Generation of periodic trains of picosecond pulses in an optical fiber: exact solutions

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Exact analytic solutions are obtained for the nonlinear Schrödinger equation which describes the transformation of a constant-amplitude cw signal in an optical fiber into a periodic train of pulses. These solutions are analyzed for various frequencies and types of initial modulation. The analysis shows that an optimum fiber length exists for such a transformation. At greater fiber lengths the field returns to the initial state. It is shown that the waveforms of the fiber output pulses depend not only on the fiber length but also on the frequency and type of the initial modulation of the cw signal. The fiber-output waveforms calculated numerically from these exact solutions are presented.

1. INTRODUCTION

It was shown in Refs. 1 and 2 that trains of picosecond pulses can be obtained in an optical fiber by exciting it with cw laser radiation. The cw signal breaks up into a periodic sequence of pulses because the refractive index of the fiber material depends linearly on the optical field and because a wave of constant amplitude has a modulational instability against periodic perturbations. In Ref. 1, however, the evolution of the solution to the problem after the initial stage of perturbation growth was investigated only by means of numerical experiment capable only of representing the solution for a finite number of initial conditions and incapable of accounting fully for the general laws that govern the phenomenon. The authors of Ref. 2 confined themselves to the initial growth stage of the periodic perturbations. In the present paper we obtain exact analytic solutions of this problem for various types of initial periodic modulation of an initial constant-amplitude wave and present the pertinent equations for the waveforms of the obtained pulses.

Mathematically speaking, the problem reduces to finding periodic solutions of the nonlinear Schrödinger equation (NSE) and is in this sense of general interest for a large class of problems of contemporary mathematical physics. The modulational instability of NSE solutions in the form of a plane wave with constant amplitude was considered in self-focusing theory,[3,4] in the problem of wave self-modulation in a nonlinear dispersive medium,[5] in the theory of waves in deep water,[6] and elsewhere, but no exact solution has been obtained to date, some attempts notwithstanding.[7] Numerical experiments[5–7] have identified an important feature of the problem, viz., that a solution initially periodically modulated in the space domine is periodic also in the time domain,[1)] so that after a certain time the initial field distribution, with weakly modulated constant amplitude, is restored, in analogy with Fermi-Pasta-Ulam restoration.[8] Our exact solutions confirm this conclusion in the case of a simple harmonic initial modulation and can determine the cases in which it holds for more complicated types of modulation.

Given the initial conditions, the NSE can be solved by the classic formalism of the inverse scattering problem.[9] Nonetheless, in the case of periodic initial conditions one encounters a number of singularities that must be taken into account by special methods using finite-band (N-band) operators.[10–12] The general explicit equations obtained[10] by this method, however, are difficult to analyze and do not solve our problem. We have searched for solutions by direct methods, partly based on the results of numerical simulation. The methods themselves are too cumbersome to describe here. We focus our attention here on the simplest analysis of the solution and also on the results and conclusions that follow from our exact solutions as applied to wave propagation in an optical fiber. The solutions themselves are relatively simply verified by direct substitution in the initial equation, and require no additional clarification in this sense.

The plan of the exposition is as follows. In Sec. 2 we formulate the problem of generating picosecond pulses in a single-mode optical fiber at wavelengths close to the absorption minimum. In Sec. 3 we obtain an exact solution of this problem for simple harmonic modulation of the input signal. In Sec. 4 we discuss problems involved in modulation by a composite signal in the presence of higher harmonics of the fundamental frequency, and a solution is presented for the relatively simple case of modulation by the first two harmonics. The waveforms of the fiber output pulses numerically calculated from our solutions and simple estimates based on them are reported in Sec. 5. In Sec. 6 we summarize the results.

2. STATEMENT OF PROBLEM

It was shown in a number of papers (see, e.g., the review by Hasegawa and Kodama,[13] Vysloukh,[14] and Mollenauer and Stolen[15]) that a wave excited in a single-mode quartz-glass optical fiber at frequencies close to the minimum absorption and to negative group dispersion, at relatively low total radiation power, should exhibit nonlinear behavior due to the nonlinear dependence of the refractive index of the quartz on the wave field:

$$n=n_0+n_2|E|^2, \tag{1}$$

where n_0 is the refractive index of the fiber material and n_2 is

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a nonlinear coefficient. Since the group dispersion is small in this frequency range, the electric-field values at which Eq. (1) must be taken into account are much lower than the self-focusing threshold, and the field itself can be approximated by the equation[13]

$$E(r,x,t)=\mathrm{Re}[\varphi(x,t)R(r)\exp i(kx-\omega t)], \tag{2}$$

where x is the longitudinal coordinate, t is the time, ω is the excitation frequency, k is the wave number, $R(r)$ is the radial eigenfunction of the linear problem, and $\varphi(x,t)$ is the envelope of the optical field. Neglecting absorption in the fiber and higher-order dispersion, the function $\varphi(x,t)$ should satisfy the NSE (Ref. 13).

$$i\frac{\partial\psi}{\partial\xi}+\frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2}+|\psi|^2\psi=0, \tag{3}$$

which has been rewritten here in dimensionless form by changing to the new variables

$$\xi=q^2\frac{x}{\lambda},\quad \tau=q(-\lambda k'')^{-1/2}\left(t-\frac{x}{v}\right),\quad \psi=\frac{(\pi n_2)^{1/2}}{q}\varphi, \tag{4}$$

where

$$\lambda=2\pi c/\omega,\qquad k''=\partial^2k/\partial\omega^2,\qquad v=\partial\omega/\partial k,$$

q is a normalization factor that determines the connection between the signal amplitude and the characteristic temporal and spatial changes of the field in the fiber.

To simplify the analysis, we introduce a function $u(\xi,\tau)$ connected with ψ by the relation

$$\psi(\xi,\tau)=u(\xi,\tau)\exp i\xi.$$

In this case only a weak dependence on the variable ξ remains the function u, and the equation for the latter takes the form

$$i\frac{\partial u}{\partial\xi}+\frac{1}{2}\frac{\partial^2u}{\partial\tau^2}-u+|u|^2u=0. \tag{5}$$

Equation (5) admits of a soliton solution

$$u=\sqrt{2}/\mathrm{ch}\,\sqrt{2}(\tau-\tau_0), \tag{6}$$

where τ_0 is the coordinate of the center of the soliton. As applied to propagation of optical pulses in a fiber, the soliton is the subject of many papers (see the literature cited in Refs. 12–15, and also Ref. 16, where solitons in a fiber were first considered).

Equation (5) has in addition a very simple solution in the form of a complex constant $u=\exp i\varphi$, which describes a stationary wave of unit amplitude and arbitrary phase φ. It is known[1–7] that this stationary solution is unstable to long-wave periodic perturbations, and that these perturbations increase exponentially as the wave propagates along the fiber (as ξ increases). In fact, let the wave entering the fiber be weakly modulated:

$$u=\left[1+\sum_{j=1}^{n}a_j(\xi)\cos j\varkappa(\tau-\tau_{0j})\right]\exp i\varphi, \tag{7}$$

where the a_j are the Fourier coefficients of the periodic modulation and are assumed to be small during the initial growth state ($|a_j|\ll1$), $\varkappa$ is the external-modulation frequency, n is the number of harmonics of the fundamental frequency in the input signal, and τ_{0j} is the initial phase of the jth harmonic. Substituting (7) in (5) and retaining the terms linear in a_j, we can show then that these coefficients are given by

$$\begin{aligned}a_j(\xi)&=A_j[j\varkappa/2+i(1-j^2\varkappa^2/4)^{1/2}]\exp(\delta_j\xi)\\&\quad+B_j[j\varkappa/2-i(1-j^2\varkappa^2/4)^{1/2}]\exp(-\delta_j\xi)\\&=A_j\exp(i\alpha_j+\delta_j\xi)+B_j\exp(-i\alpha_j-\delta_j\xi),\end{aligned} \tag{8}$$

where

$$\mathrm{tg}\,\alpha_j=2\delta_j/j^2\varkappa^2,\quad \delta_j=j\varkappa(1-j^2\varkappa^2/4)^{1/2},$$

δ_j is the growth rate of the j-th harmonic of the perturbation and is real in the frequency interval $0<j\varkappa<2$; A_j and B_j are real constant coefficients for the initial values of ξ and satisfy the conditions.

$$|A_j\exp(\delta_j\xi)|\ll1,\qquad |B_j\exp(-\delta_j\xi)|\ll1.$$

The instability growth rate δ_1 is a maximum at $\varkappa=\sqrt{2}$, and in investigations of the perturbation growth in modulational instability it is customary to retain only the fundamental harmonic of the perturbation with this value of $\varkappa$. The remaining δ_j are pure imaginary in this case. In the case of an optical waveguide it is possible to have the input cw radiation modulated beforehand at a specified period and amplitude. Taking this circumstance into account, we shall investigate below the solutions of Eq. (5) at arbitrary values of $\varkappa$. We shall see that preliminary modulation permits control of not only the pulse repetition period but also the waveform, duration, and peak amplitude.

3. SIMPLE INITIAL HARMONIC MODULATION

We consider first the simplest situation, in which $0<\varkappa<2$ and $n=1$, i.e., the initial modulation is purely harmonic and has a real growth rate δ_1. We consider for the time being only growing perturbations, and set the coefficient B_1 in (8) equal to zero. To obtain perturbations with pure exponential growth we need, as seen from (8), composite modulation, i.e., both in amplitude and phase, with a ratio α_1 of the modulation depths. This constraint permits an exact NSE solution with initial condition (7) to be found. If, however, pure amplitude or pure phase modulation is used, the solution will be close to that obtained and the waveform of the produced pulses in the end will depend little on the type of modulation. The exact (and only) solution of Eq. (5) as $\xi\to-\infty$, which has a limit (7) in which $B_1=0$ and $n=1$, is the function

$$u(\xi,\tau)=\left[1-\frac{{}^1\!/_2p\varkappa^2\,\mathrm{ch}\,\delta_1(\xi-\xi_{01})+ip\delta_1\,\mathrm{sh}\,\delta_1(\xi-\xi_{01})}{p\,\mathrm{ch}\,\delta_1(\xi-\xi_{01})-\cos\varkappa(\tau-\tau_{01})}\right]\exp i\varphi', \tag{9}$$

where

$$p=\varkappa/\delta_1,\quad \varphi'=\varphi-\arccos(1-\varkappa^2/2),$$

and the constant ξ_{01} is connected with A_1 by the relation $A_1=\delta\exp(-\delta_1\xi_{01})$. We shall not present here the method used to obtain (9), but we can verify by direct substitution that this function is indeed the solution of the NSE.

It can be seen from (9) that the initial ($\xi \to -\infty$) stationary state $\exp i\varphi$ acquires as a result of the instability development a modulation whose depth increases nonlinearly to a certain maximum at $\xi = \xi_{01}$, followed as $\xi \to \infty$ by a return to a stationary solution with a different phase $u = \exp i[\varphi + \Delta\varphi_1(\varkappa)]$, i.e., the solution (9) restores the field to its initial unit amplitude, but with a phase rotated from the initial value by $\Delta\varphi_1(\varkappa) = 2\arccos(\varkappa^2/2 - 1)$. In the concluding state of the process as $\xi \to \infty$ the linear term of the expansion of (9) in the small parameter $\exp[-\delta_1(\xi - \xi_{01})]$ again coincides with (7), but in the latter we now have $A_1 = 0$, $B = \delta_1 \exp\delta_1\xi_{01}$, and the phase has a new value $\varphi + \Delta\varphi_1(\varkappa)$. Thus, the exponentially decreasing term of (8), which has usually been neglected in papers on modulational instability, has a real physical meaning.

As $\varkappa \to 0$, the period in τ increases to infinity and in the limit we obtain from (9) an NSE solution in the form of a rational fraction:

$$u(\xi,\tau) = \left[1 - 4\frac{1 + 2i(\xi - \xi_{01})}{1 + 4(\tau - \tau_{01})^2 + 4(\xi - \xi_{01})^2}\right]\exp i\varphi'. \qquad (10)$$

For this solution, the distribution of the field in τ takes the form of a single "dark" pulse against the background of a cw signal whose form changes with changing ξ. As $\xi \to \infty$ the pulse vanishes and the initial stationary field is restored. Such a solution, however is realized as $\delta_1 \to 0$, and the initial perturbation has a power-law rather than exponential growth. This solution can hardly be obtained in pure form in optical waveguides. Nonetheless, the waveform of the generated pulses, as shown by calculations, comes close to (10) even at $\varkappa \lesssim 0.5$, and this equation can be used in practice for approximate calculations, recognizing that the pulses repeat with a frequency $\varkappa$.

We take special notice of the case $\varkappa = \sqrt{2}$, when the instability growth increment δ_1 is a maximum. This is precisely the case previously dealt with in studies of modulational instability. The solution (9) takes in this case the simpler form

$$u(\xi,\tau) = \frac{\cos\sqrt{2}(\tau - \tau_{01}) + i\sqrt{2}\,\mathrm{sh}(\xi - \xi_{01})}{\cos\sqrt{2}(\tau - \tau_{01}) - \sqrt{2}\,\mathrm{ch}(\xi - \xi_{01})}\exp i\varphi', \qquad (11)$$

and the trajectories described by this solution transform the initial state $u = \exp i(\varphi' + \pi/2)$ into the final $u = \exp i(\varphi' - \pi/2)$, so that the total phase rotation angle is exactly equal to π.

4. INITIAL MODULATION BY A COMPOSITE PERIODIC SIGNAL

In the range $0 < \varkappa < 1$ not only the fundamental, but also its harmonic with frequency $2\varkappa$, is unstable so that the first two harmonics must be taken into account in the composite modulating function. The evolution of such a perturbation leaves the signal periodic in τ and dependent both on the relative phase difference of the two harmonics and on the ratio of their initial amplitudes. The exact solution of Eq. (5) in the region $0 < \varkappa < 1$, which in the limit takes the form (7) with $n = 2$, $B_1 = 0$, $B_2 = 0$, can be written in the form

$$u(\xi,\tau) = [1 - (G + iH)/D]\exp i\varphi'', \qquad (12)$$

$$G = (\varkappa^2/4\delta_1)\,\mathrm{ch}\,\delta_1(\xi - \xi_{01})\cos 2\varkappa(\tau - \tau_{02}) + (2\varkappa^2/\delta_2)\,\mathrm{ch}\,\delta_2(\xi - \xi_{02})\cos\varkappa(\tau - \tau_{01}) + (3\varkappa^3/2\delta_1\delta_2)\,\mathrm{ch}\,\delta_1(\xi - \xi_{01})\,\mathrm{ch}\,\delta_2(\xi - \xi_{02}), \qquad (12a)$$

$$H = {}^1/_2\,\mathrm{sh}\,\delta_1(\xi - \xi_{01})\cos 2\varkappa(\tau - \tau_{02}) + \mathrm{sh}\,\delta_2(\xi - \xi_{02})\cos\varkappa(\tau - \tau_{01}) - (\varkappa/\delta_1\delta_2)[\delta_1\,\mathrm{sh}\,\delta_1(\xi - \xi_{01})\,\mathrm{ch}\,\delta_2(\xi - \xi_{02}) - \delta_2\,\mathrm{ch}\,\delta_1(\xi - \xi_{01})\,\mathrm{sh}\,\delta_2(\xi - \xi_{02})], \qquad (12b)$$

$$D = (3/4\varkappa)[\cos\varkappa(\tau + \tau_{01} - 2\tau_{02}) + {}^1/_9\cos\varkappa(3\tau - 2\tau_{02} - \tau_{01})] + (1/2\delta_1)\,\mathrm{ch}\,\delta_1(\xi - \xi_{01})\cos 2\varkappa(\tau - \tau_{02}) + (1/\delta_2)\,\mathrm{ch}\,\delta_2(\xi - \xi_{02})\cos\varkappa(\tau - \tau_{01}) - (2/3\varkappa)\{[\varkappa^2(2\varkappa^2 - 5)/2\delta_1\delta_2]\,\mathrm{ch}\,\delta_1(\xi - \xi_{01})\,\mathrm{ch}\,\delta_2(\xi - \xi_{02}) + \mathrm{sh}\,\delta_1(\xi - \xi_{01})\,\mathrm{sh}\,\delta_2(\xi - \xi_{02})\}, \qquad (12c)$$

where

$$\varphi'' = \varphi - \arccos[\delta_1\delta_2 - (\varkappa^2/2 - 1)(2\varkappa^2 - 1)],$$

and the constants ξ_{01} and ξ_{02} are connected with the initial amplitudes in the expansion of (7) by the relations

$$A_1 = \delta_1\exp(-\delta_1\xi_{01}), \qquad A_2 = \delta_2\exp(-\delta_2\xi_{02}).$$

It can easily be shown that the final state of the field determined by the solution (12) as $\xi \to \infty$ is also a wave with unit amplitude, and the total phase rotation in this case is equal to the sum of the rotations due to the elementary solutions (9):

$$\Delta\varphi_2(\varkappa) = \Delta\varphi_1(\varkappa) + \Delta\varphi_1(2\varkappa), \qquad (13)$$

so that

$$u(\xi \to \infty, \tau) = \exp(\varphi + \Delta\varphi_2(\varkappa)).$$

The quantitites ξ_{01} and ξ_{02} in solution (12) are in effect centers of two elementry solutions (9) whose nonlinear interaction leads to the solution (12). When the centers coincide, $\xi_{01} = \xi_{02} = 0$, i.e., if $A_1 = \delta_1$, $A_2 = \delta_2$, the solution (12) with phase $\varphi'' = 0$ is symmetric relative to reversal of the sign of ξ:

$$u(\xi,\tau) = u^*(-\xi,\tau).$$

If the centers of the elementary solutions are separated by a larger distance along the ξ axis, (12) breaks up into a sum of two elementary solutions (9), each with its own phase φ', just as in the case of two-soliton solutions of the NSE, when the distances between the soliton centers exceed the characteristic dimensions of both of them.

We note that at $\varkappa = 2/\sqrt{5}$ the growth rates of the two harmonics are equal, $\delta_1 = \delta_2 = \frac{4}{5}$, and solution (12) becomes simpler. We write down here only the symmetric solution ($\xi_{01} = \xi_{02} = 0$):

$$u(\xi,\tau) = -\frac{\mathrm{ch}^2\,{}^4/_5\xi + P(\tau)\,\mathrm{ch}\,{}^4/_5\xi - C(\tau) + iF(\tau)\,\mathrm{sh}\,{}^4/_5\xi}{\mathrm{ch}^2\,{}^4/_5\xi + {}^5/_4F(\tau)\,\mathrm{ch}\,{}^4/_5\xi + C(\tau)}\exp i\varphi'', \qquad (14)$$

$$P(\tau) = \frac{1}{\sqrt{5}}\left[2\cos\frac{2}{\sqrt{5}}(\tau - \tau_{01}) - \cos\frac{4}{\sqrt{5}}(\tau - \tau_{02})\right], \qquad (14a)$$

$$F(\tau)=\frac{4}{3\sqrt{5}}\left[\,2\cos\frac{2}{\sqrt{5}}(\tau-\tau_{01})+\cos\frac{4}{\sqrt{5}}(\tau-\tau_{02})\,\right], \tag{14b}$$

$$C(\tau)=\left[\frac{8}{9}+\cos\frac{2}{\sqrt{5}}(\tau+\tau_{01}-2\tau_{02}) + \frac{1}{9}\cos\frac{4}{\sqrt{5}}(3\tau-\tau_{01}-2\tau_{02})\,\right]. \tag{14c}$$

The total phase rotation effected by solution (14) is exactly equal to 2π.

In the case $\tau_{01}=\pi/\varkappa$, $\tau_{02}=0$ we can take the limit as $\varkappa\to 0$ in (12). We then obtain a second solution of the NSE, in the form of a rational fraction of order higher than (11):

$$u(\xi,\ \tau)=[1-(G_1+iH_1)/D_1]\exp i\varphi'', \tag{15}$$

$$G_1=5\xi^4+6\xi^2\tau^2+\tau^4+{}^9/_2\xi^2+{}^3/_2\tau^2-{}^3/_{16}, \tag{15a}$$

$$H_1=\xi[2\xi^4+4\xi^2\tau^2+2\tau^4+\xi^2-3\tau^2-{}^{15}/_8], \tag{15b}$$

$$D_1={}^1/_3\xi^6+\xi^4\tau^2+2\xi^2\tau^4+{}^1/_3\tau^6+{}^9/_4\xi^4 -{}^3/_2\xi^2\tau^2+{}^1/_4\tau^4+{}^{33}/_{16}\xi^2+{}^9/_{16}\tau^2+{}^3/_{64}. \tag{15c}$$

For simplicity, we have put $\xi_{01}=\xi_{02}=0$ in (15).

If the modulation frequency falls in the interval $0<\varkappa<2/n$, where n is an integer, n harmonics of the fundamental frequency of the modulation are unstable, and n-mode solutions of more complicated structure containing separatrices can exist in addition to those considered. The construction of exact analytic solutions of this type is possible in principle, but is apparently made difficult by the rapid increase, with increasing n, of the complexity of both the calculations and the final equations. A simpler approach in this case is numerical calculation of such multimode solutions on the basis of the initial equation (5). Typical solutions of this kind in numerical experiments are cited, for example, in Ref. 6. We note here, however, that account must be taken of a number of features of these solutions when it comes to their implementation, since for $n>2$ a small change of the initial conditions of this problem can alter greatly the subsequent evolution of the field, leading eventually to a complicated solution that is difficult to analyze.

1) For containing separatrices solutions, the first term of the expansion of the solution as $\xi\to\infty$ is a sum of n elementary exponentially increasing perturbations of type (7), where $B_j=0$ and the coefficients $A_j=\delta_j\exp(-\delta_j\xi_{0j})$ are arbitrary. Accordingly, as $\xi\to\infty$ the solution consists of n elementary exponentially decreasing perturbations of form (7), where $A_j=0$, $B_j\neq 0$, and the phase φ differs from its initial value.

2) The total phase rotation $\Delta\varphi_n(\varkappa)$ of a solution containing separatrices as ξ changes from $-\infty$ to ∞ is equal to the sum of rotations effected by the elementary solutions (9), i.e.,

$$\Delta\varphi_n(\varkappa)=\sum_{j=1}^{n}\Delta\varphi_1(j\varkappa) \tag{16}$$

regardless of the ratio of the coefficients A_j specified in the initial conditions.

3) If the coefficients $A_j=\delta_j$ are chosen such that all $\xi_{0j}=0$, the total solution becomes symmetric with respect to the variable ξ, i.e., a phase φ is obtained such that $u(\xi,\tau)=u^*(-\xi,\tau)$. If the value of ξ_{0j} are widely spaced along the ξ axis, the total solution breaks up into a sum of elementary solutions (9).

We have considered above a situation in which all the B_j in (7) and (8) vanish identically, so that the solutions containing separatrices can be separated. In the same case, when the initial state does not lie on the separatrix trajectory, the total solution becomes multiply periodic in the variable ξ, and in the general case the periods are incommensurate, so that the solution is in some sense chaotic and the initial state of the field is not restored. Therefore in the case of n-mode solutions with $n\geqslant 2$ restoration of the initial state, analogous to the Fermi–Pasta-Ulam restoration in excitation of a system of nonlinear oscillators, will take place only for solutions containing separatrices.

5. NUMERICAL RESULTS AND ESTIMATES

We consider first the case of simple harmonic modulation of the input signal. The pulse waveform calculated from (9) is shown in Fig. 1 for different values of $\varkappa$. The pulses have minimum widths and maximum amplitudes at $\xi=\vartheta_{01}$. The fiber length needed to obtain a specified pulse waveform is determined by the average pump power and by the ampli-

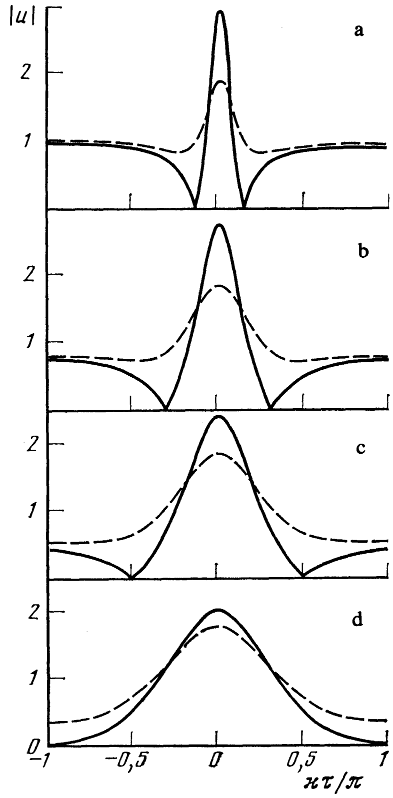

FIG. 1. Fiber output-pulse waveform for simple harmonic modulation of the input signal. The parameters are: $\xi-\xi_{01}=0$ (solid curves), ± 0.75 (dashed) $\tau_{01}=0$, $\varkappa=0.5$ (a), 1 (b), $\sqrt{2}$ (c), $\sqrt{3}$ (d).

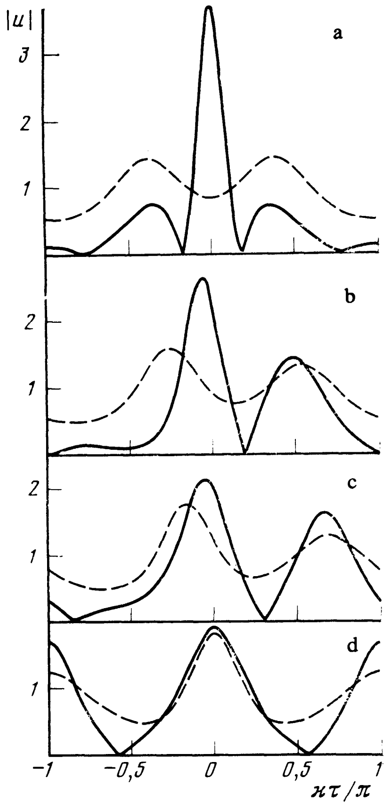

FIG. 2. Waveforms of single fiber output pulses in the limit of large modulation periods ($\varkappa \to 0$). Parameters: $\tau_{01} = 0$, $\xi - \xi_{01} = 0$ (solid curve) ± 0.5 (dashed).

tude and frequency of the initial modulation. All the estimates that follow are for a quartz single-mode fiber[13] at a radiation wavelength $\lambda = 1.55$ μm. At this wavelength quartz has $n_0 \approx 1{,}5$, $n_2 \approx 1{,}2 \cdot 10^{-22}$ $(\mathrm{m/V})^2$, a group-velocity dispersion $D = (2\pi c/\lambda^2)(\partial^2 k/\partial\omega^2) = -16$ ps/nm·km, so that $\partial^2 k/\partial\omega^2 \approx -2 \cdot 10^{-26}$ s^2/m. The depth of the initial modulation determined from the linear expansion (7) is

$$M(\xi) = A_1 \exp \delta_1 \xi = \delta_1 \exp \delta_1 (\xi - \xi_{01}).$$

The modulation depth for agent ξ obviously depends on $\varkappa$. We chose for the estimates the value $\varkappa = \sqrt{2}$, at which the growth rate is a maximum. In this case the inverse of the pulse duty cycle, i.e., the ratio of the repetition period to the width of each pulse at half-maximum peak power, is approximately 6. An initial modulation depth $M(\xi) \approx 5\%$ corresponds to $\xi - \xi_{01} \approx -3.0$, and at a pulse repetition period of, say, 6 ps we have $q \approx 1.33 \cdot 10^4$, the fiber length needed to obtain the narrowest pulse is about 260 m, and the average cw input power for a fiber with 20 μm^2 cross section is 1.9 W. The length of each pulse is in this case 1 ps. The inverse duty cycle of the pulse also increases with decreasing $\varkappa$, and is equal, for example, to 20 at $\varkappa = 0.5$. At the same time, however, the growth rate δ_1 decreases and the fiber length needed to obtain pulses of optimum waveform also increases. We note also that there exists a maximum limit ≈ 250 ps on the pulse repetition period,[1] owing to the presence of the competing stimulated Brillouin scattering in the fiber. As the modulation frequency $\varkappa$ decreases, a cw signal background

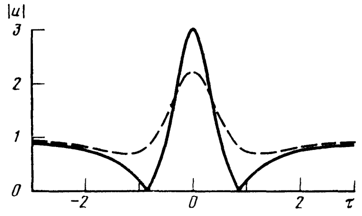

FIG. 3. Fiber output pulse waveform for initial modulation by two first two harmonics of the frequency $\varkappa = 2/\sqrt{5}$. The parameters are: $\xi = 0$ (solid curves), ± 2 (dashed), $\tau_{01} = \pi/\varkappa$, $\varkappa\tau_{02}/\pi = 0$ (a), $\frac{1}{8}$ (b), $\frac{1}{4}$ (c), $\frac{1}{2}$ (d).

is produced between the pulses and has an amplitude close to that of the pump, while the waveform of the individual pulses, to within an accuracy of $\approx 3\%$, take the form described by Eq. (10) even at $\varkappa \lesssim 0.5$. The waveform of these pulses is shown in Fig. 2. There is no background between the pulses at the fixed frequency $\varkappa = \sqrt{3}$. Thus, for simple harmonic modulation the wave form and the inverse duty cycle of the fiber output pulses can be controlled by varying the average power of the cw input and the frequency of the preliminary modulation.

If $\varkappa < 1$, the wave entering the fiber can be modulated by two harmonics. In this case it becomes possible to control the waveforms of the output signals not only by varying the modulation frequency and the cw signal power, but also by the phase shifts τ_{01} and τ_{02} of the two modulation harmonics, as well as by the parameter $\xi_{01} - \xi_{02}$ that is equivalent to the ratio of the modulation depths of the harmonics. We consider below only the dependence of the pulse waveforms on τ_{01} and τ_{02} for the case $\varkappa = 2/\sqrt{5}$. Figure 3 shows the pulse waveforms calculated from Eq. (14). It can be seen from the figure that in two cases, $\tau_{01} = \pi/\varkappa$, $\tau_{02} = 0$ and $\tau_{01} = \pi/\varkappa$, $\tau_{02} = \pi/2\varkappa$, the pulse waveform at the fiber output is symmetric about τ within each period. The highest amplitude and the smallest width of the pulse occur at $\tau_{01} = \pi/\varkappa$, $\tau_{02} = 0$.

If smaller values of $\varkappa$ are chosen, the pulse becomes more peaked its amplitude increases and its width decreases. The pulse waveform as $\varkappa \to 0$, calculated with the aid of Eq. (15), is shown in Fig. 4. The sharper maximum, however, vanishes rapidly when ξ is varied within ± 0.2 from the value at $\xi = 0$. For a given fiber length it is therefore necessary in this case to set the modulation amplitude accurately. In the case considered, the modulation depths of the two harmonics are the same:

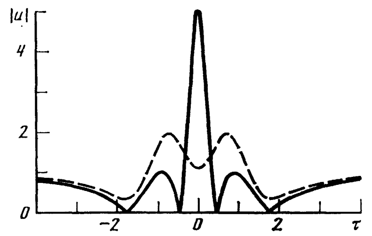

FIG. 4. Waveforms of single pulses of complex waveform as $\varkappa \to 0$. Parameters: $\xi = 0$ (solid curve), ± 0.5 (dashed).

$$M_1(\xi)=M_2(\xi)=A_1 \exp \delta_1\xi=A_2 \exp \delta_2\xi=\delta_1 \exp \delta_1\xi,$$

where $\delta_1 = \delta_2 = \frac{4}{5}$. For the pulses shown in Fig. 3a, the inverse duty cycle is 16. To maintain a pulse repetition period of 6 ps, the value of q must be $2.1\cdot10^4$. To obtain the optimum pulse waveform at the same 260 m fiber length the value of ξ should be -7.4. This yields a required cw signal power ≈ 4.7 W and a modulation depth $M_1(\xi) \approx 0.22\%$.

The present analysis shows that in the case $n = 2$, at a specified modulation frequency, variation of the average power, of the modulation depth, and of the number of harmonics can produce the three pulse types shown in Figs. 1c, 3a, and 3d. In addition, it is possible to modulate the pulse waveform by varying the parameter $\xi_{01} - \xi_{02}$ that is equivalent to the ratio of the modulation depths $M_1(\xi)$ and $M_2(\xi)$. For smaller $\varkappa$, however, when $n > 1$, the variety of fiber output pulse waveforms increases. This effect can be used, for example, in communication lines and permits alteration of the transmission code by a very slight change of the initial conditions. We note also that if necessary the length of the pulses generated can be lowered to several dozen femtoseconds.

6. CONCLUSION

This technique for converting cw radiation in an optic fiber into a train of short pulses uncovers new possibilities for developing optoelectronic devices based on optical fibers. Besides production of ultrashort pulses, we point out here the possibility of converting IR radiation into sideband frequencies that differ from the central one by the modulation frequency. The fiber can thus be used to tune the frequency of IR radiation. The results of the present paper can be used also in the theory of optical communication. The possibility of obtaining pulses having a prescribed form governed by the type of modulation of the input cw radiation makes it possible to specify the transmission code. The exact solutions (9)–(12), (14), and (15) which we obtained for the NSE may of themselves be useful also in other physical problems with periodic initial conditions, viz., in the theory of two-dimensional self-focusing, in the theory of sea waves and so on.

[1)]The spatial and temporal variables interchange roles in the case of an optic fiber.

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