, d\mathbf{k}'\, d\varkappa. \tag{5}$$

The value of $V_{\mathbf{kk}'\varkappa}^\alpha$ is given in Refs. 3 and 4 for ferromagnets and antiferromagnets, respectively. Note that the subdivision of the Hamiltonian (1) into magnetic, elastic, and magnetoelastic components is quite arbitrary, since we have omitted from the Hamiltonian component the magnetoelastic coupling which is quadratic in the amplitudes $a_{\mathbf{k}}$ and $b_\varkappa^\alpha$. As a result, the respective frequencies $\Omega_\varkappa^\alpha$ and $\omega_{\mathbf{k}}$ of the acoustic and magnon modes, and also the matrix elements $T_{\mathbf{kk}'}$ and $S_{\mathbf{kk}'}$, contain the magnetoelastic-interaction constants. Nonetheless, under the experimental conditions of Ref. 2 the values of $T_{\mathbf{kk}'}$ and $S_{\mathbf{kk}'}$ are close to those calculated in Ref. 5 with magnetostriction neglected.

3. The equations of motion for the PSW amplitudes, with allowance for relaxation, take the form[1]

$$\frac{\partial a_{\mathbf{k}}}{\partial t}+\gamma_{\mathbf{k}}a_{\mathbf{k}}+i\frac{\delta H}{\delta a_{\mathbf{k}}^*}=0. \tag{6}$$

It follows therefore that in the absence of acoustic oscillations (when $b_\varkappa^\alpha=0$), PSW pairs ($\mathbf{k}$ and $-\mathbf{k}$) appear on the surface $\omega_{\mathbf{k}}=\omega_p/2$ when $|hV_{\mathbf{k}}|>\gamma_{\mathbf{k}}$. The overall phase shift of the waves with opposite momenta is then closely related to that of the pump, but the phase difference is random. As a result, a stationary uniform distribution of the PSW is produced in the sample, except for a small region near the boundary whose thickness is approximately equal to the mean free path. The total number of the PSWs is[1]

$$N=\int \langle a_{\mathbf{k}}^* a_{\mathbf{k}}\rangle\, d\mathbf{k}=\frac{1}{|S|}(|hV|^2-\gamma^2)^{1/2}, \tag{7}$$

where S is the value of $S_{\mathbf{kk}'}$ averaged over the PSW distribution.

4. The equations of motion for the acoustic-oscillation amplitudes, with allowance for magnetostriction and damping, are

$$\frac{\partial}{\partial t}b_\varkappa^\alpha+(\Gamma_\varkappa^\alpha+i\Omega_\varkappa^\alpha)b_\varkappa^\alpha$$
$$+i\int V_{\mathbf{kk}'\varkappa}^{\alpha *}a_{\mathbf{k}}a_{\mathbf{k}'}^*\delta(\mathbf{k}-\mathbf{k}'-\varkappa)\, d\mathbf{k}\, d\mathbf{k}'=0. \tag{8}$$

Since we are considering specific acoustic oscillations such that $b_\varkappa^\alpha=\langle b_\varkappa^\alpha\rangle$, while the PSW phases are random, the sound amplitudes are determined by the oscillations of the

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correlation function $n_{\mathbf{k}}(\varkappa) = \langle a_{\mathbf{k}} a_{\mathbf{k}}^* \rangle$. In the spatially uniform case we have $n_{\mathbf{k}}(\varkappa) = n_{\mathbf{k}} \delta(\varkappa)$. By virtue of Adler's theorem, however, $V_{\mathbf{kk}0} = 0$, so that the acoustic oscillations are connected with the spatially uniform perturbations of the correlators:

$$n_{\mathbf{k}} = \langle a_{\mathbf{k}}^* a_{\mathbf{k}} \rangle, \quad \sigma_{\mathbf{k}} = \langle a_{\mathbf{k}} a_{-\mathbf{k}} \rangle.$$

We confine ourselves to the case of small sound amplitudes and weak spatial modulation of the PSW distribution. We introduce for this purpose, following Ref. 6, small perturbations of the correlation functions:

$$\tilde{n}_{\mathbf{k}}(\varkappa) = \langle \alpha_{\mathbf{k}+\varkappa} a_{\mathbf{k}}^{0*} \rangle, \quad \tilde{n}_{\mathbf{k}}^{+}(\varkappa) = \langle a_{-\mathbf{k}}^{0} \alpha_{-\mathbf{k}-\varkappa} \rangle,$$
$$\tilde{\sigma}_{\mathbf{k}}(\varkappa) = \langle \alpha_{\mathbf{k}+\varkappa} a_{-\mathbf{k}}^{0} \rangle, \quad \tilde{\sigma}_{\mathbf{k}}^{+}(\varkappa) = \langle a_{\mathbf{k}}^{0*} \alpha_{-\mathbf{k}-\varkappa}^{*} \rangle, \tag{9}$$

where $a_{\mathbf{k}}^0$ are the amplitudes of PSW with a distribution that is uniform on the average, and $\alpha_{\mathbf{k}+\varkappa}$ are small inhomogeneous corrections. We get therefore from (6), (8), and (9)

$$\gamma_{\mathbf{k}} \tilde{\sigma}_{\mathbf{k}}(\varkappa) + i P_{\mathbf{k}} \tilde{n}_{\mathbf{k}}(\varkappa) = 0, \quad \gamma_{\mathbf{k}} \tilde{\sigma}_{\mathbf{k}}^{+}(\varkappa) - i P_{\mathbf{k}}^{*} \tilde{n}_{\mathbf{k}}^{+}(\varkappa) = 0;$$
$$\Omega \tilde{N}_{\mathbf{k}}(\varkappa) - \xi_{\mathbf{k}}(\varkappa) \tilde{m}_{\mathbf{k}}(\varkappa) = 2 n_{\mathbf{k}} \int S_{\mathbf{kk}'} \tilde{m}_{\mathbf{k}'}(\varkappa) d\mathbf{k}'.$$
$$(\Omega + 2 i \gamma_{\mathbf{k}}) \tilde{m}_{\mathbf{k}}(\varkappa) - \xi_{\mathbf{k}}(\varkappa) \tilde{N}_{\mathbf{k}}(\varkappa) \tag{10}$$
$$= 2 n_{\mathbf{k}} \left[\int (S_{\mathbf{kk}'} + 2 T_{\mathbf{kk}'}) \tilde{N}_{\mathbf{k}'}(\varkappa) d\mathbf{k}' + \sum_{\alpha} F_{\mathbf{k}}^{\alpha}(\varkappa) \right],$$
$$[(\Omega + i \Gamma_{\varkappa}^{\alpha})^2 - \Omega_{\varkappa}^{\alpha 2}] F_{\mathbf{k}}^{\alpha}(\varkappa) = 2 \Omega_{\varkappa}^{\alpha} \int |V_{\mathbf{k},\mathbf{k}-\varkappa,\varkappa}^{\alpha}|^2 \tilde{N}_{\mathbf{k}'}(\varkappa) d\mathbf{k}',$$

where

$$\tilde{N}_{\mathbf{k}}(\varkappa) = \tilde{n}_{\mathbf{k}}(\varkappa) + \tilde{n}_{\mathbf{k}}^{+}(\varkappa), \quad \tilde{m}_{\mathbf{k}}(\varkappa) = \tilde{n}_{\mathbf{k}}(\varkappa) - \tilde{n}_{\mathbf{k}}^{+}(\varkappa),$$
$$F_{\mathbf{k}}^{\alpha}(\varkappa) = V_{\mathbf{kk}\varkappa}^{\alpha} b_{\varkappa}^{\alpha} + V_{\mathbf{kk}\varkappa}^{\alpha *} b_{-\varkappa}^{\alpha *}, \tag{11}$$
$$\xi_{\mathbf{k}}(\varkappa) = \omega_{\mathbf{k}+\varkappa} - \omega_{\mathbf{k}} = \mathbf{v}_{\mathbf{k}} \varkappa + \frac{1}{2} \frac{\partial^2 \omega_{\mathbf{k}}}{\partial k_\alpha \partial k_\beta} \varkappa_\alpha \varkappa_\beta,$$
$$P_{\mathbf{k}} = h V_{\mathbf{k}} + \int S_{\mathbf{kk}'} \sigma_{\mathbf{k}'} d\mathbf{k}', \quad |P_{\mathbf{k}}| = \gamma_{\mathbf{k}}.$$

We confine ourselves below to the most interesting case, that of an antiferromagnet, in which the quantities $\omega_{\mathbf{k}}$, $V_{\mathbf{k}}$, $T_{\mathbf{kk}'}$, $S_{\mathbf{kk}'}$ and the correlators $n_{\mathbf{k}}$ and $\sigma_{\mathbf{k}}$ are isotropic. We assume also the sound-wave vector and the spatial modulation $\varkappa$ of the PSW to be small, i.e.,

$$\xi_1 = v\varkappa \ll TN, \quad \xi_0 = {}^1/_2 \omega'' \varkappa^2 \ll TN. \tag{12}$$

The latter means that only the zeroth and first spherical harmonics with respect to the directions of the wave vector $\mathbf{k}$ take part in the PSW oscillations. In this approximation we get from (10) the dispersion equation

$$\Omega(\Omega + 2i\gamma) \left[1 - \frac{\xi_1^2/3}{\Omega(\Omega + 2i\gamma) - \xi_0^2} \right]^2$$
$$- \left\{ 2SN + \xi_0 \left[1 + \frac{\xi_1^2/3}{\Omega(\Omega + 2i\gamma) - \xi_0^2} \right] \right\}$$
$$\times \left\{ 2 \left[2T + S + \sum_{\alpha} \frac{2\Omega_{\varkappa}^2 |V_{\mathbf{kk}\varkappa}^{\alpha}|^2}{(\Omega + i\Gamma_{\varkappa}^{\alpha})^2 - \Omega_{\varkappa}^{\alpha 2}} \right] N \right.$$
$$\left. + \xi_0 \left[1 + \frac{\xi_1^2/3}{\Omega(\Omega + 2i\gamma) - \xi_0^2} \right] \right\} = 0. \tag{13}$$

Note that we can leave out of the expressions for the matrix elements $T_{\mathbf{kk}'}$ and $S_{\mathbf{kk}'}$ in (10) and (13) corrections that are second order in the interaction with the elastic oscillations, which are proportional to $|V_{\mathbf{kk}'\varkappa}|^2$, since we take explicitly into account the PSW interaction via Eqs. (8) and (10).

5. It is easiest to investigate the case when the dispersion of the PSW can be completely neglected by putting $\xi_1 = \xi_0 = 0$, and the damping of the magnons and of the sound can also be ignored. In addition, we take into account the interaction of the collective oscillations of the PSW with acoustic oscillations of one polarization, assuming sound frequencies of different polarizations to differ greatly from the coupled-oscillations frequency $\Omega(\varkappa)$. As a result we get from (13)

$$\Omega_{1,2}^2(\varkappa) = {}^1/_2 \{ (\Omega_{\varkappa}^2 + \Omega_0^2) \pm [(\Omega_{\varkappa}^2 - \Omega_0^2)^2 + 32 S |V_{\mathbf{kk}\varkappa}|^2 N^2]^{1/2} \}, \tag{14}$$

where $\Omega_0 = 2[S(2T+S)]^{1/2} N$ is the frequency of the homogeneous collective PSW oscillations.[1] In easy-plane antiferromagnets the matrix element S is negative,[6] so that the coupled oscillations of the PSW and of the elastic subsystem are unstable. Note that collective oscillations interacting with sound were treated in this approximation in Ref. 7, but there the transition to the normal modes of the collective oscillations is made without the appropriate transformation in the Hamiltonian H_{mel}, and the result is therefore incorrect.

If allowance is made for the damping of the spin waves and of the sound, the coupled oscillations (14) become unstable when the number N of the PSW exceeds the threshold value $N_{cr}(\varkappa)$. The minimum value of N_{cr} is reached at resonance when $\Omega_0 = \Omega_{\varkappa} = c\varkappa$:

$$(SN_{cr})^2 = \gamma_{\mathbf{k}} \Gamma_{\varkappa} \Omega_0 |S| / 2 |V_{\mathbf{kk}\varkappa}|^2. \tag{15}$$

The interaction of collective oscillations with sound is significant also far from resonance. In this case it follows from (13) that

$$\Omega_1(\varkappa) = \Omega_{\varkappa} + \frac{4 |V_{\mathbf{kk}\varkappa}|^2 SN^2}{\Omega_{\varkappa}^2 - \Omega_0^2} - i \left[\Gamma_{\varkappa} + 8\gamma_{\mathbf{k}} \frac{\Omega_{\varkappa} |V_{\mathbf{kk}\varkappa}|^2 SN^2}{(\Omega_{\varkappa}^2 - \Omega_0^2)^2} \right],$$
$$\Omega_2(\varkappa) = \Omega_0 + \frac{4 |V_{\mathbf{kk}\varkappa}|^2 SN^2}{\Omega_0^2 - \Omega_{\varkappa}^2} \frac{\Omega_{\varkappa}}{\Omega_0} \tag{16}$$
$$- i \left[\gamma_{\mathbf{k}} + 8\Gamma_{\varkappa} \frac{\Omega_{\varkappa} |V_{\mathbf{kk}\varkappa}|^2 SN^2}{(\Omega_0^2 - \Omega_{\varkappa}^2)^2} \right].$$

In the absence of magnetoelastic interaction, oscillations frequency $\Omega_1(\varkappa)$ are pure acoustic, and those with frequency $\Omega_2(\varkappa)$ are PSW collective oscillations. The damping $\Gamma_{\varkappa}$ of long-wave sound is as a rule small compared with the PSW damping $\gamma_{\mathbf{k}}$, so that at negative S the damping of the acoustic mode $\Omega_1(\varkappa)$ can become negative, which implies instability. The threshold of this instability is reached at some number N_{cr}^* of the PSW

$$S^2 N_{cr}^{*2}(\varkappa) = \Gamma_{\varkappa} |S| (\Omega_{\varkappa}^2 - \Omega_0^2)^2 / 8 \gamma_{\mathbf{k}} \Omega_{\varkappa} |V_{\mathbf{kk}\varkappa}|^2. \tag{17}$$

6. The experiments in Ref. 2 revealed excitation of an acoustic mode in the sample when a certain pump power was reached. Since the sample size was finite, the frequency $\Omega_{\varkappa}$

of the acoustic oscillations was substantially higher than the frequency Ω_0 of the collective ones. The sound-oscillation excitation threshold is determined in this case by Eq. (17) and is a minimum for the acoustic mode having the lowest frequency ($\varkappa \approx \pi/L$). An S-theory estimate of the pump amplitude h^*, at which the critical PSW number given by (17) is reached, yields for the experimental data of Ref. 2

$$h^*/h_c = [(SN_{cr}^*/\gamma)^2+1]^{1/2} \approx 5, \qquad (18)$$

where h_c is the threshold amplitude and $|h_c V| = \gamma$.

The experimental value of the ratio (18) is $(h^*/h_c)_{\exp} = 7$ to 10. Since the values of $|V_{\mathbf{kk}\varkappa}|$, $|S|$, and the damping $\gamma_{\mathbf{k}}$ are not known very accurately, the results of the theory and experiment should be regarded as in satisfactory agreement.

It must be noted that sound excitation considered here can be produced by an alternate mechanism, viz., kinetic instability (see Refs. 8 and 9), meaning instability to excitation of oscillations having random phases. Kinetic instability, however, cannot explain the experimental results of Ref. 2. Indeed, even if the kinetic-instability threshold is lower than (17), evolution of this instability cannot lead to coherent acoustic oscillations of amplitude substantially larger than thermal noise, for otherwise the coherent oscillation must be described by Eq. (8). The coherent acoustic oscillation must of necessity be caused by collective PSW oscillations and cannot be excited at $h < h^*$ given by Eq. (18).

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