Possibility of inducing the Kondo effect by a magnetic field in a ferromagnet

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We consider a metallic easy-axis ferromagnet containing easy-plane impurities and show rare earth metals as the example that at low concentrations the impurities give local levels below the bottom of the spin-wave band in the spectrum of magnetic excitations. These levels can be brought together by a magnetic field, leading at low temperatures to an anomalous "Kondo" scattering of conduction electrons. We evaluate the amplitude for impurity scattering of electrons in the weak coupling region and the contribution of this mechanism to the resistance of the metal and calculate the size of the level splitting due to the magnetostriction mechanism. Estimates of the critical fields at which the effect is expected for heavy rare earth metals give $(0.3–0.5)\cdot 10^6$ Oe. Landau quantization not only fails to suppress this effect but under certain conditions can actually enhance it.

1. INTRODUCTION

Degeneracy of the magnetic states of a paramagnetic impurity in a normal metal gives rise to anomalies in the magnetic susceptibility, specific heat, and resistivity. In particular, the Kondo effect properly refers to a minimum in the temperature dependence of the resistivity of the normal metal (see, e.g., Refs. 1 and 2). Lifting the degeneracy usually lowers the energy of the system. In the present case the degeneracy can be lifted by spin–lattice interactions and the exchange interaction of the localized spin of the impurity with conduction electrons. The first are usually small because the spin–orbit interaction is small. It is therefore in the electronic properties that anomalies are observed. At zero temperature and in the absence of magnetic field the difference in the excitation energies of an electron with spin up ($\xi_k^{\uparrow}$) and an electron with spin down pulse a change in the projection of the impurity spin ($\xi_k^{\downarrow}+\Delta E$, $\Delta E=0$) is zero. Therefore, to calculate the spectrum one must use a perturbation theory for degenerate levels. However, there is degeneracy for a large number of electrons—the phase volume of such excitations is of the order of the volume of the Fermi surface; the problem has an essentially many-particle character, and it becomes difficult to write out the secular equation and find its solutions. The problem was recently solved with the aid of the Bethe ansatz.[1,2] If, however, a magnetic field H is applied to the system, the degeneracy is lifted. The spin-flip scattering of electrons is hindered and in a sufficiently large magnetic field the Kondo effect is suppressed. In this field region the interaction between conduction electrons and impurities is weak, and one can use ordinary perturbation theory; the leading corrections to the spin-electron scattering amplitude behave as $\propto A\,(Ag_0\ln(D/H))^n$.[3] Here A is the s-d exchange constant, g_0 is the density of states at the Fermi surface, and D is a cutoff parameter ($\sim\varepsilon_F$). In the case of ordered impurities the magnetic field is replaced in $\ln(D/H)$ and in the higher orders of perturbation theory by the considerably larger Weiss field $\sim cA^2g_0J$,[4] where J is the magnitude of the impurity spin and c is the impurity concentration. Further examination shows that, as in the pure ferromagnet, the electrons are no longer scattered by the spins of individual atoms but by spin waves, and the correction to the amplitude goes as $\ln(m_{sw}/m)$,[5] where m_{sw} is the spin-wave mass and m is the electron mass. Impurities only alter the shape of the spin-wave band without qualitatively changing the answer. Therefore, in an ordinary ferromagnet and especially in a ferromagnet in an external magnetic field the Kondo effect does not exist.

Another situation can complicate the picture in anisotropic ferromagnets. In a ferromagnet with one-ion magnetic anisotropy of the easy-axis type the spin-wave excitations are separated from the ground state by a gap which widens with increasing magnetic field. If in such a ferromagnet one of the atoms is replaced by an impurity with one–ion anisotropy of the easy-plane type, then, as was shown in Ref. 6 for the particular case of a Heisenberg magnet, local levels can appear in the gap of the spin-wave spectrum for a certain relationship between the one-ion anisotropy constants of the impurity and host atoms. It is clear that at low temperatures the main contribution to the thermodynamics and the resistivity of the ferromagnetic metal is due to the scattering of conduction electrons not by spin waves but by just these local levels. It is easy to see from the following rough arguments that it is possible for local levels to appear below the bottom of the spin-wave band. The simplest Hamiltonian that models this situation is

$$H=D_{imp}(J_0^z)^2-hg_{imp}\mu_B J_0^z-\sum_t\left[D_m(S_t^z)^2+hg_m\mu_B S_t^z\right]$$
$$+H_{ex}(J,S)+H_{ex}(S,S).$$

Here both anisotropy constants D are positive, h is the magnetic field, J and S are the angular momenta of the impurity and host ions, respectively, and g is the Lande factor. Then the spectrum of the lower spin-wave branch in the absence of the impurity is given by the expression

$$\omega_k=D_m(2S-1)+hg_m\mu_B+\alpha k^2,$$

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while the energy of the $(J-1, J)$ transition of the impurity is

$$\Delta E = -D_{imp}(2J-1) + hg_{imp}\mu_B + I(0)S, \quad (1)$$

where $I(0)S$ is the mean field of the host matrix. If $\Delta E < 0$ (as is entirely possible for rare earth metals), then the ground state of the system has a moment that is smaller than the maximum possible. A case exists, however, in which ΔE is negative in small magnetic fields but positive in large fields. In this case it can happen that

$$\Delta E < \omega_{k=0}.$$

Then, by lowering the magnetic field one can reach the point at which ΔE goes to zero. Actually, of course, the criterion for the onset of quasidegeneracy is not so strict, since in a more accurate calculation (see Ref. 6) the contribution of the ansverse interactions is subtracted from (1).

There is thus a region of fields in which the situation is similar to that in which the Kondo effect occurs. The difference is that in the ordinary Kondo effect the entire spin of the impurity "precesses freely," whereas here there is only one or several degenerate transitions, with the energy of two transitions not going to zero at the same value of the magnetic field.

In the present study we consider the situation described above in an *s–f* metal. Our main purpose is to call attention to the possibility of using a magnetic field to induce quasidegeneracy of the magnetic levels of an anisotropic impurity, with all the features typical of the Kondo effect.

2. THE MODEL. LOCAL LEVELS IN THE GAP OF THE SPIN-WAVE SPECTRUM

The arguments given in the Introduction imply that in order to realize the effect one needs materials having a sufficient large one-ion anisotropy to compete with the effective exchange interaction between magnetic ions. Further, it is necessary that the impurity taken from an easy-plane magnet retains its easy-plane properties upon substitution into an easy-axis magnet. These requirements can evidently be satisfied by embedding one rare earth element in the metallic matrix of another. As a matter of fact, the crystalline field in rare earth metals is $\sim 10^2$ cm^{-1}, and the magnetic ordering temperature is of the same order of magnitude. They have a one-ion magnetic anisotropy,[7] i.e., the level splitting of the magnetic ions is due mainly to the Coulomb interaction with neighboring ions. The impurity and host ions differ only in the number of f electrons, and both have charge $+3$. The lattice parameters of heavy rare earth metals differ only slightly; the dimension of the f shell is 0.5–1 Å, and the parameters of the hexagonal lattice in Er, for example, are $a \approx 3.56$ Å, $c \approx 5.59$ Å. The wave functions of the f electrons of neighboring ions have almost no overlap; nevertheless, one rare earth metal exhibits easy-axis anisotropy, the others easy-plane (see Sec. 6 for details). It is therefore understandable that the mechanism responsible for one-ion anisotropy of one type or the other is mainly of an intra-atomic nature: of the same Coulomb field

$$V(r) = \sum_t \frac{-Ze^2}{|r-R_t|},$$

created by the ions of neighboring sites R_t, the f electrons of ions of different rare earth metals "feel" only the part which is projected onto the basis of their own many-electron functions with a specified total angular momentum J:

$$V(r) \to \sum_{MM'} (J, M|V(r)|J, M')X^{MM'}, \quad X^{MM'} = |J, M)(J, M'|.$$

Owing to the large value of the spin–orbit interaction ($\sim 10^4$ cm^{-1}), the different J multiplets are well-separated from one another. The angular momentum J is determined by the number of f electrons. Thus, if the hexagonal symmetry of the lattice is preserved, the numbers $(J, M|V(r)|J, M')$ for f ions of different rare earth metals can differ in sign. For this reason it can be hoped that the rare earth impurity will retain "its own" anisotropy upon substitution into another rare earth metal.

The Hamiltonian describing this model of the rare earth metals, with the hexagonal symmetry taken into account, is of the form

$$H = H_f^0 + H_s^0 + H_{s-f},$$

$$H_f^0 = \sum_t \left\{ \sum_M \bar{E}_M p_t X_t^{MM} + \sum_m \bar{\varepsilon}_m Y_t^{mm}(1-p_t) \right\},$$

$$H_s^0 = \sum_{k\sigma} \tilde{\xi}_k{}^\sigma c_{k\sigma}{}^+ c_{k\sigma}, \qquad \tilde{\xi}_k{}^\sigma = \varepsilon_k - \mu - \frac{1}{2} g_s \mu_B h\eta(\sigma),$$

$$H_{s-f} = -\sum_{kqf\alpha\beta} e^{iqt}[Ap_t \mathbf{J}_t + B(1-p_t)\mathbf{S}_t]\sigma_{\alpha\beta} c_{k\alpha}{}^+ c_{k+q\beta}. \quad (2)$$

Here

$$\bar{E}_M = D_2^0 M^2 + D_4^0 M^4 + D_6^0 M^6 - g_{imp}\mu_B hM,$$

$$\bar{\varepsilon}_m = d_2^0 m^2 + d_4^0 m^4 + d_6^0 m^6 - g_M \mu_B hm,$$

D_{2n}^0 and d_{2n}^0 are the one-ion anisotropy constants of the impurity and host ion, respectively, in a hexagonal field (the quantization axis is chosen along the c axis of the crystal); p_t is the projection operator ($p_t^2 = p_t$), which is equal to one at a site occupied by an impurity atom and to zero at a site occupied by a host atom; J is the total angular-momentum operator of the impurity, S is that of the host, σ are the Pauli matrices, $c_{k\alpha}^+$ is the creation operator for a conduction electron with quasimomentum $\mathbf{k}$ and spin projection α. The g factors are assumed to be included in the s-f exchange constants: $A = A_1(g_{imp} - 1)$, $B = B_1(g_M - 1)$. We will be interested in the region of low concentration c. The s-f coupling constants are assumed small: $Ag_0, Bg_0 \ll 1$; g_0 is the density of states at the Fermi surface. In addition, we will consider the region of magnetic fields in which the impurity and the matrix of the heavy rare earth metal are already in a ferromagnetic state, i.e., the helical structures characteristic of heavy rare earth metals have been "squeezed out" by the magnetic field (see estimates below) and the x and y components of the mean angular momentum $\langle \mathbf{J}_t p_t \rangle$ of the impurity are equal

to zero. To evaluate the correlation functions of interest we use the cross technique,[8] averaging each term of the diagram series with the distribution function

$$\Phi(p_1, p_2, \ldots, p_N) = \prod_{t=1}^{N} (c\delta(1-p_t) + (1-c)\delta(p_t)), \qquad (3)$$

i.e., the impurities are assumed to be uncorrelated and uniformly distributed.

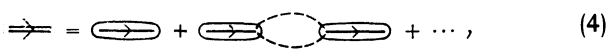
We note first of all that since we are considering the ferromagnetic state, the conduction electrons are magnetically biased, and we include the mean field created by these electrons in (2):

$$\bar{E}_M \to E_M = \bar{E}_M - AR_cM, \quad \bar{\varepsilon}_m \to \varepsilon_m = \bar{\varepsilon}_m - BR_cm,$$
$$\tilde{\xi}_{\mathbf{k}}^{\sigma} \to \xi_{\mathbf{k}}^{\sigma} = \tilde{\xi}_{\mathbf{k}}^{\sigma} - [(1-c)\langle S^z\rangle B + c\langle J^z\rangle A]\eta(\sigma).$$

Let us now consider the perturbation series for the correlation-function matrix of the transverse components of the angular momenta

$$F_{tt'}(\tau-\tau') = \begin{bmatrix} \langle T p_t J_t^+(\tau) p_{t'} J_{t'}^-(\tau')\rangle & \langle T p_t J_t^+(\tau)(1-p_{t'}) S_{t'}^-(\tau')\rangle \\ \langle T(1-p_t) S_t^+(\tau) p_{t'} J_{t'}^-(\tau')\rangle & \langle T(1-p_t) S_t^+(\tau)(1-p_{t'}) S_{t'}^-(\tau')\rangle \end{bmatrix}$$

with the goal in mind of finding the spectrum of magnetic excitations. In the presence of anisotropy it is convenient to use the diagram technique for the Hubbard operators. We cannot immediately write down the Larkin equations for F (see Ref. 5) before carrying out the average procedure. Averaging term-by-term, we see that the zeroth approximation here is the random phase approximation:



(4)

for the smallness of the concentration and coupling constants is compensated by large denominators near the transition energies $\omega \sim \Delta E_{M+1,M}$, $\Delta\varepsilon_{m+1,m}$. Here an oval enclosing an arrow and the dashed loop denote, respectively,

$$\begin{bmatrix} p_t K_t(i\nu) & 0 \\ 0 & (1-p_t)P_t(i\nu) \end{bmatrix}, \quad \begin{bmatrix} A^2 & AB \\ BA & B^2 \end{bmatrix} \chi(\mathbf{q}, i\nu), \qquad (5)$$

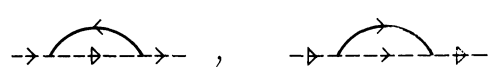
$$K_t(i\nu) = -\sum_{M=-J}^{J-1} \frac{|J^+_{M+1,M}|^2 \Delta N^{(J)}_{M+1,M}}{i\nu - \Delta E_{M,M+1}},$$

$$P_t(i\nu) = -\sum_{m=-S}^{S-1} \frac{|S^+_{m+1,m}|^2 \Delta N^{(S)}_{m+1,m}}{i\nu - \Delta\varepsilon_{m,m+1}},$$

$$\chi(\mathbf{q}, i\nu) = \frac{1}{N}\sum_{\mathbf{k}} \frac{n_{\mathbf{k}}^{\uparrow} - n_{\mathbf{k}+\mathbf{q}}^{\downarrow}}{\xi_{\mathbf{k}+\mathbf{q}}^{\downarrow} - \xi_{\mathbf{k}}^{\uparrow} - i\nu},$$

$$n_{\mathbf{k}}^{\sigma} = [\exp\{\beta\xi_{\mathbf{k}}^{\sigma}\}+1]^{-1}, \quad \Delta a_{ij} = a_i - a_j. \qquad (6)$$

Here N is the number of cells in the crystal. The spectrum of a pure anisotropic ferromagnet is analyzed in Ref. 9. Here the electron scattering by the impurities gives rise to additional terms in the vertex and in the mass operator of the electron Green function (the notation used below is analogous to that of Ref. 9). For example, the corrections to the mass operator from the transverse part of the interaction



a b

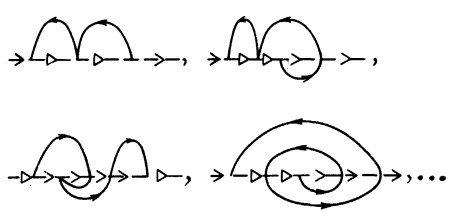
contain large logarithms of the form a:

$$\sum_M A^2\sigma \left[\ln\frac{\max\{\Delta E_{M+1,M}, \xi^{\uparrow}, T\}}{D} + i\pi \operatorname{sign}\xi^{\uparrow}\right] \times |J^+_{M+1,M}|^2 \Delta N^{(J)}_{M+1,M}$$

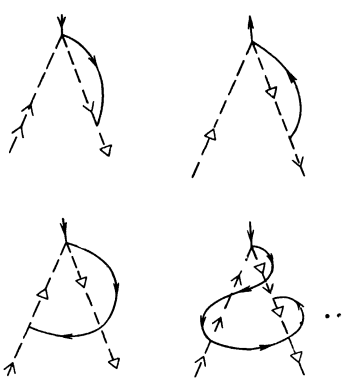
and analogously for b; D is the cutoff parameter ($\sim\varepsilon_F$). At sufficiently low temperatures only the lowest transition survives:

$$N_J = 1, \quad N_{J-1} = N_{J-2} = \ldots = N_{-J} = 0$$

and at small $\Delta E_{J-1,J} \equiv \Delta E$ the logarithm is compensated by the smallness of the coupling constant Ag_0. The corrections to the mass operator in the leading logarithmic approximation are shown below; for simplicity, the ovals are not shown—each solid line refers to one impurity:



The vertex function contain analogous contributions



The leading corrections are eliminated through the familiar (see, e.g., Ref. 4) parquet principle. We shall return to them in the next section; here we shall first consider the region of magnetic fields that are so large that the logarithmic correc-

tions are unimportant. Then there remains only the averaged series (4). Summing this series and performing an analytical continuation to the upper half of the ω plane, we get

$$F(\mathbf{q},\omega)=\frac{1}{\Delta(\mathbf{q},\omega)}$$

$$\times\begin{bmatrix}(1-P(1-c)B^2\chi)Kc & KcP(1-c)AB\chi\\ P(1-c)Kc\bar{B}A\chi & (1-KcA^2\chi)P(1-c)\end{bmatrix}$$

$$\Delta(\mathbf{q},\omega)=1-KcA^2\chi-P(1-c)B^2\chi. \tag{7}$$

Here the function $\chi(\mathbf{q},\omega)$, $K(\omega)$, and $P(\omega)$ are given by expressions (5) and (6) with the replacement $i\nu\to\omega+i\delta$. The zeros of function (7) determine the magnetic-excitation spectrum of the system. At low temperatures ($T\to 0$)

$$P(\omega)=\frac{-2S}{\omega-\Delta\varepsilon_{S-1,S}+i\delta},\quad K(\omega)=\frac{-2J}{\omega-\Delta E_{J-1,J}+i\delta}.$$

The mean-field separation of the electron spin subbands $\xi_k{}^{\downarrow}-\xi_k{}^{\uparrow}=\Delta_c$ is much larger than the energy $\omega_{\mathbf{q}}$ of the low-lying spin-wave branches, and we therefore replace $\chi(\mathbf{q},\omega)$ by $\chi(\mathbf{q},0)$. We thereupon obtain the spectrum of the lower branches:

$$\omega_{1,2}(\mathbf{q})=\{\omega_0(\mathbf{q})+\Delta E+c\chi(\mathbf{q},0)\lambda^2\pm\nu_{\mathbf{q}}\}/2,$$

$$\nu_{\mathbf{q}}=\{[\omega_0(\mathbf{q})-\Delta E+c\chi(\mathbf{q},0)\lambda^2]^2+8A^2\chi(\mathbf{q},0)Jc(\Delta\varepsilon-\Delta E)\}^{1/2}, \tag{8}$$

where $\omega_0(\mathbf{q})$ is the spectrum of the lower spin-wave branch in the ferromagnet without the impurities,

$$\omega_0(\mathbf{q})=\Delta\varepsilon-2SB^2\chi(\mathbf{q},0),\quad \lambda^2=2B^2S-2A^2J$$

and for brevity we have dropped the transitions subscripts of $\Delta\varepsilon$ and ΔE. It follows from (8) that in the region of fields and concentrations where

$$\Delta E<2cg_0A^2J[1+2A^2g_0S/\Delta_{sw}], \tag{9}$$

the purely ferromagnetic state is unstable. Here the s–f exchange constants have (for purposes of estimation) been taken equal ($A=B$), and $\Delta_{sw}=\omega_0(\mathbf{q}=0)$ is the gap in the spin-wave spectrum. This instability is analogous to that found in a Heisenberg magnet.[6] Usually, however, $\omega_0(\mathbf{q})\gg\Delta E$ (see estimates below). Therefore, in the region of low concentrations, and specifically for

$$c\ll\Delta_{sw}/2A^2g_0S \tag{9a}$$

the spectrum is of the form

$$\omega_1(\mathbf{q})=\omega_0(\mathbf{q})+c\cdot 2B^2\chi(\mathbf{q},0)S[1+2A^2g_0J\gamma_{\mathbf{q}}],$$
$$\omega_2(\mathbf{q})=\Delta E-c\cdot 2A^2\chi(\mathbf{q},0)J[1+2B^2g_0S\gamma_{\mathbf{q}}],$$
$$\gamma_{\mathbf{q}}=\chi(\mathbf{q},0)/[(\omega_0(\mathbf{q})-\Delta E)g_0]. \tag{10}$$

The excitation spectrum is this region thus consists of the nearly unchanged spin waves of the ferromagnetic matrix plus quasilocal excitations of the impurity. At fields and concentrations where

$$\Delta E\gg 2cA^2g_0J, \tag{11}$$

the dispersion in (10) can be neglected. It is clear that at low concentrations there is an interval of magnetic fields in which

$$0<\Delta E<\Delta_{sw},$$

i.e., quasilocal impurity levels can exist below the bottom of the spin-wave band. Thus, the region of system parameters in which the concept of quasilocal levels and our treatment of the problem remain valid is given by inequalities (9) and (11).

Let us now discuss the role of these levels in the region of weaker magnetic fields, where the logarithmic corrections become important.

3. AMPLITUDE FOR THE SCATTERING OF ELECTRONS BY LOCAL LEVELS

The temperature region of interest in this section is $T<\Delta_{sw}$. In this region the spin waves give a small contribution to the scattering processes. In particular, the contribution to the resistance is proportional to $T^2\exp\{-\Delta_{sw}/T\}$.[10] We can therefore simplify the model further and consider the scattering of electrons by only the two lowest local levels. The Hamiltonian of this subsystem is

$$H=H_s{}^0+\sum_{\mathbf{f}}(E_JX_{\mathbf{f}}{}^{JJ}+E_{J-1}X_{\mathbf{f}}{}^{J-1,J-1})p_{\mathbf{f}}$$
$$-\frac{A}{N}\sum_{\mathbf{kfq}}\sum_{\alpha\beta}e^{i\mathbf{qf}}p_{\mathbf{f}}\Big[\Big(JX_{\mathbf{f}}{}^{JJ}+(J-1)X_{\mathbf{f}}{}^{J-1,J-1}\Big)\sigma_{\alpha\beta}{}^z$$
$$+\frac{(2J)^{1/2}}{2}(X_{\mathbf{f}}{}^{J,J-1}\sigma_{\alpha\beta}{}^-+X_{\mathbf{f}}{}^{J-1,J}\sigma_{\alpha\beta}{}^+)\Big]c_{\mathbf{k}\alpha}{}^+c_{\mathbf{k}+\mathbf{q}\beta}. \tag{12}$$

Let us now introduce the pseudospin-1/2 operators, requiring that

$$X_{\mathbf{f}}{}^{JJ}+X_{\mathbf{f}}{}^{J-1,J-1}=1,$$
$$X_{\mathbf{f}}{}^{JJ}=\frac{1}{2}+s_{\mathbf{f}}{}^z,\qquad X_{\mathbf{f}}{}^{J-1,J-1}=\frac{1}{2}-s_{\mathbf{f}}{}^z,$$
$$X_{\mathbf{f}}{}^{J-1,J}=s_{\mathbf{f}}{}^-,\qquad X_{\mathbf{f}}{}^{J,J-1}=s_{\mathbf{f}}{}^+.$$

We now write the s pseudospin operators in terms of the fermion operators d_γ:

$$s=s_{\gamma\delta}d_\gamma{}^+d_\delta.$$

The Hamiltonian (12) is then

$$H=\sum_{\mathbf{k}\sigma}\xi_{\mathbf{k}}{}^{\sigma}c_{\mathbf{k}\sigma}{}^+c_{\mathbf{k}\sigma}-\Delta E\sum_{\mathbf{f}}s_{\mathbf{f}}{}^zp_{\mathbf{f}}$$
$$-\frac{1}{N}\sum_{\mathbf{kqf}}\sum_{\alpha\beta}e^{i\mathbf{qf}}p_{\mathbf{f}}\Big[A_{\parallel}s_{\mathbf{f}}{}^z\sigma_{\alpha\beta}{}^z+\frac{1}{2}A_{\perp}(s_{\mathbf{f}}{}^+\bar{\sigma}_{\alpha\beta}+s_{\mathbf{f}}{}^-\sigma_{\alpha\beta}{}^+)\Big]$$
$$c_{\mathbf{k}\alpha}{}^+c_{\mathbf{k}+\mathbf{q}\beta}, \tag{13}$$

where

$$\xi_k^{\sigma}=\tilde{\xi}_k^{\sigma}-Ac\left(J-\frac{1}{2}\right)\eta(\sigma), \qquad A_{\parallel}=A, \qquad A_{\perp}=\sqrt{2J}A$$

and we have used the relation

$$\frac{1}{N}\sum_t p_t \exp\{i\mathbf{fq}\}=c\delta(\mathbf{q}).$$

In this paper we shall consider only the weak coupling region. Then by writing the Hamiltonian in the form (13) we can use the calculations of Abrikosov[3] for our case. The only differences is that Abrikosov considered an iostropic exchange interaction, whereas our effective Hamiltonian (13) is anisotropic. The anisotropic case with $A_{\parallel}>A_{\perp}$ is solved in Ref. 11. For our case, $A_{\parallel}<A_{\perp}$, the exact solution unfortunately remains unknown.[1,12] As we have said, the evaluation of the spin–electron scattering amplitude Γ in the leading logarithmic approximation coincides almost completely with the calculation of Ref. 3 and leads to the following parquet equations:

$$\Gamma_{\alpha\alpha'}^{\beta\beta'}=\Gamma_{\alpha\alpha'}^{(0)\beta\beta'}+\int_{\max(\Delta E,\omega)}^{D}\frac{d\omega_1}{\omega_1}\left[\Gamma_{\alpha\alpha''}^{\beta\beta''}(\omega_1)\Gamma_{\alpha''\alpha'}^{\beta''\beta'}(\omega_1)-\Gamma_{\alpha\alpha''}^{\beta''\beta'}(\omega_1)\Gamma_{\alpha''\alpha'}^{\beta\beta''}(\omega_1)\right],$$

$$\Gamma_{\alpha\alpha'}^{(0)\beta\beta'}=\frac{1}{N}\{A_{\parallel}\sigma_{\alpha\alpha'}^{z}(s^z)^{\beta\beta'}+{}^1/_2A_{\perp}[\sigma_{\alpha\alpha'}^{+}(s^-)^{\beta\beta'}+\sigma_{\alpha\alpha'}^{-}(s^+)^{\beta\beta'}]\}.$$

It is easy to see that the substitution

$$\Gamma_{\alpha\alpha'}^{\beta\beta'}=\Gamma_0\delta_{\alpha\alpha'}\delta^{\beta\beta'}+\Gamma_{\parallel}\sigma_{\alpha\alpha'}^{z}(s^z)^{\beta\beta'}+\frac{1}{2}\Gamma_{\perp}[\sigma_{\alpha\alpha'}^{+}(s^-)^{\beta\beta'}+\sigma_{\alpha\alpha'}^{-}(s^+)^{\beta\beta'}]$$

reduces the system to

$$\Gamma_0=0, \qquad \frac{d\Gamma_{\parallel}}{dx}=-\Gamma_{\perp}^2, \qquad \frac{d\Gamma_{\perp}}{dx}=-\Gamma_{\perp}\Gamma_{\parallel} \tag{14}$$

with boundary conditions

$$\Gamma_{\parallel}(|\omega|=D)=A_{\parallel}g_0, \ \Gamma_{\perp}(|\omega|=D)=A_{\perp}g_0;$$

$$x=\ln\{D/\max\{\Delta E,\ \omega\}\}.$$

From (14) we find

$$\Gamma_{\parallel}=A_{\perp}g_0\gamma_1 \operatorname{ctg}\{\arcsin\gamma_1+A_{\perp}\gamma_1g_0x\},$$
$$\Gamma_{\perp}=A_{\perp}g_0\gamma_1\sin^{-1}\{\arcsin\gamma_1+A_{\perp}\gamma_1g_0x\},$$
$$\gamma_1=[1-A_{\parallel}^2/A_{\perp}^2]^{1/2}. \tag{15}$$

In the formal limit $A_{\perp}\to A_{\parallel}$, one has $\gamma_1\to 0$, and Eqs. (15) go over to Abrikosov's solution.[3] The value of max $\{T, \Delta E\}$ is conveniently written as $(T^2+\Delta E)^{1/2}$ (this is admissible in the logarithmic approximation). Then from solution (15) we have the following lucid expression for the temperature T_K at which the amplitudes (15) have a Kondo singularity:

$$T=T_K=\left[D^2\exp\left\{-2\frac{\arcsin\gamma_1}{A_{\perp}\gamma_1g_0}\right\}-\Delta E^2\right]^{1/2}. \tag{16}$$

In the case under discussion we have $\gamma_1=(1-1/2J)^{1/2}$. If the angular momentum of the impurity is large, then

$$\gamma_1\approx 1-\frac{1}{4J}, \quad \arcsin\gamma_1=\frac{\pi}{2}\pm\frac{1}{8J}.$$

Equations (14) for the amplitudes can also be obtained from renormalization-group arguments,[14] by analogy with Refs. 15 and 16. These equations have the integral of motion

$$\gamma^2=\Gamma_{\perp}^2-\Gamma_{\parallel}^2, \ \gamma^2=A^2g_0^2\gamma_1^2 \tag{17}$$

for $A_{\perp}^2>A_{\parallel}^2$ and $\gamma^2=\Gamma_{\parallel}^2-\Gamma_{\perp}^2$ for $A_{\perp}^2<A_{\parallel}^2$. The behavior of the invariant charges according to (14), i.e., as obtained from the first-order RG corrections, is shown in Fig. 1. From the form of the trajectories [and from Eqs. (15)] we find that the result of this approximation can be used only in the region where the temperatures and fields are so high that the amplitudes are far from the poles. We could attempt to improve the calculation by evaluating the Gell-Mann–Low function in the next order of perturbation theory. The equations are of the form[17]

$$\frac{d\Gamma_{\parallel}}{dx}=-\Gamma_{\perp}^2-\frac{1}{2}\Gamma_{\parallel}\Gamma_{\perp}^2, \quad \frac{d\Gamma_{\perp}}{dx}=-\Gamma_{\parallel}\Gamma_{\perp}-\frac{1}{2}\Gamma_{\perp}\Gamma_{\parallel}^2. \tag{18}$$

It is easy to see that the integral of motion (17) has been conserved; therefore, allowance for the next order has not fundamentally changed anything—the amplitudes as before grow with decreasing magnetic field or temperature. The invariant charges in powers of which the physical quantities are expanded in the weak-coupling region are given in this approximation by the equations:

$$\Gamma_{\perp}^2=\gamma^2+\Gamma_{\parallel}^2, \ F(\Gamma_{\parallel})=F(A_{\parallel}g_0)-x(\gamma^2+4),$$
$$F(\Gamma)=\ln\frac{(1+\Gamma/2)^2}{\gamma^2+\Gamma^2}+\frac{4}{|\gamma|}\operatorname{arctg}\frac{\Gamma}{|\gamma|}. \tag{19}$$

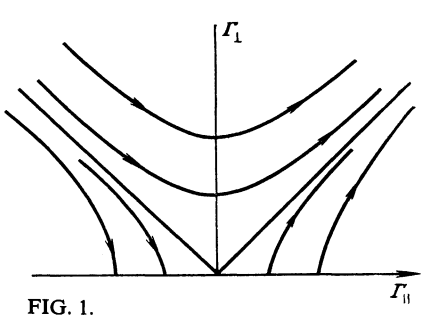
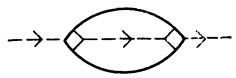


FIG. 1.

In Ref. 17 the solution of Eqs. (18) is given in somewhat different form—in terms of the isotropic (at $A_\parallel = A_\perp$) charge.

4. CONTRIBUTION OF KONDO SCATTERING TO THE RESISTIVITY

To evaluate the physical characteristics of the system we must know the electron Green function. The procedure for finding this function reduces to evaluating the contribution



to the mass operator. This contribution was evaluated for the isotropic case in Ref. 3, and so the answer in our case can be immediately given as

$$\Sigma(\omega) = -i \operatorname{sign} \omega \frac{c}{8g_0} [\Gamma_\parallel^2(\omega) + 2\Gamma_\perp^2(\omega)]. \tag{20}$$

If we use the "parquet" solution (15), which is less accurate than (19), the answer can be written in analytical form:

$$\Sigma(\omega) = -i \operatorname{sign} \omega \frac{A^2 g_0 c}{8} \times \left[3 \sin^{-2} \left\{ \arcsin \gamma_1 + A_\perp \gamma_1 \ln \frac{D}{\max\{\omega, T, \Delta E\}} \right\} - 1 \right]. \tag{21}$$

Let us now consider the static part of the resistance. Because the one-impurity scattering potential is independent of the momentum there are no corrections to the current vertex, and the transport relaxation time is equal to the electron lifetime

$$\tau^{-1}(\omega) = 2 \operatorname{Im} \Sigma(\omega). \tag{22}$$

In this case we have the well-known formulas[3] for the current $\mathbf{j}$ and conductivity σ:

$$\mathbf{j} = \frac{i\sigma \nu \mathbf{A}}{c}, \quad \sigma = \frac{ne^2}{m} \int_0^\infty \frac{d\omega \tau(\omega)}{2T \operatorname{ch}^2(\omega/2T)}. \tag{23}$$

Here n is the electron density and ν is the frequency. In the parquet approximation the exchange scattering of electrons by the impurity spin transition $(J-1, J)$ gives the following contribution to the conductivity:

$$\rho_{ex} = c \frac{A(2J)^{1/2} g_0 m}{4ne^2} \times \left[3 \sin^{-2} \left\{ \arcsin \gamma_1 - A g_0 \gamma_1 \ln \frac{D}{(T^2 + \Delta E^2)^{1/2}} \right\} - 1 \right]. \tag{24}$$

We recall that for $J = 1/2$ the one-ion anisotropy is absent altogether and there is accordingly nothing to discuss. If the more exact expressions for the amplitudes from (19) are used, the integration of expressions (2) and (22) in (23) reduces in the approximation to the replacement $\omega \to 2T$, but since (19) is transcendental in this case we cannot write down an analytical expression for j.

5. THE ROLE OF MAGNETOSTRICTION

It was mentioned in the Introduction that a second mechanism which can lift the magnetic-field-induced degeneracy of the impurity levels is magnetostriction. In the rare earths the magnetostriction is mainly due to a one-ion mechanism.[7] In our case this is the Jahn-Teller mechanism: the displacement of the equilibrium position of nearest ligands alters the symmetry of the Coulomb field acting on the impurity ion. In materials based on transition elements the spin-orbit coupling is weaker than the crystal field, and the deformation mixes the orbital parts of the wave functions. In rare earth metals the spin-orbit coupling is 10^2 times stronger than the crystal field, and the magnetic moment of the ion is compelled to follow the variation of the orbital angular momentum, and therefore the crystal field and the interaction of the impurity ion with Jahn–Teller distortions of the environment are written in terms of the total angular momentum of the ion. Thus, in our case the ordinary Jahn–Teller effect is magnetostrictive; in addition, the quasidegeneracy of the impurity levels is induced by an external magnetic field. If the splitting is sufficiently large, specifically,

$$\Delta E > D \exp\{-\arcsin \gamma_1 / A_\perp g_0 \gamma_1\}, \tag{25}$$

then the Kondo effect will be suppressed. At the present time we have a rigorous theory of the formation of elastic forces by interacting electrons and ions only for simple metals such as tin.[18] Therefore, we shall attempt to estimate the role of magnetostriction in the framework of a simple model of local deformations of the crystal field. We first assume that condition (25) holds, and we evaluate the magnetostrictive splitting ΔE_{ms} without allowance for the Kondo effect. Then, by decreasing the external magnetic field, we determine the minimum possible value of ΔE_{ms} allowed by the magnetostriction. Our calculation follows Ref. 19, a study of magnetostriction phenomena in regular magnets. The Hamiltonian for strains interacting with an impurity, with allowance for the hexagonal symmetry of the lattice, is of the form

$$\begin{aligned} H = {}& {}^1/_2 C_{11}(u_{xx}^2 + u_{yy}^2) + {}^1/_2 C_{33} u_{zz}^2 + C_{12} u_{xx} u_{yy} + C_{13} u_{zz}(u_{xx} + u_{yy}) \\ & + 2C_{44}(u_{xz}^2 + u_{yz}^2) + (C_{11} - C_{12}) u_{xy}^2 \\ & + G_1[u_{xx}(J_t^x)^2 + u_{yy}(J_t^y)^2 + u_{xy}(J_t^x J_t^y + J_t^y J_t^x)] \\ & + G_2[u_{yz}(J_t^y J_t^z + J_t^z J_t^y) + u_{xz}(J_t^x J_t^z + J_t^z J_t^x)] \\ & + G_3 u_{zz}(J_t^z)^2 + G_4(u_{xx} + u_{yy})(J_t^z)^2. \end{aligned} \tag{26}$$

Here $u_{\alpha\beta}$ is the strain tensor, and $C_{pq} = C_{\alpha\beta\gamma\delta}$ is the elastic-constant tensor in the Voigt notation.[20] We assume thus that an impurity ion with the same charge as the hot ions does not perturb the elastic constants. The mass difference and the exchange and electronic magnetostrictions will be neglected, since they cannot give a larger effect than the one-ion mechanism (see Ref. 21).

We rewrite the interaction Hamiltonian in (26) in the Hubbard-operator representation

$$H_{me}^{(imp)}=\sum_M\Big\{\frac{1}{2}G_1(u_{xx}+u_{yy})\,[J(J+1)-M^2]$$
$$+[G_3u_{zz}+G_4(u_{xx}+u_{yy})\,]M^2\}X_f^{MM}$$
$$+\sum_{MM'}\frac{1}{2}G_2(M+M')\,(u_z^+J_{M'M}^-+u_z^-J_{M'M}^+)X_f^{MM'},$$
$$u_z^\pm=u_{xz}\pm iu_{yz}. \qquad (27)$$

We are interested in the renormalization of only the lowest quasidoublet. Keeping only these levels in (27), we find the one-site Hamiltonian of the impurity:

$$H_{0f}^{(imp)}=E_J{}^0X_f^{JJ}+E_{J-1}^0X_f^{J-1,J-1}+\lambda_1u_z^+X_f^{J-1,J}+\lambda_1u_z^-X_f^{J,J-1}, \qquad (28)$$

where

$$E_M{}^0=E_M+\frac{1}{2}G_1(u_{xx}+u_{yy})\,(J(J+1)-M^2)$$
$$+(G_3u_{zz}+G_4(u_{xx}+u_{yy}))M^2,$$
$$M=J,J-1;\quad \lambda_1{}^2=\frac{1}{2}G_2J(2J-1)^2.$$

The introduction of new Hubbard operators

$$Z^{\mu\nu}=|\mu\rangle\langle\nu|$$

on the eigenstates of Hamiltonian (28)

$$|\mu\rangle=|1\rangle,|2\rangle;\quad |1\rangle=\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{pmatrix},\quad |2\rangle=\begin{pmatrix}-\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\end{pmatrix}, \qquad (29)$$

$$\cos\frac{\theta}{2}=\Big(\frac{\eta+1}{2\eta}\Big)^{1/2},\quad \eta^2=1+\frac{4\lambda_1{}^2R^2}{\Delta E_0{}^2},\quad R^2=|u_z^+|^2,$$

$$\Delta E_0=|E_{J-1}-E_J{}^0|$$
$$=|\Delta E+(2J-1)\,[\,(1/2G_1-G_4)\,(u_{xx}+u_{yy})-G_3u_{zz}]\,|,$$

allows us to write (28) in diagonal form:

$$H_{0f}^{(imp)}=\frac{\Delta E_0}{2}\eta(Z_f^{22}-Z_f^{11}). \qquad (30)$$

Here the operators X and Z are related by the formulas:

$$X_f^{JJ}=\cos^2\frac{\theta}{2}\cdot Z_f^{11}-\frac{1}{2}\sin\theta\cdot(Z_f^{12}+Z_f^{21})+\sin^2\frac{\theta}{2}\cdot Z_f^{22},$$
$$X_f^{J,J-1}=\frac{1}{2}\sin\theta\cdot(Z_f^{11}-Z_f^{22})+\cos^2\frac{\theta}{2}\cdot Z_f^{12}-\sin^2\frac{\theta}{2}\cdot Z_f^{21}. \qquad (29')$$

By differentiating the effective Hamiltonian of the strains (which we assume to be classical)

$$H_{eff}(u_{\alpha\beta})=-T\ln \mathrm{Sp}_M\exp\{-\beta H^{(imp)}\} \qquad (31)$$

with respect to $u_{\alpha\beta}$, we find the equations for the equilibrium displacements:

$$C_{11}u_{xx}+C_{12}u_{yy}+C_{13}u_{zz}+(2J-1)\,(G_4-1/2G_1)\langle X^{JJ}\rangle_M=-A_1, \qquad (32a)$$

$$A_1=1/2G_1(3J-1)+G_4(J-1)^2;$$
$$2C_{13}u_{xx}+C_{33}u_{zz}+(2J-1)\,G_3\langle X^{JJ}\rangle_M=-G_3(J-1)^2, \qquad (32b)$$
$$2C_{44}u_z^-+\lambda_1\langle X^{J-1,\,J}\rangle_M=0. \qquad (32c)$$

The averaging in (31) and (32) is only over the magnetic states, as is indicated by the subscript M. It follows from (29) and (30) that the splitting ΔE_{ms} is governed mainly by the quantity R. An equation for R can be found by substituting (29′) into (32c) and averaging with Hamiltonian (30):

$$u_z^-=-\frac{\lambda_1}{4C_{44}}\sin\theta\cdot\langle Z^{11}-Z^{22}\rangle_M. \qquad (33)$$

Here we have used the axial symmetry of the hexagonal crystal and set $R=u_{xz}$. It obviously follows from (30) that for $T=0$ the most favorable state is realized for $\langle Z^{11}\rangle_M=1$ and at the maximum rhombohedral distortion R. The latter is easily found from (29) and (33)

$$R_{max}=\pm\frac{\Delta E_0}{2\lambda_1}(\varkappa-1)^{-1/2},\quad \varkappa=\lambda^2/(8C_{44}\Delta E_0). \qquad (34)$$

Then the maximum level splitting due to magnetostriction is

$$\Delta E_{ms}=\frac{\Delta E_0}{2}\Big(\frac{\varkappa}{\varkappa-1}\Big)^{1/2}. \qquad (35)$$

Expressions (34) and (35) give somewhat exaggerated values for the distortion and the splitting, since in our naive calculation we ignored the fact that the displacement of the nearest neighbors of an impurity is unfavorable for the next-nearest neighbors, since the spin-wave spectrum of the host matrix has a large energy gap. Such an estimate is the most pessimistic for the realizability of the Kondo effect. For estimates we take the parameters for erbium: $C_{44}\approx 5\cdot 10^4\ \mathrm{cm}^{-1}$ (Ref. 10), $\approx 3\cdot 10^4\ \mathrm{cm}^{-2}$ (Ref. 19), $R\approx 10^{-3}$ (corresponding to the characteristic values of the giant magnetostriction), and $J=15/2$. Then $\Delta E_{ms}\approx 1$–$3\ \mathrm{cm}^{-1}$. This estimate cannot be considered totally reliable since the necessary experiments for determining the magnetoelastic coupling constants in pure rare earth metals have not been done.

6. ESTIMATES OF THE CRITICAL FIELDS

Let us ascertain the values of the magnetic fields at which the impurity levels would intersect in the absence of conduction electrons when an impurity of one rare earth metal is substituted into the matrix of another. The resonant

impurity could be Tb, Dy, or Ho and the matrix could be Er or Tm. The value of the field depends to a large extent on the anisotropy constants of the particular impurity. The crystal-field parameters A_{2n}^0, which determine the anisotropy constants D_{2n}^0 (see Ref. 22), were calculated in Ref. 23 with allowance for only the nearest neighbors, while in Ref. 22 the contributions of the next coordination spheres were also taken into account. The results of these calculations are given in Table I. It is difficult to give preference to one or the other set of constants since the screening radius of the point charges of the rare earth ions are unknown. Following Ref. 22, we take the effective exchange to be the same for all rare earth metals. This is reasonable since the paramagnetic Curie temperature of rare earth metals is described well by the de Gennes function

$$T_c \sim \tilde{A}^2 g_0 G, \quad G=(g_f-1)^2 J(J+1).$$

More exact estimates of the exchange parameter would be pointless in view of the uncertainty in the anisotropy constants. The latter are also taken from Ref. 22. Thus $\tilde{A}^2 g_0 = 43.05\ \text{cm}^{-1}$, and the anisotropy constants and g factors are given in Table II. Supppose we substitute the Tb ion into the Tm matrix. In the ferromagnetic phase the impurity levels are shifted with respect to their position in the paramagnetic phase:

$$E_M = E_M^{\text{pm}} - \tilde{A}\tilde{B} g_0 \langle S^z \rangle (g^{\text{Tb}}-1)(g^{\text{Tm}}-1) M - g_f^{\text{Tb}} \mu_B h M;$$

here E_M^{pm} is the energy of the level with projection M in the paramagnetic phase. In a sufficiently strong magnetic field the Tb level with $M = J = 6$ will be the ground state, but as the field is decreased the levels $E_{M=J}$ and $E_{M=J-1}$ cross at the point $h \approx 590$ kOe. An analogous estimate for Dy substituted into Tm gives $h \approx 370$ kOe. Measurements of the electrical conductivity in fields of such a size involve considerable experimental difficulties. A further decrease in the field, however, leads to the crossing of levels with other projections—levels which are lower lying in these fields. We have not considered this situation, but apparently the picture is qualitatively the same. Then the effect could be observed in weaker fields. For example, in the case of Tb in Tm the crossing of the $M = 3$ and $M = 4$ levels occurs in a field $h \approx 132$ kOe. The helical structure in such fields is still "squeezed out" in rare earth ferromagnets.

7. THE INFLUENCE OF LANDAU QUANTIZATION

The estimates made in the previous section show that the magnetic field in which the effect is expected to occur is extremely large, in the range 100–500 kOe. If the effective mass m^* of the conduction electrons is of the order of the free electron mass m, the cyclotron frequency of the conduction electrons is ~ 10–$10^2\ \text{cm}^{-1}$. It is therefore clear that Landau quantization is important for electrons with $m^* \lesssim m$. In large fields the cyclotron radius becomes much greater than the distance between impurities [which are assumed to have a low concentration obeying inequality (9a)], and a fraction of the electrons do not "notice" the impurities in the transverse direction, since their wave functions do not overlap. This should lead to a decrease in the Kondo amplitude. On the other hand, the motion of the electrons becomes quasi-one-dimensional, a situation that usually enhances scattering effects. In order to reach an understanding of the direction of the net change in the magnitude of the effect, we need only evaluate the elementary graphs of type (a) or (b) that make up the whole parquet graph.

Let us evaluate, for example, an element of type (a). We write the "bare" electron Green function in the momentum representation in the Landau gauge (see Ref. 24)

$$G_{\mathbf{pp}'}^{\sigma}(i\omega) = \int d\pi_x\, d\pi_z \sum_{n=0}^{\infty} \frac{\Phi_{n\pi_x\pi_z}(\mathbf{p})\,\Phi_{n\pi_x\pi_z}(\mathbf{p}')}{i\omega - \xi_n^{\sigma}(p_z)}, \tag{36}$$

where Φ denotes the wave functions of the electron in the Landau representation:

$$\Phi_{n\pi_x\pi_z}(\mathbf{p}) = l^{1/2} e^{il^2 p_x p_z} \psi_n(p_y l)\,\delta(\pi_x - p_x)\,\delta(\pi_z - p_z),$$

$$l^2 = \frac{c}{eB}, \quad \xi_n^{\sigma}(p_z) = \varepsilon_n^{\sigma}(p_z) - \mu,$$

$$\varepsilon_n^{\sigma}(p_z) = \frac{p_z^2}{2m^*} - \left(n+\frac{1}{2}\right)\omega_c - \frac{1}{2}\eta(\sigma)\,(\mu_B g h + 2B\langle S^z\rangle),$$

$\omega_c = eB/mc$ is the cyclotron frequency, B is the magnetic induction,

$$\psi_n(\xi) = e^{-\xi^2/2} H_n(\xi),$$

is the Hermite function (normalized to unity), and $H_n(\xi)$ is a Hermite polynomial. Green function (36) now depends on pairs of momenta, since momentum is not conserved in the direction transverse to the field. Substituting function (36) into (a) and integrating over all the intermediate momenta (the rather unwieldly calculation is not shown), we obtain the following answer at $T = 0$:

TABLE I. Crystal-field constants (in cm^{-1})

	Ref. 23	Ref. 22
A_2^0	−150	−240
A_4^0	−45	94
A_6^0	43	133

TABLE II. Anisotropy constants (in cm^{-1})

	D_2^0	D_4^0	D_6^0	f / g
Tb	0,405	0,211	−2,67	$^3/_2$
Dy	6,96	−0,19	1,97	$^4/_3$
Ho	−3,25	0,219	−2,45	$^5/_4$
Er	4,62	−0,31	3,87	$^6/_5$
Tm	−13,54	0,64	−10,5	$^7/_6$

$$\sum_{n_1}\frac{\psi_{n_1}(k_y l)\psi_{n_1}^{*}(k_y' l)}{(2\pi)^3(\omega-\xi_{n_1}{}^{\uparrow}(k_z)+i\delta)^2}\delta(k_x-k_x')\delta(k_z-k_z')e^{il^2k_x(k_y-k_y')}$$

$$\times\Big\{A^2\sum_{n_2}\Big(\frac{m}{2}\Big)^{1/2}$$

$$\times[\xi_{n_1}{}^{\uparrow}(k_z)+\Delta E-\xi_{n_2}{}^{\downarrow}(0)]^{-1/2}\ln\frac{\xi_n{}^{\uparrow}(k_z)+\Delta E}{\quad}\Big\}. \qquad (37)$$

The expression in braces is the correction to the mass operator, showing that the logarithmic correction has been preserved. We note, however, that a new effect has appeared: for

$$\omega_c(n_1-n_2)+\frac{k_z^2}{2m}=\Delta_c+\mu_B g h-\Delta E \qquad (38)$$

the correction diverges as a square root. Here we have only set out to verify that the Landau quantization does not suppress the Kondo effect. The features which appear in the electron subsystem due to fulfillment of resonance condition (38) require separate study. We note only that, as can be seen from (37), there can exist a magnetic field in which the logarithmic and square-root singularities in (37) add together: the inequality

$$\Delta E+\xi_n{}^{\uparrow}(k_z)=0$$

can be satisfied by an appropriate choice of the magnitude of the quasimomentum, i.e., at any value of the magnetic field (but for sufficiently small ΔE), while the equation

$$\xi_{n_2}{}^{\downarrow}(0)=0$$

can be satisfied by choosing the magnetic field. We thus see that at a certain value of the field the conduction electrons that have quantized orbital motion can enhance the Kondo singularity.

CONCLUSION

Let us discuss these results. The Kondo effect is due to the presence in the crystal of closely lying local levels which scatter anomalously strong by the conduction electrons. In our case we propose to create the local levels by introducing easy-plane impurities into an easy-axis ferromagnet. In a Heisenberg ferromagnet the local levels can be created by introducing a small number of negative exchange bonds. This leads[26] to the formation of a bound state of the impurity and a spin wave, to the appearance of an oscillatory effective interaction between impurities (through the spin waves), and, as a result, to low-temperature effects of the "glass" type. The question of such effects exists both in Heisenberg ferromagnets and in metallic ferromagnets upon formation of local levels due to anisotropy. For the symmetry consideration here, the z projection of the total angular momentum is conserved; this also requires the formation of a bound state in the case when the lowest state of the impurity in the crystalline field without the transverse exchange interaction is $|M\rangle$, with $M<J$ (a situation that we have not considered in this paper). However in a metallic ferromagnet the electrons can not only create an oscillatory interaction but can also screen the impurity. Therefore, in the case $M<J$ it is not clear in advance which type of manifestation—Kondo or glass—will be predominant.

Further, in our model we have ignored the difference between the volumes of the impurity host ions, and this can affect the degree to which the impurity is of the easy-plane type. We believe that allowance for this circumstance can only shift somewhat the parameter region in which the local levels exist without fundamentally altering the basic picture.

Finally, we note that the square-root singularity which arises when Landau quantization is taken into account is due to the quadratic dependence of the electron energy on the z projection of the momentum. For example, for a dispersion relation of the form

$$\xi_n{}^{\sigma}(p_z)=\omega_c(n+{}^1/_2)-{}^1/_2\eta(\sigma)(\mu_B g h+\Delta_c)+v_F(p_z-p_z{}^0) \qquad (39)$$

this singularity does not arise—only the Kondo singularity remains. This means that for spectrum (39) the results of Secs. 3 and 4 will apparently remain unchanged. Therefore, the specific geometry of the Fermi surface and the relation position of the c axis of the crystal and the direction of the external magnetic field take on a special significance. At the same time, this fact suggests that the effect should be anisotropic and should therefore be less pronounced in polycrystalline samples.

It is also clear that singularities of type (37) should also be exhibited by crystalline films on account of size-effect quantization.

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