Oscillations of the chemical potential and the energy spectrum of electrons in the inversion layer at a silicon surface in a magnetic field

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Quantum oscillations of the chemical potential of electrons in the inversion layer of an MOS structure on the (001) surface of silicon are detected and studied at temperatures of 0.4–1.5 K and in fields up to 90 kOe. The oscillations are of order 10 K in size. From an analysis of the shape of the oscillations it is established that the width of the Landau levels oscillates in a magnetic field, increasing in cases when the Fermi level is found to lie midway between energy levels. The values of the valley and spin splitting and their dependence on the magnetic field are determined, and it is shown that this splitting also oscillates when the magnetic field changes, in qualitative agreement with the theory.[1]

1. INTRODUCTION

The intensive study of two-dimensional conductors on semiconductor surfaces in recent years has led to substantial progress toward understanding their properties. It has turned out that in spite of the fundamental simplicity of the effect, the description of real systems, even at the level of constructing their energy spectrum, is an extremely complicated problem that is still far from being solved.[1] This is true of both the energy levels for the motion of electrons perpendicular to the surface (the so-called electrical-quantization levels) and the spectrum describing the motion along the surface, including the levels in a magnetic field directed normal to the surface.

The theoretical construction of the carrier spectrum is complicated by the need to simultaneously take into account various factors such as the effect of a random defect potential, the influence of external electric and magnetic fields, and the interaction of the carriers with the surface and with one another. On the other hand, the extraction of the spectrum parameters from the experimental results can only be done in the framework of some sort of theoretical model. This is equally true both for measurements of the kinetic parameters and for resistance experiments, where different frequency shifts and strong scattering can arise.[1]

Since here we shall be interested in the spectrum in a magnetic field, let us discuss this question in somewhat more detail. For definiteness we shall discuss only the case of an n-type inversion layer at the (001) surface of p-type silicon, the object that has been studied in greatest detail[1] and is the subject of the present paper. The main results known, specifically, the level scheme (Fig. 1) and the characteristic energy splitting, have been obtained from the Shubnikov–de Haas effect: the temperature dependence of the oscillation amplitude was used to determine[2] the effective mass m^* and its dependence on the electron surface density n_s, and the achievement of "spin zero" conditions, i.e., the exact equality of the spin splitting to half of the cyclotron splitting, in a magnetic field inclined to the surface of the sample was used[3] to measure the spin splitting $\Delta_s = g_{\text{eff}}\mu_B H$ and its dependence on the carrier density. It was found that the effective g factor is substantially (by two or more times) greater than the value $g = 2$ which is characteristic for conduction electrons in bulk silicon. This effect has been explained successfully by considering the interaction between electrons, but at the same time the theory also implied that the g factor should oscillate from $g = 1$ at a total system spin moment of zero (i.e., when the same number of levels are filled for spins pointing along the field as against it) to a certain value $g_{\max} > 2$ (Ref. 1). As a result, one had to be resigned to a very large ($\sim 50\%$) uncertainty[4] in the experimental value of $g_{\max}$.

Besides the cyclotron and spin splittings, the electrons in a (001)Si–MOS structure also have a valley splitting Δ_v caused by the lifting of the valley degeneracy associated with the symmetry of the crystal lattice of bulk silicon because of the breaking of the inversion symmetry at the surface. The valley splitting has been detected in a study of the Shubnikov–de Haas effect,[5] but until recently the quantitative values of Δ_v have been known even less reliably than the spin

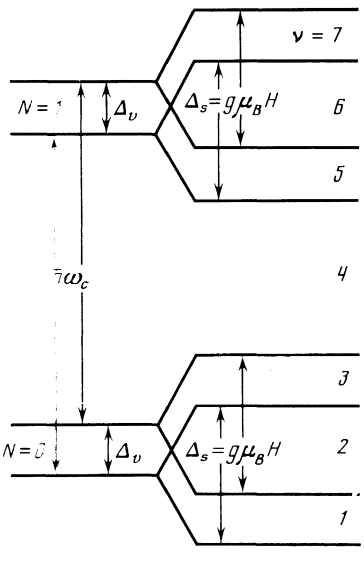

FIG. 1. Energy-level scheme of the n-inversion layer in a Si–MOS structure in a quantizing magnetic field.

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splitting; suffice it to say that a determination of the valley splitting by analysis of the line shape of the quantum oscillations of the conductivity assumes knowledge of the spin splittings (see the review by Ando *et al.*[1]).

It is clear that such a complex phenomenon as the conductivity in a magnetic field is far from the best means of studying the energy spectrum. One would rather use a knowledge of the energy spectrum as a basis for analyzing the conductivity. Attempts have therefore been undertaken to study the electron dynamics by other methods, primarily by studying the cyclotron resistance[6] and the tunneling in a magnetic field.[7] Both of these studies yielded values of the cyclotron splitting $\hbar\omega_c$ and, consequently, of the effective mass m^*, which agree to within 5% with the values of m^* determined in Ref. 2. For the spin and valley splittings, however, no information has been obtained.

In addition to Ref. 7, other experimental studies of the thermodynamic properties of two-dimensional electron systems have recently appeared. In Ref. 8, for example, an attempt was made to observe the oscillations of the magnetic moment, but as yet without meaningful results. More encouraging results have been obtained in studies of the quantum oscillations of the chemical potential of the two-dimensional electrons; such oscillations were detected in Refs. 9 and 10. In particular, measurements of these oscillations have yielded the value of the valley splitting.[11] The results of our systematic study of this effect are presented below.

2. PRINCIPLES OF THE METHOD

Suppose that a voltage V_g is applied to the electrode structure shown in Fig. 2. As a result, a layer of electrons with a density $n_s \approx V_g \varkappa / 4\pi ed$ will form in a surface layer of the semiconductor, causing the gate field to be screened at large distances (here $\varkappa$ is the dielectric constant of the SiO_2 layer, d is its thickness, and e is the electron charge; for simplicity we are ignoring the charge $\sim 10^{11}$ cm^{-2} of the depletion layer). If all the electrons belong to the same electical-quantization level (and in our case this condition always holds), they will form a two-dimensional system with a Fermi energy (measured from the bottom of their band)[1)]

$$E_F^{(0)} = 2\pi\hbar^2 n_s / pm^*, \quad (1)$$

where p is the degeneracy of the levels [$p = 4$ for (001)Si], and m^* is the mass of the carriers: $m^* \approx 0.22\, m_e$.

If the voltage source is disconnected at low temperatures, where there is practically no leakage, a set charge of the two-dimensional layer will persist indefinitely. The imposition of a magnetic field H perpendicular to the surface, however, can change the Fermi energy. The Fermi energy for an ideal sample in a quantizing field at absolute zero is determined from the condition

$$E_F = E_i = (N + {}^1/_2)\hbar\omega_c \pm \Delta_s \pm \Delta_v, \quad (2)$$

where i is the number of the last completely or partially filled sublevel (see Fig. 1), so that

$$(i-1) n_H < n_s \leqslant i n_H, \quad (3)$$

$n_H = eH/ch$ is the density of states per sublevel, Δ_s and Δ_v are the spin and valley splittings, which can depend on H and n_s, and N is the number of the Landau level (the integer part of $i/4$).

We see from (2) and (3) that for a constant value of n_s the energy E_F oscillates when the field H changes, decreasing abruptly each time a successive level is destroyed by the increasing field and then returning approximately to the original level as the field is increased further. (Similar behavior has been discussed repeatedly in textbooks on solid-state physics for the case of nondegenerate Landau levels; see, e.g., Ref. 12.)

The characteristic amplitude of the variations δE_F should be equal in order of magnitude to $\sim \hbar eH/m^*c$ and should amount to several milli-electron-volts in fields $H \sim 10^5$ Oe. It is a straightforward matter to measure the corresponding changes in the potential using (for example) an electrometer with a high input resistance connected across the gate and the contact to the inversion layer (see Fig. 2). In doing this, however, one must be certain that the measured variations of the potential actually reflect variations of the Fermi level. Let us examine this question in a little more detail.

1) The arrangement in Fig. 2 is essentially the classical scheme for measuring the variations of the contact potential difference of two metals by the Kelvin method. Here, since the magnetic field should not have a noticeable effect on the properties of the gate (an aluminum film), what is measured is just the change in the chemical potential of the two-dimensional layer. In principle this value can differ somewhat from the changes in E_F given by formula (2) because of a possible shift in the reference point (the electrical-quantization level). However, since the position of this level is determined by the total voltage across the gate, $V_g \sim 10$ V, the relative variations cannot in any case exceed $\delta V_g / V_g \approx 10^{-4}$, which is negligible.

2) It can be supposed that when the energy of the electrons changes they will redistribute themselves between the two-dimensional layer and some other states, e.g., localized states in the depletion layer. This could distort the results, but at liquid-helium temperatures the time required to establish equilibrium between the $2D$ layer and states outside the layer is extremely long and these processes can be ignored. Without dwelling on general considerations, let us point out a direct experiment which confirms this circumstance: We have determined[9] that the change in the charge entering the MOS structure agrees to within the accuracy of the measurement ($\approx 2\%$) with the change in the charge contained in the two-dimensional layer; the latter change was determined from the period of the Shubnikov–de Haas oscillations. Furthermore, the experiments described in that same paper[9]

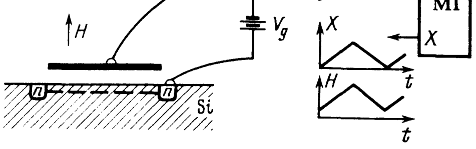

FIG. 2. Schematic of the measurements of the oscillations of the gate voltage: A) amplifier, MI) multichannel integrator.

showed that the carrier density in the two-dimensional layer remains practically unchanged when the magnetic field changes at a fixed total charge of the MOS structure. From this we see that formula (2) can be used quite safely.

3) It has been pointed out[13] that the magnetostriction plays a large role in the quantum oscillations of the chemical potential of metals. At a conduction electron density of 1 e/at., where the elastic properties of metals are largely determined by the electrons, the "feedback" due to the magnetostriction can partially compensate the effect. In our case the characteristic value of the surface density $n_s \sim 10^{12}$ cm^{-2} corresponds (with allowance for the layer thickness $\sim 10^{-7}$ cm) to a volume density $\sim 10^{-3}$–10^{-4} e/at. Therefore, the contribution of the electrons to the elastic properties is insignificant. Allowance for the fact that the electrons are located only in a thin layer in the interior of a microscopic solid will apparently only weaken their influence still further.

It can thus be concluded that the measured values of the potential variations δV_g for the arrangement shown in Fig. 2 correspond to variations of the Fermi energy of a two-dimensional layer of electrons, $\delta V_g = \delta E_F/e$. Naturally, under actual conditions the fact that the temperature and level widths are finite causes the values of δE_F to differ from the values given by Eq. (2). They can be obtained from the solution of the integral equation

$$n_H \sum_i \int_0^\infty D_i(\varepsilon) f(\varepsilon, E_F, T)\, d\varepsilon = n_s, \tag{4}$$

where $D_i(\varepsilon)$ describes the distribution of the density of states at the ith sublevel (here the summation is over all the energy levels), and $f(\varepsilon, E_F, T) = 1/[\exp((\varepsilon - E_F)/T) + 1]$ is the Fermi function. The solution of this equation is described in Sec. 4.

3. EXPERIMENTAL ARRANGEMENT

We studied MOS structures on the (001) surface of p-type silicon. The structures had dimensions of 5×8 mm and a gate-channel capacitance $C \approx 700$ pF. Figure 3 shows how the mobility of the carriers depends on their density at different temperatures. The substrate material had a resistivity $\sim 20\Omega\cdot$cm at room temperature; on cooling to liquid-helium temperatures the carriers were completely frozen out and the bulk conductivity vanished. The experiments showed that the mobility of the electrons of the inversion layer was a maximum if a positive voltage ~ 10 V was applied to the gate during the cooling, i.e., if the two-dimensional layer was

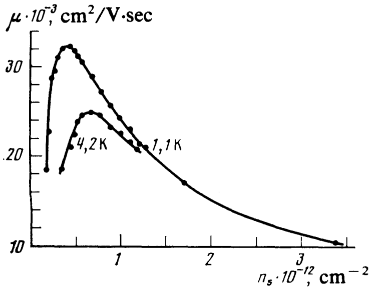

FIG. 3. Mobility of the electrons as a function of their density n_s.

created beforehand, at room temperature. At low temperatures the state of the inversion layer, specifically the electron density and mobility, changed reversibly on changes in the voltage between the gate and the contact to the layer.[2)]

The threshold voltage for metallic conductivity was $V_t \lesssim 2$ V. The carrier density n_s in the two-dimensional layer was set by the voltage V_g applied between the contact to the two-dimensional layer and the gate through a switch which shorted the input of the electrometer (Fig. 2). The absolute value of n_s was determined from the position of the quantum oscillations in a magnetic field. At the time of the measurements the key was opened and the electrometer measured the deviation δV_g from the value originally set.

The sample was placed inside a copper shield filled with a heat-exchange medium of gaseous ^{3}He. The shield was held in thermal contact with a bath of liquid ^{3}He. A magnetic field of up to 90 kOe was produced by a solenoid. The absence of overheating of the sample on account of the magnetic-field sweep was ensured by checking that the amplitude and shape of the observed oscillations did not depend on the sweep rate, which was usually not more than 100 Oe/sec. The magnetic field was swept in accordance with the output voltage of a multichannel integrator, which is proportional to the number of the channel; the output signal of the electrometer was fed to the measuring input of the integrator. The electrometer was a U5-9 amplifier with an input resistance $> 10^{14}\ \Omega$. Thus the time constant for the discharge of the MOS structure was $\sim 10^5$ sec, which corresponds to a change in the charge of $\lesssim 0.1\%$ over the time of the measurements. We note that repeated detection of the oscillations over the course of $\sim$0.5–1 h did not show any appreciable change in the picture due to a drift of the charge on the structure. The capacitance of the coaxial line joining the sample and electrometer was 120 pF and was taken into account in the processing of the experimental results.

4. EXPERIMENTAL RESULTS

The direct result of the experiment is the $V_g(H)$ curve recorded at a set value of the density n_s. Examples of the chart recordings for $T = 1$ K are shown in Fig. 4a. The $V_g(H)$ curves exhibit clearly visible potential jumps corresponding to transitions of the Fermi level resulting from changes in $N(\nu = 8, 12, 16)$ and changes in the spin at a fixed N ($\nu = 6, 10$). As the temperature is lowered, V_g exhibits jumps corresponding to the valley splitting in the spectrum. One of the recordings of such a jump is shown in Fig. 4b ($\nu = 3$).

The parameters of the spectrum were determined according to the best agreement of the $E_F(H)$ curve obtained by numerical solution of integral equation (4) with the experimental curve of $V_g(H)$. Equation (4) contains the initially unknown functions $D_i(\varepsilon)$. We assume for simplicity that all the sublevels at given values of H and n_s are characterized by the same Gaussian distribution (see below)

$$D_i(\varepsilon) = (\Gamma\sqrt{\pi})^{-1} \exp[-(\varepsilon - \varepsilon_i)^2/\Gamma^2]. \tag{5}$$

With this choice, four adjustable parameters formally remain in Eq. (4): m^*, Δ_s, Δ_v, and Γ. Actually, however, to describe the shape of each individual jump we need only two

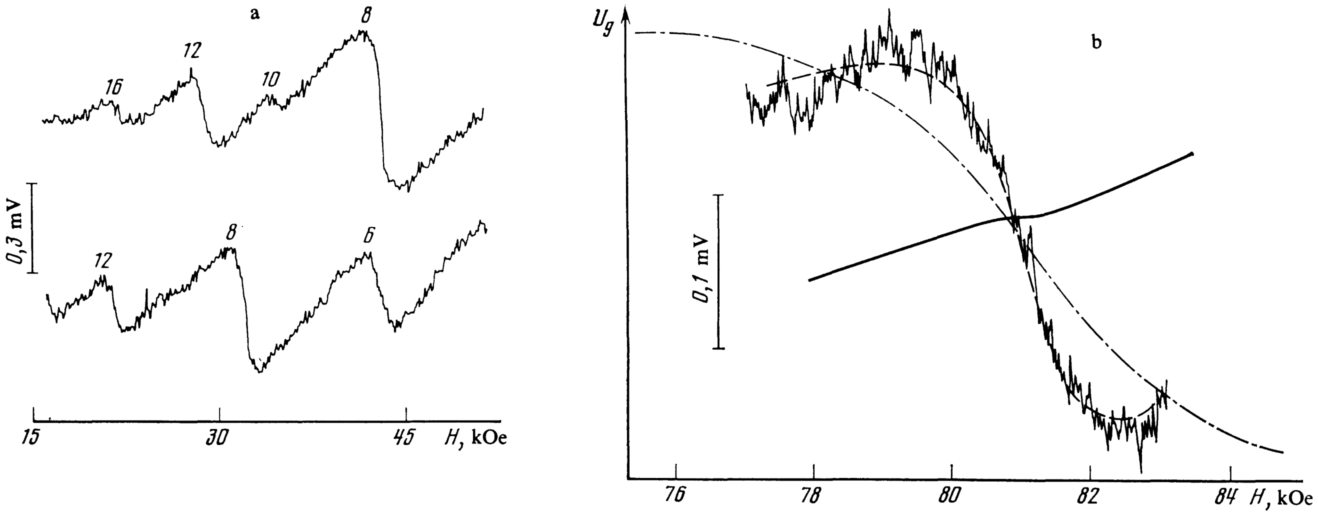

FIG. 4. a) Examples of the recordings of the oscillations of the gate voltage at two different electron densities: upper curve) $8.4\cdot10^{11}$ cm^{-2}; lower curve) $6.3\cdot10^{11}$ cm^{-2}. b) Experimental recording of the jump in the gate potential for $\nu = 3$ ($T = 0.8$ K) and the calculated curves for different values of the parameters: dashed curve) $\Gamma = 0$, $\Delta_v = 8.0$ K; dot-and-dash curve) $\Gamma = 5$ K, $\Delta_v = 12$ K; solid curve) $\Gamma = 5$ K, $\Delta_v = 8.0$ K.

parameters: the energy splitting Δ_ν of the two closest levels lying above and below E_F, and Γ. The calculated shape of the jump is nearly independent of the position and width of the remaining levels [making it perfectly acceptable to use a single function (5) to describe them] because their populations are close to the exact value n_H or to zero.

In a qualitative sense it is clear that for $kT, \Gamma \ll \Delta_\nu$ the amplitude of a jump, i.e., the distance from the minimum to the maximum along the V_g scale, is determined primarily by the value of Δ_ν, while the width of the jump in the magnetic field is determined primarily by the temperature and the smearing of the levels, or, more precisely, by the ratios kT/Δ_ν and Γ/Δ_ν. Therefore, it is possible to determine both the exact value of Δ_ν and the value of Γ and, moreover, to ascertain the magnetic-field dependence of Γ.[14] Since this circumstance is of independent interest, let us consider the determination of Γ separately.

As a first step in determining Γ it would be natural to attempt to calculate its value by calling upon other experiments, for example, measurements of the electron mobility μ in the two-dimensional layer. According to the existing theory (see Ref. 1) the density of states at each level, which would be a δ function for an ideal sample, has a shape which is close to a Gaussian distribution and depends in general on the type of scatterers. For scattering by a short-range potential, Ando obtained the following expression for μ:

$$\Gamma^2 = (\hbar\omega_c/\pi)(\hbar/\tau_f), \tag{6}$$

where $\tau_f = \mu m^*/e$ is the relaxation time determined from the carrier mobility at $H = 0$. In our case $\mu \approx 3.5\cdot10^4$ cm^2/V·sec, and for $H = 80$ kOe Eq. (6) gives $\Gamma = 5$ K. However, for this value of Γ it turns out to be impossible to choose values of Δ_ν such that the $E_F(H)$ curves calculated for reasonable values of the remaining parameters are close to the experimentally measured curves of $V_g(H)/e$ for the jumps with $\nu = 3$ (Fig. 4b) and $\nu = 5$. Thus, since we cannot use formula (6), there is only one way left to determine Γ: to choose the two parameters so as to get the best agreement between $V_g(H)$ and the curve calculated from Eq. (4). Complete agreement can be achieved for the jumps with $\nu = 3, 5, 7$; here one must take $\Gamma \lesssim 1.0$ K in the density region corresponding to the maximum mobility (see Fig. 3), and when n_s increases to $\sim 10^{12}$ cm^{-2} one gets the estimate 1 K $\lesssim \Gamma \lesssim$ 2 K for the level width; this correlates with the corresponding decrease in the mobility. The uncertainty in the determination of Γ of course leads to an increase in the uncertainties in Δ_3, Δ_5, and Δ_7, but these last uncertainties are small compared to the value of Δ_v obtained in this way (~ 7 K at $H = 80$ kOe; see Fig. 5).

The situation is more complicated in the case of the jumps with $\nu = 2, 6, 10$ and especially $\nu = 4, 8, \ldots$, which are associated with significantly larger values of Δ_v. It turns out

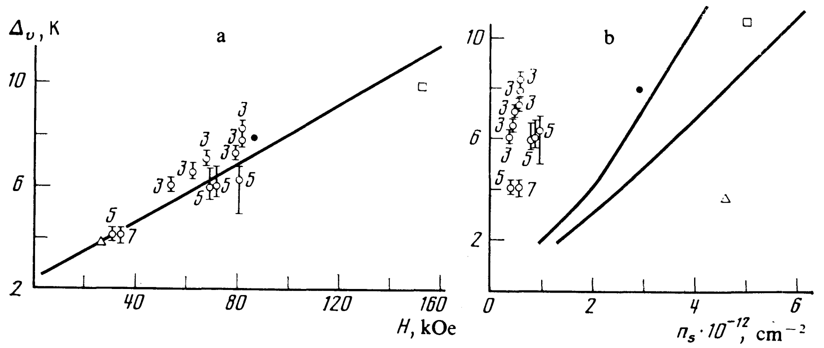

FIG. 5. Valley energy-splitting versus the magnetic field H (a) and versus the electron density n_s (b). The numbers labeling the points give the values of ν. The points Δ and $\Box$ are taken from Ref. 1, Fig. 150; the point ● is from Ref. 17. The solid curves in Fig. b are the theoretical results of Refs. 22 (upper curve) and 23 (lower curve).

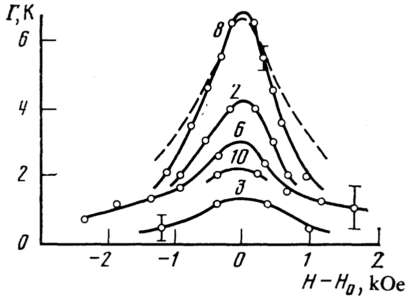

FIG. 6. Level width Γ versus the magnetic field for different ν (the numbers labeling the curves); $H_0 \approx 80$ kOe, $T = 1.34$ K (for $\nu = 3$, $T = 0.45$ K). The dashed curve corresponds to the result of Ref. 16 for $\nu = 4$.

that for these jumps one cannot choose values of Γ such that the $E_F(H)$ and $V_g(H)/e$ curves agree within the error of the measurements. The shape of the line $V_g(H)$ in this case corresponds to a level width Γ which changes depending on the position of the Fermi level relative to the corresponding energy levels: Γ attains a maximum when E_F lies exactly halfway between the levels (the center of the jump) and falls off as the Fermi level approaches the center of the Landau level (Fig. 6).

This behavior of the level widths is due, we believe, to the dependence of the screening of the random potential on the filling of the levels. We note, first of all, that the fivefold or greater difference between the measured values of Γ for the jumps with $\nu = 3, 5, 7$ and the values calculated using Eq. (6) indicates that the role of scattering by the short-range potential is small and that random variations of the potential on a scale much greater than the magnetic length are important. The screening of these fluctuations of the potential cannot involve the electrons belonging to completely filled sublevels, since their density is exactly the same (equal to $n_H = eH/ch$) at any point in the two-dimensional layer. Therefore, the leveling of the potential involves only the electrons on partially filled sublevels. One naturally expects that the screening will weaken when the conductivity and longitudinal resistivity ρ_{xx} go to zero, and this occurs whenever the Fermi level lies midway between two sublevels. As we see from Fig. 7, for our samples (with a measurement accuracy of 10^{-2}–10^{-3} Ω/□) ρ_{xx} goes to zero at $\nu = 2, 4, 6, 8, \ldots$, which correlates with an increase in Γ, but remains perfectly finite at $\nu = 3, 5, 7$, i.e., in those cases when Γ remains small.

The density of thermally excited current carriers involved in the screening (and in the conduction) is roughly $n_T \sim n_H \exp(-|\varepsilon_i - E_F|/T)$; it is minimum for $|\varepsilon_i - E_F| = \Delta_\nu/2$ and increases as the Fermi level approaches an energy sublevel. Accordingly, the conductivity increases and Γ decreases. This kind of behavior of Γ has been discussed earlier,[1] and it is apparently this effect that was observed in a study[15] of the quantum oscillations of the heat capacity of GaAs-based heterojunctions. The proposed picture correlates with the results of a study[10] of transient processes in the charging of MOS structures. The results of that study[10] imply that for $n_T \to 0$ the energy of the electrons within each level develops a scatter that reaches tens of degrees, with a characteristic scale for the variations of $\sim 10^{-5}$–10^{-4} cm—much larger than the magnetic length.

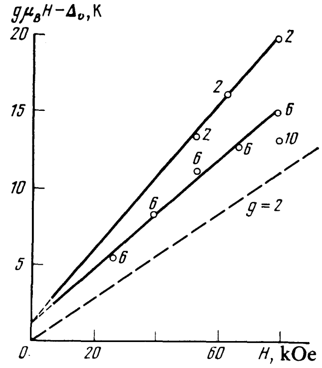

FIG. 8. Values of Δ_ν for $\nu = 2, 6, 10$ as a function of the magnetic field H.

An interesting question in the context of the present study is the extent to which one can check the values of the energy splitting obtained in such a complicated analysis which requires one to allow for the dependence of Γ on $(E_F - \varepsilon_i)$. We note that since this splitting is actually determined from the difference δV_g between the maximum and minimum values of the voltage V_g, for which Γ is small and, as is clear from what we have said, cannot exceed the values of Γ for $\nu = 3, 5, \ldots$, the indeterminancy in Γ has practically no effect on the accuracy with which the values of Δ_ν are determined, the error in the latter being due primarily to the noise in the detection of δV_g. The values of the splitting Δ_ν determined in this way are given in Fig. 8 for $\nu = 2, 6$, and 10 and in Fig. 9 for $\nu = 4$ and 8.

It would be tempting, based on the considerations set forth above, to describe the curves shown in Fig. 6 by a universal function $\Gamma(n_T)$. Unfortunately, it turns out that for different ν the same values of Γ are reached at values of n_T which differ by a factor of 2–3. This circumstance is possibly due to a deviation of the state density from a Gaussian on the tails of the distribution or to the influence of scattering by the short-range potential.

The energy distribution $\tilde{D}(\varepsilon)$ [Eq. (5)] of the carriers at the Landau level in the case of a field-dependent width

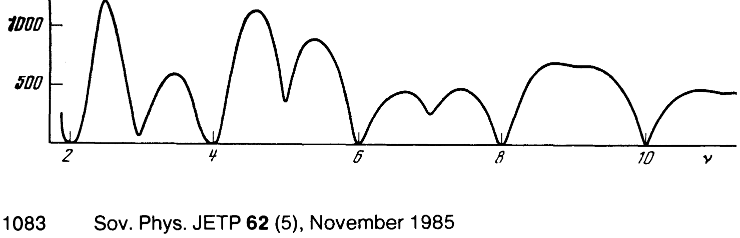

FIG. 7. Resistivity ρ_{xx} versus the occupation number ν of the levels for $H = 90$ kOe, $T = 1$ K ($n_H = 2.16 \cdot 10^{11}$ cm^{-2}).

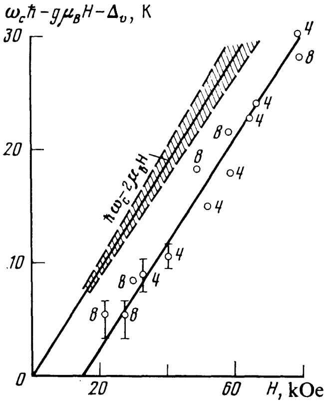

FIG. 9. Values of Δ_ν for $\nu = 4$ and 8 versus the magnetic field H.

Γ(Fig. 6) have "tails" which fall off considerably more slowly than in the case of a Gaussian function with constant Γ. In fact, it is the function $\widetilde{D}(\varepsilon)$ that determines the behavior of the kinetic characteristics of ρ_{xx} and ρ_{xy}. In an analysis[16] of the shape of the wings of the $\rho_{xy}(V_g)$ plateau, the form of $\widetilde{D}(\varepsilon)$ was recovered for the particular case $H = 79.3$ kOe near $\nu = 4$. The dashed curve in Fig. 6 shows how H depends on the width of the Gaussian in accordance with the analytical dependence $\widetilde{D}(\varepsilon)$ obtained in Ref. 16. As we see, there is qualitative agreement between the results of the measurement of the shape of the ρ_{xy} plateau and the results of the measurements of the jump in the potential.

In concluding this section we note the following. It can be shown that our result—a magnetic-field-dependent Γ—is related to the choice of a particular distribution function, and by replacing the distribution with some other kind, e.g., a Lorentzian, one can eliminate the oscillations of the level widths. But this is not the case, since a distribution with fixed parameters cannot simultaneously give sufficient overlap of the levels for $\Delta_\nu = 20$–30 K to describe the finite slope of the $V_g(H)$ curve near the center of the corresponding jumps and the relatively small overlap for $\Delta_\nu \lesssim 10$ K that is a necessary condition for the very existence of both the jumps with $\nu = 3, 5, 7$ and the jumps with other values of ν in weak fields $H \sim 15$–30 kOe.

5. DISCUSSION OF EXPERIMENTAL RESULTS FOR THE ENERGY SPLITTING

Valley splitting

The direct results of the experiment are the values of the energy splitting at times when the Fermi level lies within the corresponding energy interval. For this splitting, the cases corresponding to $\nu = 4, 8, \ldots$ are governed by three parameters of spectrum (2), those with $\nu = 2, 6, \ldots$ by two, and only the splitting for $\nu = 3, 5, 7$ is determined by a single parameter—the valley splitting. It is therefore natural to begin with a discussion of the experimental results for this last group. The data shown in Fig. 5a show that our measured values are satisfactorily approximated by the function $\Delta_\nu = (2.4 + 6 \cdot 10^{-5} H)$ K. We do not see any substantial dependence of Δ_v on n_s: the energy splitting obtained at approximately the same values of the field is nearly the same for $\nu = 3$ and 5 (i.e., when the densities are in a ratio of 3:5) or $\nu = 5$ and 7 (density ratio 5:7). The absence of a connection between Δ_v and n_s is particularly apparent when our measured values are plotted on the $\Delta_v - n_s$ plane (Fig. 5b). It is in just such a form that the results of other measurements of Δ_v have been presented (they are also given in Fig. 5b), because the existing theoretical predictions (reviewed in Ref. 1) imply that Δ_v should grow approximately in proportion to n_s. Thus we can state with certainty that the experiment does not confirm any actual dependence of this kind. If the values of Δ_v measured previously are plotted as a function of H, then all the points are found to lie near a single straight line (Fig. 5a) regardless of the density n_s, which varies for a given value of H (with allowance for the data of Refs. 1 and 17) by 4–5 times. The dependence of Δ_v on H is possibly due to a renormalization of the valley splitting because of the electron–electron interaction. A calculation of this effect is given in Ref. 18, but unfortunately, only for a single value of the field, $H = 140$ kOe, so that we cannot compare our results with the theoretical field dependence because the latter is not known. As to the numerical value for $H \approx 140$ kOe, the renormalized value Δ_v calculated in Ref. 18 is almost an order of magnitude larger than the value implied by Fig. 5a. This may be because the calculation of Ref. 18 used the value of the splitting at $H = 0$, which is significantly larger than the value $\Delta_v \approx 2.5$ K implied by our results.

Spin splitting

As we see from Figs. 5 and 8, the spin splitting $\Delta_s = \Delta_v + \Delta_\nu$ for $\nu = 2$ and 6 does not extrapolate to zero for $H \to 0$. A nonzero spin splitting has also been observed for the electrons of a two-dimensional layer in a GaAs-based heterojunction by extrapolation of the EPR data[19] to $H = 0$. However, GaAs has a strong spin-orbit interaction that is responsible for this result.[20] For the electrons in Si the spin-orbit interaction is small, and this result is unexpected.

The spin splitting of the Landau levels of the electrons of the inversion layer is usually written in the form[1]

$$\Delta_s = g_{\text{eff}}\, \mu_B H, \tag{7}$$

where g_{eff} depends on n_s. One can try to extract the values of g_{eff} by combining the data compiled in Figs. 5 and 8. To do this one has to assume that Δ_v does not depend on the position of the Fermi level. The values of g_{eff} determined in this way are given in Fig. 10. These values agree well with the results obtained in Ref. 4 for the maximum value of g.

This is not the only possible approach, however. From a theoretical point of view the large value of g_{eff} is due to a strong electron-electron interaction.[1] But this renormalization depends substantially on the position of the Fermi level: it is large when one is observing transitions 2,6, ... , etc., and vanishes for $\nu = 4, 8, \ldots$, i.e., in this case $\nu = 2$ and the experiments[21] appear to confirm this. An anomalous effect is expected to occur for the renormalization of the valley splitting as well.[18] To find out whether this is so we can use the measured level splitting for $\nu = 4, 8$ (Fig. 9). It is clear that

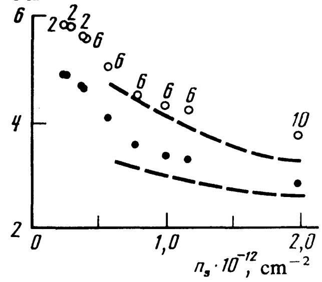

FIG. 10. Experimental dependence of the effective g factor on the density n_s. The points ○ correspond to g_{eff}, the points ● to g'_{eff} (see text). The dashed curves show the boundaries of the range of possible values of g_{max} from Ref. 4.

if there are no oscillations of g_{eff} and Δ_v, it should be true that

$$\sum_{i=1}^{4} \Delta_{k+i} = \hbar\omega_c,$$

if the corresponding splitting is measured at the same value of the field. After summing Δ_2, Δ_3, Δ_4, and Δ_5 for $H = 80$ kOe, we obtain $\hbar\omega_c = 64 \pm 4$ K, which corresponds to an effective mass $m^* = (0.16 \pm 0.01)\, m_e$. This is substantially smaller than the known value $m^* = (0.225 \pm 0.015) m_e$ obtained at $n_s \approx 5 \cdot 10^{11}$–$10^{12}$ cm^{-2} by different methods.[2,6,7]

One can estimate the minimum values of Δ_s and Δ_v by comparing the quantity $\hbar\omega_c - 2\mu_B H = \hbar eH/m^*c - 2\mu_B H$ with the splitting Δ_4 or Δ_8. As we see from Fig. 9, these cases, although differing in details, are rather similar and their magnetic-field dependence is nearly linear, with a slope that is practically equal to that of the straight line $\hbar\omega_c - 2\mu_B H$. The distance between these two straight lines along the energy scale is $\sim 6 \pm 2$ K, i.e., it agrees, if the uncertainty in the measurements is taken into account, with the sum of the remanent valley and spin splitting extrapolated to $H = 0$: $\Delta_2(0) + \Delta_3(0) + \Delta_5(0) = 7 \pm 1$ K. We assume that this result indicates that the unrenormalized value $\Delta_v^{(0)}$ is equal to $\Delta_v(0)$ and is most likely independent of n_s, while the linear dependence of Δ_v on the field H is due to the renormalization on account of the electron-electron interaction; this latter renormalization arises at different occupation numbers for different valleys. Accordingly, the unrenormalized value is $\Delta_s = \Delta_s(0) + 2\mu_B H$, while the renormalized value is $\Delta_s = \Delta_s(0) + g^*\mu_B H$, where according to Fig. 8 we have $g^* = 3.6$, 2.6, and 2.3 for $\nu = 2$, 6, and 10, respectively. If we accept such an interpretation, we must acknowledge that the points for g_{eff} in Fig. 10 do not reflect the actual behavior of the g factor. In view of the oscillatory dependence of Δ_v on the number of filled levels, it should be preferable to take the values g'_{eff} determined from Fig. 8 with $\Delta_v(0)$. It is easy to see that, thanks to the linear dependence $\Delta_v(H)$, this results simply in a decrease in the g factor by 0.9 (Fig. 10). In any case there is not much difference between the values of either g_{eff} or g'_{eff} and the results of the previous studies. We note in this regard that the theory developed by Ando and Uemura (see Ref. 1) predicts that the renormalization of the spin splitting depends substantially on μ. The near agreement of our results with those of other investigators who used samples with $\mu = (10$–$15) \cdot 10^3$ cm^2/V·sec, i.e., 3–4 times smaller, casts doubt on this conclusion of the theory. We note that the values Δ_4 and Δ_8 are somewhat different, possibly in part because of a dependence on n_s of the effective mass, for which we know of no reliable values from other experiments at densities $n_s \lesssim 5 \cdot 10^{11}$ cm^{-2}, the region in which the majority of the points in Fig. 9 fall. On the other hand, according to Fig. 8 the splittings Δ_6 and Δ_{10} are smaller than Δ_2, so that if the renormalization does not vanish altogether, one can expect that Δ_8 will be somewhat larger than Δ_4. At present, however, there is no justification for discussing these small differences.

6. CONCLUSION

Let us list the main results of this study.

By studying the quantum oscillations of the chemical potential of the n-inversion layer at the surface of an (001)Si–MOS structure, we have measured the valley splitting Δ_v and the spin splitting Δ_s. Extrapolation of the experimental results leads to the conclusion that both types of splitting remain present at $H = 0$, the valley splitting with a value $\Delta_v(0) \approx 2.4$ K and the spin splitting with $\Delta_s(0) \approx 4$ K. In a quantizing field this splitting undergoes a renormalization which is proportional to the field and which is maximum when the Fermi level lies between the corresponding energy levels and apparently vanishes when the Fermi level lies outside these intervals. We also established that the motion of the levels on changes in the field is accompanied by oscillations of the level widths (see Fig. 6).

It must be noted that there are still some questions that require further elucidation. First of all, is there a renormalization of Δ_4 and Δ_8 like that observed for the valley and spin splitting? Another circumstance that must be mentioned in this connection is that the renormalized valley splitting is independent of the density n_s, in contrast to the explicit density dependence of the spin splitting. It is possible that this circumstance arises because for Δ_v the decrease in the renormalization with increasing n_s can be compensated by the growth of Δ_v due to the growth in the density. Further experiments are needed to resolve these questions. In particular, it would be interesting to use a study of the quantum oscillations of the chemical potential to follow the transformation of the spectrum when the magnetic field is inclined to the surface of the sample or when the temperature is varied over wide ranges.

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[1)] If valley or spin splitting remains present in the absence of magnetic field, the reference point should be taken midway between the split levels.

[2)] Contact regions with n-type conductivity were formed on the surface of the crystal during the fabrication of the structure.

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