Temperature dependences of the Hall and magnetoresistance coefficients of vanadium and tantalum single crystals: Anisotropy of electron-phonon scattering

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The Hall coefficient $R_H(T)$, transverse magnetoresistance $\Delta\rho_c(T)$, and the temperature-dependent component $\Delta\rho_c(T)$ of the impurity electrical resistance are measured as functions of temperature $T = 4.2$–300 K for vanadium and tantalum single crystals with $\rho_{273.2}/\rho_{4.2} = 1350$ and 500, respectively. The curves $R_H(T)$ have minima at $T_0 = 33$ and 24 K for vanadium and tantalum, respectively, which indicates that the electron-phonon scattering is anisotropic. The anisotropy is caused by "intersheet" electron-phonon processes in which the charge carriers are scattered between the open hole surface $\Gamma H\ 3h$ and the closed hole ellipsoids $N\ 3h$. The curves $\Delta r_H(T)$ and $\Delta\rho_c(T)$ have maxima for T close to T_0. The observed extrema have a common physical origin and can be explained by the Kagan-Zhernov-Flerov theory, which postulates that the nonequilibrium part of the electron distribution function is anisotropic. The observed dependence $R_H(T)$ for $T > T_0$ agrees with calculations of $R_H(T)$ for vanadium based on realistic models for the electron and phonon spectra, including the anisotropy for the electron-phonon interaction matrix element.

INTRODUCTION

According to the Kagan-Zhernov-Flerov theory devloped first in Refs. 1 and 2 and subsequently in Refs. 3 and 4, the anisotropic scattering of the conduction electrons in metals causes a pronounced anisotropy $\delta f_{\mathbf{k}}$ of the nonequilibrium part of the electron distribution function. The anisotropic scattering is caused primarily by Peierls (umklapp) processes and by the anisotropy of the Fermi surface and phonon spectrum. In turn, the angular dependence of $\delta f_{\mathbf{k}}$ should give rise to characteristic maxima in the impurity resistance $\Delta\rho_c(T)$ and magnetoresistance $\Delta\rho_H(T)$, and to extrema (a minimum or a maximum) in the Hall coefficient $R_H(T)$ as functions of temperature for $T \ll \Theta_D$, where Θ_D is the Debye temperature. The magnitude of these extrema is comparable to the magnitude of the kinetic coefficients for the case of an isotropic distribution function and is determined by the degree of anistropy of the function $\delta f_{\mathbf{k}}$ at a given temperature. Even for metals with a spherical Fermi surface (sodium, potassium), the anisotropy in $\delta f_{\mathbf{k}}$ due to umklapp processes is so pronounced that there is a range of temperatures over which the electrical resistance ρ_c varies severalfold.[2] By tending to make $\delta f_{\mathbf{k}}$ isotropic, elastic scattering of the conduction electrons by impurities suppresses these effects; moreover, and unlike the case for $\Delta\rho_H(T)$ and $\Delta\rho_c(T)$, the temperatures at which $R_H(T)$ has an extremum depend hardly at all on the impurity concentration.[4]

The Kagan-Zhernov-Flerov theory is supported by much experimental evidence. For example, $\Delta\rho_c(T)$ has been found to have peaks for copper, silver, magnesium, cadmium, indium and other metals (cf. the bibliography in Refs. 1 and 2); $\Delta\rho_H(T)$ is known to have maxima for aluminum and indium[4]; $R_H(T)$ has extrema for both the simple (Na, Cd, Cu, cf. Ref. 4) and the transition metals (Mo, W, Re, Ru, cf. Refs. 5–9). However, no experiment has yet been done in which $\Delta\rho_c(T)$, $\Delta\rho_H(T)$, and $R_H(T)$ are all observed to have extrema in the same metal. Since the Kagan-Zhernov-Flerov theory plays a key role in understanding the low-temperature behavior of the kinetic coefficients of metals, such an experiment would be of fundamental importance since it would demonstrate directly that these extrema all have a common physical origin.

The purpose of this paper is to study how the anisotropy of the electron-phonon scattering influences $\Delta\rho_c(T)$, $\Delta\rho_H(T)$, and $R_H(T)$ for vanadium and tantalum crystals.

Vanadium and tantalum are interesting for the following reasons. First, unlike the case for most transition metals, the electron-phonon contribution to the electrical resistance of vanadium[10,11] and tantalum[12] in the limit $C \to 0$ of vanishing impurity concentration is proportional to T^4 for $T \ll \Theta_D$. This anomalous $\rho(T)$ dependence may be attributed to phonon-induced jumps of the conduction electrons by an amount Δk where the different sheets of the Fermi surface approach one another closely (we call this intersheet scattering). Indeed, $\rho(T)$ was calculated in Ref. 13 by using realistic electron and phonon spectra and allowing for the angular dependence of the electron-phonon interaction matrix element. The results agreed quantitatively with the observed $\rho(T)$ dependence for vanadium and showed that intersheet electron-phonon jumping between the $\Gamma H\ 3h$ and $N\ 3h$ sheets of the Fermi surface (cf. Fig. 1) contributes more to $\rho(T)$ than intrasheet scattering, and this is the reason for the overall dependence $\rho_{\text{eff}}(T) \propto T^4$. The situation is presumably the same for tantalum, whose electronic structure is analogous to that of vanadium.

We thus anticipate that the anisotropy of the nonequilibrium part of the electron distribution function in V and Ta may be due to electron transitions between sheets, which

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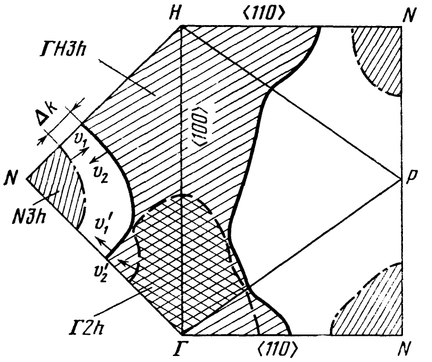

FIG. 1. Model for the Fermi surface of vanadium : $\Gamma H = 2.081$ Å^{-1}; $\Gamma N = 1.472$ Å^{-1}; $\Gamma P = 1.802$ Å^{-1}.

(like umklapp scattering) is a highly anisotropic process. The detection of extrema in the Hall and magnetoresistance coefficients as functions of temperature would yield more direct evidence for the existence of intersheet transitions.

A second reason why V and Ta are of interest is that in this case the interplane electron-phonon scattering in a magnetic field may be deterministic[14] in the sense that the phonon-induced electron transitions take place between sheets with the same carrier sign (all the sheets are p-type for V and Ta). According to Ref. 14, the probability for a conduction electron to jump a distance Δk from one sheet to another tends to unity. By contrast, for tungsten, molybdenum, and rhenium,[6-9] the intersheet scattering is random and the electron transitions occur between sheets (orbits) with opposite carrier signs and have probability less than 0.5. The effect of the anisotropic intersheet scattering in V and Ta should therefore be more pronounced than for metals with random scattering.

Third, our model Fermi surface (Fig. 1) suggests that intersheet scattering in V and Ta should occur for electron states that cover a significant fraction of the area of the Fermi surface, $\delta S_F \lesssim S_F$, so that the kinetic properties should be more strongly influenced than for metals (e.g., W, Mo, Re) in which the scattering is localized in small regions of the Fermi surface ($\delta S_F \ll S_F$).

Finally, vanadium and tantalum belong to the class of uncompensated metals, for which $\Delta\rho_c(T)$, $\Delta\rho_H(T)$, and $R_H(T)$ can all have extrema. [No maximum in $\Delta\rho_H(T)$ is observed in compensated metals.]

Studies of vanadium and tantalum are also of independent interest, because they can show the extent to which the Kagan-Zhernov-Flerov theory, which was developed for simple metals, also applies to the transition metals.

MODEL FERMI SURFACE FOR VANADIUM AND TANTALUM

Vanadium and tantalum are transition $3d$- and $5d$-metals with atomic numbers $Z = 23$ and $Z = 73$, respectively, and have body-centered cubic lattices. The electron spectrum has been calculated by the augmented plane-wave (self-consistent, nonrelativistic approximation)[15,16] and LCAO[17] methods, and by a combination of the orthogonal plane-wave and close-coupling techniques.[18] The results found by these methods are mutually consistent and show that the Fermi surface for vanadium (Fig. 1) has three kinds of sheets: 1) closed hole sheets ("ellipsoids") $N3h$ which are centered at point N in the Brillouin zone and contain states in the third band; 2) a multiply connected hole sheet (jungle gym) $\Gamma H3h$, which is also composed of states in the third band and forms a system of mutually coupled rippled cylinders with axis parallel to ΓH, i.e., to the ⟨100⟩ axis; 3) the closed hole octahedron sheet $\Gamma 2h$, which consists of states in the second band and is centered at the point Γ. The model Fermi surface for vanadium was corrected to accommodate data from measurements of the galvanomagnetic properties,[21] magnetothermal oscillations,[19] and de Haas–van Alphen effect.[19,20]

Tantalum is a structural and electronic analog of vanadium, and according to the calculation in Ref. 22, which was carried out by the augmented plane-wave method corrected for relativistic effects, the Fermi surfaces for Ta and V differ only in scale. Because of the significant relativistic effects for Ta, the linear dimensions of the $\Gamma 2h$ and $\Gamma H3h$ sheets are 50% greater than for V, while there is a corresponding decrease (by 33%) in the dimensions of the $N3h$ sheet. The model Fermi surface for Ta in Ref. 22 was refined to yield quantitative agreement with data from measurements of the de Haas–van Alphen effect[23] and galvanometric properties.[24]

The enhanced relativistic effects for Ta displace the branches of the electron energy spectrum $\varepsilon(\mathbf{k})$ and the Fermi level from their positions for vanadium and significantly alter the dimensions of the sheets but leave the gap Δk between the $\Gamma H3h$ and $N3h$ sheets almost unchanged (to within $\sim 10\%$). The calculations in Refs. 15–18, 22 and the experimental results on the de Haas–van Alphen effect[19,20,23] indicate that the gap Δk is slightly anisotropic (by 10%) in the plane ΓNH and is narrowest ($\Delta k \approx 0.2$ Å^{-1}) along NH. Both V and Ta are uncompensated metals in which the total number of sheets of the Fermi surface corresponds to one hole per atom.

MEASUREMENT RESULTS AND DISCUSSION

We tested wafers of dimensions $0.60 \times 1.30 \times 10.00$ and $0.70 \times 2.70 \times 9.05$ mm^3 cleaved from single crystals by ap-

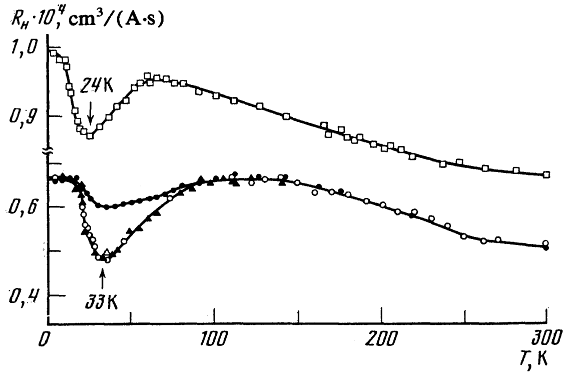

FIG. 2. Temperature dependence of the Hall coefficient for tantalum and vanadium single crystals ($H = 13.5$ kOe): □, Ta-3; ●, V-6; ○, △, and ▲ correspond to different series of measurements for sample V-4.

TABLE I. Properties of vanadium and tantalum samples.

Metal (sample)	$\rho_{273,2}/\rho_{4,2}$ [1]	$\rho_{273,2}$, 10^{-6} $\Omega\cdot$cm	ρ_0, [2] 10^{-6} $\Omega\cdot$cm	a, [3], 10^{-11} $\Omega\cdot\text{cm}\cdot\text{K}^2$	c, [3] 10^{-13} $\Omega\cdot\text{cm}\cdot\text{K}^{-4}$	T_0, K	$\rho_{\text{eff}}(T_0)/\rho_0$	Θ_D, K [4]
V-4	1350	19,8	0,0147	3,3	1.00	33±2	6,7	397
V-6	170	19,8	0,117	4,2	1,00	33±3	1,1	397
Ta-3	600	13,8	0,023	0,4	2,50	24±2	3,6	238

[1] The values $\rho_{4.2}$ were found from measurements in a magnetic field.

[2] The electrical resistance $\rho(T)$ of vanadium and tantalum for $T < \Theta_D/10$ is described by the expression $\rho(T) = \rho_0 + aT^2 + cT^4$.

[3] The values of a and c were measured in Refs. 10 and 12.

[4] The Debye temperatures Θ_D are taken from Ref. 25.

plying an electrical arc. The wafers were then buffed and electrolytically polished. The magnetic field was applied along the ⟨110⟩ and ⟨100⟩ axis for the V and Ta samples, respectivley. The Hall and magnetoresistance effects were measured for specimens cleaved from the same single crystals[1] whose electrical resistance ρ was measured in Refs. 10 and 12. We measured $R_H(T)$ and the transverse magnetoresistance $\Delta\rho_H(T)$ for $4.2 \leqslant T \leqslant 300$ K and magnetic fields $\leqslant 85$ kOe. The temperature during the measurements was kept constant to within ± 0.01 K by an electronic thermostat.

Figure 2 shows the temperature dependences $R_H(T)$ for vanadium and tantalum single crystals. Clearly, the $R_H(T)$ curves for the two metals are similar. The values R_H are positive at $T = 300$ K and are equal to $(0.51 \pm 0.02)\cdot 10^{-4}$ cm^3/A·s for V and $(0.81 \pm 0.02)\cdot 10^{-4}$ cm^3/A·s for Ta. As T decreases, R_H at first increases monotonically but then drops to well-defined minima at $T_0 \approx 33$ K for V and Ta, respectively. Table I shows that for the purest specimens (V-4 and Ta-3), T_0 lies within the "zero-impurity-concentration limit" for the electrical resistance, i.e., for $T \approx T_0$ electron-phonon scattering is the dominant scattering mechanism for the conduction electron for both V and Ta, and the scattering is small-angle ($T/\Theta_D \ll 1$). According to Ref. 4, this suggests that the observed extrema in $R_H(T)$ are caused by anisotropy of the electron-phonon scattering.

This anisotropy could in principle be caused by electron-phonon umklapp processes which change the electron wave vector by $\mathbf{G}$, where $\mathbf{G}$ is a reciprocal lattice vector. Nevertheless, the values of T_0, the dimensions of the sheets, and the structure of the Fermi surfaces and phonon spectra for V and Ta imply that umklapp processes cannot be responsible for the minima in $R_H(T)$ for these metals. Indeed, these processes become "frozen-out" at temperatures T^* for which $\mathbf{q} < \mathbf{G} - 2\mathbf{k}_F$, where $\mathbf{q}$ is the wave vector of the phonon, $\mathbf{G}$ is a reciprocal lattice vector, and $\mathbf{k}_F$ is the Fermi wave vector. Estimates based on the phonon spectra indicate that T^* is equal to ~ 300 and ~ 200 K for the electron states on the closed sheets $\Gamma 2h$ and $N3h$, respectively, for both V and Ta. These values are nearly ten times greater than the experimentally observed T_0. On the other hand, the umklapp processes for the electron states on the open $\Gamma H3h$ surface do not "freeze-out" for any nonzero T; they therefore cannot account for the observed minimum in $R_H(T)$.

The anisotropy of the electron-phonon scattering responsible for the minimum in $R_H(T)$ is most probably caused by phonon-induced transitions of electrons between the $N3h$ closed hole sheets and the open surface $\Gamma H3h$, which are separated by a gap Δk (Fig. 1). As T decreases, these intersheet transitions become localized where $N3h$ and $\Gamma H3h$ approach each other most closely in the plane ΓNH. That is, they are localized with "hot spots" which drain the momentum of the electron subsystem more effectively than elsewhere on the Fermi surface, because intersheet scattering changes the Fermi velocity v_F of the carriers by an amount $\sim 2v_F$ in the hot spots.[2] Since the Δk gap is slightly anisotropic in the plane ΓNH, the area δS_F of the hot spots is comparable to the total area of the $N3h$ sheets, and there are 12 hot spots per Brillouin zone. Neither intraband (normal) electron-phonon scattering nor $\Gamma 2h$–$\Gamma H3h$ intersheet transitions can alter v_F so substantially elsewhere on the Fermi surface (cf. Fig. 1).

The anisotropy of the intersheet scattering is greatest at the temperature T_0 for which $|\mathbf{q}| \approx \Delta k$ holds. For $T < T_0$, i.e., when $|\mathbf{q}| < \Delta k$, intersheet scattering is "frozen-out" and the anisotropy drops abruptly. Indeed, experiments show that the values of R_H for V and Ta are similar and lack extrema both at liquid helium temperatures ($T < T_0$), where isotropic large-angle impurity scattering dominates, and at liquid nitrogen temperatures ($T > T_0$), for which isotropic scattering by phonons with large wave vectors dominates.

The above interpretation is supported by the results of additional experiments. First, measurements for sample V-6 (Table I and Fig. 2) show that the minima in $R_H(T)$ for vanadium become much shallower when there is an increase in the concentration of impurities that scatter the condition electrons elastically and thus tend to make $\delta f_{\mathbf{k}}$ isotropic. In contrast to the situation for $\Delta\rho_H(T)$ and $\Delta\rho_c(T)$, the minimum in $R_H(T)$ occurs at nearly the same temperature for both "pure" (V-4) and "dirty" (V-6) vanadium samples, even though the contribution of impurity scattering to $\rho(T)$ is almost ten times greater for V-6. These findings are in full agreement with the theory.[2,4]

Second, the transverse magnetoresistance

$$\Delta\rho_H(T)/\rho_0 = [\rho(H, T) - \rho(H=0, T)]/\rho(H=0, T=0)$$

is found to have characteristic peaks as a function of temperature for both V and Ta (Fig. 3). These peaks disappear as the impurity concentration increases. According to Ref. 4, these findings demonstrate unambiguously that the electron-phonon scattering is anisotropic. Indeed, it was shown

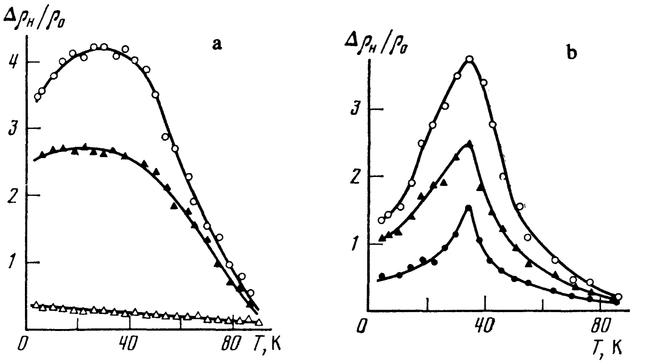

FIG. 3. Temperature curves for the transverse magnetoresistance $\Delta\rho_H(T)/\rho_0$: a) for vanadium (V-4); b) for tantalum (Ta-3). The points ○, ▲, and ● correspond to $H = 85$, 50, and 13.5 kOe, respectively; the values △ were recorded for sample V-5 at $H = 85$ kOe.

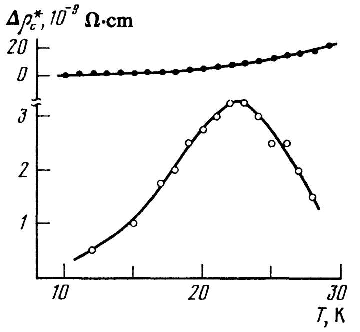

FIG. 4. Departure of the temperature-dependent components of the impurity resistance for samples V-6 (○) and V-5 (●) from the values of V-4.

in Ref. 4 that for an uncompensated metal

$$\Delta\rho_H(T)/\rho_{H=0}(T)=\eta(T)/[1-\eta(T)], \quad \eta(T)=P_{12}^2/P_{11}P_{22},$$

where the P_{ij} are the matrix elements of the electron-phonon scattering operator. The experimental data on $\Delta\rho_H(T)$ and $\rho_{H=0}(T)$ then yield $\eta \approx 0.6$–0.8 for V and Ta at $T = 30$ K and $H = 85$ kOe.

Third, if the observed behavior is in fact caused by anisotropic electron-phonon scattering, then the calculations in Ref. 2 indicate that in addition to producing extrema in $R_H(T)$ and $\Delta\rho_H(T)$ for metals, the anisotropy should also cause the temperature-dependent component

$$\Delta\rho_c(T)=\rho(T, C)-\rho(T, C=0)-\rho(T=0, C)$$

of the impurity resistance to have a maximum for sufficiently low impurity concentrations C. This maximum should disappear for large C. This prompted us to measure $\rho_c(T)$ for samples V-4, V-6, and V-5 ($\rho_{273.2}/\rho_{4.2} = 37$), for which the residual resistance varied by a factor of 35 from $1.5\cdot10^{-8}$ to $5.5\cdot10^{-7}$ Ω·cm. Indeed, Fig. 4 shows that $\Delta\rho_c$ for sample V-6 had a peak at $T/\Theta_D = 0.056$. No peak was present for sample V-5, for which C was large.

Finally, according to Ref. 4 the asymmetry of the minima in $R_H(T)$ for V and Ta, combined with the characteristic slow change in R_H for $T \gtrsim T_0$, also indicates that anisotropic electron-phonon scattering is responsible for the peaks.

The gap Δk between the $N\,3h$ and the $\Gamma H\,3h$ sheets has been calculated for V and Ta by using realistic phonon spectra and experimental measurements of T_0 and the speed of sound. The results unambiguously support the hypothesis that phonon-induced $N\,3H$–$\Gamma H\,3h$ intersheet electron transitions are responsible for the minima in $R_H(T)$.

ESTIMATES OF Δk FOR VANADIUM AND TANTALUM

Analysis of the electron-phonon interaction matrix element shows that unlike normal (intrasheet) scattering of conduction electrons from the $\mathbf{k}_1$ to the $\mathbf{k}_2$ state (where $\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q} = 0$), transverse phonons[27] play a decisive role in intersheet scattering ($\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q} = K$), just as in umklapp processes ($\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q} = \mathbf{G}$) (here $|\mathbf{K}|$ is the distance between the states on the sheets of the Fermi surface between which the scattering occurs). Figure 1 shows that for electron states in the $N\,3h$ and $\Gamma H\,3h$ sheets, phonon-induced intersheet scattering is most likely to occur where these sheets cross the plane ΓNH. Transverse phonons moving along the $\langle 110\rangle$ axis ($\mathbf{q}\|\langle 110\rangle$) will play a dominant role in this scattering. In order to estimate Δk one must therefore use the branch of the phonon spectrum that corresponds to transverse phonons with $\mathbf{q}$ of the form ($\xi\xi 0$) and use the speed of transverse sound along the $\langle 110\rangle$ direction.

The speed of sound s in V and Ta was measured by the rf resonance technique ($f = 10$ MHz) to within ±0.2% in Ref. 28. The results agree to within the experimental error with the results of echo measurements[29,30] and imply that $s = 2.677\cdot10^5$ cm/s and $2.224\cdot10^5$ cm/s for V and Ta, respectively, for transverse sound propagating along the $\langle 110\rangle$ axis.[28] Since $T_0/\Theta_D \ll 1$ for both V and Ta (cf. Table I), we are justified in using the approximation $\Delta k = q_0 = kT_0/\hbar s$ in the estimates.

TABLE II. Gap width Δk (in Å^{-1}) between the $\Gamma H\,3h$ and the $N\,3h$ sheets in the plane ΓHN for vanadium and tantalum.

Metal	Δk_{exp} 1)	Δk_{exp} 2)	Δk_{th}	
V	0,16±0,01	0,14±0,02	0,165 0,17 0,21	[18] [17] [16]
Ta	0,18±0,015	0,15±0,02	0,18	[23]

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1)Calculated from T_0 and the speed of sound s.

2)Calcualted from T_0 and the phonon spectra.

The phonon spectra for V and Ta were found by x-ray diffraction in Ref. 31 and by inelastic neutron scattering in Ref. 32, respectively. These results show clearly that the dispersion $\nu(\mathbf{q})$ for the transverse phonons is linear for $q \lesssim 0.2\Gamma H (\nu \leqslant 2 \cdot 10^{12}$ Hz), which corresponds to temperatures $T \lesssim 100$ K. We are therefore justified in using the relation $\Delta k = kT_0/\hbar s$ to estimate Δk from the speed of sound. The phonon spectra can be used to find $\Delta k = q_0$ directly form the thermal excitation energy $k_B T_0$.

Table II shows the gap sizes Δk calculated for V and Ta from measurements of T_0, s, and the phonon spectra. We see that the speed-of-sound estimates for Δk agree with the results found directly from the phonon spectra. Nevertheless, we prefer the speed-of-sound values Δk because the error in the phonon spectra for V and Ta was at least ten times greater than the error in measuring s.

Our values for the gap Δk separating the multiply connected hole surface $\Gamma H\, 3h$ from the closed hole ellipsoids $N\, 3h$ along the direction NH in the Brillouin zone are thus in quantitative agreement with various theoretical calculations which include corrections for the de Haas–van Alphen effect and magnetothermal oscillations.[19,20,23] This shows that intersheet electron-phonon transitions are responsible for the anisotropy of electron-phonon scattering in V and Ta which causes the characteristic minima in $R_H(T)$.

It is striking that $R_H(T)$ has extrema even when electron-phonon and elastic electron-impurity scattering contribute equally to the resistance (for example, $\rho_{\text{eff}}(T_0)/\rho_0 = 1.1$ for V-6, while the extrema in $R_H(T)$ for tungsten[6] and rhenium[9] disappear completely for $\rho_{\text{eff}}(T_0)/\rho_0 \lesssim 5$). These experimental findings indicate that intersheet phonon-electron transitions occur particularly readily in V and Ta, possibly because of the following two factors. First, the relative area $\delta S_F/S_F$ of the hot spots in V and Ta is much greater than for W and Re. Second, a deterministic situation apparently holds for these transitions,[14] so that the transition probability approaches unity for $\omega_c \tau \sim 1$.

The anisotropy of the intersheet electron-phonon scattering is thus responsible for the observed extrema in $R_H(T)$, $\Delta\rho_H(T)$, and $\Delta\rho_c(T)$ for vanadium and tantalum. In order to understand the reason for the strong temperature dependence of R_H for $T > T_0$, we used realistic electron and phonon spectra to calculate $R_H(T)$ for vanadium, which we will discuss next.

CALCULATED RESULTS AND COMPARISON WITH EXPERIMENT

We calculated the Hall coefficient $R_H(T)$ of vanadium for $20 \leqslant T \leqslant 293$ K[3)] from the expression

$$R_H(T) = \frac{12\pi^3}{ce} \int \tau^2(\mathbf{k},T) v_{\mathbf{k}}^2 \left(\overline{\frac{1}{\rho_{\mathbf{k}}}}\right) dS_{\mathbf{k}} \left[\int \tau(\mathbf{k},T) v_{\mathbf{k}}\, dS_{\mathbf{k}}\right]^{-2},$$

which follows from the kinetic equation in the weak effective field approximation $\omega_c \tau \ll 1$ if we assume that the relaxation time $\tau(\mathbf{k},T)$ does not depend on the direction of the momentum vector. Here $v_{\mathbf{k}}$ is the electron velocity in a state $\mathbf{k}$ on the Fermi surface, and $2\,\overline{(1/\rho_{\mathbf{k}})} = 1/r_1 + 1/r_2$ is the mean curvature of the Fermi surface at the point $\mathbf{k}$ in reciprocal space (r_1 and r_2 are the principal radii of curvature at $\mathbf{k}$).

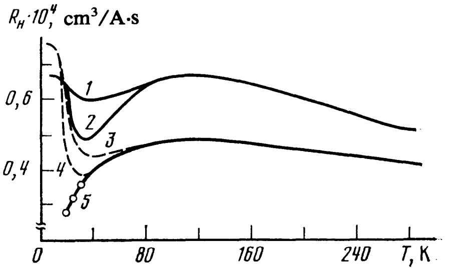

FIG. 5. Calculated temperature curves of the Hall coefficient for vanadium: 3) for sample V-6; 4) for sample V-4; 5) behavior of $R_H(T)$ without impurity scattering; 1, 2) experimental values for samples V-6 and V-4, respectively.

Figure 5 shows the calculated curve $R_H(T)$. We see that it agrees qualitatively with the experimental curve for $T \geqslant 30$ K but gives values that are $\approx 30\%$ lower. The calculated curve, which treats only electron-phonon scattering, continues to decrease until $T \approx 20$ K, whereas the experimental curve has a distinct minimum at 33 K. It was shown in Ref. 10 that for sample V-4 the impurity resistance is comparable to the contribution from electron-phonon scattering at $T = 15$ K. Since we know the geometry of the Fermi surface for vanadium and the carrier dynamics on the surface, we therefore calculated the average relaxation time corresponding to the impurity component and included it in the calculation (cf. Fig. 5). The corrected curve also has a minimum at 33 K and agrees qualitatively with the experimental curve.

CONCLUSIONS

We have shown experimentally that the transverse magnetoresistance and the temperature-dependent part of the impurity resistance have maxima as functions of temperature, while the Hall coefficient has a minimum. Experimental results for the transition metals (vanadium and tantalum, in our case) have not been previously reported in which all three effects are simultaneously present. The extrema are sensitive to the impurity concentration and have a common physical origin—the anisotropy of the nonequilibrium correction to the electron distribution function caused by intersheet electron-phonon transitions.

The Kagan-Zhernov-Flerov theory, in which anisotropy of the electron distribution function plays a key role, is thus useful in understanding the kinetic properties of the transition as well as the simple metals.

We thank A. P. Zhernov for a helpful discussion of topics touched upon in this paper.

[1)] The vanadium single crystals were grown by V. M. Azhazha and G. P. Kovtun at the Khar'kov Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR; R. Sh. Nasyrov grew the Ta single crystal at the Institute of Metal Physics, Ural Science Center, Academy of Sciences of the USSR.

[2)] According to the calculation in Ref. 26, for vanadium v_F is equal to $0.51 \cdot 10^8$, $0.28 \cdot 10^8$, and $0.35 \cdot 10^8$ cm/s on the sheets $N\, 3h$, $\Gamma 2h$, and $\Gamma H\, 3h$, respectively.

[3)] The computational method became inaccurate for T below 20 K.

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