Tangential molecular forces in a nonequilibrium fluctuating electromagnetic field

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It is shown that, in the presence of gyrotropy, tangential molecular forces due to a fluctuating electromagnetic field can exist between media heated to different temperatures and filling half-spaces separated by a plane-parallel vacuum gap. A general expression for such a force is obtained in terms of the surface-impedance tensors of the media, and the particular case of weak gyrotropy, which demonstrates patently the presence of the tangential force, is considered in detail.

The fluctuating magnetic field due to polarizability and magnetization fluctuations in substances is the cause of a number of important physical effects. These include the molecular attraction forces (Van der Waals forces) first investigated in Ref. 1. Another effect of practical importance is heat exchange between bodies heated to different temperatures.[2] If the gap between them is small, the heat flux deviates anomalously from the value given by classical theory of thermal radiation. The resultant dependence of the heat flux on the gap width is quite unexpected (it can have a minimum). The present paper deals with one more interesting manifestation of the nonequilibrium nearby fluctuating field, whereby tangential forces are produced between the bodies in the presence of gyrotropy. Just as in Refs. 1 and 2, we consider very simple geometric conditions (Fig. 1). The half-spaces $z<0$ and $z>a$ are filled respectively with media 1 and 2. The plane-parallel gap between them is assumed to be a vacuum, and the properties of the media are taken to be independent of the tangential coordinates $x_1 \equiv x$ and $x_2 \equiv y$. We shall show that under certain conditions there can exist forces directed along the boundaries of the media and tending to set them in motion relative to each other. We call them tangential molecular forces. It will be shown below that the conditions referred to are, first, that at least one of the media be gyrotropic in the presence of a constant (time-independent) external magnetizing field **B**, hereafter assumed to be uniform; second, the temperatures of the media must be different.

1. GENERAL EXPRESSION FOR THE TANGENTIAL FORCE

We consider first the case when media 1 and 2 that fill the half-spaces $z<0$ and $z>a$ are nongyrotropic but have an arbitrary anisotropy. The spectral density (in the positive frequencies) of the tangential force [designated $F_\alpha(\omega)$] acting on medium 1 per unit surface is obviously determined by the αz components (the Greek subscripts take on here and elsewhere the values 1 and 2) of the electromagnetic stress tensor $T_{ik}(\mathbf{r},\omega)$, i, $k=1,2,3$ taken at an arbitrary point **r** inside the gap (clearly, the force should not depend on **r**). We have thus for the spectral density of the tangential force the expression[1)]

$$F_\alpha(\omega)=\frac{1}{4\pi}\langle E_\alpha^*(\mathbf{r},\omega)E_z(\mathbf{r},\omega)\rangle+\frac{1}{4\pi}\langle H_\alpha^*(\mathbf{r},\omega)H_z(\mathbf{r},\omega)\rangle+\text{c.c.} \qquad (1)$$

where the asterisk denotes complex conjugation and the angle brackets statistical averaging. The experimentally observed quantity is not $F_\alpha(\omega)$ but the total force obtained by integrating $F_\alpha(\omega)$ with respect to frequency from 0 to $+\infty$.

Since the fluctuating-electromagnetic-field sources distributed in media 1 and 2 are statistically independent of one another, each of the correlators $\langle ...\rangle$ in Eq. (1) can be written as a sum of the correlators $\langle ...\rangle_1$ and $\langle ...\rangle_2$ of the fields generated by media 1 and 2 respectively. $F_\alpha(\omega)$ takes then, in accordance with (1), the form

$$F_\alpha(\omega)=\frac{1}{4\pi}\langle E_\alpha^*(\mathbf{r},\omega)E_z(\mathbf{r},\omega)\rangle_1+\frac{1}{4\pi}\langle E_\alpha^*(\mathbf{r},\omega)E_z(\mathbf{r},\omega)\rangle_2+\frac{1}{4\pi}\langle H_\alpha^*(\mathbf{r},\omega)H_z(\mathbf{r},\omega)\rangle_1+\frac{1}{4\pi}\langle H_\alpha^*(\mathbf{r},\omega)H_z(\mathbf{r},\omega)\rangle_2+\text{c.c.}$$

The problem of finding the tangential force is thus reduced to that of finding of the correlators of the fields generated by media 1 and 2.

A general theory of equilibrium thermal fluctuations of an electromagnetic field was developed in Refs. 3 and 4. The exposition in Ref. 4, which leads to a generalized Kirchoff's law, besides physically more satisfactory, simplifies substantially the calculations.

To determine the fluctuating-electromagnetic-field correlator generated by medium 1 at a point **r** it is necessary, according to the generalized Kirchoff's law, to calculate the thermal losses of the fields in medium 1 due to auxiliary fields located at the point **r** and having a definite orientation. The correlators are then obtained by familiar means.[4] Similarly, to determine the correlators of the fields generated by medium 2 we must know the thermal losses in medium 2.

The quantities most suitable for the determination of the thermal losses in media 1 and 2 are their surface-impedance tensors that relate, on the boundaries of the media, the tangential components of the electric and magnetic fields (see, e.g., Refs. 2 and 5). Expanding the fields $\mathbf{E}(t,\mathbf{r})$ and

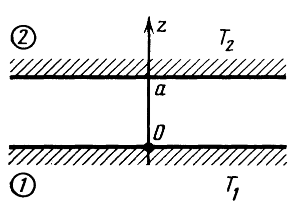

FIG. 1

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$\mathbf{H}(t,\mathbf{r})$ in Fourier integrals with respect to time and to the tangential coordinates $x = \{x_1, x_2\}$:

$$\begin{Bmatrix} \mathbf{E}(t,\mathbf{r}) \\ \mathbf{H}(t,\mathbf{r}) \end{Bmatrix} = \int d\omega \int d^2\varkappa \begin{Bmatrix} \mathbf{E}(\omega,\varkappa;z) \\ \mathbf{H}(\omega,\varkappa;z) \end{Bmatrix} e^{i\varkappa x - i\omega t},$$

where $\varkappa = \{\varkappa_1, \varkappa_2\}$ is a two-dimensional wave vector, the impedance relation on the boundary of medium 1 takes the form

$$E_\alpha(\omega,\varkappa) = \zeta_{2\alpha\beta}(\omega,\varkappa)[\mathbf{H}(\omega,\varkappa)\times\mathbf{n}]_\beta,$$

and on the boundary of medium 1

$$E_\alpha(\omega,\varkappa) = -\zeta_{1\alpha\beta}(\omega,\varkappa)[\mathbf{H}(\omega,\varkappa)\times\mathbf{n}]_\beta, \tag{2}$$

where $\zeta_{1\alpha\beta}$ and $\zeta_{2\alpha\beta}$ are the two-dimensional surface-impedance tensors of media 1 and 2, $\mathbf{n}$ is a unit vector directed along the positive z axis, and summation over repeated indices is carried out as usual. A minus sign is contained in the boundary condition (2) because the surface impedance is defined here relative to a unit normal vector directed to the interior of the medium.

To shorten the equations we use hereafter matrix notation. We use $\hat{\zeta}_1$ and $\hat{\zeta}_2$ to denote 2×2 matrices with respective elements $\zeta_{1\alpha\beta}$ and $\zeta_{2\alpha\beta}$, with the subscripts α and β numbering the rows and columns, respectively.

The use of the generalized Kirchoff's law leads to the following expression for the spectral density of the tangential force $F_\alpha(\omega)$ acting on the medium 1, in terms of the tensors of the two surface impedances (we omit the straightforward but unwieldy algebra):

$$F_\alpha(\omega) = \frac{\Pi(T_1) - \Pi(T_2)}{2\pi^3\omega} \int d^2\varkappa\, \varkappa_\alpha \mathscr{M}(\omega,\varkappa), \tag{3}$$

where

$$\Pi(T) \equiv \hbar\omega[\exp(\hbar\omega/k_B T) - 1]^{-1},$$

k_B is the Boltzmann constant, T is the absolute temperature,

$$\mathscr{M}(\omega,\varkappa) = \mathrm{Sp}\{\hat{\xi}^+ \hat{R}^+ (\hat{\zeta}_1 + \hat{\zeta}_1^+) \hat{R}\hat{\xi}(\hat{\zeta}_2 + \hat{\zeta}_2^+)\}, \tag{4}$$

R stands for the matrix

$$\hat{R}^{-1} = (1 + \hat{\xi}\hat{\zeta}_2)(1 + \hat{\xi}\hat{\zeta}_1)e^{qa} - (1 - \hat{\xi}\hat{\zeta}_2)(1 - \hat{\xi}\hat{\zeta}_1)e^{-qa},$$

the superscript + denotes Hermitian conjugation, $q = (\varkappa^2 - k^2)^{1/2}$, $\varkappa$ is the modulus of the wave vector $\varkappa$, and $k = \omega/c$. Finally, $\hat{\xi}$ is the vacuum surface-impedance tensor (see Ref. 2):

$$\xi_{\alpha\beta} = \frac{iq}{k}\left(\delta_{\alpha\beta} - \frac{\varkappa_\alpha\varkappa_\beta}{\varkappa^2}\right) + \frac{k}{iq}\frac{\varkappa_\alpha\varkappa_\beta}{\varkappa^2}.$$

We note first that it can be seen from (3) that nonzero tangential forces can exist only if $T_1 \neq T_2$, i.e., in a nonequilibrium thermodynamic system. This is of course natural, for otherwise a perpetual-motion engine of the second kind would be feasible. Next, there can be no tangential forces at all between nongyrotropic media (if the latter have arbitrary anisotropy). In fact, using the temporal reversibility of the microequations of motion, we can show in the usual manner (see, e.g., Ref. 6, § 96) that in the absence of gyrotropy the surface-impedance tensors of media 1 and 2 should satisfy the following symmetry relations:

$$\zeta_{1\alpha\beta}(\omega,\varkappa) = \zeta_{1\beta\alpha}(\omega,-\varkappa), \quad \zeta_{2\alpha\beta}(\omega,\varkappa) = \zeta_{2\beta\alpha}(\omega,-\varkappa).$$

It can be easily verified that this leads to

$$\mathscr{M}(\omega,\varkappa) = \mathscr{M}(\omega,-\varkappa). \tag{5}$$

Therefore the tangential force is zero, since the integral in (3) vanishes.

Let us examine now the situation in the case when media 1 and (or) 2 are gyrotropic and are located in a magnetizing field $\mathbf{B}_0$. To calculate the correlators of the thermal electromagnetic field in this case we must, according to the generalized Kirchoff law, calculate the thermal losses for the reversed magnetizing field $(-\mathbf{B}_0)$. The expression for the tangential forces takes then, as before the form (3), but the quantity $\mathscr{M}$, which depends now on $\mathbf{B}_0$ must be taken for the reversed magnetizing field. Thus, in the presence of gyrotropy we have for the tangential force the expression

$$F_\alpha(\omega;\mathbf{B}_0) = \frac{\Pi(T_1) - \Pi(T_2)}{2\pi^3\omega} \int d^2\varkappa\, \varkappa_\alpha \mathscr{M}(\omega,\varkappa;-\mathbf{B}_0). \tag{6}$$

If media 1 and 2 are gyrotropic, their surface-impedance tensors have the following symmetry properties

$$\zeta_{1\alpha\beta}(\omega,\varkappa;\mathbf{B}_0) = \zeta_{1\beta\alpha}(\omega,-\varkappa;-\mathbf{B}_0),$$

$$\zeta_{2\alpha\beta}(\omega,\varkappa;\mathbf{B}_0) = \zeta_{2\beta\alpha}(\omega,-\varkappa;-\mathbf{B}_0),$$

while $\mathscr{M}$ has the property

$$\mathscr{M}(\omega,\varkappa;\mathbf{B}_0) = \mathscr{M}(\omega,-\varkappa;-\mathbf{B}_0). \tag{7}$$

Therefore, reversing the sign of the integration variable in (6) and using Eq. (7), we can rewrite (6) in the form

$$F_\alpha(\omega;\mathbf{B}_0) = -\frac{\Pi(T_1) - \Pi(T_2)}{2\pi^3\omega} \int d^2\varkappa\, \varkappa_\alpha \mathscr{M}(\omega,\varkappa;\mathbf{B}_0). \tag{8}$$

From a comparison of (8) and (6) it follows that

$$F_\alpha(\omega;\mathbf{B}_0) = -F_\alpha(\omega;-\mathbf{B}_0).$$

The tangential force thus reverses sign together with $\mathbf{B}_0$, i.e., it is an odd function of the magnetizing field.

Of course, the foregoing analysis does not mean that tangential forces must exist in the case of gyrotropic media. It only shows that the general symmetry properties of the kinetic coefficients (in this case, of the surface-impedance tensors of the media) do not forbid their existence, as they do in the absence of gyrotropy. To verify the existence of tangential forces, it is simpler to consider some particular case.

2. TANGENTIAL FORCES IN THE PRESENCE OF WEAK GYROTROPY

Let, in the absence of a magnetizing field, the media 1 and 2 be homogeneous and isotropic, with respective permittivities and permeabilities $\varepsilon_1(\omega)$, $\mu_1(\omega)$, and $\varepsilon_2(\omega)$, $\mu_2(\omega)$.

We confine ourselves in the calculation of the tangential forces to the first order in the field $\mathbf{B}_0$.

As shown above, in the absence of a magnetizing field the tangential molecular forces are zero. Thus, the term of first order in $\mathbf{B}_0$ will be the first term of the power-law expansion. In this approximation, the dielectric tensors of media 1 and 2 are known [see Ref. 6, § 101] to be of the form

$$\varepsilon_{1jk}(\omega;\ \mathbf{B}_0)=\varepsilon_1(\omega)\delta_{jk}+ib_1(\omega)e_{jkl}B_{0l}, \quad (9)$$

$$\varepsilon_{2jk}(\omega;\ \mathbf{B}_0)=\varepsilon_2(\omega)\delta_{jk}+ib_2(\omega)e_{jkl}B_{0l}, \quad (10)$$

where e_{jkl} is a fully antisymmetric unit tensor ($e_{123}=1$), and the coefficients $b_1(\omega)$ and $b_2(\omega)$ determine the electric gyrotropies of the media. To simplify (without fundamentally changing) somewhat the calculations we assume that media 1 and 2 have only electric gyrotropy, so that the magnetic permeabilities of the media remain unchanged when the magnetizing field $\mathbf{B}_0$ is applied.

If the magnetizing field is perpendicular to the boundaries of media 1 and 2, i.e., is directed along the z axis, it follows from symmetry considerations that there are no tangential forces in this case. We direct therefore the magnetizing field along the boundaries of the media.

We express the surface impedances of the media, accurate to first order in $\mathbf{B}_0$, in the form

$$\hat{\zeta}_1=\hat{\bar{\zeta}}_1+\delta\hat{\zeta}_1,\quad \hat{\zeta}_2=\hat{\bar{\zeta}}_2+\delta\hat{\zeta}_2.$$

Here $\hat{\bar{\zeta}}_1$ and $\hat{\bar{\zeta}}_2$ are the surface-impedance tensors in the zeroth approximation, i.e., the impedances of the homogeneous and isotropic half-spaces $z<0$ and $z>a$ with constants ε_1,μ_1 and ε_2,μ_2. The expressions for these tensors are

$$\bar{\zeta}_{1\alpha\beta}=\zeta_{1t}(\delta_{\alpha\beta}-\varkappa_\alpha\varkappa_\beta/\varkappa^2)+\zeta_{1l}\varkappa_\alpha\varkappa_\beta/\varkappa^2, \quad (11)$$

$$\zeta_{1t}=k\mu_1/iq_1,\quad \zeta_{1l}=iq_1/k\varepsilon_1,\quad q_1=(\varkappa^2-k^2\varepsilon_1\mu_1)^{1/2}, \quad (12)$$

$$\bar{\zeta}_{2\alpha\beta}=\zeta_{2t}(\delta_{\alpha\beta}-\varkappa_\alpha\varkappa_\beta/\varkappa^2)+\zeta_{2l}\varkappa_\alpha\varkappa_\beta/\varkappa^2, \quad (13)$$

$$\zeta_{2t}=k\mu_2/iq_2,\quad \zeta_{2l}=iq_2/k\varepsilon_2,\quad q_2=(\varkappa^2-k^2\varepsilon_2\mu_2)^{1/2}. \quad (14)$$

The branches of the roots are chosen such that $\mathrm{Re}\, q_{1,2}>0$.

To find the corrections $\delta\hat{\zeta}_1$ and $\delta\hat{\zeta}_2$ we must solve, accurate to first order in $\mathbf{B}_0$, the equation for the surface-impedance tensor of a medium with the constants (9) and (19) [see Ref. 5]. As a result we get

$$\delta\zeta_{1\alpha\beta}=-\frac{\zeta_{1t}}{2q_1}\frac{b_1}{\varepsilon_1}(g_\alpha\varkappa_\beta+g_\beta\varkappa_\alpha)+\frac{b_1}{q_1\varepsilon_1}(\zeta_{1t}-\zeta_{1l})(\varkappa\mathbf{g})\frac{\varkappa_\alpha\varkappa_\beta}{\varkappa^2} \quad (15)$$

$$\delta\zeta_{2\alpha\beta}=\frac{\zeta_{2t}}{2q_2}\frac{b_2}{\varepsilon_2}(g_\alpha\varkappa_\beta+\varkappa_\alpha g_\beta)-\frac{b_2}{q_2\varepsilon_2}(\zeta_{2t}-\zeta_{2l})(\varkappa\mathbf{g})\frac{\varkappa_\alpha\varkappa_\beta}{\varkappa^2}, \quad (16)$$

where $\mathbf{g}=\mathbf{n}\times\mathbf{B}_0$. Using expressions (10)–(16) we obtain from (4):

$$\mathscr{M}(\omega,\varkappa;\ \mathbf{B}_0)=\tilde{\mathscr{M}}(\omega,\varkappa)+\delta\mathscr{M}(\omega,\varkappa;\ \mathbf{B}_0),$$

where $\tilde{\mathscr{M}}$ is the zeroth-order term, and the first-order correction is

$$\delta\mathscr{M}=-2(\varkappa\mathbf{g})\,\mathrm{Re}\left\{\beta_2|q|^2\left(\frac{q_1}{\varepsilon_1}-\frac{q_1^*}{\varepsilon_1^*}\right)\frac{W_\varepsilon}{\Delta_\varepsilon|\Delta_\varepsilon|^2}-(1\leftrightarrow2)\right\}. \quad (17)$$

Here($\leftrightarrow$) denotes the expression obtained from the first term by interchanging the subscripts 1 and 2; in addition, we have introduced the notation

$$W_\varepsilon\equiv\left(q+\frac{q_1}{\varepsilon_1}\right)\left(q+\frac{q_2^*}{\varepsilon_2^*}\right)e^{qa}-\left(q-\frac{q_1}{\varepsilon_1}\right)\left(q-\frac{q_2^*}{\varepsilon_2^*}\right)e^{-qa},$$

$$\Delta_\varepsilon\equiv\left(q+\frac{q_1}{\varepsilon_1}\right)\left(q+\frac{q_2}{\varepsilon_2}\right)e^{qa}-\left(q-\frac{q_1}{\varepsilon_1}\right)\left(q-\frac{q_2}{\varepsilon_2}\right)e^{-qa},$$

$$\beta_2\equiv\frac{b_2}{\varepsilon_2^2}.$$

By virtue of the property (5), $\tilde{\mathscr{M}}$ is an even function of the wave vector and makes therefore no contribution to the tangential force. We have therefore, according to (8) the expression

$$F_\alpha(\omega;\mathbf{B}_0)=-\frac{\Pi(T_1)-\Pi(T_2)}{2\pi^3\omega}\int d^2\varkappa\,\varkappa_\alpha\delta\mathscr{M}(\omega,\varkappa;\mathbf{B}_0).$$

Substituting here $\delta\mathscr{M}$ from (17) we obtain after simple transformations the following final expression for the tangential force acting on the medium 1:

$$\mathbf{F}(\omega;\ \mathbf{B}_0)=\mathbf{g}\{\Pi(T_1)-\Pi(T_2)\}\mathscr{L}(\omega), \quad (18)$$

where

$$\mathscr{L}(\omega)=\frac{1}{\pi^2\omega}\mathrm{Re}\int_0^\infty d\varkappa|q|^2\varkappa^3\left\{\beta_2\left(\frac{q_1}{\varepsilon_1}-\frac{q_1^*}{\varepsilon_1^*}\right)\frac{W_\varepsilon}{\Delta_\varepsilon|\Delta_\varepsilon|^2}-(1\leftrightarrow2)\right\}. \quad (19)$$

It can be seen from (18) that the tangential force is directed along the vector $\mathbf{g}=\mathbf{n}\times\mathbf{B}_0$, i.e., it is parallel to the boundary planes of media 1 and 2 and is perpendicular to the magnetizing field $\mathbf{B}_0$.

The result (18), (19) shows that even in the particular case of weak gyrotropy the tangential force is a complicated function of the gap width a. Its behavior at small a can, however, be easily understood. Putting $a=0$ in (19), we can easily show that the integral diverges at the upper limit. We change therefore in (19) to a new variable $x=a\varkappa$. As a result we find that as $a\to0$ the function $\mathscr{L}(\omega)$ takes the form

$$\mathscr{L}(\omega)\approx\frac{1}{\pi^2\omega a^3}\mathrm{Re}\int_0^\infty dx\,x^2\left\{\beta_2\left(\frac{1}{\varepsilon_1}-\frac{1}{\varepsilon_1^*}\right)\frac{\bar{W}_\varepsilon}{\bar{\Delta}_\varepsilon|\bar{\Delta}_\varepsilon|^2}-(1\leftrightarrow2)\right\}, \quad (20)$$

where

$$\bar{W}_\varepsilon=(1+1/\varepsilon_1)(1+1/\varepsilon_2^*)e^x-(1-1/\varepsilon_1)(1-1/\varepsilon_2^*)e^{-x},$$

$$\bar{\Delta}_\varepsilon=(1+1/\varepsilon_1)(1+1/\varepsilon_2)e^x-(1-1/\varepsilon_1)(1-1/\varepsilon_2)e^{-x}.$$

It follows thus from (20) that at small gap widths the tangential force is proportional to $1/a^2$. Actually the validity of Eq. (20) begins at $a\ll\lambda$, where λ is the smallest of the characteristic wavelengths in the absorption spectra of media 1 and 2.

Further simplification is obtained when the dielectric constants of the media differ little from unity:

$$\varepsilon_1(\omega)=1+\Delta\varepsilon_1(\omega), \quad \varepsilon_2(\omega)=1+\Delta\varepsilon_2(\omega).$$

Then, retaining in (29) only the terms linear in $\Delta\varepsilon_{1,2}$, we get

$$\mathscr{L}(\omega)\approx(\varepsilon_1''b_2''-\varepsilon_2''b_1'')/32\pi^2\omega a^3. \qquad (21)$$

We write here $\varepsilon''_{1,2}$ in place of $\Delta\varepsilon''_{1,2}$, since $\varepsilon''_{1,2}=\Delta\varepsilon''_{1,2}$.

The foregoing consideration of the tangential forces in the case of weak gyrotropy was aimed at proving their existence. From the experimental standpoint, however, interest attaches to the opposite case of strong gyrotropy, when the effect is expected to be noticeable.

Nonetheless, at least a rough estimate of the tangential force in the case of weak gyrotropy is of interest. Assume that the temperature of one of the bodies is significantly higher than that of the other, say $T\equiv T_1\gg T_2$. Assume also that only one of the bodies is gyrotropic (say, 2). At small gap widths a we then obtain from (18) and (21) for the modulus of the total (integrated over all frequencies from 0 to ∞) tangential force

$$F\approx\frac{\hbar}{32\pi^2a^3}\int_0^\infty d\omega\frac{\omega}{\exp(\omega/\omega_T)-1}\frac{\varepsilon_1''(\omega)}{\omega}(b_2''(\omega)B_0),$$

$$\omega_T\equiv\frac{k_BT}{\hbar}.$$

To estimate the integral, the function $(\varepsilon_1''(\omega)/\omega)\times(b_2''(\omega)B_0)$ can be taken outside the integral sign and assumed equal to its value at the characteristic frequency ω_T of the Planck distribution. Let $T=1000$ K. Then

$$F=a^{-3}\varepsilon_1''(\omega_T)(b_2''(\omega_T)B_0)\cdot10^{-15}.$$

We have used equations for media with permittivities and permeabilities close to unity. Let, for example $\varepsilon_1''(\omega_T)\sim10^{-1}$. Since the gyrotropy is small by definition, the quantity $(b_2''(\omega_T)B_0)$ that describes the additional thermal losses due to gyrotropy should be small compared with the constant $\varepsilon_1''(\omega_T)$ that determines the main thermal losses in the system. We put $b_2''(\omega_T)B_0\sim10^{-2}$. We have then for F the estimate

$$F\sim10^{-18}/a^3;$$

at $a\sim1\ \mu$m we get $F\sim10^{-6}$ dyn. We note for comparison that, for example, the Van der Waals attraction force between absolutely cold ideal metals at $a\sim1\ \mu$m is of the order of 10^{-2}. We emphasize once more that this rough estimate, under the assumption of weak gyrotropy, is of little interest for experimental observations, where it is natural to strive for maximum-gyrotropy conditions.

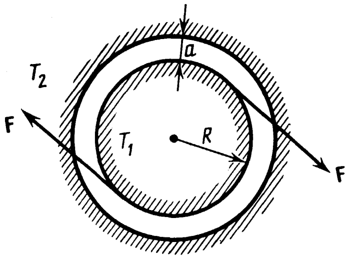

FIG. 2

Expression (18) for the tangential component was obtained assuming weak gyrotropy, but if the media are isotropic at $\mathbf{B}_0=0$ the forces take the same form also for an arbitrary magnetizing field. This follows from the fact that $\mathbf{g}=\mathbf{n}\times\mathbf{B}_0$ is the only true vector that can be performed in this case. Then, of course, $\mathscr{L}$ is a function of $\mathbf{B}_0$, and is furthermore even.

For the direction indicated, it is apparently more convenient to measure the tangential force in the cylindrical system shown in Fig. 2, with the magnetizing field $\mathbf{B}_0$ directed along the axis of the cylinders. Of course, at $R\gg a$ the surface density of the tangential forces can be estimated using the equations for a planar gap. It can be seen from (18) that in this case the tangential forces are directed in the plane of the figure tangent to the surfaces of the cylinders and can thus produce a torque.

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[1)] In contrast to Ref. 2, the expansion here is in Fourier integrals with respect to time, with a factor $\exp(i\omega t)$.

[1]E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **29**, 94 (1955) [Sov. Phys. JETP **2**, 73 (1955)].
[2]M. L. Levin, V. G. Polevoĭ, and S. M. Rytov, ibid. **79**, 2087 (1980) [**52**, 1054 (1980)].
[3]S. M. Rytov, Theory of Electric Fluctuations and of Thermal Radiation [in Russian], USSR Acad. Sci. Press, 1953.
[4]M. L. Levin and S. M. Rytov, Theory of Equilibrium Thermal Fluctuations in Electrodynamics [in Russian], Nauka, 1967.
[5]V. G. Polevoĭ, Izv. Vyssh. Uche. Zav., Radiofizika **27**, 1316 (1984).
[6]L. D. Landau and E. M. Lifshitz, Quantum Electrodynamics, Pergamon, 1984.

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