High-frequency hopping electrical conductivity of disordered two-dimensional systems

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Theoretical formulas that take the interaction of the electrons into account are obtained for the high-frequency hopping electrical conductivity of the relaxation type. Two-dimensional systems with a comparatively small localization length are considered. It is shown that the existence of the Coulomb gap has a substantial effect on the real and imaginary parts of the electrical conductivity. Measurement of the electrical conductivity in certain frequency and temperature ranges makes it possible to determine the localization length.

1. According to current ideas, the electron states in disordered two-dimensional systems are localized at sufficiently low temperatures. In the case of weak disorder (weak localization) the localization length is large in comparison with the electron wavelength. In this case the localization affects the electrical conductivity only in the region of very low temperatures and very low frequencies, while outside this region the Drude formula is valid. In the case of strong disorder (or strong localization) the wavefunction contains one charcteristic length, called the localization length, and the electrical conductivity has a hopping character in a wide range of frequencies and temperatures.

The present paper is devoted to the theory of the high-frequency hopping electrical conductivity for strong disorder. The term "high-frequency conductivity" is used here only in the sense that $\mathrm{Re}\sigma(\omega) \ll \mathrm{Im}\sigma(\omega)$. For the hopping conductivity this condition is fulfilled when $\mathrm{Re}\sigma(\omega) \gg \sigma_{\mathrm{st}}$, where σ_{st} is the static hopping conductivity.

High-frequency hopping conduction[1] occurs by electronic transitions between localized states with close energies. The states that are optimal for such transitions form compact pairs lying at a considerable distance from each other. There are no transitions between pairs, so that the pairs cannot give rise to transport of current in a static field, although a high-frequency field effects transitions within pairs, thereby producing polarization. Transitions within pairs can occur both with and without the help of phonons. In the former case, called the relaxation case, the energy Ω required for the transition of an electron within a pair is on the order of kT, while in the latter, no-phonon case, this energy is equal to the energy $\hbar\Omega$ of a quantum of the field. At frequencies $\omega < \omega_{\mathrm{ph}}$ and $\hbar\omega < kT$, where ω_{ph} is the characteristic phonon frequency order of 10^{12}–10^{13} sec^{-1}, relaxation absorption dominates, and we shall be discussing precisely this case. It should be kept in mind, however, that the no-phonon absorption does not lead to substatially different results.

In relaxation absorption the distance r_ω between localized states within one pair is determined from the condition that the field frequency ω is of the order of the frequency $\omega_{\mathrm{ph}} \exp(-2r_\omega/a)$ of the transitions within the pair. It follows from this that

$$r_\omega = \frac{a}{2} \ln \frac{\omega_{ph}}{\omega}, \tag{1}$$

where a is the localization length. As shown in Ref. 2, when calculating the conductivity one must take into account the interaction between the electrons. If $e^2/\varkappa r_\omega > \Delta$, where Δ is the width of the Coulomb gap, it is sufficient to take into account the intrapair interaction, which changes the result substantially for $e^2/\varkappa r_\omega > kT$. (Here e is the electron charge and κ is the dielectric permittivity.) But if $e^2/\varkappa r_\omega < \Delta$, the states responsible for the conduction are found inside the Coulomb gap and it is necessary to take into account the interaction of all the electrons with energies close to the Fermi level. For a three-dimensional system the conductivity in the presence of electron-electron interaction was considered in Refs. 2–5. For two-dimensional systems, to our knowledge, such an analysis has not been carried out.

2. The dipole moment per unit area can be represented in the form

$$P(\omega) = \int \mathbf{d}(\omega, \Omega, \mathbf{r}) F(\Omega, r)\, d^2r\, d\Omega, \tag{2}$$

where $\mathbf{d}(\omega,\Omega,\mathbf{r})$ is the dipole moment of a pair in the electric field $\mathbf{E}$, ω is the frequency of the field, Ω is the energy necessary for an electronic transition within the pair, $\mathbf{r}$ is the two-dimensional moment arm linking the centers of the localized states of the pair (the separation of the pair), and $F(\Omega,r)$ is the distribution function of the number of pairs per unit area with respect to Ω and r. According to Ref. 2,

$$\mathbf{d}(\omega, \Omega, \mathbf{r}) = e\mathbf{r} \frac{e\mathbf{E}\mathbf{r}}{kT} \mathrm{ch}^{-2} \frac{\Omega}{2kT} \frac{1}{1+i\omega\tau}, \tag{3}$$

$$\frac{1}{\tau} = \omega_{ph} \exp\left\{-\frac{2r}{a}\right\}. \tag{4}$$

We shall calculate the function $F(\Omega,r)$ in the two-dimensional case. For this it is necessary to specify the system to be considered. We shall assume that, when the electron-electron interaction is not taken into account, the density of electron states does not depend on the energy and is equal to g_0 in an energy range greater than the width Δ of the Coulomb gap. Then, when the Coulomb interaction is taken into account, the density of states $g(\varepsilon)$ has the form (see Ref. 6)

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$$g(\varepsilon)=\frac{2}{\pi}|\varepsilon|\frac{\varkappa^2}{e^4}, \quad |\varepsilon|\ll\Delta,$$

$$g(\varepsilon)=g_0, \quad |\varepsilon|\gg\Delta. \tag{5}$$

Here ε is the energy of the localized states, measured from the Fermi level. The dielectric permittivity $\varkappa$ describes the medium in which the two-dimensional system is placed; it is assumed that the metallic surface is far from the system (in comparison with r_ω, as will be seen from the following). The Coulomb-gap width satisfies

$$\Delta=(e^4/\varkappa^2)g_0. \tag{6}$$

We note that the form of $g(\varepsilon)$ for $|\varepsilon|\ll\Delta$ does not depend on the model of the disordered system. The function $F(\Omega, r)$ at zero temperature is calculated from the formula[7]

$$F(\Omega,r)=\int_0^\infty g(\varepsilon_1)\,d\varepsilon_1\int_{-\infty}^0 g(\varepsilon_2)\,d\varepsilon_2\,\delta\left(\varepsilon_1-\varepsilon_2-\frac{e^2}{\varkappa r}-\Omega\right). \tag{7}$$

Substituting (5) into (7), we obtain for $\Omega+e^2/\varkappa r\ll\Delta$

$$F(\Omega,r)=\frac{2}{3\pi^2}\left(\frac{\varkappa^2}{e^4}\right)^2\left(\Omega+\frac{e^2}{\varkappa r}\right)^3, \tag{8}$$

and for $\Omega+e^2/\varkappa r\gg\Delta$

$$F(\Omega,r)=g_0^2\left(\Omega+\frac{e^2}{\varkappa r}\right). \tag{9}$$

Formulas (8) and (9) are valid if $\Omega+e^2/\varkappa r\gg kT$. In the calculation of the integrals in formula (2) values of Ω of the order of kT and values of r of the order of r_ω are important. Therefore, we can use formulas (8) and (9) under the condition

$$e^2/\varkappa r_\omega\gg kT. \tag{10}$$

3. Substituting (3) into (2) and replacing $F(\Omega, r)$ by $F(0, r)\equiv F(r)$, we perform the integration over Ω and the angles of the vector $\mathbf{r}$. We obtain

$$\sigma(\omega)=i\frac{\pi}{2}\omega e^2\int_0^\infty\frac{F(r)r^3dr}{1+i\omega\tau}. \tag{11}$$

It should be noted that it is possible to add the contributions from all pairs only if the pairs interact weakly with each other. We shall check that this condition is fulfilled in the most important case, when the separation r of the pairs satisfies the condition $e^2/\varkappa r\ll\Delta$. According to (8), the number of pairs per unit area that have separation of order r and energy of order kT can be estimated as $kT\varkappa/e^2r$. The average distance between them is

$$\bar{R}=(e^2r/kT\varkappa)^{1/2},$$

and $r/\bar{R}=(kT\varkappa r/e^2)^{1/2}\ll 1$. Then their interaction energy is

$$e^2r^2/\varkappa\bar{R}^3=kT(kT\varkappa r/e^2)^{1/2},$$

which, in the case under consideration, is smaller than the pair energy kT. We note that in the three-dimensional case the interaction energy is of order of kT, and this gives rise in the theory to the serious difficulties to which attention was drawn in Ref. 4.

Using formula (4) to replace r by the variable τ in the integral (11), one can see without difficulty that the product $F(r)r^3$ can be taken outside the integral for $r=r_\omega$. In the calculation of $\mathrm{Im}\sigma$ the integration over τ must be restricted to the region $\tau>\omega_{\mathrm{ph}}^{-1}$, and so it is not possible to determine the number under the logarithm. To within this number, we obtain

$$\mathrm{Re}\,\sigma=\frac{\pi^2}{8}e^2a\omega r_\omega^3F(r_\omega), \tag{12}$$

$$\mathrm{Im}\,\sigma=\frac{\pi}{4}e^2a\omega r_\omega^3F(r_\omega)\ln\frac{\omega_{ph}}{\omega}. \tag{13}$$

Substituting into (12) and (13) the expressions (8) and (9), we find that, for $e^2/\varkappa r_\omega\ll\Delta$, when the states responsible for the conduction lie inside the Coulomb gap,

$$\mathrm{Re}\,\sigma=\varkappa\omega a/2, \tag{14}$$

$$\mathrm{Im}\,\sigma=\varkappa\omega r_\omega/6\pi, \tag{15}$$

and the loss angle ψ is small:

$$\psi=\frac{\mathrm{Re}\,\sigma}{\mathrm{Im}\,\sigma}=\frac{\pi}{2}\left[\ln\frac{\omega_{ph}}{\omega}\right]^{-1}. \tag{16}$$

At a nonzero temperature the regime with small loss angle is destroyed at sufficiently low frequencies, when r_ω becomes comparable to the hopping length r_T of the static hopping conductivity. At still lower frequencies the hopping occurs not through isolated pairs but along percolation paths (see Ref. 6) penetrating the entire system. The conductivity in such conditions ceases to depend on the frequency and reaches its static limit. According to Ref. 6, in Coulomb-gap conditions the length r_T is of the order of $a(T_0/T)^{1/2}$, where $T_0=e^2/\varkappa a$, so that $r_\omega=r_T$ for $\omega=\omega_{\mathrm{ph}}\exp\{-T_0/T\}^{1/2}$. According to (14), at this frequency $\mathrm{Re}\sigma\propto\exp\{-T_0/T\}^{1/2}$.

For sufficiently high frequencies, $e^2/\varkappa r_\omega\gg\Delta$. In this regime, according to (9) and (12), (13),

$$\mathrm{Re}\,\sigma=\frac{\pi^2}{8}\frac{a\omega e^4}{\varkappa}r_\omega^2g_0^2=\frac{\pi^2}{8}(\varkappa\omega a)\left(\frac{\Delta\varkappa r_\omega}{e^2}\right)^2, \tag{17}$$

$$\mathrm{Im}\,\sigma=\frac{2}{\pi}\ln\frac{\omega_{ph}}{\omega}\mathrm{Re}\,\sigma, \tag{18}$$

and the loss angle is determined by formula (16).

Finally, in the high-temperature case ($kT\gg e^2/\varkappa r_\omega$, $kT>\Delta$), the electron-electron interaction turns out to be unimportant. According to Ref. 2, in this case

$$F(\Omega,r)=\Omega g_0^2\,\mathrm{cth}^{-1}\frac{\Omega}{2kT}. \tag{19}$$

Substituting (3) and (19) into (2), we obtain

$$\mathrm{Re}\,\sigma=\frac{\pi^4}{16}e^2\omega g_0^2kTr_\omega^3a, \tag{20}$$

and the loss angle is determined, as before, by formula (16). Formula (20) is the two-dimensional analog of the Austin-Mott formula.[8] Formulas (17)–(20), unlike formulas (14), (15), depend on the model of the disordered system, since they contain the quantity g_0. If the spread of the levels is associated entirely with the Coulomb interaction at distances large in comparison with the lattice constant there are no grounds to suppose that the density of states will be independent of the energy in any range of energies. In this case

the entire spread of the levels is of the order of the Coulomb-gap width Δ. Nevertheless, the formulas (17) and (18) remain valid in order of magnitude. The quantity g_0 must be regarded as of order $n\varkappa e^2/n^{1/2}$, where n is the surface density of charged centers. However, the formula (20), which is valid in the case $kT \gg \Delta$, has no region of applicability if the spread of the level is of the order of Δ.

The most interesting of the regimes considered is the Coulomb-gap regime, which is described by formulas (14)–(16). Measurements in this regime make it possible to check ideas about the Coulomb gap and to determine the localization length directly. It is probable that transition to this regime, i.e., fulfillment of the condition $e^2/\varkappa r_\omega < \Delta$, can be realized more easily by increasing the localization length (as a result of an increase in the concentration of electrons in the structure) than by lowering the frequency.

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