Interaction of an electron beam with highly turbulent plasma

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An analysis is given of the interaction of an electron beam with plasma for $v_{\text{beam}}/v_T = (m_i/m_e)^{1/2}$, when the principal nonlinear process is the development of modulation instability. A detailed numerical simulation has been carried out of the beam-plasma interaction in one-dimensional geometry. Simple physical estimates that can be used to determine the turbulence parameters are reported. It is shown that they agree well with the results of numerical calculations. Similar estimates are then used to investigate beam relaxation in the three-dimensional situation.

There are many experimental situations involving plasmas, both in the laboratory and in space, in which the relaxation of an electron beam cannot be described either by the quasilinear theory or the weak-turbulence approximation (see, for example, Ref. 1). The development of the modulation instability of beam- excited Langmuir oscillations leads to the spatial modulation of turbulence and to the appearance of collapsing Langmuir cavitons.[2] This collapse leads to additional damping of the oscillations, and the influence of this effect on beam relaxation has been examined in a considerable number of papers (see, for example, Refs. 1 and 3).

Apart from the additional energy dissipation, modulation instability broadens the spectrum of excited oscillations. Since the growth rate of beam instability is not sign-definite, this fact may substantially alter the energy flux flowing into the plasma.[4–6] We shall investigate this effect by considering the interaction of a beam of sufficiently fast electrons with a plasma.

Most of this paper is concerned with the numerical simulation of the beam-plasma interaction in a one-dimensional model. Simple estimates are given for the rate of energy dissipation, the level of turbulence, and other spectrum parameters. The fact that the estimates are in good agreement with numerical calculations is a justification for using these ideas in the description of the beam-plasma interaction in the three-dimensional situation.

1. Consider the excitation of oscillations by an electron beam with relative velocity spread $\Delta v/v_0 > (n_b/n_0)^{1/3}$, for which the instability may be regarded as kinetic (Δv is the velocity spread in the beam and n_b its density). Let the beam velocity v_0 be sufficiently high, i.e.,

$$v_0/v_T > 3(M/m)^{1/2}. \tag{1}$$

This condition is satisfied, for example, in type-III solar flares, when the electron velocity v_0 is of the order of $c/3$. When condition (1) is satisfied, long-wave Langmuir oscillations with $k_0 r_d = v_T/v_0 < (1/3)(m/M)^{1/2}$ are excited in the plasma. In this wave-vector range there are no effective mechanisms that would restrict the oscillation amplitude, other than modulation instability; this is therefore important even when the threshold is only just exceeded.

Let us begin with the one-dimensional problem. Since nonlinear effects restrict the amplitude of the oscillations to a level much lower than that predicted by quasilinear theory, we shall neglect the change in the beam distribution function. The electric field in the Langmuir oscillations excited by the electron beam is described by

$$iE_t + \frac{3}{2}\omega_p r_d^2 E_{xx} + \omega_p \frac{|E|^2}{32\pi nT} E = i(\gamma_b - \nu_{ei})E. \tag{2}$$

The first term on the right-hand side of this equation describes the interaction with the beam, which assumes a particularly simple form in the k-representation

$$(\hat{\gamma}_b E)_k = \gamma_b(k) E_k, \quad \int \gamma_b(k)\,dk = 0, \quad \gamma_b \sim \omega_p \frac{n_b}{n}\left(\frac{v_0}{\Delta v}\right)^2. \tag{3}$$

The second term in (2) describes collisional damping.

Let us suppose that the beam and plasma parameters are such that the growth rate of the modulation instability $\gamma_{\text{mod}} \sim \omega_p W/nT$ is much greater than that of the beam instability, where $W \sim |E|^2/8\pi$ is the oscillation energy density. The structure of the steady (albeit only on average) distribution of the oscillations does not then depend on the properties of the beam. Figure 1 illustrates schematically the distribution of energy over a range of scales, i.e., the turbulence spectrum. It is clear that the spectrum can be characterized by three parameters: the width Δk, the oscillation energy density W, and the position p of the spectrum peak. In strong Langmuir turbulence, the spectrum width Δk is related to W by[1–3]

$$W/nT \sim (\Delta k r_d)^2. \tag{4}$$

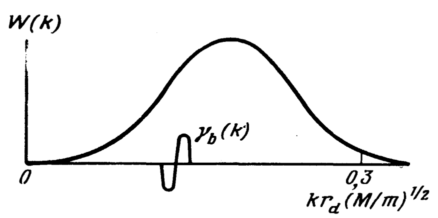

FIG. 1. Growth rate of beam instability and schematic distribution of oscillation energy in k-space for $\gamma_{\text{mod}} \gg \gamma_b$.

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To determine the remaining two parameters, it is sufficient to have the energy-momentum balance equation

$$\int \gamma_b(k) W_k\, dk = \nu_{ei} \int W_k\, dk = \nu_{ei} W,$$
$$\int \gamma_b(k)\, k W_k\, dk = \nu_{ei} \int k W_k\, dk. \quad (5)$$

In a previous paper by one of the present authors,[6] these equations were obtained as the consistency conditions for (2), and were analyzed for solutions consisting of a set of solitons. However, it will be shown below that the results do not depend on the details of the spectral distribution.

Let us suppose that the growth-rate width $\delta k \sim \Delta v k_0/v_0$ is much smaller than the width Δk of the spectrum. Expanding W_k into a series around zero growth rate

$$W(k) = W(k_0) + W'(k_0)(k - k_0),$$

we find from (5) that

$$\nu_{ei} W = \int (k - k_0) \gamma_b(k)\, dk\, W'(k_0), \quad (6)$$
$$p - k_0 = W(k_0)/W'(k_0). \quad (7)$$

Since $W'(k_0) = W(k_0)/\Delta k$, we see that, as the intensity of oscillations increases and, consequently, the spectrum width increases, the peak of the spectrum shifts to the right:

$$(p - k_0) r_d \sim \Delta k r_d \sim (W/nT)^{1/2}. \quad (8)$$

The energy flux into the plasma is then given by

$$Q \sim \int \gamma_b W_k\, dk \sim \gamma_b W (\delta k/\Delta k)^2 \sim \gamma_b n T (\delta k r_d)^2. \quad (9)$$

It is clear that the broadening of the spectrum produces an appreciable reduction in absorption, and the instability can be stabilized even by weak linear damping. (We recall that there is no collapse in the one-dimensional problem.)

We note further that the energy flux into the plasma does not depend on the dissipation mechanism. Finally, the level of the oscillations is described by

$$\frac{W}{nT} = \frac{\gamma_b}{\nu_{ei}} (\delta k r_d)^2. \quad (10)$$

We must now examine the range of validity of our results. First of all, the growth rate of the modulation instability $\gamma_{\mathrm{mod}} \sim \omega_p W/nT$ must exceed the beam value. Using (10), we obtain

$$(\delta k r_d)^2 > \nu_{ei}/\omega_p \quad \text{or} \quad \Delta v/v_0 > (v_0/v_T)(\nu_{ei}/\omega_p)^{1/2}. \quad (11)$$

This condition leads thus to a restriction on the minimum velocity spread in the beam, and does not depend on density.

Moreover, the level of turbulence must be sufficiently low: $W/nT < m/M$ or $\gamma_b/\nu_{ei} < m/M (\delta k r_d)^2$. Effective excitation of ion-acoustic oscillations begins when this condition is violated. Furthermore, the characteristic group velocity of the oscillations must not be close to the thermal velocity since, otherwise, intensive interaction between the oscillations and ions will begin.[7]

We note further that it is shown in Ref. 8 that solitons are accelerated and retarded when the pump is off and on, respectively. They should therefore bunch near $k = k_0$, which is in conflict with the above shift of the spectrum toward greater k. However, this process is due to the emission of sound by solitons. We, however, are considering here the static approximation in which sound is not generated.

2. In deriving the above estimates for turbulence parameters, we have actually introduced the assumption that the oscillation spectrum was smooth and stationary on average. Since it was not clear to what extent this assumptions was valid, we carried out a number of numerical calculations.

We note that the problem involves three appreciably different time scales $\nu_{ei}^{-1} > \gamma_b^{-1} > \gamma_{\mathrm{mod}}^{-1}$ and two spatial scales $\delta k^{-1} > \Delta k^{-1}$. The necessity for allowing the smallest of these means that, although the problem is one-dimensional, the calculations become nontrivial because the solutions must be stable over long time intervals.

We have solved numerically the dynamic equation (2) with periodic and stochastic initial conditions. The number of points on the sampling interval was 512 and the computation time was $\gtrsim 30\gamma_b^{-1}$. To solve (2), we used a conservative difference scheme of the second order of accuracy in space and time, which has integral conservation laws for (2).

The resulting turbulence consists largely of a set of solitons that scatter each other, and is essentially nonstationary. However, the average turbulence parameters reach stationary values relatively rapidly. The turbulence spectrum averaged over several γ_b^{-1} is also stationary.

Figure 2 shows the time-averaged distribution of the oscillations. It is clear that the numerical results are in good agreement with the qualitative representation of Fig. 1.

For a quantitative verification, we investigated the level of turbulence as a function of beam width [formula (10)]. Figure 3 shows that the quadratic dependence of W on δk which follows from (10) is in fact satisfactorily confirmed, and the numerical coefficient in (10) turns out to be close to unity: $W/nT \approx \gamma_b (\delta k r_d)^2/\nu_{ei}$. In this case, the beam-instability growth rate was taken in the form of a cubic parabola.

The character of the spectrum changes sharply (Fig. 4) when condition (11) is violated. It can be said that it consists of a quasimonochromatic wave, whose wave vector corresponds to the maximum growth rate, and a smooth broad spectrum. It was precisely this shape of the spectrum that was proposed in Ref. 1.

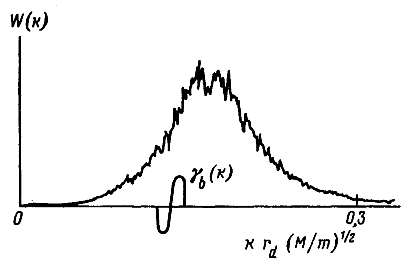

FIG. 2. Average distribution of oscillations in k-space for $\gamma_{\mathrm{mod}} > \gamma_b$, obtained as a result of numerical calculations with $\gamma_b = 3\nu_{ei}$, $\delta k r_d = 0.06\,(m/M)^{1/2}$. The computation time was $36\gamma_b^{-1}$.

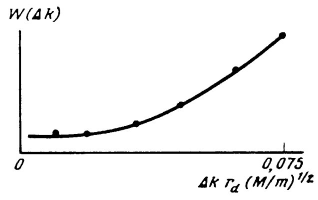

FIG. 3. Steady oscillation energy density as a function of the instability growth rate $\gamma_b = 7\nu_{ei}$. The first two points correspond to $\gamma_b > \gamma_{\rm mod}$.

The energy flux into the plasma and the level of turbulence can be estimated as follows. Let the oscillation energy in the region of growth rate W_0 be much smaller than the total oscillation energy W. In equilibrium, the energy flux into the resonant oscillation $Q = \gamma_b W_0$ is compensated by outflow over the spectrum due to modulation instability $Q = \gamma_{\rm mod} W_0$. When $W > W_0$, the growth rate of the modulation instability is determined by the wide spectrum $\gamma_{\rm mod} = \omega_p W/nT$. The liberated energy is finally absorbed by collisions, $Q = \nu_{ei} W$, so that we have

$$W/nT \sim \gamma_b/\omega_p, \quad W_0/nT \sim \nu_{ei}/\omega_p. \tag{12}$$

The energy flux into the plasma is given by

$$Q = \gamma_b \nu_{ei} nT/\omega_p. \tag{13}$$

It follows that, as the growth-rate width is reduced (at a constant value of the growth rate), the energy flux decreases and then reaches the constant value given by (13) for $(\delta k r_d)^2 \sim \nu_{ei}/\omega_p$. Numerical calculations have confirmed this (see Fig. 3).

Figure 5 shows Q, W and W_0 as functions of the growth rate when its width is small. It is clear that (12) and (13) are confirmed quite well by the numerical calculations.

3. So far, we have assumed a one-dimensional turbulence. However, it is well-known that, in isotropic plasma, solitons, and consequently the entire turbulence, are unstable against transverse perturbations. It would appear that the turbulence can be made one-dimensional by placing the plasma in a magnetic field. However, when $\omega_H > \omega_p$, the dispersion relation for the Langmuir oscillations is

$$\omega_k = \omega_p |k_z|/k = \omega_p |\cos\theta|.$$

It is clear that these oscillations can decay into oscillations propagating at a large angle to the magnetic field, so that it is only in systems with small transverse dimensions, for which they decay instability can be suppressed by effectively extending the oscillations beyond the walls of the chamber, that the one-dimensional turbulence can be produced. This was done in the well-known experiments reported in Ref. 9. When $\omega_H < \omega_p$, the dispersion relation for the Langmuir oscillations is

$$\omega_k = \omega_p \left(1 + \frac{3}{2} k^2 r_d^2 + \frac{1}{2}\left(\frac{\omega_H}{\omega_p}\right)^2 \frac{k_\perp^2}{k^2}\right). \tag{14}$$

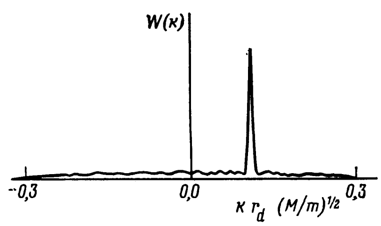

FIG. 4. Distribution of the oscillations in k-space for $\gamma_{\rm mod} < \gamma_b$, $\gamma_b = 4\nu_{ei}$, $\delta k r_d = 0.025\,(m/M)^{1/2}$.

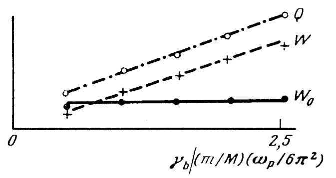

FIG. 5. Oscillation energy density W, the energy of resonant waves W_0, and the energy flux Q into the plasma as functions of the beam growth rate: $\delta k r_d = 0.01\,(m/M)^{1/2}$.

It is clear that the onset of transverse modulation leads to an increase in frequency, and is energetically unprofitable. However, the question remains open for long-wave $k_\perp \ll k$ perturbations such that $\delta\omega \sim (\omega_H^2/\omega_p)(k_\perp^2/k_0^2) \lessdot \gamma$.

The equation describing the evolution of oscillations (14) has the following form in the static limit:

$$\Delta\left(i\Psi_t + \frac{3}{2}\omega_p r_d^2 \Delta\Psi\right) - \frac{\omega_H^2}{\omega_p}\Delta_\perp \Psi + \omega_p \,\mathrm{div}\left(\frac{|\nabla\Psi|^2}{32\pi nT}\nabla\Psi\right) = 0. \tag{15}$$

We shall confine our attention to the case where instability results in the excitation of a narrow packet of oscillations with $\Delta k < k_0 = \omega_p/v_0$. We can then transform in (15) to the equation for the envelopes which, in terms of the dimensionless variables, is

$$i\Psi_t + {}^1/_2(1 + \omega_H^2/\omega_p^2(k_0 r_d)^2)\Delta_\perp\Psi + {}^1/_2\Psi_{zz} + |\Psi|^2\Psi = 0.$$

This differs from the usual Schrödinger equation, in which one-dimensional solitons are unstable against transverse perturbations,[10] only by the stretching of the transverse coordinates (relative to the magnetic field). It may therefore be considered that the magnetic field does not influence the instability itself and its maximum growth rate. The magnetic field only leads to an increase in the transverse dimensions of the perturbations. The increase in the characteristic transverse dimensions during the development of modulation instability in a weak magnetic field has already been mentioned in Ref. 11. The dimensions of the unstable perturbations should exceed those of a soliton by the factor $\omega_H/\omega_p k_0 r_d$. Since $k_0 r_d < (1/3)(m/M)^{1/2}$ in a strong magnetic field $\omega_H \lessgtr \omega_p$, this ratio is of the order of 100, and one-dimensional turbulence can be observed in specially designed laboratory experiments.

4. We must now consider beam relaxation in isotropic plasmas. As before, we shall confine our attention to the case where $k r_d < (m/M)^{1/2}$. Let us suppose that the broadening of the spectrum by modulation instability exceeds the growth-rate width both in the longitudinal and transverse directions.

In the three-dimensional case, the development of modulation instability leads to the collapse of Langmuir oscillations. The turbulence spectrum due to collapses falls rapidly with increasing k. Thus, in the inertial interval, we have $W_k \propto k^{-5/2}$, so that the collapsing cavitons do not contribute to the energy balance (5), and the energy flux into the plasma can be estimated as in Sec. 2. If we now introduce the longitudinal and transverse growth-rate widths $\delta k_{\parallel}, \delta k_{\perp}$, we can write

$$Q\sim\gamma_b(\delta k_{\parallel}\delta k_{\perp}/(\Delta k)^2)^2W\sim\gamma_b(\delta k_{\parallel}\delta k_{\perp}r_d^2)^2nT(nT/W). \tag{16}$$

We shall suppose that the collapse is subsonic, so that, provided the condition[2]

$$\nu_{ei}>\nu_{eff}\sim\omega_p(W/nT)^{3/2}(M/m)^{1/2}$$

is satisfied, absorption is governed by collisions, and

$$W/nT\sim(\gamma_b/\nu_{ei})^{1/2}(\delta k_{\parallel}\delta k_{\perp}r_d^2). \tag{17}$$

The condition for the validity of our analysis, $\gamma_{\rm mod}>\gamma_b$, now becomes more stringent:

$$\gamma_b\nu_{ei}/\omega_p^2<(\delta k_{\parallel}\delta k_{\perp}r_d^2)^2,$$

than in the one-dimensional case:

$$\nu_{ei}/\omega_p<(\delta k_{\parallel}r_d)^2.$$

When $\nu_{\rm eff}>\nu_{ei}$, absorption is determined by the collapse:

$$W/nT\sim(\gamma_b/\omega_p)^{2/7}(m/M)^{1/7}(\delta k_{\parallel}\delta k_{\perp}r_d^2)^{4/7}. \tag{18}$$

The turbulence spectra become modified when the instability growth rate becomes greater than $\gamma_{\rm mod}$.

The spectra now consist of a quasimonochromatic wave with energy W_0, a wide spectrum W, and a rapidly falling tail due to the collapsing cavitons. When $\nu_{\rm eff}<\nu_{ei}$ and dissipation is determined by collisions, we have, as in the one-dimensional case,

$$\gamma_bW_0=\nu_{ei}W,\quad \gamma_b=\gamma_{mod}=\omega_pW/nT,$$

so that (12) and (13) are valid. When energy dissipation is determined by collapse, the level of turbulence can be found from the condition $\gamma_b=\gamma_{\rm mod}$, and is given by

$$W/nT=\gamma_b/\omega_p. \tag{19}$$

The oscillation intensity in the region of the growth rate can be found from the energy balance

$$W_0/nT=(\gamma_b/\omega_p)^{3/2}(M/m)^{1/2}\ll W/nT. \tag{20}$$

The energy flux into the plasma turns out to be

$$Q=\gamma_bnT(\gamma_b/\omega_p)^{3/2}(M/m)^{1/2}. \tag{21}$$

We note once again that our results are valid only for $W/nT<m/M$ or for $\gamma_b/\omega_p<m/M$, i.e., we have the limiting relation $Q\sim\omega_p nT(m/M)^2$. Intense ion-acoustic oscillations are excited as the beam density increases, and the physical picture becomes very much more involved.

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