Pinch effect in an electron-hole plasma with extrinsic conductivity

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A theoretical study is made of the pinch effect in a semiconductor plasma when the electron density n_0 exceeds the hole density p_0 by N ionized donors. An exact solution is obtained for a plasma detached from the walls of the current channel in the case when the temperatures of the current carriers differ strongly on account of heating by the applied electric field ($T_n \gg T_p$). The critical current for detachment of the plasma is given by the expression $I_k = I_B\, p_0/(p_0+N)$, where I_B is the Bennett current [see Eq. (1)], and when a current $I = I_B\,(p_0+N)/p_0$ is attained the spatial distribution of the carriers, as in the Bennett problem, degenerates into a δ function. It is found that a plasma with an "excess" electron density creates a magnetic field which increases from the plasma pinch to the periphery of the current channel and causes additional constriction of the bipolar plasma.

1. The main criterion for the appearance of the pinch effect—the constriction of a bipolar plasma in the self-magnetic field of a flowing current—is the so-called Bennett condition,[1] which determines the critical current for the occurrence of an appreciable redistribution of current carriers:

$$I \geqslant I_B = \frac{2c^2(T_n+T_p)}{e(v_n+v_p)}, \qquad (1)$$

where $v_{n,\,p}$ and $T_{n,\,p}$ are the drift velocities and temperatures of the electrons and holes. The Bennett problem has a steady-state solution at currents below I_B, and for $I \geqslant I_B$ the spatial distribution of the plasma is a δ function.

Condition (1) has turned out to be convenient for identifying the pinch in both gaseous and electron-hole plasmas and is often used to determine the region of current values corresponding to the onset of the pinch.[2] Bennett's critical current (1), like the solution of the Bennett problem itself, was obtained under the assumption of an intrinsic bipolar plasma ($n = p$). In the actual situation in a semiconductor, however, the electron and hole densities can differ substantially on account of a finite density of an uncompensated impurity. One therefore wonders whether it is correct to use relation (1) to identify the pinch in a solid. Furthermore, the Bennett problem does not provide for a nonisothermal plasma ($T_n/T_p \neq 1$), which is present in practically all the pinch-effect experiments on account of the heating of the current carriers by the electric field.

In the present study we show for the example of the electron-hole plasma of a semiconductor that allowance for the extrinsic (impurity) component of the current substantially alters the character of the formation of the pinch and leads to a new criterion for the onset of the pinch.

2. A detailed treatment of the pinch problem follows from the joint solution of Maxwell's equations and the continuity equations for electrons and holes. As in the Bennett problem, we neglect generation and recombination in the interior and on the surface of the crystal. The scattering will be assumed to be so insignificant that there is no temperature gradient over the cross section of the current channel. The remaining approximations are analogous to the familiar ones.[3,4] The only important difference between our treatment and the usual treatment ($n = p$) is, first of all, that the extrinsic component of the current is taken into account. For definiteness, we consider an n-type semiconductor. The condition of neutrality in this case will be of the form $n = p + N$, where N is the density of uncompensated donors, and the extrinsic character of the material will be rendered by the parameter $f_N = N/p_0$, which gives the ratio of the density of "excess" electrons to the equilibrium density of the bipolar plasma. Second, allowance for these excess electrons enables us to take into account not only the difference between the temperatures of the current carriers but also the ratio of these temperatures, in contrast to the Bennett problem, where the temperatures appear only additively [see (1)].

Let us introduce the dimensionless quantities

$$f(\rho) = \frac{p(r)}{p_0}, \quad \rho = \frac{r}{R}, \quad h(\rho) = \frac{e(v_n+v_p)}{c(T_n+T_p)} R H_\varphi,$$

$$b_T = \frac{T_p}{T_n+T_p}, \quad b_v = \frac{v_n}{v_n+v_p}, \quad \varepsilon = \frac{8I}{I_B(1+b_v f_N)},$$

where p_0 and $p(r)$ are the equilibrium and local densities of the plasma, r is the coordinate, R is the radius of the sample, $h(\rho) \propto H_\varphi$ is the self-magnetic field, I is the total current, and ε is the dimensionless total current.

The equations describing the steady-state distributions of the current carriers and the magnetic field over the cross section of a cylindrical crystal in our approximation can be written in the form

$$\frac{df}{d\rho} + h(\rho) f \frac{f+f_N}{f+b_T f_N} = 0, \qquad (2)$$

$$\frac{1}{\rho}\frac{d}{d\rho}(h(\rho)\rho) = \varepsilon (f + b_v f_N). \qquad (3)$$

System (2), (3) must be supplemented by the condition that the number of particles in the interior of the crystal is conserved:

$$\int_0^1 f(\rho)\rho\, d\rho = \frac{1}{2}. \qquad (4)$$

Equations (2)–(4) cannot be solved exactly. Let us construct an approximate solution for the case of very different carrier velocities ($v_n \gg v_p, b_v \approx 1$); this is a realistic condition which is easily satisfied in narrow-gap semiconductors, which are widely used in pinch-effect studies.

For a well-developed pinch, when $f(\rho\approx 0) \gg f_N$, b_T, F_N, v_v, F_N, the problem is solved by the well-known Bennet solution:

$$f(\rho\approx 0)=\frac{8}{\varepsilon}\frac{C}{(\rho^2+C)^2}, \quad h(\rho\approx 0)=\frac{4\rho}{\rho^2+C}, \tag{5}$$

where C is an integration constant. Substituting these relations into (2) and (3), we find an expression for the spatial distributions of the magnetic field and plasma density:

$$h(\rho)\approx 4\rho/(\rho^2+C)+\varepsilon b_v f_N \rho/2, \tag{6}$$

$$f^{b_T}(f+f_N)^{1-b_T}\approx\frac{8}{\varepsilon}\frac{C}{(\rho^2+C)^2}\exp(-\varepsilon b_v f_N \rho^2/4). \tag{7}$$

The applicability condition for (6) and (7) is

$$C(\varepsilon)\ll 8/\varepsilon f_N,\ 8/\varepsilon b_v f_N. \tag{8}$$

Let us now discuss some concrete cases.

a) Isothermal plasma ($T_n = T_p = T$, $b_T = 1/2$). In this case solution (7) becomes

$$f(\rho)=-\frac{f_N}{2}+\left(\frac{f_N^2}{4}+A(\rho)\right)^{1/2}, \quad A(\rho)=\frac{8C}{\varepsilon(\rho^2+C)^2}\exp(-\varepsilon b_v f_N \rho^2/4). \tag{9}$$

Using the particle conservation condition (4) we find that C has a positive definite value for $\varepsilon < 8$. This means that criterion (1) now becomes

$$I\geqslant I_B(1+b_v f_N). \tag{10}$$

It follows from (10) that the condition for "separation" of the steady-state solution is weakened and that Bennett's critical current increases by an amount related to the residual monopolar conductivity of the crystal, i.e., to the conductivity of the excess electrons, which are not subject to pinching.

b) Nonisothermal plasma ($T_n \gg T_p$, $b_T \ll 1$). Such a situation is possible in semiconductors with highly different carrier mobilities ($v_n \gg v_p$) on heating by an electric field. In solving equation (2) we can neglect $b_T f_N$, considering it to be small compared to f. In this case the problem admits an exact solution, since $b_v \approx 1$ ($v_n \gg v_p$):

$$f(\rho)=\frac{8}{\varepsilon}\frac{C}{(\rho^2+C)^2}-f_N. \tag{11}$$

One is easily satisfied that for the values of ε under consideration, the approximate solution (7) leads to this same result.

We see from (11) that at the point

$$\rho_k^2=(8C/\varepsilon f_N)^{1/2}-C$$

the plasma density is zero [$f(\rho_k)=0$]. Thus, solution (11) describes a plasma which is "detached" from the boundaries of the crystal, and coordinate ρ_k gives the radius of the plasma pinch. We note that in a purely bipolar plasma (the Bennett problem) the steady-state solution does not entail the concepts of "detachment" of the plasma and radius of the plasma pinch.

We can determine the critical current at which detachment of the plasma occurs. Using the balance equations (4) and (11), we find that detachment sets in at $\varepsilon_k = 8/(1+f_N)^2$ or

$$I_k=I_B p_0/(p_0+N). \tag{12}$$

We see that $I_k \leqslant I_B$, and the higher the density of excess electrons, the smaller the critical current for pinching. It is easily seen from (12) that in a purely bipolar plasma ($N=0$) detachment occurs only when the Bennett current (1) is reached. For $0\leqslant\varepsilon\leqslant\varepsilon_k$ the plasma density at the center of the crystal is

$$f(0)=(1+f_N)/[1-{}^1/_8\varepsilon(1+f_N)]-f_N. \tag{13}$$

At $\varepsilon=\varepsilon_k$ we have $f(0)=2+1/f_N$.

In the case $\varepsilon>\varepsilon_k$ ($I>I_k$), expressions (4) and (11) can be used to determine the radius of the pinch ($\rho_k = r_k/R$) and the plasma density at the center of the crystal:

$$\rho_k^2=f_N^{-1}[(8/\varepsilon)^{1/2}-1], \tag{14}$$

$$f(0)=f_N\{[1-(\varepsilon/8)^{1/2}]^{-2}-1\}. \tag{15}$$

Using expressions (10) and (12), we can give the end points of the current region in which there exists a plasma detached from the boundaries:

$$I_B p_0/(p_0+N)=I_k\leqslant I\leqslant I_B(p_0+N)/p_0. \tag{16}$$

Thus, in a significant range of currents the bipolar component of the plasma is not in contact with the walls of the crystal; the radius of the pinch is smaller for larger N, and only at $I = I_B(1+N/p_0)$ does the spatial distribution of the bipolar plasma degenerate into a δ function, as in the Bennett problem.

Let us use (7) to estimate how small the plasma density is beyond the boundaries of the pinch for a finite value of b_T. For $f<f_N$ we find from (7) that

$$f(\rho)\approx\left[\frac{8C}{\varepsilon(\rho^2+C)^2}f_N^{b_T-1}\right]^{1/b_T},$$

i.e., for very different temperatures ($b_T \ll 1$) we can to good accuracy regard the plasma as completely detached (a quantity much smaller than unity is raised to the power b_T^{-1}).

3. Let us discuss the physical interpretation of these results. The numerical solution of problem (2)–(4) is plotted in Figs. 1 and 2, which demonstrate the spatial distribution of the self-magnetic field and the density of electron-hole pairs in dimensionless units. Let us first consider the case of an isothermal plasma (Fig. 1). Curves 1 and 1′ in Fig. 1a, calculated for light doping ($f_N = 0.2$), are analogous to the familiar solution of the problem for a bipolar plasma: the radial magnetic field falls off practically in inverse proportion to the distance from the boundary of the plasma pinch. In the case of a highly extrinsic semiconductor ($f_N \gtrsim 1$) the magnetic-field distribution takes on a qualitatively different shape. The magnetic field has a minimum beyond the boundary of the pinch and then increases monotonically toward the periphery (curve 3′). With increasing density of excess electrons (1′–3′) the coordinates which determine the position of the minimum and maximum of $h(\rho)$ move toward the center, and their ratio decreases. The solution of the problem also includes (for certain values of the currents) a magnetic field which is strictly increasing toward the bound-

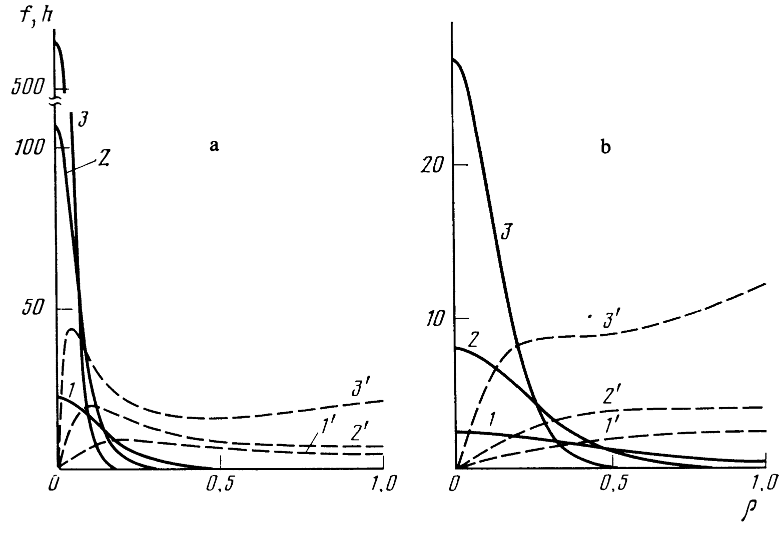

FIG. 1. Spatial distribution of the self-magnetic field h (dashed curves) and density f (solid curves) of an isothermal ($T_n = T_p$) bipolar plasma for the cases: a) $I = \frac{7}{8} I_B(1+f_N)$; b) $I = \frac{1}{2} I_B(1+f_N)$ and for various values of f_N: 1,1′) 0.2; 2,2′) 1; 3,3′) 5.

ary of the crystal. Interestingly, such a magnetic-field configuration satisfies the well-known "min B" principle[5] (Fig. 1b). The nature of such an unusual h distribution can be explained as follows. The self-magnetic field in the crystal is produced by two plasma components: the bipolar electron-hole component, which is subject to constriction (and creates a magnetic field that decays toward the periphery), and a monopolar component, due to excess electrons ionized from donor levels. The electronic component does not suffer any appreciable constriction and thus creates a magnetic field which increases linearly toward the boundary of the crystal. It is the sum of these two components that creates the behavior of H_φ described above. In a bipolar plasma the magnetic-field configuration always remains unchanged ($H_\varphi \propto r^{-1}$).

As we have said, in the case of a nonisothermal plasma ($T_n \gg T_p$)—the situation which actually obtains in experiment because of the heating of the electrons by the electric field—the plasma is detached from the boundaries of the current channel (Fig. 2). As we see, with increasing ratio T_n/T_p the density of the plasma on the axis of the crystal and the degree of its localization both increase. In fact, at the center of the crystal ($\rho = 0$) the density of the bipolar plasma corresponds to more than 90% of its limiting value ($T_n/T_p = \infty$), while outside the plasma pinch (the boundaries of the pinch are indicated by arrows in Fig. 2) it is close to zero.

Another important result is that the degree of constriction of a plasma with "excess" electrons is much stronger than in a bipolar plasma (curves 1 and 6 in Fig. 2a). In contrast to the Bennett problem, here the bipolar plasma is constricted by a combined magnetic field (the field of the plasma and the field of the extrinsic electrons). Therefore, as the number of excess electrons increases, the constriction of the plasma is enhanced, i.e., the degree of localization of the pinch remains appreciable at larger densities of excess electrons, under otherwise equal conditions. Interestingly, at a constant density of excess electrons ($f_N = \text{const}$) the degree of localization of the plasma depends importantly on the ratio of the carrier temperature, and for $T_N/T_p > 5$ the degree of nonisothermicity has practically no effect on the final result (Fig. 2a).

The constriction of the plasma can also be treated formally from a different point of view. By neglecting the bipolar plasma density f in (2), one can see that the constriction of the plasma occurs solely on account of the magnetic pressure of the current of extrinsic electrons. However, even for $f \sim f_N$ the problem becomes a self-consistent one, and here we must allow for the self-magnetic field of the plasma. Thus

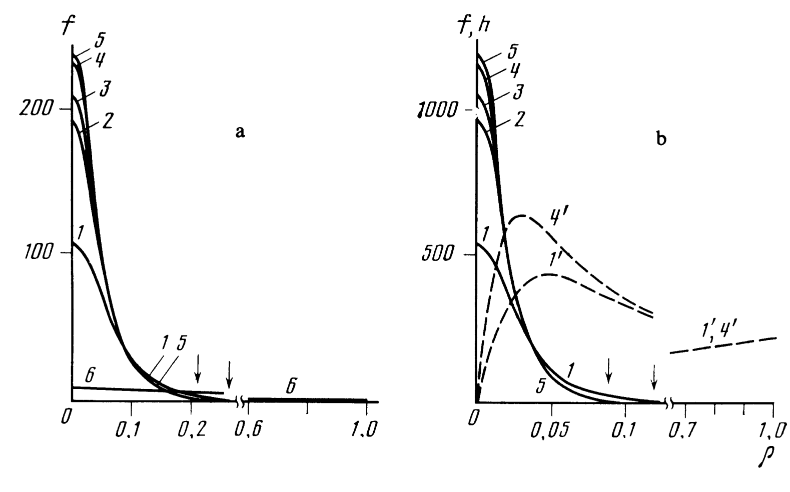

FIG. 2. Spatial distribution of the self-magnetic field h (dashed curves) and density f (solid curves) of a nonisothermal ($T_n \neq T_p$) bipolar plasma for the cases $I = \frac{7}{8} I_B(1+f_N)$ at $f_N = 1$ (a) and $f_N = 5$ (b) for various values of T_n/T_p: 1,1′) 1; 2) 5; 3) 10; 4,4′) 50; 5) ∞. The arrows indicate the boundary of the plasma pinch for curves 1 and 5; 6) $f_N = 0$ (bipolar plasma).

we have a sort of combination of two effects—the pinch effect and the magnetoconcentration effect, which is due to the extrinsic current. We note that the constriction of the plasma with allowance for the "extrinsic" magnetoconcentration effect is stronger than the pinch effect [see Eqs. (5) and (7)]; this apparently explains the detachment of the plasma from the boundaries of the crystal.

4. Let us now estimate the plasma pinching current and compare it with the experimental results. The plasma pinching current for InSb that was determined in Ref. 2 and has since been regarded as established is around 4 A. However, it has turned out that I_B (1) has a somewhat overstated value in comparison with many experimental results. For example, measurements with a microwave probe[6,7] have revealed that the spatial distribution of the current carriers is substantially inhomogeneous at a current of only ≈ 0.8 A. These measurements were made at the very beginning of impact ionization, when the plasma density was still small ($p \lesssim N$). Estimates show that the magnetic field produced by the extrinsic electrons played the leading role in the constriction of the plasma that was formed.

This hypothesis is also supported by the majority of the experimental observations of the helical instability in the pinch effect.[8,9] Those studies revealed a splitting of the current-voltage characteristics in a longitudinal magnetic field due to the anomalous diffusion of the plasma out of the pinch channel. Here it is important to emphasize that the splitting of the current-voltage characteristics in a magnetic field, like the observation of the pinch in Refs. 7 and 10, occurred practically right at the impact ionization threshold, when the density of the bipolar plasma was still small ($p \lesssim N$). This behavior of the current-voltage characteristics indicated a strong manifestation of the pinch effect, since otherwise the helical instability would not have appeared because a density gradient is necessary for the existence of the instability.

Another important circumstance which is not taken into account in the study of the pinch effect is the heating of the electron gas prior to impact ionization. In narrow-gap semiconductors the heating of the current carriers occurs even in rather weak electric fields (≈ 50 V/cm), while in fields corresponding to impact ionization the rise in the temperature above the equilibrium temperature has increased to a factor of several times. For example, in n-InSb crystals at 77 K the electron temperature increases to 250 K when the electric field reaches the impact ionization threshold, and is subsequently limited by the temperature of the optical phonons.[11] At the same time, because of the large difference in mobility ($\mu_n \gg \mu_p$), the hole temperature undergoes practically no change, but remains equal to the lattice temperature (≈ 80 K).

Let us now estimate the departure from thermal equilibrium. For the electron and hole temperature indicated above we obtain a temperature ratio $T_n/T_p \approx 3$ ($b_T = 0.25$), and we can therefore treat the plasma as essentially nonisothermal. For the actual carrier densities at the impact ionization threshold, we find from (12) that the detachment of the plasma from the boundaries (i.e., the formation of a strong spatial inhomogeneity) occurs at currents ≈ 0.5 A, in good agreement with the experimental data. We can thus state with confidence that the constriction of the plasma occurs in the initial stage of its formation and that the constriction conditions are determined by the conductivity of the extrinsic current carriers.

In conclusion we note that allowance for the nonlinear recombination of current carriers does not alter the qualitative picture of the formation of the pinch—all the features of the problem remain as before, and in the limit of high densities of "excess" electrons ($f_N > 1$) the solutions agree with the analogous solution for a nonrecombining plasma.

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