Autosolitons in a hot semiconductor plasma

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It is shown that, besides static autosolitons (AS), there can be excited in a stable homogeneous generated electron-hole plasma (EHP) heated in the process of Auger recombination a pulsating or moving undamped AS in the form of a striation or bunch of hot plasma. The amplitudes and sizes of the static AS in one-, two-, and three-dimensional systems are determined, and the stability of these AS is analyzed. The evolution of the transition from a static to a pulsating and moving AS as the EHP generation rate is varied is followed. The characteristic values of the AS-pulsation frequency and the velocity of the moving AS are found. It is found that under certain conditions it is possible to excite in a nonequilibrium EHP a static and a pulsating AS of the complex-domain-wall type, as well as moving AS in the form of reversible switching waves that convert a "cold" EHP into a "hot" one, and vice versa. Other experimentally realizable cases in which the various AS can be excited in a semiconductor plasma are discussed, and the AS parameters for some typical semiconductors are estimated.

I. INTRODUCTION. THE PHYSICS OF AUTOSOLITON EXISTENCE

The electron-hole plasma (EHP) is one of the examples of active systems with diffusion whose general nonlinear theory[1–6] predicts that, with the aid of an external short-lived perturbation, we can excite in such systems in the region of stability of their homogeneous state a solitary state whose stationary shape does not depend on the form of the initial perturbation, but is determined only by the parameters of the specific system. It is natural to call such self-sustaining localized eigenstates of nonequilibrium systems autosolitons (AS). In Refs. 1, 2, 7, and 8 the static AS produced in a nondegenerate EHP heated in the process of photogeneration or by electromagnetic radiation are investigated. In the present paper we study the situation in which we can generate in a homogeneous stable EHP not only static, but also traveling undamped, and pulsating, AS whose volume or shape varies periodically in time.

Let us consider a homogeneous semiconductor film in which there occurs uniform photogeneration of an EHP of such density that the carriers are degenerate and their collision time $\tau_e \ll \tau_\varepsilon$, the characteristic time of the relaxation—in energy terms—of the hot carriers on the phonons. In a degenerate hot EHP the thermocurrent is suppressed in comparison with the diffusional current because of the smallness of the ratio of the temperature T of the carriers to their Fermi level energy F. We shall assume that the electrons and holes have identical parameters (i.e., that the plasma is symmetric), and that, because of the high density of the EHP, the rate R of recombination of the carriers is determined by the Auger processes. As a result of the electron-electron collisions, the Auger-recombination-generated carriers with energy of the order of forbidden-band width E_g of the semiconductor heat up the EHP.[9–11] The power that goes into heating up the carriers is equal to $W = bE_gR$, with $b \approx 0.5$ when $\tau_e(E_g) \lesssim \tau_\varepsilon(E_g)$. In the case under consideration the density and temperature of the homegeneous EHP can be determined from the equations

$$G=R(n,\ T)\equiv n/\tau_r(n,\ T), \tag{1}$$

$$R(n,\ T)\,bE_g=P(n,\ T)=n(T-T_l)\tau_\varepsilon^{-1}, \tag{2}$$

where T_l is the temperature of the lattice, G is the rate of carrier generation, and $P(n, T)$ is the power transferred from the electron system to the phonons. It follows from an analysis[9,10] of Eqs. (1) and (2) that, as G increases, the temperature $T = T_h$ of the homogeneous plasma, as a rule, increases monotonically, and the G dependence of $n = n_h$ is N-shaped (Fig. 1a). The rise of T_h with increasing G is due to the increase of the rate R at which the Auger processes occur, i.e., the rate of generation of hot carriers. The decrease of the concentration n_h of the homogeneous plasma in the region $G > G_0$ (Fig. 1a) occurs when R increases with increasing T, which is characteristic of the Auger-recombination process.[12] Let us emphasize that, in the case under consideration, to a given value of G, correspond one value n_h and one value of T_h (Fig. 1a). In spite of this, spontaneous excitation of uniform relaxational oscillations occurs in EHP with $G > G_0$,[9,10] which is due to the very large time constant of the n variation in comparison with the T variation ($\tau_r \gg \tau_\varepsilon$). When $G < G_0$, the homogeneous EHP state is stable. At the same time, if a small region (of dimension smaller than the carrier-diffusion length L) of the semiconductor is irradiated in addition by a short light pulse, then a stable AS can appear in the form of a static, pulsating, or moving bunch of hot EHP.

The existence of a static AS (see Sec. 3) in a stable EHP is due to the fact that the carrier diffusion length $L \gg l$, the relaxation length of their energy ($L^2/l^2 \approx (F\tau_r/T\tau_\varepsilon) \gg 1$), while R increases rapidly with increasing T. Rapid recombination of the carriers occurs at the AS core (the high-temperature region (Figs. 2a–2c); nevertheless, the carrier concentration in a core of dimension $\mathscr{L}_s < L$ decreases to a significantly lesser degree than R increases, owing to a rapid diffusional inflow of carriers from the peripheral region of the AS into the core region. The carriers that enter the AS core as a result of diffusion from the periphery rapidly re-

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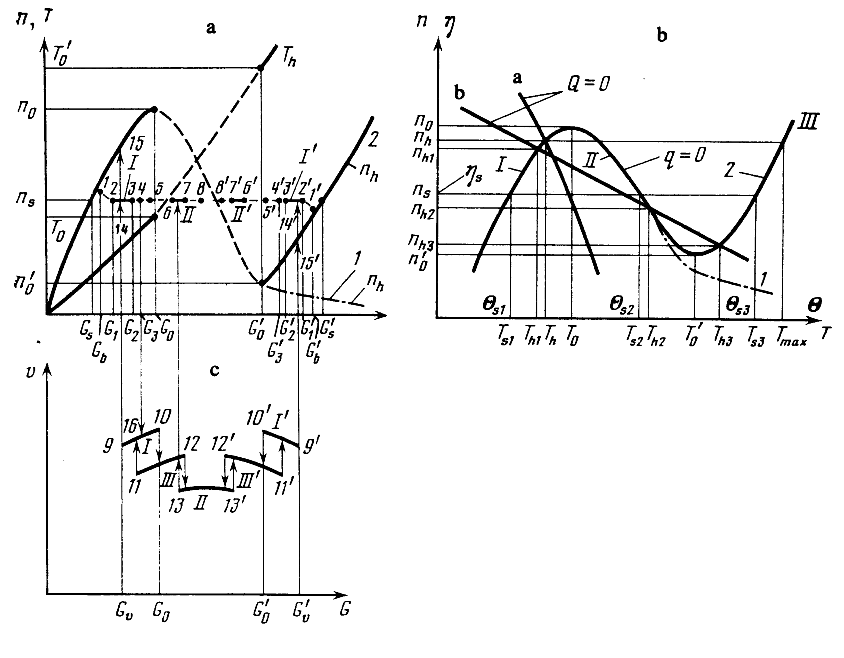

FIG. 1. Evolution of an AS as the generation rate G is varied: a) dependence of the concentration n_h and the temperature T_h of the homogeneous plasma on G; b) form of the local relation (curves 1 and 2) and of the curve of states (curves a and b); c) G dependence of the traveling-AS velocity. In Figs. a) and c) the curves I depict the dependence on G of the concentration in the wall of a broad static AS (a) and of the traveling-AS velocity (c); the curves II and III depict the corresponding dependences for multiautosoliton states of period $\mathscr{L}_{p1}$ and $\mathscr{L}_{p2} > \mathscr{L}_{p1}$. The dashed lines indicate the unstable sections. The Arabic numerals indicate the points of disappearance of the solutions in the form of a hot AS (1, 5, 8–13), or the points where stability is lost (2, 3, 6, and 7). The numbers I′–III′ and 1′–13′ correspond to cold AS. The arrows between Figs. a) and c) indicate possible transformations of a static or pulsating AS into a traveling AS, or vice versa, upon the loss of stability by the AS or the disappearance of a solution in the form of the AS in question. The curve b in Fig. b) corresponds to the flip-flop regime, in which there is realized at a given G three homogeneous states, two of which, n_{h1}, T_{h1} and n_{h3}, T_{h3}, are stable.

combine there, producing in the Auger process carriers with a high energy of the order of E_g, which, in turn, maintain a high temperature at the AS core. When $L \gg \mathscr{L}_s$, the concentration $n = n_s$ at the core practically does not change, and, because $\mathscr{L}_s \gg l$, we can assume that the energy-balance equation (2) is satisfied locally. From (2) it formally follows that the T dependence of n is, as a rule, N-shaped, i.e., to the value $n = n_s$ correspond three temperature values T_{s1}, T_{s2}, and T_{s3} (Fig. 1b). The AS core is a stable "phase," with $T \approx T_{s3}$, surrounded by a stable plasma with $T = T_h$. The unstable

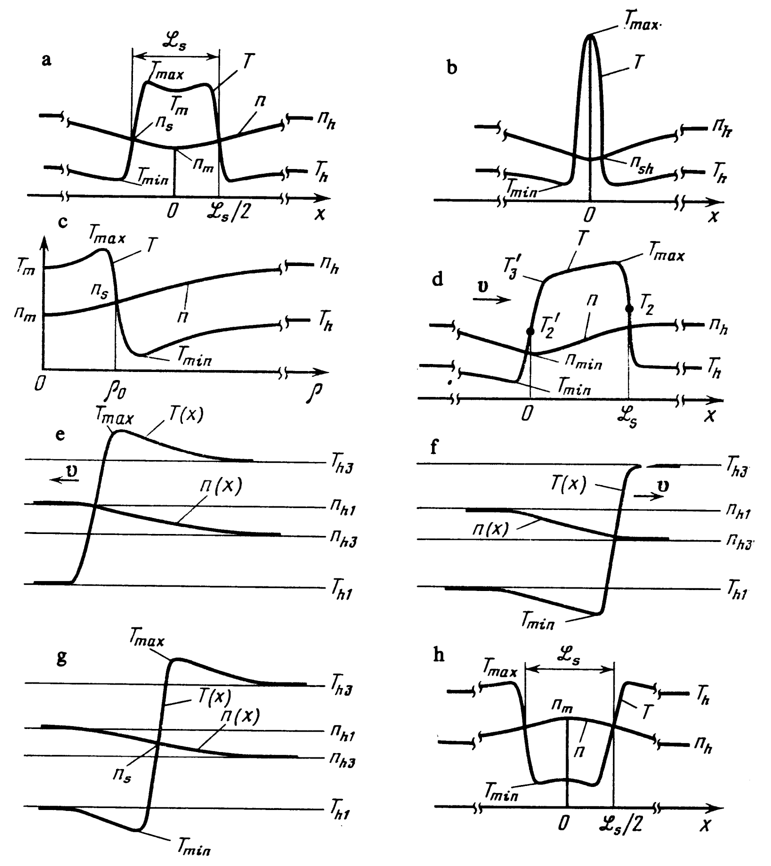

FIG. 2. Electron-concentration (n) and effective-electron-temperature (T) distributions in an AS in the form of: a), b) a hot broad solitary, and a hot narrow solitary, striation; c) a radially symmetric hot bunch of large radius; d) a moving broad striation; e), f) a traveling AS of the type of a complicated domain wall: cold state↔hot state switching wave; g) an AS of the static-complicated-domain-wall type; h) a broad solitary "cold" striation.

state with $T \approx T_{s2}$ is concentrated only in the walls of the AS, i.e., in the regions of rapid variation of $T(x)$ (Fig. 2a).

The stability of the static AS (see Sec. 4) is due to the fact that the wall of the AS has a small dimension of the order of $l \ll L$, and that a rise in temperature in the AS walls is damped out by a corresponding decrease in the carrier concentration. This damping is realized only in some G-value range, at the ends of which the static AS is, as a result of the instability, transformed into a pulsating AS. The appearance of the latter is due to the fact that $\tau_r \gg \tau_\varepsilon$. Therefore, for temperature fluctuations that vary with some frequency $\omega = \omega_c$ ($\omega_c \tau_r > 1$, $\omega_c \tau_\varepsilon \ll 1$), the time constant of the T variation does count, and, because of the extreme sluggishness, the concentration is less effective in damping out the growing T fluctuations localized in the walls of the AS.

At some $G < G_0$, besides the static or pulsating AS, there can be excited an AS that travels in any of the directions (see Sec. 5), or an AS in the form of a spiral wave. Unlike, for example, the Gunn domain, a traveling AS occurs in an isotropic EHP, in which there are no macroscopic (heat, concentration) fluxes. A traveling AS in an EHP is in many respects similar to a pulse excited in a nerve fiber, and have properties in common with it.[13] Let us elucidate the appearance of a traveling AS. It follows from the energy-balance equation (2) that two stable states of the plasma, i.e., the states with $T = T_h$ and $T = T_{\max}$, correspond to a given concentration value $n = n_h$ (Fig. 1b). Because $\tau_r \gg \tau_\varepsilon$, a brief (duration $t_p \ll \tau_r$) local heating of the EHP by, say, radiation absorbed by the free carriers will take the plasma at that place into the state with $T = T_{\max}$ and $n \approx n_h$. Through thermal conduction, the resulting hot carriers will heat up the neighboring regions, taking them, one after another, into the state with $T = T_{\max}$, i.e., a $(T = T_h)$-state—into—a $(T = T_{\max})$-state switching wave appears. Over a period of time $\tau \approx \tau_\varepsilon T/F$ the electron-heat flux propagates over a distance $\sim l$; therefore, this wave has velocity $v \sim l/\tau$. Behind this switching wave (i.e., behind the leading wall of the AS (Fig. 2d)), the temperature and, hence, the Auger-recombination rate are high; therefore, the carrier concentration decreases with a characteristic time $\sim \tau_r$, i.e., it falls off over a distance $\sim \tilde{L} = v\tau_r \sim l\tau_r F/T\tau_\varepsilon$. This depletion of the EHP concentration stops at some value $n = n_{\min}$ (Fig. 2d), at which the velocity of the trailing wall is equal to that of the leading wall. Behind the trailing wall the concentration is restored to the $n = n_h$ value in a region of dimension $\sim \tilde{L}$ (Fig. 2d).

Conditions can be realized in semiconductors[9,10] under which to a given value of G correspond two stable homogeneous states with $n = n_{h1}$, $T = T_{h1}$ (the "cold phase") and $n = n_{h3}$, $T = T_{h3}$ (the "hot phase") (Fig. 1b, curve b). In such an EHP it is possible to excite an AS in the form of reversible switching waves that take the EHP from the cold phase into the hot phase (Fig. 2e), or, conversely, from the hot phase into the cold phase (Fig. 2f), it being possible for the reversible waves to move in the same direction. The possibility of alternately switching the EHP from the cold into the hot phase and vice versa allows the excitation in the plasma of quite an arbitrary sequence of traveling AS of different widths. We can also excite a static or a pulsating AS in the EHP in question. As $G \to G_k$ (Subsec. 3.4), the static-AS dimension $\mathscr{L}_s \to \infty$, and an AS in the form of a domain wall of irregular shape is produced (Fig. 2g).

2. THE BASIC EQUATIONS

In a symmetric quasineutral EHP, the equations for the carrier density and temperature distributions have the form

$$\partial n/\partial t = e^{-1}\,\mathrm{div}\,\mathbf{j}_e + G - R, \qquad (3)$$

$$\partial (n\bar{\varepsilon})/\partial t = -\mathrm{div}\,\mathbf{j}_\varepsilon + W - P, \qquad (4)$$

in which the current density and the electron-energy flux are equal to

$$\mathbf{j}_e = D\nabla n, \quad \mathbf{j}_\varepsilon = -\varkappa \nabla T, \qquad (5)$$

$$D = 2e\tau_p F/3m, \quad \varkappa = (\pi^2 T n \tau_p/3m)\,\zeta(n, T),$$

where D and $\varkappa$ are the coefficients of diffusion and thermal conductivity of the carriers; $\bar{\varepsilon}$, m, and τ_p are respectively the mean energy, the effective mass, and the characteristic time of relaxation of the momentum of the electrons; the quantity $\zeta = 1$ when $\tau_p \ll \tau_e \ll \tau_\varepsilon$ and $\zeta \sim \tau_e/\tau_p \ll 1$, when $1 \gg \tau_e/\tau_p > (T/F)^2$. In (5) we have taken account of the fact that the field $\mathbf{E} = 0$ in a symmetric EHP, and that, when

$$\tau_e/\tau_r \ll T/F \ll (\zeta\tau_e/\tau_r)^{1/2}, \qquad (6)$$

we can neglect the thermocurrent and the energy flux due to the electron current (see the Appendix). Equations (3)–(5) reduce to the following:

$$\tau_r^0 \partial n/\partial t = L^2 \nabla (\Phi^{-1} \nabla n) - Q(n, T, G) \qquad (7)$$

$$(F^0 n T/F)\,\tau \partial T/\partial t = l^2 \nabla [(\varkappa/\varkappa^0) \nabla T] - q(n, T), \qquad (8)$$

$$Q(n, T, G) = R(n, T) G_0^{-1} - G, \qquad (9)$$

$$q(n, T) = [P(n, T) - bE_g R(n, T)]\,\tau_\varepsilon^0 (n_0 T_0)^{-1}, \qquad (10)$$

where

$$\Phi = F^0 \tau_p^0 / F(n) \tau_p(n), \quad \tau = \tau_\varepsilon^0 \pi^2 T_0 / 2F^0, \quad l = (\varkappa^0 \tau_\varepsilon^0 / n_0)^{1/2},$$

$$L = [{}^2/{}_3 F^0 \tau_p^0 \tau_r^0 / m]^{1/2}.$$

Here and below n, T, and G are measured in units of n_0, T_0, and G_0, respectively, and a zero superscript on a quantity indicates that it is taken at $T_h = T_0$ and $n_h = n_0$, which correspond to the point $G = G_0$ where $dn_h/dG = 0$ (Fig. 1a). The temperature T_h and concentration n_h of a homogeneous plasma are determined from the equations (1) and (2), from which we find that

$$\frac{dT_h}{dG} = \frac{T_h}{G}(\nu - \delta)(\nu\xi - \Gamma\delta)^{-1}, \quad \frac{dn_h}{dG} = \frac{n_h}{G}(\xi - \Gamma)(\nu\xi - \Gamma\delta)^{-1}, \qquad (11)$$

where

$$\nu = \frac{\partial \ln R}{\partial \ln n}, \quad \delta = \frac{\partial \ln P}{\partial \ln n}, \quad \xi = \frac{\partial \ln P}{\partial \ln T}, \quad \Gamma = \frac{\partial \ln R}{\partial \ln T}. \qquad (12)$$

Normally, in the case of Auger recombination, $\nu > \delta$ (Ref. 12). Therefore, the condition $\Gamma > \xi\delta/\nu$ may not be fulfilled even when $\Gamma > \xi$. Then, according to (11), T_h increases monotonically with increasing G, and $n_h(G)$ is Λ- or N-shaped (Fig. 1a). The possible appearance of a second extremum on the $n_h(G)$ curve at $G = G_0' > G_0$ is due to the fact that, at high T, the quantity R ceases to increase rapidly with increasing T (Ref. 12) in comparison with the function $P(T)$.

If, on the other hand, the quantity $\Gamma \geqslant \xi\delta/\nu$ in some G-value range, then, according to (11), the $T_h(G)$ curve is S-shaped, and there exist[9,10] two stable states of the homogeneous EHP (Fig. 1b, curve b).

It follows from the expressions given above that

$$\varepsilon^2 = l^2/L^2 = {}^2/{}_3\alpha\xi < \alpha = (\pi^2 T_0/2F^0)(\tau_\varepsilon^0/\tau_r^0) \ll \xi \leqslant 1, \quad (13)$$

$Q'_n \equiv \partial Q/\partial n > 0$, and the quantity

$$q_T' = W\tau_\varepsilon^0(\xi - \Gamma)(n_0 T_0 T_h)^{-1}$$

becomes negative when $G > G_0$. This allows the use, in the analysis of the EHP, of the results of the general theory,[1–6] from which it follows that the shape of the AS depends essentially on the form of the local $n(T)$ relation corresponding to $q(n, T) = 0$ and the curve $n(T)$ of states that corresponds to $Q(n, T, G) = 0$ (Fig. 1b). Analysis shows that the local relation may be N- or Λ-shaped (Fig. 1b).

3. THE STATIC AUTOSOLITON

1. Using the ideas of the theory of singular perturbations,[14] we can show (see the Appendix) that the concentration and temperature distributions in a one-dimensional AS in the case of an N-shaped local relation (Fig. 1b, curve 2) can, when allowance is made for the symmetry of the AS about its center ($x = 0$, Fig. 2a), be written up to terms of the order of $\varepsilon \ll 1$, (13), in the form

$$T(x) = T_{sh} + \begin{cases} T_{\rm III} - T_{s3} \\ T_{\rm I} - T_{s1} \end{cases}, \quad n(x) = \begin{cases} n_{\rm III}, & 0 \leqslant x \leqslant \mathscr{L}_s/2 \\ n_{\rm I}, & \mathscr{L}_s/2 \leqslant x < \infty \end{cases}. \quad (14)$$

Here $T_{sh}(x)$, a sharp distribution describing the AS wall, corresponds to the separatrix of the equation[2–4]

$$l^2 d^2\Theta/dx^2 = q(T(\Theta), n_{sh}), \quad n_{sh} = \text{const}, \quad (15)$$

while $n_{\rm I,\,III}(x)$ and $T_{\rm I,\,III}(x)$, smooth distributions describing $n(x)$ and $T(x)$ outside the AS walls, are those solutions to the equations[2–4]

$$L^2 d^2\eta_j/dx^2 = Q_j \equiv Q(T_j(\eta), \eta, G), \qquad q(T_j, n) = 0, \quad j = \text{I, III}, \quad (16)$$

which correspond to the boundary conditions

$$\eta_{\rm I}(\infty) = \eta_h(n_h), \quad \eta_{\rm I}(\mathscr{L}_s/2) = \eta_s(n_s),$$

$$\eta_{\rm III}(\mathscr{L}_s/2) = \eta_s, \quad d\eta_{\rm III}/dx|_{x=0} = 0.$$

In these equations

$$\Theta(T) = \int^{T} \frac{\varkappa(T', n_{sh})}{\varkappa^0} dT', \qquad \eta(n) = \int^{n} \Phi^{-1}(n')\, dn'$$

are single-valued functions of T and n; the subscripts I and III indicate that the relation between T and n corresponds to the branch I ($T \leqslant T_0$) or III ($T \geqslant T'_0$) of the single-valued function $T(n)$ on the local relation (Fig. 1b).[2–4] In (15) the value $n_{sh} = n_s$ of the concentration in the wall of a broad AS is found from the equations[2–4]

$$\int_{T_{s1}}^{T_{s3}} q(T, n_s)\, dT = 0, \quad q(T_{si}, n_s) = 0, \quad i = 1, 2, 3, \quad (17)$$

which determine the minimum $T_{\min} = T_{s1}$ and maximum $T_{\max} = T_{s3}$ temperatures in the AS (Fig. 2a), as well as the temperature at the point $x = \mathscr{L}_s/2$: $T_{sh}(\mathscr{L}_s/2) = T_{s2}$. Integrating (16) with allowance for the continuity of the carrier flux through the AS wall (at the point $x = \mathscr{L}_s/2$, Fig. 2a), we obtain equations for the determination of the concentration $n(0) = n_m$ and the temperature $T(0) = T_m$ at the center of a broad AS (Fig. 2a), as well as the width $\mathscr{L}_s$ of the AS:

$$\int_{\eta_s}^{\eta_h} Q_{\rm I}\, d\eta + \int_{\eta_m}^{\eta_s} Q_{\rm III}\, d\eta = 0, \quad \mathscr{L}_s = \sqrt{2}L \int_{\eta_m}^{\eta_s} \left[\int_{\eta_m}^{\eta} Q_{\rm III}\, d\eta \right]^{-1/2} d\eta, \quad (18)$$

$$q(n_m, T_m) = 0.$$

Thus, the principal AS parameters can be determined up to quantities of the order of ε (see the Appendix) from the simple algebraic equations (17) and (18) without having to solve the complex problems for $n(x)$ and $T(x)$. It follows from (10) and (17) that the principal AS parameters $T_{\max}$, $T_{\min}$, and $n = n_s$ (Fig. 2a) are G independent in practically the entire region of existence of the AS (Fig. 1a). This result is due to the fact that the local EHP-energy balance (2), which essentially gives the distribution $T(x)$ in the AS wall, does not depend on G. Only the AS width $\mathscr{L}_s$ and the values $n = n_m$ and $T = T_m$ at the center of the AS depend on the quantity G (Fig. 2a). These parameters are determined by the integrated carrier-number balance in the AS. It follows from the equation (18) for this balance that the quantity Q should have difference signs at the core of the AS ($Q_{\rm III}$) and outside it ($Q_{\rm I}$). Therefore, (18) can be fulfilled only when $n_h > n_s$, i.e., when $G > G_s$, where G_s is the generation rate at which $n_h > n_s$ (Fig. 1a). As G decreases, the quantity $\mathscr{L}_s$ decreases, and the solution in the form of an AS vanishes at some G_b slightly greater than G_s, and located in the vicinity of the point where $dn_{sh}/dG = \infty$ (Fig. 1a). Thus, an AS in the form of a bunch of hot EHP (Fig. 2a) can exist only in the region $G_b \leqslant G \leqslant G_0$. As $G \to G_0$, the monotonic transition of $T(x)$ and $n(x)$ at the periphery of the AS to their homogeneous values T_h and n_h (Fig. 2a) may be replaced by a regime in which these quantities are damped in amplitude, but oscillate with period $\sim (lL)^{1/2}$.

In accordance with the general results obtained in Refs. 2–4, an AS in the form of a bunch of cold EHP with a slightly elevated carrier concentration (Fig. 2h) can be excited in a hot homogeneous EHP with $T_h > T'_0$ ($G > G'_0$, Fig. 1a). The parameters of this homogeneous AS can be determined from Eqs. (17) and (18) if the subscript I is replaced by III and III is replaced by I in the latter equation. Besides the simplest AS (Figs. 2a and 2h), there exist AS of complicated shape, as well as multi-autosoliton states in the form of periodically or randomly disposed AS. The latter can exist in the region $G_0 < G < G'_0$ as well (Fig. 1a).

2. Only narrow hot AS can be realized in the region $G < G_0$ in an EHP with Λ-shaped local relation (Fig. 2b). The amplitude of such an AS is not restricted by the nonlinearities of the system, but is determined by the balance of the energy entering the EHP during the Auger recombination of the carriers at the core of the AS and the energy that leaves as a result of thermal conduction (i.e., as a result of the diffusional spreading of the temperature). We find up to terms of the order of ε^2 (see the Appendix) that in a narrow hot AS

$$T(x) = T_{sh}(x) - T_i + T_{\rm I}(x), \quad n(x) = n_{\rm I}(x), \quad 0 \leqslant x < \infty, \quad (19)$$

where $T_{sh}(x)$ is determined by that separatrix of Eq. (15) which passes through the saddle point $\Theta_i(T_i)$ (in Fig. 2b the quantity $T_{\min} \approx T_i$), $x = 0$ being the value to which corresponds the point where $d\Theta/dx = 0$ and $\Theta = \Theta_{\max}$; $T_{\rm I}(x)$ and $n_{\rm I}(x)$ are given by the solution to Eq. (16) with $j = {\rm I}$ (for $T \leqslant T_0$, Fig. 1b) and the boundary conditions $\eta(\infty) = \eta_h(n_h)$ and $\eta(0) = \eta_{sh}(n_{sh})$. The values T_i and n_{sh} correspond to the branch I of the local relation (Fig. 1b), and are determined from the condition $q(T_i, n_{sh}) = 0$ and the integrated carrier-number balance in the AS:

$$\int_0^\infty Q_{\rm I}(n_{\rm I}, T_{\rm I}, G)\,dx = -\int_0^\infty \{Q(n_{sh}, T_{sh}(x), G) - Q(n_{sh}, T_i, G)\}\,dx. \quad (20)$$

As G decreases, the AS amplitude decreases, and at the point $G = G_b$, where $dn_{sh}/dG = \infty$, the AS with $T_{\max} - T_0 \gtrsim T_0$ suddenly disappears.

3. The concentration and temperature distributions, $n(\rho)$ and $T(\rho)$, in a radially symmetric AS with a large radius $\rho = \rho_0 \gg l$, which is realized in the case of an N-shaped local relation, is given up to quantities of the order of ε by the expressions (14) with x replaced by ρ and $\mathscr{L}_s/2$ by ρ_0 (Fig. 2c). The function $T_{sh}(\rho)$ describes the AS wall, and corresponds to the separatrix of Eq. (15) with $n_{sh} = n_s$ and x replaced by ρ; the functions $n_{\rm I,\,III}(\rho)$ and $T_{\rm I,\,III}(\rho)$, which describe $n(\rho)$ and $T(\rho)$ outside the AS wall, are the solutions to the equations (16) with the operator d^2/dx^2 replaced by $\rho^{-1-s}(d/d\rho)\times(\rho^{1+s}d/d\rho)$ ($s = 1$ or 0 according as the AS is spherically or cylindrically symmetric). Up to quantities of the order of ε, the values $T_{\min} = T_{s1}$, $T_{\max} = T_{s3}$, and $T_{sh}(\rho_0) = T_{s2}$, where the T_{si} and n_s are determined from (17). Taking account of the continuity condition on the carrier flux through the AS wall (in the case when $\rho = \rho_0$), we obtain the following equations for the determination of ρ_0, $n(0) = n_m$, and $T(0) = T_m$ (Fig. 2c):

$$\int_0^{\rho_0} Q_{\rm III}\rho^{1+s}\,d\rho + \int_{\rho_0}^\infty Q_{\rm I}\rho^{1+s}\,d\rho = 0, \qquad q(n_m, T_m) = 0,$$

$$\eta_s - \eta_m = L^{-2}\int_0^{\rho_0}\left[\rho^{-1-s}\int_0^{\rho_0} Q_{\rm III}\rho^{1+s}\,d\rho\right]d\rho. \quad (21)$$

Another radially symmetric AS is the state in the form of a spherically or cylindrically symmetric layer of hot EHP with $T \approx T_{s3}$, outside which (at the center and the periphery) the state of the EHP is close to its homogeneous state. In the case of an Λ-shaped local relation, a hot radially-symmetric AS of radius $\rho_0 \sim l$ is realized (see Appendix).

4. It follows from an analysis of the formulas given in Subsec. 3.1 that there exists in an EHP with two stable homogeneous states (Fig. 1b, curve b) a hot static AS when $G < G_k$ (Fig. 2a) and a cold one when $G > G_k$ (Fig. 2h). The value $G = G_k$ at which $\mathscr{L}_s = \infty$ can be determined up to a quantity of the order of ε from the equation

$$\int_{\eta_{h1}}^{\eta_s} Q_{\rm I}(\eta, T_{\rm I}(\eta), G_k)\,d\eta = \int_{\eta_{h3}}^{\eta_s} Q_{\rm III}(\eta, T_{\rm III}(\eta), G_k)\,d\eta,$$

$$q(\eta, T_{\rm I,III}) = 0. \quad (22)$$

4. STABILITY OF THE STATIC AUTOSOLITON

Linearizing (7) and (8) with respect to the fluctuations of the form

$$\delta\Theta = \delta\Theta(\mathbf{r})e^{-\gamma t}, \quad \delta\eta = \delta\eta(\mathbf{r})e^{-\gamma t}, \quad (23)$$

and using the requirement that the variation of $n(x)$ be smooth, we arrive at the system of equations

$$(\hat{H}_\eta - \alpha^{-1}\Phi\gamma)\delta\eta = -Q_\Theta'\delta\Theta, \quad \hat{H}_\eta = -\varepsilon^{-2}\Delta + V_\eta,$$
$$V_\eta = Q_\eta'(\eta(\mathbf{r}), \Theta(\mathbf{r}), G), \quad (24)$$

$$(\hat{H}_\Theta - \varphi\gamma)\delta\Theta = -q_\eta'\delta\eta, \quad \hat{H}_\Theta = -\Delta + V_\Theta,$$
$$V_\Theta = q_\Theta'(\eta(\mathbf{r}), \Theta(\mathbf{r})), \quad (25)$$

in which length and time are measured in units of l and τ, and $\varphi = nT\varkappa^0F^0/F\varkappa$. By letting the operator ∇ act on the equations (7) and (8) for the stationary case, and then multiplying by an arbitrary unit vector $\mathbf{n}$, we can easily verify that $\delta\Theta \propto \mathbf{n}\nabla\Theta$ and $\delta n \propto \mathbf{n}\nabla\eta$ are the eigenfunctions of the problem (24), (25) that correspond to the eigenvalue $\gamma = 0$.[3] This result is a consequence of the translational symmetry of the problem. It clearly follows from this fact and the oscillation theorem that, in problems whose characteristics are described by a single equation of the type (8) with $n = \text{const}$ (such problems arise in the theory of combustion,[15] as well as in the investigation of current cords or field domains in semiconductors with non-single-valued CVC[16]), only the monotonic solution, for which $d\Theta/dx$ (or $d\Theta/d\rho$) does not have a single node can be stable.[15,16] In the case of the complicated problem under consideration (the system of equations (24), (25) is of fourth order) the oscillation theorem does not apply, and it can be expected[1–6] that complicated states, including the one in the form of randomly disposed AS, for which $d\Theta/dx$ has a set of nodes,[3,17] can be stable in it.

To investigate the stability of the AS, let us expand $\delta\Theta$ and $\delta\eta$ in series in terms of the eigenfunctions $\delta\Theta_n$ and $\delta\eta_l$ of the self-adjoint problems

$$\hat{H}_\Theta\delta\Theta_n = \varphi\lambda_n\delta\Theta_n, \quad \hat{H}_\eta\delta\eta_l = \Phi\mu_l\delta\eta_l, \quad (26)$$

for which the functions $\delta\Theta_n$ and $\delta\eta_l$ are normalized with weight $\varphi > 0$ and $\Phi > 0$, and satisfy cyclic boundary conditions in the case when the system's dimensions $\mathscr{L}_{x,y,z} \to \infty$. By substituting these series into (24) and (25), we can easily verify[1] that the γ-fluctuation spectrum is determined by Eq.

(35) in Ref. 5. According to Ref. 5, the condition for stability of an AS against fluctuations with $\operatorname{Im}\gamma = \omega = 0$ reduces to

$$\lambda_n + \sum_{l=0}^{\infty} \mathscr{P}_{lnn}\mu_l^{-1} > 0,$$

$$\mathscr{P}_{lnn} = -\int_V \delta\Theta_n^{*} q_\eta' \delta\eta_l \, d\mathbf{r} \int_V \delta\eta_l^{*} Q_\Theta' \, \delta\Theta_n \, d\mathbf{r} \geqslant 0, \qquad (27)$$

while the condition for the appearance of a pulsating AS has the form

$$\lambda_n = -\sum_{l=0}^{\infty} \mathscr{P}_{lnn}\mu_l (\mu_l^2 + \alpha^{-2}\omega_c^2)^{-1},$$

$$\sum_{l=0}^{\infty} \mathscr{P}_{lnn} (\mu_l^2 + \alpha^{-2}\omega_c^2)^{-1} = \alpha, \qquad (28)$$

where ω_c is the frequency of the critical fluctuation (pulsation).

1. To investigate the stability of a one-dimensional broad AS (Fig. 2a), let us write $\delta\Theta(\mathbf{r})$ and $\delta\eta(\mathbf{r})$ in (23) in the form

$$\delta\Theta(\mathbf{r}) = \delta\Theta(x) \exp(i\mathbf{k}_\perp \mathbf{r}_\perp),$$
$$\delta\eta(\mathbf{r}) = \delta\eta(x) \exp(i\mathbf{k}_\perp \mathbf{r}_\perp). \qquad (29)$$

Substituting (29) into (24), (25), we arrive at a system of equations of the type (24), (25), in which the Laplacian in the operators $\hat{H}_\Theta$ and $\hat{H}_\eta$ should be replaced by d^2/dx^2, and for λ_n and μ_l in (27), (28) we should substitute

$$\lambda_n = \lambda_p + ak_\perp^2, \quad \mu_l = \mu_k + \varepsilon^{-2} g k_\perp^2, \qquad (30)$$

where

$$k_\perp^2 = k_y^2 + k_z^2, \quad a = \int \delta\Theta_p^2 \, dx \sim 1, \quad g = \int \delta\eta_k^2 \, dx \sim 1,$$

$\delta\Theta_p$, $\delta\eta_k$ and λ_p, μ_k are the eigenfunctions and eigenvalues of the one-dimensional problems (26).

In the AS walls (Fig. 3a) the temperature varies from T_{s1} to T_{s3} (Subsec. 3.1), and concentrated in them is a region of potentially unstable EHP with $T \approx T_{s2}$ corresponding to the branch II of the local relation (Fig. 1b), for which $V_\Theta \equiv q_\Theta' \propto \xi - \Gamma < 0$ (see Sec. 2). It follows from this that $V_\Theta(x)$ has the form of two narrow potential wells with $V_{\min} < 0$ (Fig. 3b). Outside the AS walls, i.e., in the regions of smooth distributions, where $T(x) \approx T_{\rm I}(x)$ or $T_{\rm III}(x)$ (Subsec. 3.1), the magnitude of the potential $V_\Theta \equiv q_\Theta' \gtrsim 1$. Therefore, the ground-state function $\delta\Theta_0^{(0)}$ in each isolated well is highly localized in a region of dimension of the order of l, and to it corresponds an eigenvalue $\lambda^{(0)} < 0$. It follows from Refs. 2–4 and 6 that the spectrum λ_p of the potential V_Θ for a static AS having the form of two wells located at a distance $\mathscr{L}_s \gg l$ apart (Fig. 3b) contains only two negative values λ_0 and λ_1, to which correspond $\delta\Theta_0$ and $\delta\Theta_1$ (Figs. 3d and 3e): the coupling and anticoupling combinations of the ground-state functions $\delta\Theta_0^{(0)}$ of each of the isolated wells.[18] The values of λ_0 and λ_1 for $\mathscr{L}_s < L$ are then roughly equal to

$$\lambda_0 \sim -\varepsilon\mathscr{L}_s/L - \exp(-\mathscr{L}_s/l), \quad \lambda_1 \sim -\varepsilon\mathscr{L}_s/L. \qquad (31)$$

It can be seen from (25) and (26) that $\delta\Theta_0$ and $\delta\Theta_1$ are growing—with increments of $-\lambda_0$ and $-\lambda_1$—temperature fluctuations in the case when $\delta\eta = 0$. At the same time, the temperature variation is accompanied by a corresponding concentration variation, the damping effect of which is taken into account in (27) and (28) by the terms containing the coefficients $\mathscr{P}_{lnn} \geqslant 0$, (27). The local rise $\delta\Theta_{0,1}$ in temperature in the AS walls leads to a change in the EHP concentration in regions of dimension of the order of L (Figs. 3d and 3e).

The potential

$$V_\eta \equiv Q_\eta' = \nu\Phi R(\eta, \Theta) n^{-1} G_0^{-1}$$

has the form of two wells of depth $\gtrsim 1$ and width $\sim L$ (i.e., ε^{-1}), located at a distance $\mathscr{L}_s$ apart, and separated by a high potential barrier (Fig. 3c). In this case it is natural to suppose[18] that the eigenfunctions $\delta\eta_0$ and $\delta\eta_l$ of the problem (26) are localized in the region of the AS (Figs. 3d and 3e). It follows from the form and the symmetry of the functions $\delta\Theta_p$ and $\delta\eta_k$ about the center of the AS (Fig. 3) and the equality of the number of zeros of the function $\delta\eta_k$ to the index "k" that $\mathscr{P}_{kpp} = 0$ when $k+p$ is an odd number, and $\mathscr{P}_{kpp} \ll \mathscr{P}_{ppp}$ when $k+p$ is an even number. Using this, we find from (27), (28), and (30) for $p = 0$ and 1 that, approximately,

$$\lambda_p + ak_\perp^2 + \mathscr{P}_{ppp}(\mu_p + gk_\perp^2\varepsilon^{-2})^{-1} < 0, \qquad (32)$$

$$\lambda_p + \alpha\mu_p < 0, \quad \omega_c^{(p)} = \alpha^{1/2}(\mathscr{P}_{ppp} + \lambda_p\mu_p)^{1/2}. \qquad (33)$$

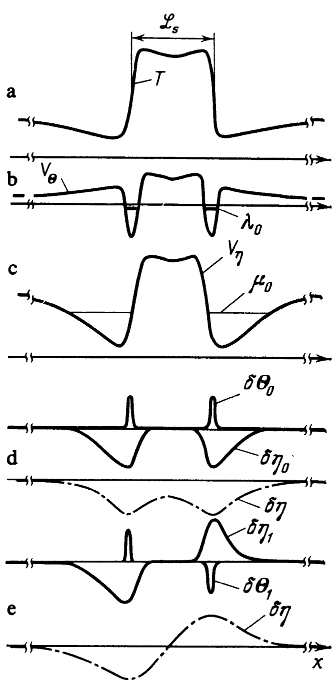

FIG. 3. For the analysis of the stability of the static AS: a) form of the dependence $T(x)$; form of the potentials V_Θ (b) and V_η (c), of the critical fluctuations $\delta\Theta_{0,1}$, and of the perturbations $\delta\eta_{0,1}$ or $\delta\eta$ damping these fluctuations (d and e).

Taking account of the fact that the functions $\delta\Theta_0$ and $\delta\Theta_1$ are localized in the regions of the AS walls, whose dimension is of the order of l, while $\delta\eta_0$ and $\delta\eta_1$ are localized in regions of dimension of the order of L (Fig. 3), we can, following Refs. 1, 5, and 6, find from the formulas (27), (9), and (10), as well as (39) in Ref. 6 that

$$\mathscr{P}_{000}\sim\mathscr{P}_{111}\sim-\varepsilon\langle q_\eta'\rangle_{sh}\langle Q_\Theta'\rangle_{sh}\sim\varepsilon\Gamma R(T_{s3},\ n_s)G_0^{-1}\gtrsim\varepsilon,$$
$$\mu_0\sim\Gamma R(T_{s3},\ n_s)G_0^{-1},$$

where $\langle\cdots\rangle_{sh}$ denotes averaging of the function over the region of the AS wall.

1.1. In the one-dimensional case $k_\perp=0$, and the condition (32) practically determines the point $G=G_b$ (see Fig. 1a) where the solution in the form of a hot AS vanishes, and $dn_{sh}/dG=\infty$.[5,17] Using (31), we can easily verify that, at the boundary where the condition (32) is fulfilled, the AS width

$$\mathscr{L}_s=\mathscr{L}_b\sim l\ln\varepsilon^{-1}.$$

It follows from (31) and (33) that the AS is stable against the growth of the fluctuation $\delta\Theta_0\cos(\omega_c^{(0)}t)$ if its width $\mathscr{L}_s$ lies in the range

$$\mathscr{L}_1=\mathscr{L}_s(G_1)<\mathscr{L}_s<\mathscr{L}_2=\mathscr{L}_s(G_2),$$
$$\mathscr{L}_1\sim l\ln(\mu_0\alpha^{-1}),\quad \mathscr{L}_2\sim 3l\,\mu_0/2\zeta. \tag{34}$$

It follows from the estimates given in (34) that the condition for the existence of a static AS is more easily fulfilled in an EHP for which $\zeta\sim\tau_e/\tau_p\ll1$. Since $\alpha\ll\varepsilon$ (13), $\mathscr{L}_1>\mathscr{L}_b$, i.e., $G_1>G_b$. In other words, it follows from (34) that the AS goes over into a pulsating AS when G is decreased (the point 2 in Fig. 1a), as well as when it is increased (the point 3 in Fig. 1a). From the form of the fluctuation $\delta\Theta_0$ (Fig. 3d) it follows[5] that a static AS goes over into a pulsating AS at the corresponding bifurcation points $G=G_1$ and G_2, the pulsation frequency in the case of a mild excitation regime being, according to (33), given by

$$\omega_c^{(0)}\sim\varepsilon^{1/2}(\tau\tau_r^0)^{-1/2}.$$

It follows from (31) that the condition (33) with respect to $p=1$ can be fulfilled only when the system is heated, the corresponding bifurcation point $G=G_3$ (the point 4 in Fig. 1a), which corresponds to the solution $\Theta(x)\pm\delta\Theta_1\cos(\omega_c^{(1)}t)$, being located close to $G=G_2$, since for a broad AS the quantity $\lambda_1\sim\lambda_0$, (31). From the form of $\delta\Theta_1$ (Fig. 3d) it follows that a solution in the form of an AS whose walls oscillate in phase with frequency $\omega_c^{(1)}\simeq\omega_c^{(0)}$ branches out from the solution in the form of a static AS at $G=G_3$. Thus, as G increases, the expanding static AS can, at $G>G_2$, spontaneously go over not into a pulsating AS, but into an AS traveling with velocity

$$v\sim\omega_c^{(1)}\mathscr{L}_s\sim\alpha^{1/2}\varepsilon^{-1/2}l/\tau$$

(the 4→16 jump in Figs. 1a and 1c).

When the potential $V_\eta\equiv Q_\eta'$ depends weakly on x, and it can be assumed that $V_\eta=Q_\eta'(\eta_h,\Theta_\eta)\equiv V_0$, and $\Phi=1$, then it follows from (24) that

$$\delta\eta(x)=-\frac{\varepsilon^2}{2w}\left[e^{-wx}\int_{-\infty}^{x}e^{w\xi}Q_\Theta'\delta\Theta\,d\xi+e^{wx}\int_{x}^{\infty}e^{-w\xi}Q_\Theta'\delta\Theta\,d\xi\right],$$
$$w=\varepsilon(V_0-\alpha^{-1}\gamma)^{1/2}. \tag{35}$$

Substituting (35) into (25), we obtain

$$\det[(\lambda_p-\gamma)\delta_{pm}+\mathscr{P}_{pm}(V_0-\alpha^{-1}\gamma)^{-1/2}]=0, \tag{36}$$

where

$$\mathscr{P}_{pm}=-\frac{\varepsilon}{2}\left[\int_{-\infty}^{\infty}\delta\Theta_m q_\eta'\left\{e^{-wx}\int_{-\infty}^{x}e^{w\xi}Q_\Theta'\delta\Theta_p\,d\xi+e^{wx}\int_{x}^{\infty}e^{-w\xi}Q_\Theta'\delta\Theta_p\,d\xi\right\}dx\right]. \tag{37}$$

Taking only the functions $\delta\Theta_0$ and $\delta\Theta_1$ into account in (36), we have

$$\Gamma_p(\gamma)\equiv\lambda_p-\gamma+\mathscr{P}_{pp}(V_0-\alpha^{-1}\gamma)^{-1/2}=0,\quad p=0,\ 1. \tag{38}$$

The functions $\delta\Theta_0$ and $\delta\Theta_1$ are localized in the AS walls, which are of dimension l (Figs. 3d and 3e), and the perturbations $\delta\eta(x)$ induced by them (the dot-dash curves in Fig. 3) have, according to (35), forms close to the functions $\delta\eta_0$ and $\delta\eta_1$ respectively (Figs. 3d and 3e). It follows from (35) that, outside the AS walls, $\delta\eta(x)$ falls off exponentially with characteristic length of the order of $L\gg l$. Taking this into account, we find from (37) that

$$\mathscr{P}_{00}\approx\mathscr{P}_0[1+\exp(-w\mathscr{L}_s)],\quad \mathscr{P}_{11}\approx\mathscr{P}_0[1-\exp(-w\mathscr{L}_s)], \tag{39}$$
$$\mathscr{P}_0=(-\varepsilon/2)\langle q_\eta'\rangle_{sh}\langle Q_\Theta'\rangle_{sh}.$$

Substituting (39) into (38), we find from an analysis of the zeros of the function $\Gamma_p(\gamma=i\Omega)$ in the upper half-plane of Ω that the AS loses its stability against the $\delta\Theta_0$ fluctuation that varies with frequency

$$\omega_c=\alpha^{1/2}\left[\frac{r^2}{2}\mathscr{P}_0^2+\alpha V_0^2\lambda_0\right]^{1/2}|\lambda_0|^{-1/2},$$
$$r=\begin{cases}1, & \mathscr{L}_s\gg L(\varepsilon^{-1}\alpha)^{1/3},\\ 2, & \mathscr{L}_s\ll L(\varepsilon^{-1}\alpha)^{1/3},\end{cases} \tag{40}$$

when

$$\lambda_0+\alpha[V_0+(V_0^2+\alpha^{-2}\omega_c^2)^{1/2}]<0. \tag{41}$$

These expressions essentially coincide with those that follow form (33) for $p=0$. From (40), (41), and (31) it follows that

$$\omega_c\sim\varepsilon^{1/2}(\varepsilon\alpha^{-1})^{1/6}(\tau\tau_r^0)^{-1/2},$$

and the static AS becomes a pulsating AS at $G=G_1$ and G_2 (Fig. 1a), when its dimension

$$\mathscr{L}_s(G_1)=\mathscr{L}_1\sim l\ln(\varepsilon^2\alpha)^{-1/3}$$
$$\text{or}\quad \mathscr{L}_s(G_2)=\mathscr{L}_2\sim L(\varepsilon/\zeta)^{1/3}\approx L(\alpha\varepsilon^{-1}).^{1/3}$$

1.2. The conclusions drawn in Subsec. 4.1.1 about the AS stability hold true in the two- and three-dimensional cases, when we cannot set $k_\perp=0$. An exception is an EHP with $\zeta<\alpha^{1/3}\mu_0^{2/3}$, when the condition (33) is more rigid than

(32), which is fulfilled at

$$\lambda_0 < -\varepsilon^2\mu_0 a g^{-1} - 2\varepsilon[a(\mathscr{P}_{000}+\lambda_0\mu_0)g^{-1}]^{1/2} \sim -\varepsilon^{3/2}\mu_0^{1/2} \quad (42)$$

with respect to fluctuations with

$$k_\perp = k_c = \varepsilon^{1/2}[(\mathscr{P}_{000}+\lambda_0\mu_0)/ag]^{1/4} \sim \varepsilon^{3/4}\mu_0^{1/4}, \quad (43)$$

which strive to stratify the one-dimensional AS into smaller regions—bunches—in the plane yz of its walls.[3] It follows from (31) and (42) that the AS stratifies both as G decreases (with the stratification occurring at some $G = G_3$), and as it increases (at $G = G_4$), when its width

$$\mathscr{L}_s(G_3) = \mathscr{L}_3 \sim l \ln(\varepsilon^{-3/2}\mu_0^{-1/2})$$

or

$$\mathscr{L}_s(G_4) = \mathscr{L}_4 \sim (lL)^{1/2}\mu_0^{1/2}.$$

Taking the results obtained in Subsec. 4.1.1 into account, we can conclude that in the general case a broad static AS is stable when its width $\mathscr{L}_s$ lies in the range $\mathscr{L}_1$, $\mathscr{L}_3 < \mathscr{L}_s < \mathscr{L}_2$, $\mathscr{L}_4$, where the quantities $\mathscr{L}_1$ and $\mathscr{L}_3$ are given in Subsec. 4.1.1.

For a one-dimensional AS (Fig. 2b), which is realized in an EHP with a Λ-shaped local relation (Subsec. 3.2), the quantity $\lambda_0 \sim -1$. Therefore, according to (42), such an AS in a two- or three-dimensional sample is unstable against its division into bunches of small radii.[3]

2. The potential $V_\Theta(\rho)$ for a radially symmetric AS is a single potential well in which other negative eigenvalues

$$\lambda_0^{(\beta)} \sim -\varepsilon\rho_0/L - [1+s-\beta(s+\beta)](l/\rho_0)^2 < 0,$$

corresponding to radially asymmetric fluctuations with $\beta \neq 0$ may occur besides $\lambda_0^{(0)}$.[3] Similarly, in the potential well V_η (Refs. 3 and 4)

$$\mu_0^{(\beta)} \sim \mu_0^{(0)} + \varepsilon^{-2}\beta(s+\beta)(l/\rho_0)^2.$$

It follows from an analysis of the conditions (27) and (28) with $\lambda_n = \lambda_0^{(\beta)} < 0$ and $\mu_l = \mu_k^{(\beta)}$ that a static AS in an EHP to which corresponds an N-shaped local relation can be stable when $\zeta < \alpha^{1/2}\mu_0^{3/2}$. In this case the condition (27) for $\lambda_n = \lambda_0^{(\beta)}$ with $\beta \neq 0$ may be fulfilled as G is varied, i.e., a static bunch can become unstable against a radially nonsymmetric fluctuation.

An AS in the form of a radially symmetric layer (Subsec. 3.3) enclosed between spherical (cylindrical) surfaces of radii $\rho_1 > L$ and ρ_2 turns out to be more stable that a bunch. Such an AS is stable in roughly the same range of $\mathscr{L}_s \equiv \rho_2 - \rho_1$ values as a one-dimensional AS (Subsec. 4.1.2).

5. THE TRAVELING AUTOSOLITON

The properties of a traveling AS are described by Eqs. (7) and (8) if we go over to the self-similar variable $x \to x - vt$ in them. This adds the term $\tilde{L}\partial n/\partial x$, where $\tilde{L} = v\tau_r^0$, to the right-hand side of (7) and the term $(v\tau F^0 Tn/F)\partial T/\partial x$, to (8). We can, by going through a procedure similar to the one expounded in the Appendix, verify that we have, when the condition (6) is fulfilled,[1)]

$$T(x) = T_{sh_1} + T_{sh_2} + \begin{cases} T_{\rm I} - T_{\min} - T_3 \\ T_{\rm III} - T_3 - T_3', \\ -T_3' \end{cases}$$

$$n(x) = \begin{cases} n_{\rm I}, & x \leqslant 0 \\ n_{\rm III}, & 0 \leqslant x \leqslant \mathscr{L}_s, \\ n_h, & x \geqslant \mathscr{L}_s \end{cases} \quad (44)$$

where the T_{sh_i} are the solutions corresponding to the separatrices of the equation

$$l^2 d^2\Theta/dx^2 + v\tau\varphi\, d\Theta/dx - q(\Theta, n_{sh}) = 0,$$

$$n_{sh_1} = n_h, \qquad n_{sh_2} = n_{min}, \quad (45)$$

that go from the saddle point $\Theta_3(T_3)$ to the saddle point $\Theta_1(T_1)$ for $i = 1$ and from the saddle point $\Theta_1'(T_1')$ to the saddle point $\Theta_3'(T_1')$ for $i = 2$; $n_{\rm I,\,III}(x)$ and $T_{\rm I,\,III}(x)$ describe $n(x)$ and $T(x)$ outside the AS walls, and are the solutions to the equations

$$\tilde{L}\, dn_j/dx - Q_j = 0, \qquad q(T_j, n) = 0, \quad j = {\rm I,\ III} \quad (46)$$

with the boundary conditions $n_{\rm I}(0) = n_{\min}$ and $n_{\rm III}(\mathscr{L}_s) = n_h$. The values of T_1, $T_1' < T_0$ and T_3, $T_3' > T_0'$ at the saddle points are determined from the equations

$$q(T_k, n_h) = 0, \quad q(T_k', n_{min}) = 0, \quad k = 1, 2, 3, \quad (47)$$

which also give the minimum $T_{\min} = T_1'$ and maximum $T_{\max} = T_3$ temperatures in the AS (see Fig. 2d) and the values of the temperature at the points $x = 0$ and $\mathscr{L}_s$: $T_{sh_1}(\mathscr{L}_s) = T_2$ and $T_{sh_2}(0) = T_2'$. The magnitude of the AS velocity and the value of $n_{\min}$ in the AS can be determined from the equations

$$v = \left[\int_{\Theta_{max}}^{\Theta_h} q(T(\Theta), n_h)\, d\Theta\right]\left[\tau\int_{-\infty}^{\infty}\varphi\left(\frac{d\Theta_{sh_1}}{dx}\right)^2 dx\right]^{-1}$$

$$= \left[\int_{\Theta_1'}^{\Theta_3'} q(T(\Theta), n_{min})\, d\Theta\right]\left[\tau\int_{-\infty}^{\infty}\varphi\left(\frac{d\Theta_{sh_2}}{dx}\right)^2 dx\right]^{-1}. \quad (48)$$

Integrating Eq. (46) for $j = {\rm III}$, we find

$$\mathscr{L}_s = \tilde{L}\int_{n_{min}}^{n_h} Q_{\rm III}^{-1}\, dn. \quad (49)$$

It follows from an analysis of Eqs. (48) and (49) that, as $G \to G_0$ ($n_h \to n_0$, Fig. 1a), the quantities v, $\mathscr{L}_s$, and $T_{\max}$ attain their maximum values ($v_{\max} \sim l/\tau$), while $T_{\min}$ and $n_{\min}$ attain their minimum values. On the other hand, as $G \to G_s$ ($\eta_h \to \eta_s$), the velocity $v \to 0$. At the same time, according to Ref. 6, for $\alpha \gg \varepsilon^4$, (13), a solution in the form of a traveling AS can be constructed only whe $v \gg \alpha^{1/2} l/\tau$. From this it follows that a traveling AS disappears suddenly at $n_h > n_s$, i.e., at some $G = G_v > G_s$ (the 9→15 jump in Fig. 1), or else it is transformed into a static (the 9→14 jump in Fig. 1), or a pulsating, AS. The traveling AS disappears at $G > G_0$, but there exist both a periodic and a nonperiodic sequence of traveling AS. Therefore, in the region $G > G_0$ a solitary AS can stimulate the appearance of a traveling-AS sequence (the jump at the point 10 in Fig. 1c), or else there can arise in the EHP the uniform relaxational oscillations considered in Refs. 9 and 10. The evolution of a cold AS in a hot EHP, as G is varied, proceeds in similar fashion (in the region $G > G'_0$, Figs. 1a and 1c).

Since the AS has a velocity $l/\tau \gtrsim v \gg \alpha^{1/2} l/\tau$, it is according to Ref. 6, stable in the one-dimensional case in the entire region of its existence. It is shown in the Appendix that the condition for the stability of a traveling AS in the two- three-dimensional cases ($k_\perp \neq 0$) reduces to

$$\lambda_0 + a k_\perp^2 + \tilde{L}^{-1}[-\langle q_n' \rangle_{sh} \langle Q_\Theta' \rangle_{sh}] > 0. \tag{50}$$

It follows from (50) that the stability of a traveling AS in a three- or two-dimensional sample against fluctuations with $k_\perp \neq 0$ follows from the stability of the AS in the one-dimensional case,[6] i.e., in the $k_\perp = 0$ case.

6. ON THE CONDITIONS FOR THE OBSERVATION OF AN AS IN SOME SEMICONDUCTORS

1. The above-investigated simplest model of a symmetric EHP is realized in, for example, PbTe, in which $m_e^* = m_h^* \approx 0.02 m_0$, and the carriers turn out to be degenerate even at low concentrations (for $T \approx 10$ K, at $n > n_d = 2 \times 10^{15}$ cm^{-3}).[2)] It is known from experiment[21,22] that the Auger-recombination process in PbTe is the dominant process in the temperature region $4.2 \lesssim T \lesssim 77$ K when $n \gtrsim 10^{16}$ cm^{-3}. Using Ref. 23, we can find that the Auger-recombination rate in PbTe is equal to

$$R[\text{cm}^{-3}\cdot\text{sec}^{-1}] \equiv n/\tau_r = 4\cdot 10^{19} (T/4.2 \text{ K})^5 \exp[2(F-\varepsilon_i)/T] \tag{51}$$

for a concentration $n_d < n < 4 \times 10^{16}$ cm^{-3} and a carrier temperature $T < F \lesssim \varepsilon_i = m_t E_g / 4 m_l \approx 50$ K (m_t and m_l are the transverse and longitudinal effective masses). At a lattice temperature $T_l = 4.2$ K the carriers with $T < 10$ K dissipate their energy on the acoustic phonons ($\tau_\varepsilon \sim 3 \times 10^{-8}$ sec); those with $T > 10$ K, on the optical phonons.[24] Therefore, the local relation is N-shaped (see Fig. 1b), with $T_0 = 5$ K, $T'_0 = 10$ K, $G_0 = 7 \times 10^{20}$ cm$^{-3}\cdot$sec^{-1}, $n_0 = 2.5 \times 10^{16}$ cm^{-3}, and the pump power corresponding to G_0 being equal to $w_0 = G_0 E_g \mathscr{L}_z \approx 0.14$ W$\cdot$cm^{-2} for a film thickness of $\mathscr{L}_z = 10^{-2}$ cm. Analyzing the local relation together with (17), we obtain the following estimates for the parameters of the static AS (Fig. 2a): $T_{\min} \approx T_{s1} = 4.5$ K, $T_{\max} \approx T_{s3} = 11$ K, $n_{sh} \approx n_s = 2.1 \times 10^{16}$ cm^{-3}. Taking account of the fact that the carrier mobility $\mu \approx 8 \times 10^5$ cm$^2\cdot$V$^{-1}\cdot$sec^{-1} in PbTe,[24] we find that $l \approx 4 \times 10^{-3}$ cm and $L \sim 10^{-1}$ cm. According to (13), $\alpha \approx 2 \times 10^{-3}$, i.e., we can, in accordance with the results obtained in Secs. 4 and 5, excite in an EHP in PbTe a pulsating AS with pulsation frequency $\omega \sim 10^4$ sec^{-1}, or a traveling AS with velocity $v \sim 4 \times 10^4$ cm$\cdot$sec^{-1}, in which $\tilde{L} \approx 1$ cm.

The conditions for AS excitation are fulfilled in broad-band semiconductors, such as Si and GaAs, even at room temperatures ($T_l \approx 300$ K).

2. AS's can occur not only in degenerate, but also in nondegenerate EHP, in which the Auger-recombination rate, as a rule, increases most rapidly with increasing T. Analysis shows that the properties of a traveling AS in a nondegenerate EHP are the same as for a degenerate EHP (Sec. 5).

3. Static AS's can occur in a nondegenerate EHP heated in the electric field of both polar and nonpolar semiconductor at $T_l \approx 300$ K. In polar semiconductors (InSb, PbTe, GaAs) at $T > \Theta_D$, where Θ_D is ther Debye temperature, the carriers dissipate their momentum and energy on the polar optical phonons. The product of $\tau_p \propto T^\alpha$ and $\tau_\varepsilon \propto T^s$ then increases with increasing $T(\alpha + s > 0)$, i.e., the conditions for spontaneous appearance of static AS are fulfilled.[25,26] It is precisely under these conditions that static AS in the form of bunches of hot EHP were experimentally detected and studied in GaAs.[27] In Si and Ge at $T > \Theta_D$ the carriers dissipate their energy on the nonpolar optical phonons ($s = 1/2$) and their momentum on the acoustic and optical phonons ($\alpha = -1/2$). Such an EHP with $p \approx n$ will stratify in the direction of the applied field at

$$T > T_0 = T_l (2+\alpha+s)(1+\alpha+s)^{-1} = 2T_l \sim 600-700 \text{ K}.$$

Thus, if we apply to an n-Ge (or n-Si) sample an electric field of intensity $E > E_0$ close to the value at which the electron drift velocity begins to saturate ($E_0 \sim 10^3$ V/cm), and produce with the aid of, say, photogeneration an EHP with $p \approx n \sim 10^{17}$–10^{18} cm^{-3}, then there will spontaneously appear in the sample static or moving—in the direction of the field—AS in the form of hot EHP bunches (see Fig. 1 in Ref. 26). The smaller the quantity $\varepsilon = l/L \approx (\tau_\varepsilon/\tau_r)^{1/2} \ll 1$ is, the higher the maximum value of the carrier temperature in the AS ($T_{\max}$) will be. Even for $\varepsilon \sim 0.1$, the quantity $T_{\max} \sim 100 T_l$, i.e., there should occur as the center of the AS intense impact ionization, which will limit the value of $T_{\max}$.

Thus, in a relatively weak electric field ($E \sim 10^3$ V/cm) a homogeneous EHP can undergo strification that leads to the spontaneous appearance of local regions of intense impact ionization of the carriers. It is possible that such an effect was observed in the experiment reported in Ref. 27.

For the EHP in the semiconductors under consideration here the critical field intensity (i.e., the intensity at which the AS still exists) $E = E_b$ is proportional to the value $\varepsilon = l/L \ll 1$ ($E_b \ll E_0$). In other words, a stable static AS, at the core of which $T_{\max} \gg T_l \approx 300$ K, can be excited in a very weakly heated EHP with $T_h - T_l \ll T_l$.

4. AS's can occur not only in a hot EHP, but also in an EHP that has thermalized with the lattice. In this case, in a degenerate EHP, there can be excited static, pulsating, and traveling AS; in a nondegenerate EHP, as a rule, static AS.[1] In a nondegenerate EHP the $T(\mathbf{r})$ and $N(\mathbf{r})$ distributions in an AS vary in phase and with the same characteristic length l_T that characterizes the variation of the temperature $T(\mathbf{r}) = T_l(\mathbf{r})$ in a semiconductor film, i.e., the AS is a bunch of high-temperature, high-density EHP.[1] The temperature at the center of the AS in the case of relatively small l_T/L values is higher than the melting point of the semiconductor; therefore, the formation of an AS can lead to local melting of the film.

APPENDIX

Derivation of the expressions (14)–(22) and (50) in the main text

Adding to the expressions (5) for $\mathbf{j}_e$ and $\mathbf{j}_\varepsilon$, respectively, the terms $(\pi^2 ne\tau_p/3m)(T/F)\nabla T$ and $(F/e)\mathbf{j}_e$, which were discarded in (5), we obtain from (3) and (4) the following equations, which describe the distributions $T(\mathbf{r})$ and $n(\mathbf{r})$ in the steady-state case:

$$l^2\Delta T+l^2a_1(\nabla T)^2+l^2a_2(\nabla\eta\nabla T)+l^2a_3(\nabla\eta)^2-(\varkappa^0/\varkappa)(q-\alpha a_4Q)=0, \quad (A.1)$$

$$L^2\Delta\eta+L^2M\Delta T+L^2M_\eta'(\nabla\eta\nabla T)+L^2M_T'(\nabla T)^2-Q=0, \quad (A.2)$$

where

$$a_1=\frac{1}{\varkappa}\frac{\partial\varkappa}{\partial T},\quad a_2=\frac{1}{\varkappa}\frac{\partial\varkappa}{\partial\eta}+\frac{2\Phi}{3\zeta n},$$

$$a_3=\frac{4\Phi^2}{3\pi^2Tn^2\zeta}\left(\frac{F}{T_0}\right)^2,$$

$$a_4=\frac{2F^0F}{\pi^2T_0^2},\quad M=\frac{\pi^2nT\tau_p}{2\tau_p^0}\frac{T_0^2}{FF^0}.$$

1. For the analysis of the one-dimensional static AS (see Fig. 2a) in the case of an N-shaped local relation (Subsec. 3.1) we introduce the notation

$$X_1\equiv T,\quad X_2\equiv\varepsilon dT/dx,\quad X_3\equiv\eta,\quad X_4\equiv d\eta/dx,$$

and write (A.1) and (A.2) in the form of a system of equations:

$$\varepsilon dX_i/dx=f_i,\quad i=1,2;\quad dX_i/dx=f_i,\quad i=3,4, \quad (A.3)$$

where

$$f_1=X_2,\quad f_2=(\varkappa^0/\varkappa)(q-\alpha a_4Q)-a_1X_2^2-\varepsilon a_2X_2X_4-\varepsilon^2a_3X_4^2,$$
$$f_3=X_4,\quad f_4=Q-M\varepsilon^{-2}f_2-\varepsilon^{-1}M_\eta'X_2X_4-\varepsilon^{-2}M_T'X_2^2; \quad (A.4)$$

here and below (except in Sebsec. 5) x is measured in units of L. Let us divide the semi-infinite axis $x\geqslant0$ into two sections:

$m=1$, where $0\leqslant x\leqslant x_0\equiv\mathscr{L}_s/2$ и $m=2$, where $x\geqslant x_0$.

The boundary conditions for the functions $X_i^{(m)}(x)$ in each of the sections $m=1,2$ have the form

$$X_2^{(1)}(0)=X_4^{(1)}(0)=0,\quad X_1^{(2)}(\infty)=T_h,\quad X_3^{(2)}(\infty)=\eta_h,$$
$$X_i^{(1)}(x_0)=X_i^{(2)}(x_0).$$
$$i=1,\ldots,4. \quad (A.5)$$

Let us, in accordance with the theory of singular perturbations,[14] write the solutions to the system (A.3) in the form

$$X_i^{(m)}(x)=\tilde{X}_i^{(m)}(x,\varepsilon)+\overline{X}_i^{(m)}(\xi,\varepsilon),\quad i=1,\ldots,4,\quad m=1,2, \quad (A.6)$$

where the outer $\tilde{X}_i^{(m)}(x,\varepsilon)$ and inner (boundary) $\overline{X}_i^{(m)}(\xi,\varepsilon)$ solutions will be sought in the form of series in powers of ε:

$$\tilde{X}_i(x,\varepsilon)=\tilde{X}_{i,0}(x)+\varepsilon\tilde{X}_{i,1}(x)+\ldots+\varepsilon^k\tilde{X}_{i,k}(x)+\ldots,$$
$$\overline{X}_i(\xi,\varepsilon)=\overline{X}_{i,0}(\xi)+\varepsilon\overline{X}_{i,1}(\xi)+\ldots+\varepsilon^k\overline{X}_{i,k}(\xi)+\ldots, \quad (A.7)$$

where $\xi=(x-x_0)/\varepsilon$. Let us substitute (A.6) and (A.7) into (A.3). Next, expanding the functions $f_j(X_i)$ in series in powers of ε, and equating the coefficients of the same powers of ε (separately depending on x and separately depending on ξ (Ref. 14)), we obtain in the zeroth approximation in ε the system of equations

$$\tilde{X}_{2,0}^{(m)}=0,\quad f_2(\tilde{X}_{i,0}^{(m)}(x))=0,\quad d\tilde{X}_{3,0}^{(m)}/dx=\tilde{X}_{4,0}^{(m)}, \quad (A.8)$$

$$d\tilde{X}_{4,0}^{(m)}/dx=f_4(\tilde{X}_{i,0}^{(m)}(x)) \quad (A.9)$$

$$d\overline{X}_{1,0}^{(m)}/d\xi=\overline{X}_{2,0}^{(m)},\quad d\overline{X}_{2,0}^{(m)}/d\xi=\bar{f}_{2,0},\quad d\overline{X}_{3,0}^{(m)}/d\xi=0,$$
$$d\overline{X}_{4,0}^{(m)}/d\xi=\varepsilon\bar{f}_{4,0},$$
$$\bar{f}_{j,0}=f_j(\tilde{X}_{i,0}^{(m)}(x_0)+\overline{X}_{i,0}^{(m)}(\xi))-f_j(\tilde{X}_{i,0}^{(m)}(x_0)). \quad (A.10)$$

When

$$\alpha a_4,\quad \varepsilon^2a_3\ll1,\qquad M\ll\varepsilon, \quad (A.11)$$

which is equivalent to the condition (6), we can, taking (A.8) into account, neglect in (A.9) and (A.10) those terms entering into the functions f_j and $\bar{f}_{j,0}$ which are proportional to a_3, a_4, M, M'_T, and M'_η, and also assume that $\varepsilon\bar{f}_{4,0}=0$. Next, taking into account the boundary conditions

$$\overline{X}_i(\pm\infty)=0 \quad (A.12)$$

for the boundary functions[14] and the conditions (A.5), and also setting

$$\overline{X}_{1,0}^{(1)}(0)=T_{s2}-T_{s3},\quad \overline{X}_{1,0}^{(2)}(0)=T_{s2}-T_{s1}, \quad (A.13)$$

where the T_{si} satisfy (17), we obtain the results presented in Subsec. 3.1 in the main text.

2. For the analysis of the radially symmetric AS (see Fig. 2c) in the case of an N-shaped local relation (Subsec. 3.3) we introduce the notation

$$X_1\equiv T,\quad X_2\equiv\varepsilon dT/d\rho,\quad X_3\equiv\eta,\quad X_4\equiv d\eta/d\rho$$

and write (A.1) and (A.2) in the form of a system of equations:

$$\varepsilon dX_1/d\rho=f_1,\quad \varepsilon dX_2/d\rho=f_2-\varepsilon(1+s)\rho^{-1}X_2,$$
$$dX_3/d\rho=f_3,\quad dX_4/d\rho=f_4-(1+s)\rho^{-1}X_4, \quad (A.14)$$

where the $f_j(X_i)$ are given by (A.4) and ρ is measured here and below in units of L. Let us divide the semi-infinite axis $\rho\geqslant0$ into two sections: $m=1$, where $0\leqslant\rho\leqslant\rho_0$, and $m=2$, where $\rho\geqslant\rho_0$. If we carry out the same iterative procedure as in Subsec. 1, we arrive, in the zeroth approximation in ε, at the system of equations (A.8)–(A.10) with x and ξ replaced respectively by ρ and $\xi=(\rho-\rho_0)/\varepsilon$ and with the term $-(1+s)\rho^{-1}\tilde{X}_{4,0}^{(m)}(\rho)$ added to the right-hand side of (A.9). Then, taking (A.11), (A.13), and (A.5) into account, we obtain the results given in Subsec. 3.3

3. Let us consider the one-dimensional AS in the case of a Λ-shaped local relation. This form of the relation (Fig. 1b) is characteristic of an EHP in which $\Gamma\gg1$ (12), and therefore it is possible for the following condition to be fulfilled in the AS-core region:

$$R(T_{max})G_0^{-1}\sim\varepsilon^{-2}\gg1, \quad (A.15)$$

i.e., $|q(T_{\max})|$, $|Q(T_{\max})|\sim\varepsilon^{-2}$. Let us introduce the notation

$$\mu=\varepsilon^2,\quad X_1=T,\quad X_2=\mu dT/dx,\quad X_3=\eta,\quad X_4=d\eta/dx, \quad (A.16)$$

and write (A.1) and (A.2) in the form

$$\mu dX_i/dx=f_i,\quad i=1,\,2;\quad dX_i/dx=f_i,\quad i=3,\,4, \quad (A.17)$$

where

$$f_1=X_2,\quad f_2=\mu(\varkappa^0/\varkappa)(q-\alpha a_4Q)-a_3X_2^2-\mu a_2X_2X_4-\mu^2a_3X_4^2,$$
$$f_3=X_4,\quad f_4=Q-M\mu^{-2}f_2-\mu^{-1}M_\eta'X_2X_4-\mu^{-2}M_T'X_2^2. \quad (A.18)$$

The boundary conditions for the functions $X_i(x)$ have the form

$$X_2(0)=X_4(0)=0,\quad X_1(\infty)=T_h,\quad X_3(\infty)=\eta_h. \quad (A.19)$$

Substituting into (A.17) and (A.18) the functions (A.6) and (A.7), which are defined for all $x\geqslant0$, and setting $\xi=x/\mu$, we arrive in the zeroth approximation in μ at the system of equations (A.8)–(A.10) (without the superscript m). Taking (A.12) into acount at $\xi=\infty$, we obtain from (A.8)–(A.10), (A.19), and (A.11) the results given in Subsec. 3.2 and the equation

$$d\bar X_{4,0}/d\xi=\mu\bar f_{4,0},$$

which describes the variation of the quantity dn/dx at the AS core.

4. Let us consider the radially symmetric AS in the case of a Λ-shaped local relation. Using (A.16) with x replaced by ρ, we can write (A.1) and (A.2) in the form of the system (A.14) if we replace in it ε by μ and as $f_j(X_i)$ we use (A.18). As a result we arrive, in the zeroth approximation in μ, at the system (A.8)–(A.10), in which we must set $\xi=\rho/\mu$, and add to the right-hand side of the last of the equations (A.8) and the right-hand sides of the equations (A.9) and (A.10) the following terms respectively:

$$-(1+s)\rho^{-1}\bar X_{4,0}(\rho),\quad -(1+s)\xi^{-1}\bar X_{2,0}(\xi),\quad -(1+s)\xi^{-1}\bar X_{4,0}(\xi).$$

From the equations obtained we find, taking (A.11), (A.12), and (A.19) into account, that $T(\rho)$ and $n(\rho)$ can be represented in the form (19) with x replaced by ρ, i.e., $T(\rho)$ and $n(\rho)$ are qualitatively similar to the distributions $T(x)$ and $n(x)$, depicted in Fig. 2b, in the region $x\geqslant0$. The distributions $T_{sh}(\rho)$ and $T_I(\rho)$, $n_I(\rho)$ satisfy the boundary conditions given in Subsec. 3.2, and can respectively be determined from Eqs. (15) and (16) with the operator d^2/dx^2 replaced by $\rho^{-1-s}(d/d\rho)(\rho^{1+s}d/d\rho)$. The values of the T_i and n_{sh} can be determined from the conditions $q(T_i, n_{sh})=0$,

$$\int_0^\infty Q_I\rho^{1+s}\,d\rho=-\int_0^\infty\{Q(n_{sh},T_{sh}(\rho),G)-Q(n_{sh},T_i,G)\}\rho^{1+s}\,d\rho.$$

5. Derivation of the criterion for stability of a traveling AS. Let us linearize in terms of perturbations $\delta\Theta$ and δn of the form (29) Eqs. (7) and (8) with respect to the above-considered self-similar solution depicted in Fig. 2d. As a result, we obtain

$$(\hat H_\Theta+k_\perp^2-\gamma\varphi)\delta\Theta=-\hat q_n\delta n,\qquad \hat H_\Theta=-d^2/dx^2-v\varphi d/dx+\hat q_\Theta. \quad (A.20)$$

$$(-\mathcal{L}d/dx-\alpha^{-1}\gamma+Q_n'+\Phi^{-1}k_\perp^2\varepsilon^{-2})\delta n=-Q_\Theta'\delta\Theta, \quad (A.21)$$

$$\hat q_n=\tilde q_n'-v\varphi_n'd\Theta/dx,\qquad \hat q_\Theta=\tilde q_\Theta'-v\varphi_\Theta'd\Theta/dx,\qquad \tilde q=(\mu_2^0/\mu_2)q,$$

$$\varkappa(n,T)=\mu_1(T)\mu_2(n),\qquad \Theta=\int^T(\mu_1(T)/\mu_1^0)\,dT,$$

where x and t are measured in units of l and τ respectively. Let us solve Eq. (A.21) for δn with the boundary conditions that follow from $n(\pm\infty)=n_h$, and let us substitute this solution into (A.20). Going over in the latter from the $\delta\Theta$ functions to $\delta\tilde\Theta=\delta\Theta\exp(v/2\int^x\varphi\,dx)$, we obtain

$$(\tilde H_\Theta+k_\perp^2-\gamma\varphi)\delta\tilde\Theta=\mathcal{L}^{-1}\exp\left(\frac{v}{2}\int^x\varphi\,dx\right)$$
$$\times q_n'\left\{\exp\left(\int^x S\,dx\right)\int_x^\infty Q_\Theta'\delta\tilde\Theta\exp\left[-\int^\xi\left(S+\frac{v\varphi}{2}\right)dx\right]d\xi\right\}, \quad (A.22)$$

where

$$\tilde H_\Theta=-d^2/dx^2+V_\Theta,\qquad V_\Theta=\hat q_\Theta+{}^1/_4v^2\varphi^2+{}^1/_2v\,(d\varphi/dx),$$
$$S=(Q_n'+k_\perp^2\varepsilon^{-2}\Phi^{-1}-\alpha^{-1}\gamma)\mathcal{L}^{-1}. \quad (A.23)$$

Expanding $\delta\tilde\Theta$ in a series in terms of the eigenfunctions $\delta\tilde\Theta_n$ of the problem $\hat H_\Theta\delta\tilde\Theta_n=\lambda_n\varphi\delta\tilde\Theta_n$, and substituting it into (A.22), we obtain, after appropriate transformations, the equation

$$\det[(\lambda_p-\gamma+ak_\perp^2)\delta_{pm}+\mathscr{P}_{pm}]=0, \quad (A.24)$$

where

$$\mathscr{P}_{pm}=-\mathcal{L}^{-1}\int_{-\infty}^{\infty}\delta\tilde\Theta_m\hat q_n\exp\left\{\int^x\left(S+\frac{v\varphi}{2}\right)dx\right\}$$
$$\times\left[\int_x^\infty Q_\Theta'\delta\tilde\Theta_p\exp\left\{-\int^\xi\left(S+\frac{v\varphi}{2}\right)dx\right\}d\xi\right]dx. \quad (A.25)$$

Owing to the asymmetry of the traveling AS, the λ_n spectrum for $\alpha\ll\varepsilon$ contains only one negative value:

$$\lambda_0\sim-|\Delta\eta_{sh}|\sim-\mathcal{L}^{-1}\equiv-\alpha v^{-1},$$

and the function $\delta\tilde\Theta_0$ corresponding to it is localized in the leading wall (of dimension $\sim l$) of the traveling AS (in the vicinity of the point $x=\mathscr{L}_s$ in Fig. 2d).[6] Retaining only the function $\delta\tilde\Theta_0$ in (A.24), and taking its δ-function character into account, we obtain in the case when $k_\perp<l^{-1}$ and $\omega<\tau^{-1}$ the condition (50).

[1)] The problem under consideration is mathematically similar to the problem of pulse propagation in a nerve fiber,[13] and the results presented in the present section can be considered to be a generalization of the results obtained for models of the Fitz-Hugh–Nagumo type.[19,20]

[2)] The estimates in Subsec. 6.1 were made in collaboration with N. Yu. Mizerina.

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