Parametric excitation of nuclear spin waves under conditions of strong modulation of their spectrum

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Theoretical and experimental investigations were made of parametric excitation of magnons under conditions of strong modulation of their spectrum by an external rf field $H_m \cos \omega_m t$. Antiferromagnets $CsMnF_3$ and $MnCO_3$ were used in a study of the influence of the modulation field on the parametric excitation threshold of nuclear spin waves and on the phenomenon of a double parametric resonance. A theory was developed and used to calculate the pump threshold in the case of single- and double-frequency modulation, and to simulate the behavior of a signal of the parametric system above the excitation threshold. The results of the theory were in agreement with the experimental data.

I. INTRODUCTION

The method of parallel microwave pumping is usually employed in studies of parametrically excited spin waves in magnetic crystals. The essence of this method is as follows. A sample is placed in a magnetic field $\mathbf{H}_0 + \mathbf{h} \cos \omega_p t$ ($\mathbf{h} \| \mathbf{H}_0$) and when a certain threshold pump amplitude $h_{\rm th}$ is reached, a parametric instability develops in the interior of a crystal and it results in the decay of a microwave pump photon into two magnons of frequencies $\omega_{\mathbf{k}} + \omega_{-\mathbf{k}} = \omega_p$. If in addition to these an rf field $\mathbf{H}_m \cos \omega_m t$ ($\mathbf{H}_m \| \mathbf{H}_0, \omega_m \ll \omega_p$) is applied, then the magnon spectrum is subject to the following modulation:

$$\omega_k(t) = \omega_k + (\partial \omega_k / \partial H) H_m \cos \omega_m t,$$

which—as shown by Suhl[1]—should increase the threshold for the excitation of parametric spin waves because of the departure on the average from the parametric resonance condition. This prediction was confirmed experimentally by Hartwick, Peressini, and Weiss[2] in a study of pumping of ferrimagnetic yttrium iron garnet (YIG).

Zakharov, L'vov, and Starobinets[3] developed a model theory of turbulence of parametric spin waves (it is called the S theory) and this made it possible to study parametric spin waves in a much more thorough way. This theory was applied to experiments with field modulation by Zautkin *et al.*,[4] who carried out a detailed theoretical and experimental investigation of the influence of rf modulation on the threshold $h_{\rm th}$ and the nonlinear susceptibility of YIG above the threshold.

Similar investigations of parametric excitation of magnons under conditions of modulation of their spectrum have been carried out also on antiferromagnets.[5–10] The most interesting results were reported in Refs. 7 and 8 for nuclear spin waves: an oscillatory dependence of the threshold amplitude $h_{\rm th}$ on H_m and ω_m was discovered for these waves[7] and strong anomalies in the behavior of $h_{\rm th}$ under conditions of two-frequency modulation of the field $H_{m1} \cos \omega_{m1} t + H_{m2} \cos \omega_{m2} t$ were reported in Ref. 8. A simple theoretical interpretation of these effects was given in Ref. 11, where it was shown that pumping of parametric spin waves at combination frequencies (pump frequency in combination with modulation frequencies) is possible at high amplitudes H_m. However, the approach of Ref. 11 is not valid above the threshold. In the present study an analysis of the effects which appear in the case of strong modulation of the magnon spectrum is made using the S theory. In contrast to the treatment in Ref. 4, we shall allow for parametric instabilities of modes with various effective pump frequencies and for the interaction between these modes. In addition to the development of the theory, we shall also report some experimental results on the influence of modulation on the near-threshold and above-threshold behavior of parametric nuclear spin waves in antiferromagnets.

PRINCIPAL EQUATIONS

The system of equations for N_k, representing the number of parametric spin waves with a fixed wave vector $\mathbf{k}$, and for Θ_k, representing their phase "mismatch" relative to the pump wave ($\Theta_k \equiv \pi/2 - \Psi_k$, where Ψ_k is the time-dependent phase of a parametric magnon pair), can be obtained from Ref. 3 and represented in the form

$$d\Theta_k/d\tau + b \sin \Theta_k = \Delta + a \cos \Omega\tau + (2r_T + r_S) N_k, \qquad (1a)$$

$$dN_k/d\tau = N_k (b \cos \Theta_k - 1). \qquad (1b)$$

The following notation is adopted above:

$$\tau \equiv 2\gamma_k t, \quad b \equiv h/h_{\rm th0} \quad \Omega \equiv \omega_m/2\gamma_k;$$

$$a \equiv -(\partial\omega_k/\partial H)(H_m/\gamma_k), \quad \Delta \equiv (\omega_p/2 - \omega_k)/\gamma_k;$$

$$r_T \equiv -T_k/\hbar\gamma_k\mathcal{N}, \quad r_S \equiv -S_k/\hbar\gamma_k\mathcal{N};$$

γ_k is the rate of relaxation of parametric spin waves; ω_p, ω_m, and ω_k are the frequencies of the pump, modulation, and spin waves, respectively; T_k and S_k are the coefficients representing nonlinear interactions of parametric spin waves[3]; $\mathcal{N}$ is the number of magnetic cells in a sample; $h_{\rm th0}$ is the pumping threshold in the absence of the modulating field.

If there is no modulating field ($H_m = 0$), the main parameters of a system of parametric spin waves above the threshold are described by the equilibrium values $\Theta_k^{(0)}$ and $N_k^{(0)}$ (Ref. 3). The modulating field creates forced oscilla-

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tions of the quantities Θ_k and N_k near these equilibrium values. The regime of weak modulation ($a/\Omega \ll 1$) was studied in Ref. 10, where good agreement between the theory and experiment was achieved. An increase in the amplitude H_m of the modulating field increased considerably the pumping threshold and parametric effects associated specifically with the modulating field appeared above the threshold.[4,6,8] Forced oscillations of Θ_k and N_k observed in this case were complex because of the nonlinearity of the initial system of equations (1).

It is convenient to make the following substitution of the variables in the system (1):

$$\Theta_k=\theta_k+\Delta\tau. \quad (2)$$

We then obtain

$$d\theta_k/d\tau+b\sin(\theta_k+\Delta\tau)=a\cos\Omega\tau+(2r_T+r_S)N_k, \quad (3a)$$

$$d\ln N_k/d\tau=b\cos(\theta_k+\Delta\tau)-1. \quad (3b)$$

We shall now consider a class of solutions of the system (3), which conserve the quantities

$$\langle\theta_k(\tau)\rangle, \quad \langle N_k(\tau)\rangle,$$

where

$$\langle f\rangle\equiv\frac{1}{\tau_m}\int_0^{\tau_m} f\,d\tau$$

are the values averaged over a time τ_m that is a multiple of the modulation period: $\tau_m = 2\pi n/\Omega$, where $n = 1, 2, 3, \ldots$. These are known as the quasistationary states which satisfy

$$\theta_k(\tau)=\theta_k(\tau+\tau_m), \quad N_k(\tau)=N_k(\tau+\tau_m) \quad (4)$$

(and this is also true of the time derivatives of these quantities). Substituting $\tau\to\tau+\tau_m$ in the system (3) and using the relationships in Eq. (4), we readily obtain

$$\sin(\theta_k+\Delta\tau)=\sin(\theta_k+\Delta\tau+\Delta\tau_m) \quad (5)$$

and a similar equality for the cosine function. It readily follows from Eq. (5) that

$$\Delta\tau_m=2\pi n_1,$$

i.e.,

$$\Delta=(n_1/n)\Omega, \quad n_1=0, \pm1, \pm2, \ldots . \quad (6)$$

Therefore, in the case of quasistationary states the quantity $\Delta\equiv(\omega_p/2-\omega_k)/\gamma_k$ can assume a rational set of values, all multiples of Ω. In other words, a parametric resonance is possible when the following "combination condition" is satisfied:

$$n\omega_p-n_1\omega_m=2n\omega_k. \quad (7)$$

This condition can be regarded as the definition of those values of k at which parametric buildup of spin waves is possible. The possibility of parametric excitation of spin waves at different effective pump frequencies [see Eq. (7)] was first pointed out in Ref. 11 in connection with a calculation of the excitation threshold of parametric spin waves. Obviously, above the threshold a signal from a parametric system is formed by a set of excited modes with different values of ω_k. Then, the behavior of each mode is described by its own set of values of N_k and θ_k. However, the interaction between different parametric spin waves reduces (within the framework of the S theory) to a mutual renormalization of the spectrum.

It should be pointed out that, strictly speaking, the system (1) is valid only in the case of fast modulation of the spectrum ($\omega_m \gg \gamma_k$), when a narrow packet of magnons is excited at the pumping threshold. However, if $\omega_m \lesssim \gamma_k$, then modulation of the spectrum of parametric spin waves becomes smooth and the process of formation of a parametric instability now involves groups of magnons with different wave numbers: the amplitudes of the modes satisfying at a given moment the parametric resonance condition increase, whereas the amplitudes of those which are off resonance decrease. These circumstances can be allowed for by rewriting the resonance condition (7) in the form

$$n\omega_p-n_1\omega_m=2n\overline{\omega_k(t)}, \quad (8)$$

where

$$\overline{\omega_k(t)}\equiv\frac{1}{t_0}\int_{t-t_0}^{t}\omega_k(t_1)\,dt_1,$$

and t_0 is the characteristic time of the correlation between groups of excited magnons (essentially t_0^{-1} represents the width of a packet of parametric spin waves).

Consequently, the effective modulation field in Eq. (1a) becomes

$$\overline{\omega_k(t)}-\omega_k(t)\to a\cos\Omega\tau-a(\sin\xi/\xi)\cos(\Omega\tau+\xi)$$

$$=\tilde{a}\cos(\Omega\tau+\varphi), \quad (9)$$

where

$$\tilde{a}=a[1+(\sin\xi/\xi)^2-\sin 2\xi/\xi]^{1/2},$$
$$\cos\varphi=(a/\tilde{a})(1-\sin 2\xi/2\xi), \quad (10a)$$
$$\xi=\Omega\tau_0/2, \quad \tau_0\equiv2\gamma_k t_0.$$

We can easily show that in the limit $\Omega\gg\tau_0^{-1}$ Eq. (9) corresponds to fast modulation of the spectrum ($\tilde{a}\approx a$), whereas in the opposite limiting case a packet of parametric spin waves can follow changes in the magnetic field ($\tilde{a}\approx a\xi$). Since the conditions for the formation of a packet of parametric spin waves depend strongly on the rate of change of the magnetic field, the time t_0 depends on both H_m and ω_m, which makes it difficult to determine the quantity $\tilde{a}$ from Eq. (10a) in the range of frequencies where $\Omega\sim\tau_0^{-1}$. Therefore, in our subsequent calculations we shall use a simplified formula

$$\tilde{a}=a\xi/(1+\xi), \quad (10b)$$

which corresponds to the limiting cases.

PARAMETRIC INSTABILITY THRESHOLD

We shall now calculate the parametric instability threshold under conditions of modulation of the magnon spectrum. Since quasistationary states satisfy the condition

$$\langle d\ln N_k/d\tau\rangle=0,$$

it follows that the averaged form of Eq. (3b) can be repre-

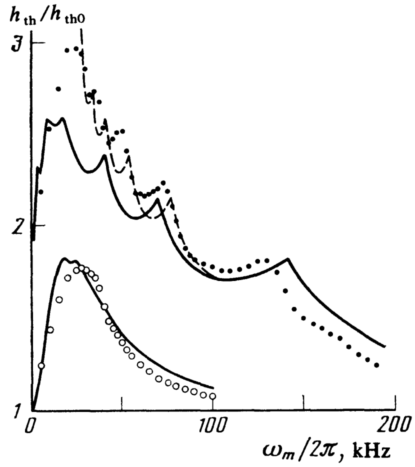

FIG. 1. Relative increase in the threshold of parametric excitation of nuclear spin waves in $CsMnF_3$ plotted as a function of the modulation frequency and amplitude (○)—$H_m = 0.28$ Oe, ●—$H_m = 0.85$ Oe) recorded at $T = 4.2$ K for $H_0 = 1.18$ kOe and $\omega_p/2\pi = 1$ GHz. The dashed curve represents a theoretical dependence from Ref. 11. The continuous curves are drawn using our formulas (14) and (16).

sented by

$$b = [\langle \cos\theta_k \cos\Delta\tau\rangle - \langle \sin\theta_k \sin\Delta\tau\rangle]^{-1}. \qquad (11)$$

In the equation for the phase θ_k near the pumping threshold we can ignore the term $(2r_T + r_S)N_k$ responsible for the magnon-magnon interactions. Consequently, allowing for Eq. (9), we obtain

$$d\theta_k/d\tau + b\sin(\theta_k + \Delta\tau) = \tilde{a}\cos(\Omega\tau + \varphi). \qquad (12)$$

A self-consistent solution of Eqs. (11) and (12) allows us to determine the parametric instability threshold $b_{th} \equiv h_{th}/h_{th0}$ at given values of $\tilde{a}$, Ω, and Δ. In general, we can find b_{th} only by a numerical calculation on a computer. An approximate solution of Eq. (12) can be represented in the form

$$\theta_k(\tau) = \theta_k(0) + \tilde{a}(\Omega^2+1)^{-1/2}\sin(\Omega\tau + \tilde{\varphi}),$$
$$\tilde{\varphi} = \varphi + \varphi_1, \quad \operatorname{tg}\varphi_1 = 1/\Omega. \qquad (13)$$

This solution is valid for all values of Ω if $\tilde{a} \ll 1$, $\Delta = 0$, and $\theta_k(0) = 0$. In the opposite case ($\tilde{a} \gg 1$), the solution (13) is valid only at high frequencies $\Omega \gg 1$.

In substituting Eq. (13) into Eq. (11), we must use the following formulas from Ref. 12:

$$\exp(ix\sin y) = J_0(x) + 2\sum_{n=1}^{\infty}\{J_{2n}(x)\cos(2ny) + iJ_{2n-1}(x)\sin[(2n-1)y]\},$$

where $J_n(x)$ are Bessel functions.

Simple transformations make it possible to derive the following from Eq. (11):

$$b_{th} = \min_l |J_l^{-1}(\tilde{a}(\Omega^2+1)^{-1/2})|; \qquad (14)$$

we then have

$$\theta_k(0) - l\tilde{\varphi} = \begin{cases} 0, & l = 0, 2, 4, \ldots \\ \pi, & l = 1, 3, 5, \ldots \end{cases}.$$

Therefore, the relative increase in the excitation threshold of parametric spin waves under strong modulation conditions is governed not only by the deviation (on the average) from the parametric resonance condition, but also by the possibility of development of a parametric instability at combination frequencies $\omega_p \pm l\omega_m$. A result similar to Eq. (14) was obtained earlier[11] in the resonance approximation. The formulas are identical for $\Omega \gg 1$, $\tau_0^{-1} = 0$ and $\partial\omega_k/\partial H \equiv 2\mu$, where μ is a free parameter in Ref. 11.

In the case of small modulation amplitudes ($\tilde{a} \ll 1$), Eq. (14) can be transformed to

$$b_{th} \approx 1 + \tilde{a}^2/4(\Omega^2+1). \qquad (15)$$

If $\tau_0^{-1} = 0$, this relationship is equivalent to the formula for the threshold field obtained in Ref. 4.

We shall now compare the results of calculations of the threshold carried out using Eq. (14) with the data obtained from an experiment involving excitation of nuclear spin waves. In this case the explicit form of the argument of the Bessel functions is

$$\frac{\tilde{a}}{(\Omega^2+1)^{1/2}} = 2\frac{\partial\omega_k}{\partial H}\frac{H_m}{(\omega_m^2+4\gamma_k^2)^{1/2}}\frac{\xi}{1+\xi},$$
$$\frac{\partial\omega_k}{\partial H} = \frac{\omega_n}{\omega_k}\left[1-\left(\frac{\omega_k}{\omega_n}\right)^2\right]^2\frac{\omega_n(2H+H_D)}{2H_\Delta^2}. \qquad (16)$$

Here, ω_n is the unshifted NMR frequency; H_D is the Dzyaloshinskii field; $H_\Delta^2 \propto 1/T$ is the hyperfine interaction parameter.

An experimental investigation of the parametric excitation threshold of nuclear spin waves was made under conditions of strong modulation of the spectrum of these waves in easy-plane antiferromagnets $CsMnF_3$ and $MnCO_3$ in the range of pump frequencies $\omega_p/2\pi = 1.0$–1.2 GHz at temperature $T = 1.6$–4.2 K. We used the method described in Refs. 7 and 8 (the values of H_m were mistakenly underestimated in Refs. 7 and 8 by a factor $\sqrt{2}$).

Figure 1 shows the relative increase in the threshold h_{th}/h_{th0} of a sample of $CsMnF_3$ as a function of the modulation frequency and amplitude. In contrast to similar curves for electron magnons, studied earlier in the same antiferromagnets[6] and in YIG crystals,[4] we observed here singularities in the form of peaks which shifted on increase in H_m toward higher frequencies. In high fields $H_m \approx 4$ Oe we were able to resolve the peaks with numbers right up to $n = 10$. A qualitative account of these singularities was proposed in Ref. 7. A more rigorous theory of Ref. 11 allowing for the pumping at combination frequencies provided a satisfactory description of the experimental curves in the frequency range $\omega_m \gtrsim 2\gamma_k$ (dashed curve in Fig. 1). However, at lower values of ω_m the ratio h_{th}/h_{th0} should rise without limit if the theory of Ref. 11 is obeyed. The continuous curves in Fig. 1 are the theoretical dependences calculated from Eqs. (14) and (16) using the parameters taken from our experiments. The free parameter was $\tau_0^{-1} = 0.09$ for the upper curve and $\tau_0^{-1} = 0.02$ for the lower curve. The assumption that $\tau_0^{-1} \neq 0$ (i.e., that the width of a packet of parametric spin waves was finite) yielded a fundamentally new result: $h_{th}/h_{th0} \to 1$ in the limit $\omega_m \to 0$.

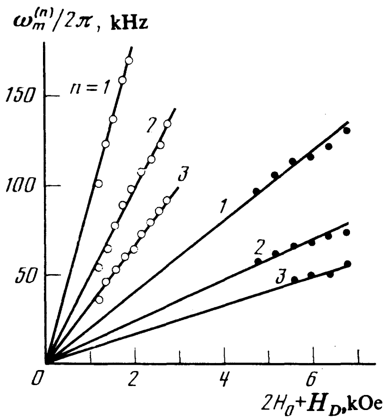

FIG. 2. Dependences of the frequencies of the peaks on an external magnetic field H_0: O) $CsMnF_3$, $H_D=0$, $T=4.2$ K, $H_m=1.4$ Oe; ●) $MnCO_3$, $H_D=4.4$ kOe, $T=1.7$ K, $H_m=1.15$ Oe, $\omega_p/2\pi=1$ GHz.

According to Eq. (14), peaks of the threshold h_{th}/h_{th0} occur when the values of the Bessel functions become equal as the argument increases: $J_{n-1}(x_n)=J_n(x_n)$. This occurs at $x_1\approx1.45$, $x_2\approx2.65$, $x_3\approx3.7$, $x_4\approx4.9$, etc. Therefore, variation of the parameters in the argument of Eq. (16) shifts similarly the threshold. Figure 2 shows the dependences of the positions ($\omega_m^{(n)}$) of the first three peaks ($n=1$, 2, 3) on the combination (sum) of the fields ($2H_0+H_D$). We can see that in the case of $CsMnF_3$ ($H_D=0$) and $MnCO_3$ ($H_D=4.4$ kOe) the positions of the peaks in the range $\omega_m>2\gamma_k$ are accurately proportional to that specific combination of the fields, as predicted by Eqs. (16) and (14). At the end of this section we shall give the results of a theoretical calculation of the threshold of parallel pumping of parametric spin waves under conditions of two-frequency modulation of the spectrum:

$$H_m(t)=H_{m1}\cos\omega_{m1}t+H_{m2}\cos(\omega_{m2}t+\varphi_{12}), \tag{17}$$

where φ_{12} is the angle of dephasing (phase mismatch) between the above two rf fields. We shall simplify the calculations by using an approximate solution for the phase θ_k in the form

$$\theta_k\approx\theta_k(0)+(a_1/\Omega_1)\sin\Omega_1\tau+(a_2/\Omega_2)\sin(\Omega_2\tau+\varphi_{12}),$$

where

$$\Omega_j\equiv\frac{\omega_{mj}}{2\gamma_k},\quad a_j\equiv-\frac{\partial\omega_k}{\partial H}\frac{H_{mj}}{\gamma_k},\quad j=1,2.$$

We can easily see that in the case of quasistationary states and two-frequency modulation by the field of Eq. (17) the mismatch Δ can be found from the relationships

$$\Omega_1\tau_m=2\pi n_1,\quad \Omega_2\tau_m=2\pi n_2,\quad \Delta=2\pi\tilde{n}/\tau_m,$$
$$n_1=1,2,3,\ldots;\ n_2=1,2,3,\ldots;\ \tilde{n}=0;\ \pm1,\pm2\ldots. \tag{18}$$

We carried out a computer calculation of the threshold using Eq. (11) in the frequency range $\omega_m>2\gamma_k$ assuming that $\varphi_{12}=0[\theta_k(0)=0$ or π, in order to ensure that $b_{th}>0]$. The minimum value of the threshold was found by selecting suitable combinations described by Eq. (18). The results are presented in Fig. 3a. Figure 3b gives the experimental data obtained in Ref. 8 (the theoretical calculation was carried out for specific values of the parameters in this investigation). We can see that there is a qualitative agreement between the theoretical and experimental curves. Some quantitative discrepancy is due to the simplifications of the model used in the calculations.

COLLECTIVE OSCILLATIONS IN A SYSTEM OF PARAMETRIC SPIN WAVES

The interaction of parametric spin waves with one another and with an external pump field gives rise to collective oscillations in the spectrum of an excited magnetic crystal[3] and these represent oscillations of N_k and Θ_k against the background of a stationary state of parametric spin waves above the threshold. As already pointed out, a new phenomenon in the form of a double parametric resonance of magnons was discovered and reported in Ref. 4. It was found that an rf field $H_m\cos\omega_m t$ excited parametrically collective oscillations of frequency $\omega_m/2$ in a system of parametric spin waves of YIG when the amplitude of the rf field exceeded a certain threshold value. This phenomenon has been observed subsequently in systems of electron[6] and nuclear[9] spin waves in antiferromagnetics $CsMnF_3$ and $MnCO_3$. The experimental results obtained were in reasonably satisfactory agreement with the theory of Ref. 4 in respect of the dependence of the threshold rf field (needed for the excitation of parametric collective oscillations) on the excess above the threshold h/h_{th0}. However, these experiments revealed also a second threshold at which a double parametric resonance was suppressed at high values of H_m and this was not explained by the theory of Ref. 4. The new threshold stimulated further experiments, which we carried out on a sample of $CsMnF_3$ using high values of H_m. The black dots in Fig. 4b represent the dependence of the relative increase in the threshold h_{th}/h_{th0} on H_m and the open circles are boun-

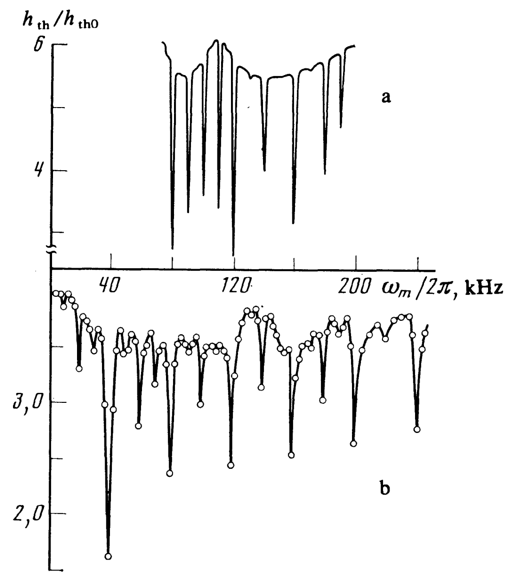

FIG. 3. Relative increase in the pumping threshold of nuclear spin waves under two-frequency modulation conditions ($\omega_{m1}\equiv\omega_m$, $\omega_{m2}=2\pi\cdot40$ kHz): a) theory; b) experimental results[8] for $CsMnF_3$; $H_{m1}=H_{m2}=1.13$ Oe, $T=4.2$ K, $H_0=1.2$ kOe, $\omega_p/2\pi=1$ GHz, $h_{th0}=0.2$ Oe, $\gamma_k\approx2\pi\cdot20$ kHz.

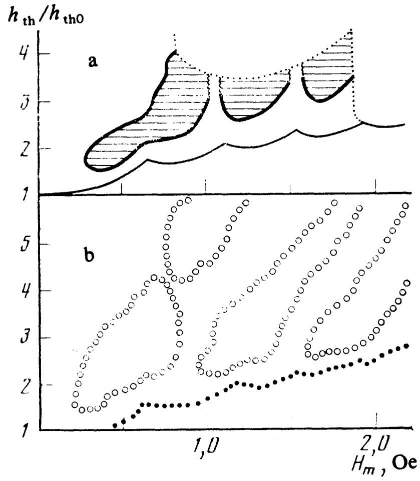

FIG. 4. Behavior of a system of parametric nuclear spin waves above the threshold. a) Theoretical results where a thin continuous line shows the dependence $h_{th}(H_m)/h_{th0}$. The dashed regions represent the subharmonic signal $\omega_m/2$. The dotted curve is the limit of the calculations. In the regions between the wavy lines the oscillation regime is unstable (stochastic). b) Experimental results obtained at $T = 4.2$ K for $H_0 = 1.2$ kOe, $\omega_p = 2\pi\cdot 1015$ MHz, $\omega_m = 2\pi\cdot 80$ kHz. The black dots represent the parametric instability threshold h_{th}/h_{th0}. The open circles are the boundaries of the regions where the $\omega_m/2$ subharmonic signal is observed.

daries of the regions in the $(H_m, h/h_{th0})$ plane inside which there is a subharmonic $\omega_m/2$ in the absorption spectrum of this sample. We can see that above the threshold of suppression of the double parametric resonance there are at least two more regions where this effect reappears and disappears, and the positions of these regions in the plane in question are correlated in a certain manner with the peaks of the lower curve. The behavior of the $\omega_m/2$ signal inside all the regions of existence of the double parametric resonance is approximately the same: a signal $\omega_m/2$ appears at the resonance excitation threshold and then, as H_m is increased (for a fixed value of h/h_{th0}) or h/h_{th0} is increased (keeping H_m fixed), the amplitude of the signal rises, reaches a maximum, and falls again to the noise level. An irregular (random) spectrum is frequently observed beyond the maximum of the $\omega_m/2$ signal. No other subharmonics of the modulation frequency, apart from $\omega_m/2$, were observed in this experiment, whereas a study of a double parametric resonance of electron magnons in the same sample revealed not only $\omega_m/2$ but also the subharmonics $\omega_m/4$ and $\omega_m/8$, indicating the possibility of realization in a system of parametric spin waves of stochastic regimes which develop after doubling of the oscillation period.[13,14]

We also simulated the experimental situation on a computer. We represented the system of equations (3) in the form

$$d\theta_j/d\tau + b\sin(\theta_j + \Delta_j\tau) = a\cos\Omega\tau + n_j + 2n, \quad (19a)$$

$$d\ln n_j/d\tau = b\cos(\theta_j + \Delta_j\tau) - 1 - \varkappa n, \quad (19b)$$

where

$$\Delta_j = j\Omega, \quad j = 0, \pm 1, \pm 2, \pm 3; \quad n_j \equiv r_S N_j;$$

$$(r_S = r_T); \quad n = \sum_j n_j.$$

The term $-\varkappa n$ in Eq. (19b) describes a positive nonlinear attenuation of parametric spin waves. A numerical solution of the system (19) was made by the Newton method (100 steps per period) with initial parameters $\theta_j = 0, n_j = 0, 1$ for all the modes. The values of the parameters were selected in accordance with the experimental conditions: $\gamma_k = 2\pi\cdot 24.3$ kHz, $a = 4.68\, H_m$ Oe, $\varkappa = 0.2$. After 150 cycles (when the transient processes were completed), the spectrum of oscillations of the number of parametric spin waves was calculated by the fast Fourier transform method. Figure 5 shows typical Fourier spectra of a system of parametric spin waves which appear on increase in the parameter $b \equiv h/h_{th0}$. We can see how chaos grows in the system as a result of a sequence of bifurcations causing doubling of the oscillation period. It should be pointed out that the suppression of the subharmonic spectrum (on further increase of the excess above the threshold) occurs in the reverse order (..., $\omega_m/4$, $\omega_m/2$, ω_m). The zeroth level in Fig. 5 corresponds to the maximum sensitivity of the apparatus. Therefore, the theoretical calculation demonstrates that it is possible to record the signal only of the subharmonic $\omega_m/2$, as indeed found experimentally. Figure 4a shows the behavior of the parametric system above the threshold. The calculated regions where the subharmonic $\omega_m/2$ appears are shown shaded. In the intervals between the boundaries of the regions (located between the wavy lines) it is difficult to carry out numerical calculations because stochastic oscillation regimes extremely sensitive to small changes in the external conditions (as naturally manifested in the form of fluctuations) appear in the corresponding ranges of the parameters. A comparison of the theoretical and experimental behavior in Fig. 4 shows that the agreement is qualitative.

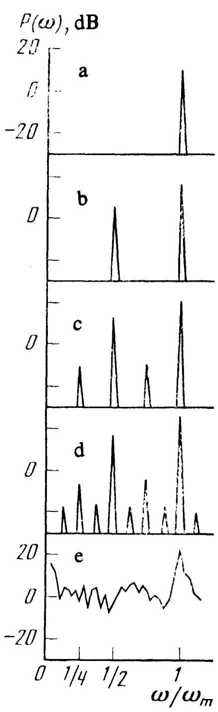

FIG. 5. Fourier spectra of a system of parametric spin waves calculated for $H_m = 0.78$ Oe and different values of $b \equiv h/h_{th0}$: a) 2.2; b) 2.36; c) 2.38; d) 2.383; e) 2.6. The zeroth-frequency signal is not shown.

CONCLUSIONS

It is shown that parametric excitation of magnons under conditions of strong modulation of their spectrum can be described using equations of the S theory. An analysis of these equations demonstrates that there is a class of solutions corresponding to parametric instabilities of magnons at combination frequencies (composed of the pump and modulation frequencies). As a result, the response of a parametrically excited system in the regime above the threshold is due to a set of groups of parametric spin waves with different effective pump frequencies. The interaction between these groups gives rise to a mutual renormalization of their spectrum and the nonlinear attenuation of parametric spin waves is governed by the total intensity of their excitation.

At low modulation frequencies the response of a parametrically excited system includes contributions not only from parametric spin waves which are in resonance with the pump wave, but also from a group of parametric spin waves off resonance. A parametric system can "follow" changes in the magnetic field, which weakens the influence of the modulating field on the threshold of excitation of parametric spin waves when the modulation frequency is lowered.

Our theoretical analysis accounts for a number of experimental results on the parametric excitation of nuclear spin waves under conditions of strong modulation of their spectrum:

1) a quantitative description is obtained of a nonmonotonic dependence of the relative threshold of parametric excitation of nuclear spin waves throughout the investigated range of modulation frequencies and amplitudes, including a dependence of the positions of the peaks on an external magnetic field;

2) a qualitative description is given of the characteristics of the excitation threshold of nuclear spin waves under conditions of two-frequency modulation of the spectrum;

3) numerical simulation is made of the behavior of a parametric system above the threshold and the results are in qualitative agreement with experiments on a double parametric resonance of nuclear spin waves.

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