Threshold for Čerenkov radiation in periodic media

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We show that in a periodic medium the particle velocity corresponding to the threshold for Čerenkov radiation decreases in comparison with that for a uniform medium. The relative amount of reduction of the threshold is proportional to the modulation amplitude δ of the dielectric constant, and the radiation spectrum (below the threshold for a uniform medium) is found to be concentrated near a specific frequency in a small region of frequencies with relative width $\sim\delta$.

Although 50 years have already elapsed since the discovery of Čerenkov radiation,[1] interest in this effect has not diminished, and the regions of its application are constantly expanding. Recently considerable attention has been given to study of Čerenkov radiation in periodic media, in which important features of the radiation appear which are not present in uniform media.[2,3]

In the present article we investigate theoretically the spectral distribution of Čerenkov radiation in periodic media with particles having velocities close to the threshold for this radiation. It is shown that the spectral density of the radiation of a particle whose velocity v is less than the threshold value $c_p = c\varepsilon^{-1/2}$ in the corresponding uniform medium (ε is the average value of the dielectric constant of the medium) and satisfies the condition $c_p - v \lesssim \delta c_p$, where δ is the amplitude of the spatial modulation of the dielectric constant, is found to be of the same order as the spectral density immediately above the threshold and considerably exceeds the spectral density of structural Čerenkov radiation.[4] Structural Čerenkov radiation (and also quasi-Čerenkov, parametric, and resonance radiation as well as transition scattering) is the name given to the radiation, not having a velocity threshold, of a charged particle moving uniformly in a straight line in a periodic medium.[2–4]

For simplicity we shall carry out the discussion for a medium with a one-dimensional harmonic spatial modulation of the dielectric constant and for motion of a particle along the direction of modulation. In other words, we shall assume that the dielectric properties of the medium are described by the expression

$$\varepsilon(z)=\varepsilon(1+2\delta\cos\tau z),\quad \tau=2\pi/d, \tag{1}$$

and that the charged particle is moving along the z axis with a velocity v close to c_p, $|v-c_p|\sim\delta c_p$ (δ is a small quantity), where d is the periodic spatial modulation of the permittivity. For the amplitude of the radiation field of a particle in the two-wave approximation of dynamical diffraction theory,[4] i.e., assuming that the spatial Fourier components of the radiation field can be represented in the form

$$\mathbf{E}(\mathbf{k}_0,\mathbf{k}_1)=\mathbf{E}_0e^{i\mathbf{k}_0\mathbf{r}}+\mathbf{E}_1e^{i\mathbf{k}_1\mathbf{r}},$$

we obtain the following system of equations:

$$\left(1-\frac{k_0^2}{\varkappa^2}\right)E_0+\delta E_1=\frac{iev\theta}{2\pi^2\omega\varepsilon}\delta(\omega-\mathbf{k}_0\mathbf{v}),$$
$$\delta E_0+\left(1-\frac{k_1^2}{\varkappa^2}\right)E_1=0, \tag{2}$$

where $\varkappa^2=\omega^2\varepsilon/c^2$, $\mathbf{k}_1=\mathbf{k}_0+\tau$, e is the charge of the particle, the small quantity θ is the opening angle of the Čerenkov cone, and ω is the frequency of the radiation.

We note that for structural Čerenkov radiation there is a lower limit of the radiated frequencies ω_e^s,[5] which to lowest order in δ is given by the expression

$$\omega_e^s=2\omega_B(1+c_p/v)^{-1},\quad \omega_B=c_p\tau/2.$$

Using the solution of the system of equations (2) and calculating the retarding (reactive) force of the radiation field on the charged particle, we obtain for the spectral density of the radiation loss for $v>c_p$ the following expressions:

$$dW/d\omega=(e^2\omega/c^2)(1-c_p^2/v^2),\quad \omega>\omega_e,$$
$$dW/d\omega=(e^2\omega\delta/2c^2)\{\nu[(\nu-q)^2+1]^{-1/2}+1\}\times\{\nu-q+[(\nu-q)^2+1]^{1/2}\}^{-1},\quad \omega<\omega_e, \tag{3}$$

where

$$\nu=2(\omega-\omega_B)/\delta\omega_B,\quad q=(1-c_p^2/v^2)/\delta,$$
$$\omega_e=\{1+[4(v-c_p)^2+\delta^2c_p^2][8c_p(v-c_p)]^{-1}\}\omega_B.$$

As in seen from Eq. (3), for a particle velocity exceeding c_p and a frequency $\omega>\omega_e$ and outside the interval $|\omega-\omega_B|\leqslant\delta\omega_B$ the spectral density of the loss to radiation is described by the Frank-Tamm formula, while at frequencies in the interval $|\omega-\omega_B|\leqslant\delta\omega_B$ it substantially exceeds the spectral density $dW_c/d\omega$ of the loss to radiation in a uniform medium (see Fig. 1a):

$$dW_m/d\omega=(dW_c/d\omega)[1+(1+q^{-2})^{1/2}]/2,$$

where $\omega=\omega_B$, i.e., where $\nu=0$.

For $v<c_p$ the loss to radiation is nonzero only for $\omega>\omega_e^s$ and is described by the formula of the second line of Eq. (3). As follows from Eq. (3), for a particle velocity satisfying the condition $1-v/c_p\lesssim\delta$, the radiation loss spectral density function $dW/d\omega$ has a clearly defined maximum at $\nu=0$ in the frequency region $|\omega-\omega_B|\sim\delta\omega_B$ (Fig. 1b), the value of $dW/d\omega$ in which is of the same order as for particle velocities greater than $c_p(v/c_p-1\sim\delta)$. For particle velocities considerably less than threshold ($1-v/c_p>\delta$), the loss to radiation drops rapidly and the spectral distribution of the radiation loss corresponds to values characteristic for structural Čerenkov radiation, i.e., $dW/d\omega\sim e^2\omega\delta^2/c^2$ and is at least δ^{-1} times less than the spectral density of Čerenkov radiation for $v/c_p-1\sim\delta c_v$.

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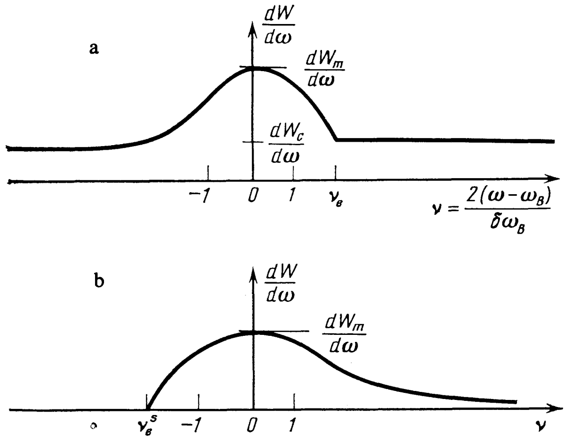

FIG. 1. Qualitative appearance of the spectral distribution of energy loss in the form of Čerenkov radiation near threshold in a periodic medium: (a)–$v > c_p$; (b)–$v < c_p$, $c_p - v \sim \delta c_p$.

Our results therefore show that in periodic media the Čerenkov radiation threshold in velocity shifts by an amount $\Delta c_p \approx \delta c_p$. However, below the threshold c_p the intensity of radiation characteristic of Čerenkov radiation near threshold for $v/c_p - 1 \sim \delta$ is realized only in a frequency interval of the order $\delta\omega_B$. Therefore, in particular, the loss through radiation integrated over frequency for $v < c_p$ obtained from Eq. (3) does not diverge even with neglect of the frequency dispersion and is found to be approximately

$$W = (e^2\omega_B{}^2\delta^2/4c^2)\,|\ln(1-v/c_p)|.$$

Here the range of frequencies allowed for the radiation broadens considerably, and the integrated loss W diverges logarithmically as v approaches c_p.

The shift in velocity of the Čerenkov radiation threshold in periodic media by an amount $\Delta v = \delta c_p$ corresponds to a reduction of the energy threshold by an amount

$$\Delta E = E_t\{1-[(\varepsilon-1)/[\varepsilon-(1-\delta)^2]]^{1/2}\},$$

where E_t is the threshold energy for radiation in a uniform medium. In the case of ultrarelativistic threshold energies E_t this reduction may be very important.

The physical cause of the shift of the Čerenkov radiation threshold is related to the diffraction of the electromagnetic field in the periodic medium. For frequencies at which the strongest diffraction occurs in the medium, the effective value of the permittivity is found to differ from ε by an amount $\sim\delta\varepsilon$, which leads to a shift of the Čerenkov radiation threshold. For the same reason the frequency distribution of the radiation intensity below the threshold for a uniform medium, which is determined by ε, turns out to differ

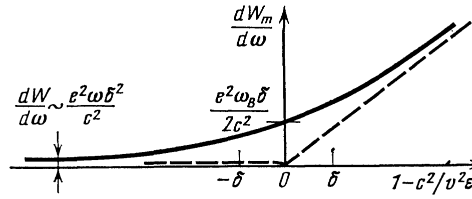

FIG. 2. The maximum of the spectral density $dW_m/d\omega$ as a function of the particle velocity at the Bragg frequency ω_B. The dashed curve corresponds to radiation in a uniform medium with a refractive index $\sqrt{\varepsilon}$ (this same curve approximately describes the spectral intensity of the radiation in a periodic medium for frequencies $|\omega - \omega_B| > \delta\omega_B$).

from the distribution above threshold and to be concentrated in a frequency region $\sim\delta\omega_B$.

These results are quite natural also from very general considerations if we take into account that as the result of the spatial modulation of ε in the frequency interval $\Delta\omega/\omega$ the phase velocity of the light to first order in δ varies in the range from $(c/\sqrt{\varepsilon})(1+\delta)$ to $(c/\sqrt{\varepsilon})(1-\delta)$. Therefore in this velocity interval the spectral intensity of Čerenkov radiation exceeds the corresponding intensity of radiation in a uniform medium with refractive index $\sqrt{\varepsilon}$ (see Fig. 2) by an amount of the order $(e^2\omega/c^2)\delta$.

Note that cholesteric liquid crystals are suitable objects for experimental observation of the threshold shift which we have discussed. As a result of the fact that for them the value of δ is rather large, the shift of the velocity threshold can be as large as 5–10%. In addition, in these crystals, by changing the temperature insignificantly[6] and transforming the liquid crystal into an isotropic liquid, it is comparatively simple to establish conditions under which the particle velocity for the isotropic phase lies below the Čerenkov radiation threshold, while in the cholesteric phase as a result of this shift it is above threshold.

[1]P. A. Čerenkov, Dokl. Akad. Nauk SSR **2**, 451 (1934).

[2]V. L. Ginzburg and V. N. Tsitovich, Perekhodnoe izluchenie i perekhodnoe rasseyanie (Transition Radiation and Transition Scattering), Nauka, Moscow, 1984.

[3]G. M. Garibyan and Shi Yan, Rentgenovskoe perekhodnoe izluchenie (X-Ray Transition Radiation), Armenian Academy of Sciences Publishing House, Erevan, 1983.

[4]V. A. Belyakov and V. P. Orlov, Trudy VIII Vsesoyuz. soveshch. po fizike vzaimodeĭstviya zaryazhennykh chastits s kristallami (Proceedings of the Eighth All-Union Conf. on the Physics of Interaction of Charged Particles with Crystals), Moscow State University Publishing House, Moscow, 1977, p. 69.

[5]B. M. Bolotovskiĭ and G. V. Voskresenskiĭ, Usp. Fiz. Nauk **94**, 377 (1968) [Sov. Phys. Uspekhi **11**, 143 (1968)].

[6]V. A. Belyakov, V. E. Dmitrienko, and V. P. Orlov, Usp. Fiz. Nauk **127**, 221 (1979) [Sov. Phys. Uspekhi **22**, 63 (1979)].

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