Spin glass in an Ising two-sublattice magnet

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Two-component magnetic systems are investigated in the framework of an infinite-range model with fluctuating exchange couplings. Depending on the sign of the average exchange-interaction energy and the relative magnitudes of the average value and standard deviation of this energy, re-entrant phase transitions to a spin glass are possible both from the ferromagnetic and from the antiferromagnetic phase. The phase diagram in a magnetic field for systems in which the average exchange energy has antiferromagnetic sign is constructed. Its characteristic feature is a nonmonotonic dependence of the temperature of the transition to the spin glass (the de Almeida–Thouless temperature) on the external field. The susceptibility in the antiferromagnetic phase is found, and it is shown that for sufficiently strong disorder the susceptibility in a region of temperatures near the Néel point increases as the temperature decreases.

Theoretical investigations of spin glasses are currently conducted mainly in the framework of the infinite-range model proposed by Sherrington and Kirkpatrick[1] (the SK model). Depending on the relative magnitudes of the parameters, this model describes both the paramagnet–spin-glass transition and successive paramagnet–ferromagnet–spin-glass transitions (the latter transition is customarily called a re-entrant transition).[2] In the SK model a spin glass is a nonergodic phase characterized by an infinite number of order parameters, viz., by a function $q(x)$ (Refs. 3–5). Although in reality the interaction energy depends in an essential way on the distance, the results obtained on the basis of the SK model give a good description of the basic properties of spin glasses (see the reviews in Refs. 6 and 7).

Sherrington and Kirkpatrick[1] proposed a variant of infinite-range model in which the re-entrant transition to the spin glass can occur only from the ferromagnetic phase. At the same time, the introduction of disorder into an antiferromagnet leads naturally to antiferromagnet–spin-glass phase transitions. Transitions of this type have been observed in many substances.[8–11]

In the present paper the SK model is modified so as to be able to describe multicomponent Ising magnets undergoing re-entrant transitions both from the ferromagnetic and from the antiferromagnetic phase (and, with an obvious change of the parameters, from the ferrimagnetic phase). We consider in detail the situation in which, as the temperature is lowered, the paramagnetic phase is succeeded by the antiferromagnetic phase and then by the spin glass. It is shown that an Ising antiferromagnet with a sufficiently large number of frustrated couplings (i.e., with about half of the couplings frustrated) possesses unusual properties. Near the Néel temperature the susceptibility in the antiferromagnetic phase does not display the usual decrease with lowering of the temperature, but, on the contrary, increases.

The phase diagram in a magnetic field is constructed. With increase of the field the Néel temperature $T_N(H)$ decreases monotonically to the value $T_0 = T_N(H_0)$ at which $T_N(H)$ intersects the de Almeida–Thouless (AT) line $T_f(H)$ on which the transition to the spin-glass state occurs. Thus, T_0 is a triple point on the phase diagram in the (T, H) plane. The AT line undergoes a striking change in comparison with that for the one-component model. In the one-component model T_f decreases monotonically as the field increases. In our case, for $H < H_0$ the temperature $T_f(H)$ passes successively through a minimum and a maximum as the field increases, and its value at the maximum exceeds $T_f(0)$.

In this paper we do not discuss the properties of the spin-glass phase. This question will be considered separately.

1. THE FREE ENERGY

We shall consider a two-component Ising magnet in which only spins of different types interact. The Hamiltonian of the system has the form

$$\mathscr{H} = -\sum_{i,j} J_{ij} S_{1i} S_{2j} - H \sum_i (S_{1i} + S_{2i}), \tag{1}$$

where S_{1i} and S_{2i} are the spins of the different components, $S_i = \pm 1$, and H is the external magnetic field. The exchange integrals will be assumed to be distance-independent random quantities with the normal distribution

$$P(J_{ij}) = \frac{N^{1/2}}{(2\pi)^{1/2} J} \exp\left\{-\frac{(J_{ij} + J_0/N)^2}{2J^2} N\right\}, \tag{2}$$

where N is the number of spins in each of the components. For $J_0 \leqslant 0$ this model differs only in inessential details from the SK model. But if $J_0 > 0$, then for small J, our model, unlike the SK model, admits the establishment of long-range antiferromagnetic order.

By making use of the method of replicas, we can represent the free energy per spin in the form[1]

$$f = -T \lim_{\substack{n\to 0\\ N\to\infty}} \frac{1}{2Nn} \left\{ \prod_{(m,n)} P(J_{mn})\, dJ_{mn}\, \mathrm{Tr} \exp\left[\sum_{\alpha=1}^{n} \sum_{i,j=1}^{N} \frac{J_{ij}}{T} S_{1i}^{\alpha} S_{2j}^{\alpha} + \frac{H}{T} \sum_{i,\alpha}^{n} \sum_{i=1}^{N} (S_{1i}^{\alpha} + S_{2i}^{\alpha}) \right] - 1 \right\}, \tag{3}$$

where α is the replica index. By integrating over J_{ij} we trans-

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form the expression for f to the form

$$f=-T\lim_{\substack{n\to 0\\ N\to\infty}}\frac{1}{2Nn}\Big\{\exp\frac{J^2}{2T^2}Nn\,\mathrm{Tr}\exp\Big[\sum_{(\alpha,\beta)}\frac{J^2}{4NT^2}\times\Big[\Big(\sum_i S_{1i}{}^{\alpha}S_{1i}{}^{\beta}+\sum_i S_{2i}{}^{\alpha}S_{2i}{}^{\beta}\Big)^2-\Big(\sum_i S_{1i}{}^{\alpha}S_{1i}{}^{\beta}\Big)^2-\Big(\sum_i S_{2i}{}^{\alpha}S_{2i}{}^{\beta}\Big)^2\Big]-\sum_{\alpha}\frac{J_0}{2NT}\Big[\Big(\sum_i S_{1i}{}^{\alpha}+\sum_i S_{2i}{}^{\alpha}\Big)^2-\Big(\sum_i S_{1i}{}^{\alpha}\Big)^2-\Big(\sum_i S_{2i}{}^{\alpha}\Big)^2\Big]+\frac{H}{T}\sum_{\alpha}\sum_i(S_{1i}{}^{\alpha}+S_{2i}{}^{\alpha})\Big]-1\Big\}. \tag{4}$$

The summation over (α,β) is performed over all different pairs of indices α and β. Following next the standard procedure, we obtain

$$f=-T\lim_{\substack{n\to 0\\ N\to\infty}}\frac{1}{2nN}\times\Big\{\exp\Big(\frac{JNn}{2T^2}\Big)\int\prod_{\alpha,p}\Big(\frac{N}{2\pi}\Big)^{1/2}dx_p{}^{\alpha}\prod_{(\alpha,\beta)p}\Big(\frac{N}{2\pi}\Big)^{1/2}dy_p{}^{\alpha\beta}\times\exp[N\Phi(x_p{}^{\alpha},y_p{}^{\alpha\beta})]-1\Big\}, \tag{5}$$

where $p=1,2$, and 3, and

$$\Phi(x_p{}^{\alpha},y_p{}^{\alpha\beta})=-\Big[\sum_{\alpha,p}\frac{1}{2}(x_p{}^{\alpha})^2+\sum_{(\alpha,\beta)p}\frac{1}{2}(y_p{}^{\alpha\beta})^2-\ln\mathrm{Tr}\exp\Big[\frac{H}{T}\sum_{\alpha}(S_1{}^{\alpha}+S_2{}^{\alpha})+i\Big(\frac{J_0}{T}\Big)^{1/2}\sum_{\alpha}(S_1{}^{\alpha}+S_2{}^{\alpha})x_3{}^{\alpha}+\Big(\frac{J_0}{T}\Big)^{1/2}\sum_{\alpha}(S_1{}^{\alpha}x_1{}^{\alpha}+S_2{}^{\alpha}x_2{}^{\alpha})+\frac{J}{T}\sum_{(\alpha,\beta)}(S_1{}^{\alpha}S_1{}^{\beta}+S_2{}^{\alpha}S_2{}^{\beta})y_3{}^{\alpha\beta}+i\frac{1}{T}\sum_{(\alpha,\beta)}(S_1{}^{\alpha}S_1{}^{\beta}y_1{}^{\alpha\beta}+S_2{}^{\alpha}S_2{}^{\beta}y_2{}^{\alpha\beta})\Big]. \tag{6}$$

In the limit $N\to\infty$ the integrals are calculated by the method of steepest descent. As a result,

$$f=-\frac{J^2}{4T^2}-\frac{T}{2}\lim_{n\to 0}\frac{1}{n}\Phi((x_p{}^{\alpha})_0,(y_p{}^{\alpha\beta})_0), \tag{7}$$

where the subscript 0 indicates the solutions of the saddle-point equations

$$\begin{aligned}
x_p{}^{\alpha}&=m_p{}^{\alpha}\Big(\frac{J_0}{T}\Big)^{1/2}=\Big(\frac{J_0}{T}\Big)^{1/2}\langle S_p{}^{\alpha}\rangle,\quad p=1,2,\\
x_3{}^{\alpha}&=2m^{\alpha}i\Big(\frac{J_0}{T}\Big)^{1/2}=i\Big(\frac{J_0}{T}\Big)^{1/2}\langle S_1{}^{\alpha}+S_2{}^{\alpha}\rangle,\\
y_p{}^{\alpha\beta}&=iq_p{}^{\alpha\beta}\frac{J}{T}=i\frac{J}{T}\langle S_p{}^{\alpha}S_p{}^{\beta}\rangle,\quad p=1,2,\\
y_3{}^{\alpha\beta}&=2q^{\alpha\beta}\frac{J}{T}=\frac{J}{T}\langle S_1{}^{\alpha}S_1{}^{\beta}+S_2{}^{\alpha}S_2{}^{\beta}\rangle.
\end{aligned} \tag{8}$$

The angular brackets signify averaging with the exponential that appears inside the trace in (6).

2. THE REPLICA-SYMMETRIC SOLUTION

Assuming that the matrix elements $q^{\alpha\beta}$ and $q_{1,2}^{\alpha\beta}$ in (8) are all the same and equal to q and $q_{1,2}$, respectively (analogously, $m^{\alpha}=m$ and $m_{1,2}^{\alpha}=m_{1,2}$), we obtain equations of the SK type:

$$\begin{aligned}
m_{1,2}&=\langle \mathrm{th}\,E_{2,1}(z)\rangle_c,\\
q_{1,2}&=\langle \mathrm{th}^2 E_{2,1}(z)\rangle_c,\\
m&=1/2(m_1+m_2),\qquad q=1/2(q_1+q_2).
\end{aligned} \tag{9}$$

Here

$$E_{1,2}(z)=T^{-1}(H-J_0m_{1,2}+Jq_{1,2}^{1/2}z).$$

The angular brackets in (9) signify averging over z with a Gaussian distribution function:

$$\langle A(z)\rangle_c=\frac{1}{\sqrt{2\pi}}\int A(z)e^{-z^2/2}dz. \tag{10}$$

In the absence of an external magnetic field we have $q_1=q_2$ and (if $J_0>0$) $m_1=-m_2$, or (if $J_0<0$) $m_1=m_2$, so that Eqs. (9) coincide with the SK equations.[1] Therefore, for $H=0$ the phase diagram determined by the system (9) is analogous to that obtained in the one-component SK model. The corresponding diagram is given in Fig. 1, in which the AT line is also shown.[2] In contrast to the SK diagram, in our model, depending on the sign of J_0, both ferromagnetic and antiferromagnetic ordering are possible, and, correspondingly, there arises a re-entrant transition to the spin-glass phase both from the ferromagnetic phase and from the antiferromagnetic phase.

It is known that the replica-symmetric solution is valid only in that region of the phase diagram which lies above the AT line on which the transition to the spin glass occurs. The equation determining this line in an external magnetic field will be derived in the next Section.

3. THE DE ALMEIDA-THOULESS LINE

In order to determine the region of stability of the replica-symmetric solution it is necessary to find the eigenvalues of the matrix

$$\Psi_{pp_1}^{\alpha\beta,\gamma\delta}=\lim_{n\to 0}\frac{\partial^2\Phi}{\partial y_p{}^{\alpha\beta}\,\partial y_{p_1}{}^{\gamma\delta}}. \tag{11}$$

Instability appears when one of the eigenvalues becomes

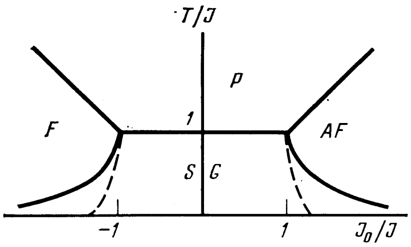

FIG. 1. Phase diagram in the (T, J_0) plane. The dashed lines are lines of re-entrant transitions in the replica-symmetric model.

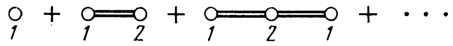

FIG. 2. Diagrammatic series for the correlator $K^{(1)}$.

negative.[2] Here Ψ is a 3×3 matrix in the space of the vectors $x=(\hat{y}_1,\hat{y}_2,\hat{y}_3)$, where $\hat{y}_p$ is the matrix with elements $y_p^{\alpha\beta}$. As shown in Ref. 2, there are three independent eigenfunctions $\varphi_p^{(i)}(i=1,2,3)$ of the matrices $\hat{\Psi}_{pp}$. To one of these functions, which we denote by $\varphi_p^{(1)}$, there corresponds an eigenvalue that changes sign at a certain temperature. It is easy to convince oneself that all the matrices $\hat{\Psi}_{pp_1}$ that are nondiagonal in the subscripts have the same symmetry in the replica space as the diagonal matrices $\hat{\Psi}_{pp}$. Therefore, the matrices $\hat{\Psi}_{pp_1}$ do not mix functions $\varphi_p^{(i)}$ with different i. As a result, the eigenfunctions of the matrix Ψ are columns with elements proportional to $\varphi_p^{(i)}$. In order to find the condition for instability of the replica symmetric solution it is sufficient to take the eigenfunction constructed entirely from $\phi_p^{(1)}$. The equation for the eigenvalues λ has the form

$$\begin{vmatrix} \lambda_{y_3}-\lambda & \lambda_{y_3y_1} & \lambda_{y_3y_2} \\ \lambda_{y_3y_1} & \lambda_{y_1}-\lambda & 0 \\ \lambda_{y_3y_2} & 0 & \lambda_{y_2}-\lambda \end{vmatrix}=0. \tag{12}$$

Here

$$\lambda_{y_3}=\frac{\partial^2\Phi}{\partial y_3^{\alpha\beta}\,\partial y_3^{\alpha\beta}}-2\frac{\partial^2\Phi}{\partial y_3^{\alpha\beta}\,\partial y_3^{\alpha\delta}}$$

$$+\frac{\partial^2\Phi}{\partial y_3^{\alpha\beta}\,\partial y_3^{\gamma\delta}}=1-\frac{J^2}{T^2}\langle \mathrm{ch}^{-4}(E_1(z))+\mathrm{ch}^{-4}E_2(z)\rangle_c, \tag{13}$$

$$\lambda_{y_{1,2}}=1+\frac{J^2}{T^2}(1-\langle \mathrm{ch}^2 E_{2,1}(z)\rangle_c), \tag{14}$$

$$\lambda_{y_3y_{1,2}}=i\frac{J^2}{T^2}\langle \mathrm{ch}^{-4}E_{2,1}(z)\rangle_c. \tag{15}$$

One of the roots of Eq. (12) is equal to unity. The other two roots are

$$\lambda_{1,2}=1\pm\frac{J^2}{T^2}[\langle \mathrm{ch}^{-4}E_1(z)\rangle_c\langle \mathrm{ch}^{-4}E_2(z)\rangle_c]^{1/2}. \tag{16}$$

This means that the line of the transition to the spin-glass state is determined by the equation

$$T^4/J^4=\langle \mathrm{ch}^{-4}E_1(z)\rangle_c\langle \mathrm{ch}^{-4}E_2(z)\rangle_c, \tag{17}$$

where $m_{1,q}$ and $q_{1,2}$ must be found from Eqs. (9).

We now show that the AT instability line can be obtained by considering the singularity of the correlator

$$K^{(r)}=\langle \chi_{ri}^2\rangle_c,\qquad r=1,\ 2. \tag{18}$$

Here the local susceptibility of the spin S_{ri} is equal to

$$\chi_{ri}=\sum_{j,r_1}\langle (S_{ri}-\langle S_{ri}\rangle_T)(S_{r_1j}-\langle S_{r_1j}\rangle_T)\rangle_T. \tag{19}$$

The brackets $\langle\dots\rangle_T$ signify thermodynamic averaging with the Hamiltonian (1), which can be represented in the form

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}_1,$$

$$\mathscr{H}_0=\frac{J_0}{N}\sum_{i,j}S_{1i}S_{2j}-\sum_{i,j}\left(J_{ij}+\frac{J_0}{N}\right)(\langle S_{1i}\rangle_T S_{2j}+\langle S_{2j}\rangle_T S_{1i})$$

$$-H\sum_i(S_{1i}+S_{2i}),$$

$$\mathscr{H}_1=-\sum_{i,j}\left(J_{ij}+\frac{J_0}{N}\right)(S_{1i}-\langle S_{1i}\rangle_T)(S_{2j}-\langle S_{2j}\rangle_T). \tag{20}$$

The term quadratic in the spin in $\mathscr{H}_0$ for the infinite-range model under consideration is equivalent, like the other two terms, to the energy of the spins in the field.

The diagrammatic series for K, obtained by means of the standard technique for the Ising model,[12] is shown in Fig. 2. The circles with indices 1 and 2 correspond to correlators $K_0^{(1,2)}$ in which the averaging is performed with the Hamiltonian $\mathscr{H}_0$:

$$K_0^{(1,2)}=\langle \mathrm{ch}^{-4}E_{2,1}(z)\rangle_c,$$

while the double lines linking the circles correspond to J^2/T^2. The system of Dyson equations for the correlators has the form

$$K^{(1)}=K_0^{(1)}+J^2T^{-2}K_0^{(1)}K^{(2)},$$
$$K^{(2)}=K_0^{(2)}+J^2T^{-2}K_0^{(2)}K^{(1)}, \tag{21}$$

and its solution is

$$K^{(1,2)}=K_0^{(1,2)}\left(1+K_0^{(2,1)}\frac{J^2}{T^2}\right)\left(1-\frac{J^4}{T^4}K_0^{(1)}K_0^{(2)}\right)^{-1}. \tag{22}$$

The condition for vanishing of the denominator in (22) coincides with the equation (17) that determines the temperature of the transition to the spin glass.

For the one-component model with $J_0=0$ and $H_0=0$ the connection between the stability condition and the position of the singularity in the correlator was noted in Refs. 13 and 14.

In the absence of an external magnetic field we have $\langle\cosh^{-4}E_1(z)\rangle_c=\langle\cosh^{-4}E_2(z)\rangle_c$, and Eq. (17) coincides with the AT equation. The corresponding phase-transition line is depicted in Fig. 1.

4. THE PHASE DIAGRAM IN A MAGNETIC FIELD

In the one-component model or in the two-component model with $J_0<0$ an external field destroys the transition from the paramagnetic to the ferromagnetic phase and leads to a monotonic decrease of T_f with increase of the field. In the two-component system with $J_0>0$ the effect of the magnetic field on the phase diagram turns out to be much more interesting.

We shall consider first the phase diagram near the triple point, when $(J_0-J)/J_0=b\ll 1$. Expanding the expression (9) in powers of m, q and H, we obtain an equation for the Néel temperature $T_N(H)$:

$$1-\frac{J_0}{T_N}+J_0\left(H-\frac{J_0m}{T_N}\right)^2+\frac{J_0J^2}{T_N^3}q-2\frac{J^4J_0}{T_N^5}q^2=0, \tag{23}$$

where

$$m=m_1=m_2=\frac{HT_N}{T_N+J_0}, \tag{24}$$

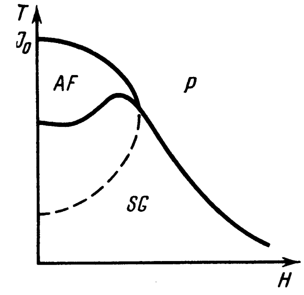

FIG. 3. Phase diagram in the (T,H) plane. The dashed line is the line of the re-entrant transition in the replica-symmetric model.

and $q(T_N) = q_1(T_N) = q_2(T_N)$ satisfies the equation

$$q\left(1-\frac{J^2}{T^2}\right)=\frac{H^2}{(T_N+J_0)^2}-2\left(\frac{J}{T}\right)^4 q^2+\frac{17}{3}\left(\frac{J}{T}\right)^6 q^3. \tag{25}$$

In very weak magnetic fields, when $T_N(H) > J$, it follows from (25) that

$$q=H^2/6J_0^2. \tag{26}$$

Substituting (26) and (24) into (23), we obtain

$$T_N=J_0(1-H^2/8bJ_0^2). \tag{27}$$

This formula is valid if $H \ll bJ_0$. With increase of H, when T_N approaches J, the term proportional to $(1 - J^2/T^2)$ in Eq. (25) can be discarded, and

$$T_N=J_0(1-H/2^{3/2}J_0). \tag{28}$$

The fields for which this formula is valid satisfy the inequality $|J_0 b - H|/H \ll 1$.

Finally, we consider the region of fields for which the temperature satisfies $T_N < J$ and the inequality $J/T_N - 1 \gg H/J$. Then

$$q=\frac{1}{2}\left(\frac{T_N}{J}\right)^2\left[1-\frac{T_N^2}{J^2}+\frac{17}{12}\left(1-\frac{T_N^2}{J^2}\right)^2 +\frac{H^2}{4}\left(1-\frac{T_N}{J}\right)^{-1}\right]. \tag{29}$$

Using (23), (24), and (29), we obtain a cubic equation for T_N:

$$(1-T_N/J)^3-3b(1-T_N/J)+3H^2/8J^2=0, \tag{30}$$

which in the physical region of parameters has two solutions if the field satisfies

$$H\leqslant H_0=J(4b^{3/2}/3^{1/2}), \tag{31}$$

and has no solutions if $H > H_0$. One of the solutions is

$$T_N^{(1)}=J\left[1-2b^{1/2}\cos\left(\frac{\alpha}{3}-\frac{2\pi}{3}\right)\right], \tag{32}$$

where

$$\cos\alpha=-H^2/H_0^2,$$

joins with (28) at $H \sim bJ_0$ and describes the decrease of the Néel temperature as the field increases with $H \gg bJ_0$ to $H = H_0$. At $H = H_0$ the Néel temperature is equal to

$$T_N^{(1)}(H_0)\equiv T_0=J(1-b^{1/2}). \tag{33}$$

The second solution

$$T_N^{(2)}=J\left[1-2b^{1/2}\cos\frac{\alpha}{3}\right] \tag{34}$$

satisfies the inequality $J/T_N - 1 \gg H$ for all magnetic fields less than H_0. It gives the temperature of the re-entrant transition from the antiferromagnetic state to the spin glass in the replica-symmetric scheme. For $H = 0$ we have $T_N^{(2)}(0) = J(1 - 3b)^{1/2})$. As the field increases, so does $T_N^{(2)}$, and at $H = H_0$ it coincides with T_0 (Fig. 3). Below we shall see that in the phase diagram the whole branch described by formula (34) falls in the region of instability of the replica-symmetric solution.

It follows from the above analysis that near the triple point of the phase diagram of Fig. 1 the Néel temperature decreases anomalously fast with increasing H. The destruction of the antiferromagnetic order by the magnetic field is entirely different from that in an ordered magnet, in which, as H increases the Néel temperature decreases to zero in fields comparable to the exchange field. In strongly disordered antiferromagnets, for which $J \cong J_0$, the minimum Néel temperature T_0 differs little from J_0, and is attained in a field $H_0 \ll J_0$. As we shall see below, the conclusion that the minimum Néel temperature is not equal to zero also remains valid far from the triple point, but in this case the antiferromagnetism vanishes at temperatures substantially lower than J_0.

We now turn to the AT instability line. It is simplest of all to obtain the dependence $T_f(H)$ for $H \geqslant H_0$. In this case, $m_1 = m_2$ and $q_1 = q_2$, and the dependence $T_f(H)$ is obtained directly from the AT formula for a one-component magnet by replacing H by $H - J_0 m = H/(1 + J_0/T) \cong H/2$:

$$T_f(H)=J[1-b^{1/2}(H/H_0)^{2/3}],\quad H\geqslant H_0. \tag{35}$$

From this is can be seen that for $H = H_0$ the temperature $T_f(H_0)$ coincides with $T_0 = T_N(H_0)$. Thus, the point (T_0,H_0) lies at the intersection of the lines $T_f(H)$ and $T_N(H)$, so that (T_0,H_0) is a triple point on the (T,H) phase diagram.

In the region $H < H_0$ on the AT line the antiferromagnetic order parameter $l(H) = \frac{1}{2}[m_1(H) - m_2(H)]$ is nonzero. As H decreases from H_0 the temperature $T_f(H)$ increases in accordance with a law close to (35), for so long as the parameter $l(H)$ is small in comparison with the sublattice magnetization, which in order of magnitude is equal to H. This is the case when $(H_0 - H)/H_0$ is of the order of unity, and formula (35) shows that $T_f(H)$ then increases by an amount of the order of $Jb^{1/2}$ in comparison with T_0. With further decrease of the field the character of the dependence changes sharply. In order to understand the general form of the dependence $T_f(H)$ we shall consider extremely weak fields.

For $H = 0$ we have $m_1(0) = m_2(0)$ and $q_1(0) = q_2(0)$, and from Eqs. (9) we obtain

$$m_1^2(0)=2b\tau-{}^2/_3\tau^3,$$
$$q_1(0)=\tau[1+{}^4/_3 b-{}^4/_9\tau^2],\quad \tau=1-T/J_0. \tag{36}$$

It follows from (36) and (17) that

$$T_f(0)=J(1-b^{1/2}). \qquad (37)$$

It can be seen from (37), (35), and (33) that to within the leading terms in $b^{1/2}$ the temperatures $T_f(0)$, T_0 and $T_f(H_0)$ coincide.

Expanding in powers of H, we obtain

$$m_{1,2}(H)=m_{1,2}(0)+H/2J_0\mp H^2/8J_0^2m_1(0),$$
$$q_{1,2}(H)=q_1(0)\pm HJ_0m_1(0)/2T^2+{}^1/_{12}H^2/T^2. \qquad (38)$$

In this region of fields the temperature of the transition to the spin glass decreases as the field increases in accordance with the law

$$T_f(H)=T_f(0)-H^2/12J_0^2, \quad H<H_0. \qquad (39)$$

According to (39), the decrease occurs very slowly and the maximum change $T_f(0)-T_f(H_0)$ is of the order of $Jb^{3/2}\ll J-T_f(0)=Jb^{1/2}$. On the other hand, as was shown earlier, at a certain H of the order of H_0 the temperature $T_f(H)$ takes a value that exceeds $T_f(0)$ by an amount of the order of $Jb^{1/2}$. Consequently, there is a region of fields of the order of H_0 in which the transition temperature $T_f(H)$ increases with increase of H (see Figs. 3).

In the one-component model that has been investigated up to now, irrespective of the relative magnitudes of the parameters the temperature $T_f(H)$ decreases monotonically with increase of the field, i.e., the magnetic field always suppresses the spin glass. In the two-component Ising model with $J_0>0$ the field suppresses the spin glass in weak and sufficiently strong fields, but there is a region of fields which, on the contrary, suppress the antiferromagnetism and facilitate the appearance of the spin glass, and the transition temperature $T_f(H)$ in these fields is higher than $T_f(0)$. This character of the dependence $T_f(H)$ is valid not only for $b\ll 1$ but for all relative magnitudes of J_0 and J, provided that $J_0>J$.

The odd behavior of $T_f(H)$ becomes understandable if we examine how the internal field acting on the sublattices in the ordered Ising ferromagnet varies as a function of the external field. From the equations

$$m_{1,2}=\operatorname{th}\left(\frac{H}{T}-\frac{J_0}{T}m_{2,1}\right)$$

it follows that

$$\frac{d}{dH}(H-J_0m_2)=\left[1-\frac{J_0}{T}(1-m_2^2)\right]$$
$$\times\left[1-\frac{J_0^2}{T^2}(1-m_1^2)(1-m_2^2)\right]^{-1}. \qquad (40)$$

The denominator in (40) is always positive. As can be easily checked, e.g., for $J_0/T\gg 1$, the numerator changes sign twice with increase of H, so that the internal field acting on the first, stiffer sublattice increases with H in weak and strong fields and decreases in intermediate fields. The increase of T_f is a reflection of this decrease in "stiffness" in intermediate fields.

We now find $T_f(H)$ for $J_0\gg J$. We shall assume that the average internal fields $H-J_0m_{1,2}$ acting on the sublattices are much greater than J. Then we have

$$T_f(H)=\frac{2}{3}\sqrt{\frac{2}{\pi}}J\exp\left\{-\frac{1}{4J^2}[(H-J_0m_1)^2+(H-J_0m_2)^2]\right\}. \qquad (41)$$

In fields $H<J_0$ and $(J_0-H)/J\gg 1$, when, to within exponentially small terms, we have $m_1=-m_2=1$, the temperature T_f decreases with increase of the field:

$$T_f(H)\sim T_f(0)\exp\left(-\frac{H^2}{2J^2}\right).$$

But if $H>J_0$ and $(H-J_0)/J\gg 1$, then $m_1=m_2=1$ and $T_f(H)$ decreases like $J\exp\{-(H-J_0)^2/2J^2\}$ with increase of the field. Thus, approaching $H\sim J_0$ from the region of strong fields we obtain $T_f\sim J$, and approaching $H\sim J_0$ from the region of weak fields we obtain $T_f\sim J\exp(-J_0^2/2J^2)$. This means that in the interval of fields $|H-J_0|\sim J$ there is a sharp increase of the temperature of the transition to the spin glass with increase of the field. The phase diagram has qualitatively the same form as that in Fig. 3.

5. MAGNETIC SUSCEPTIBILITY IN THE ANTIFERROMAGNETIC PHASE

In the preceding section we saw that for $(J_0-J)/J_0\ll 1$ fluctuations of the exchange-interaction energy strongly affect the properties of the antiferromagnetic phase. It turns out that the temperature behavior of the magnetic susceptibility in a wide range of parameters is changed qualitatively by the fluctuations.

Differentiating Eqs. (9) with respect to H, we obtain a system of equations for $\partial m_{1,2}/\partial H$ and $\partial q_{1,2}/\partial H$. Taking into account that in the limit of zero field we have $\partial m_1/\partial H=\partial m_2/\partial H=\chi$, $\partial q_1/\partial H=-\partial q_2/\partial H$ and $m_1=-m_2$, $q_1=q_2$, we have

$$\chi=\frac{\Pi}{J_0(1+\Pi)},$$
$$\Pi=\frac{J_0}{T}\left[1-q_1+\frac{(2J^2/T^2)\langle\operatorname{th}E_1(z)\operatorname{ch}^{-2}E_1(z)\rangle_c^2}{1+(J/T\sqrt{q_1})\langle\operatorname{th}E_1(z)\operatorname{ch}^{-2}E_1(z)\rangle_c}\right]. \qquad (42)$$

It follows from this that in the antiferromagnetic phase near $T_N=J_0$, i.e., for $\tau=(T_N-T)/T_N\ll 1$, the susceptibility satisfies

$$\chi=\frac{1}{2J_0}\left[1+\tau\frac{(J_0^2-J^2)(2J^2-J_0^2)}{(J_0^2+J^2)(2J^2+J_0^2)}\right]. \qquad (43)$$

Comparing (43) with the paramagnetic susceptibility $\chi=(T+J_0)^{-1}$, we see that, as usual, the susceptibility has a discontinuity at the transition point. But in contrast to the ordered antiferromagnet, if $J<J_0<2^{1/2}J$ the susceptibility in the antiferromagnetic phase near T_N does not decrease with increase of the temperature, but increases, though more slowly than in the paramagnet.

This unusual behavior of the susceptibility in the antiferromagnetic phase is due to the large number of frustrated couplings. If there are few frustrations, i.e., $J<J_0/\sqrt{2}$, the susceptibility, as usual, decreases in the interior of the antiferromagnetic phase.

In the case $b\ll 1$ it is possible to calculate the magnetic susceptibility in the entire temperature range of the antiferromagnetic phase. It is easy to see that for $b \ll 1$

$$\Pi = \frac{J_0}{T}[1 - q_1 + (J_0 m_1)^2], \tag{44}$$

and it follows from (36) that

$$\chi = \frac{1}{2J_0}\left[1 + \frac{1}{3}\left(b\tau - \frac{1}{3}\tau^3\right)\right] = \frac{1}{2J_0}\left[1 + \frac{1}{6}m_1^2(0)\right]. \tag{45}$$

The susceptibility described by formula (45) has a maximum at $\tau = b^{1/2}$, so that, to leading order in $b^{1/2}$, the temperature of the maximum coincides with T_f.

If $J_0/J \gg 1$, then $1 - q_1$ is proportional to $\exp(-J_0^2/2J^2)$, and the second term is proportional to the square of this exponential. Therefore,

$$\chi = \frac{1}{T}(1 - q_1) = \frac{1}{\sqrt{2\pi} J} \exp\left(-\frac{J_0^2}{2J^2}\right)\left(2 + \frac{1}{12}T^2 \frac{J_0^2}{J^4}\right). \tag{46}$$

The expression (46) is valid in the wide temperature range $J^2/J_0 \gg T \geqslant T_f \propto \exp(-J_0^2/2J^2)$.

Finally, we note that the nonlinear susceptibility $\chi^{(3)} = \frac{1}{2}\partial^3(m_1 + m_2)/\partial H^3$ is anomalously large in the paramagnetic phase near the triple point of the phase diagram, i.e., for $b \ll 1$:

$$\chi^{(3)} = -3/8J_0^4 b. \tag{47}$$

CONCLUSION

The question of whether the results obtained here are applicable to Heisenberg antiferromagnets of the "easy-axis" type requires additional analysis. But it may be anticipated that they are applicable at least for so long as the external field, directed along the easy axis, is smaller than the sublattice-reversal field.

In the experimental study of disordered antiferromagnets one should keep in mind the random-field effects that can occur if the external magnetic field is switched on along the easy axis.[15,16] In order that these effects not lead to destruction of the long-range antiferromagnetic order it is necessary to apply not-too-strong external fields or to switch on the external field after the transition to the antiferromagnetic phase.[16]

The model investigated here can be generalized easily to describe more complicated magnets. In particular, one could include the interaction within the subsystems with spins S_1 and S_2. However, this should not qualitatively alter the results obtained here.

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