Peculiarities of optical spectra of thin layers

V. S. Pekar

Institute of Semiconductors, Ukrainian Academy of Sciences
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Emission and absorption of light by a layer whose thickness is comparable with or smaller than the light wavelength in the medium are considered theoretically. The factor by which the spectrum of a thin layer differs from the spectrum of an infinite homogeneous medium is calculated. It is shown that the nonequilibrium emission and absorption of light incident on the surface of a strongly scattering layer proceed predominantly via waveguide modes. The radiation-brightness spectral density integrated over the directions turned out to be a sawtooth function of the frequency and of the thickness. The conditions are found under which no equilibrium in the photon subsystem is established for thermal radiation, and the radiation distribution becomes essentially non-Planckian. It is shown that incandescent lamps can be improved by suppressing the IR part of the emission spectrum. The results differ quantitatively from those obtained in classical interference theory, since account is taken of the dependence of the phototransitions on the layer thickness, a dependence of importance for thin layers. It is shown that as the layer becomes thinner its transparency and rate of cooling can decrease greatly.

In many cases it is necessary to know the changes that occur in the emission or absorption spectrum of a body whose dimensions become comparable with or smaller than the wavelength of the light. This question arises, for example, in investigations of the optical properties of dust, colloids, and clusters distributed in liquid or solid phases, in the construction of thin-layer light sources, solar-energy absorbers, thermal screens, and others.

The small-size model considered in the present paper is an optically thin layer, for which the problem has the simplest quantitative solution.

Such layers were considered theoretically in many studies dating back apparently to Refs. 1–3. In most studies, right up to the present (see, e.g., Ref. 4), the dependences of the emission (absorption) spectra on the observation angle and on the thickness were determined only by interference of waves multiply reflected from the surfaces of the layer, i.e., it was assumed that the probabilities of the phototransitions of the emitting centers (or of the volume elements of the medium if the radiation is not produced by impurities) are the same as in an infinite homogeneous medium. Yet the spectral, angular, and polarizational dependences of the probability of phototransition of a center in an inhomogeneous medium can be entirely different from those in a homogeneous one. It is known that the probabilities of the phototransitions depend on the amplitude and form of the electromagnetic modes, which are not plane waves in inhomogeneous media (e.g., in a layer bordering on vacuum). Therefore the phototransition probabilities also differ, generally speaking, from those in the case of emission in a homogeneous medium.[5,6] The layer thickness can then exceeed considerably the dimensions of the center (of the radiating complex).

In Refs 6–8 were determined the phototransition probabilities, and the fields were quantized, for arbitrary one-dimensionally inhomogeneous media, open planar resonators, and waveguides. It was proved that correct field quantization is possible only if the mode-orthogonality relation is of the form

$$\int \mathbf{A}_\nu^*(r)\,\varepsilon(x)\,\mathbf{A}_{\nu'}(r)\,d^3r=C\delta_{\nu\nu'}, \tag{1}$$

where $\varepsilon(x)$ is the dielectric constant; ν is the mode number and includes the frequency, polarization index, and the two-dimensional wave vector in the yz plane. Here $\mathbf{A}_\nu$ are the stationary solutions of Maxwell's equations for the vector potential.

It has also been shown that relation (1) holds only for real ε, i.e., if the absorption and emission of the light is neglected in the zeroth approximation. Another necessary condition is that there be no energy flux through the surface that encloses the orthonormalization volume in (1). The last condition is not satisfied by the customarily employed Fox-Lee modes, which are essentially nonstationary for thin layers because of the outflow of energy from the layer to the vacuum. In our earlier studies[6–8] as well as below we have therefore introduced absolutely stationary superresonator modes[6–10] that do not attenuate in vacuum (outflowing modes) and those that attenuate in vacuum (waveguide modes).

The quantization procedure using (1) was independently confirmed by Ujihara[11–14] for a specific $\varepsilon(x)$ dependence.

Bykov[5] has considered an excited atom in a medium in which ε is a periodic step function of x. It was shown that the radiative lifetime of the atom is then substantially different than in a homogeneous medium.

There are also experimental data on the radiative lifetime of an excited center in inhomogeneous media. For example, Draxhage, Kuhn, and Schäfer[15] have shown that the lifetime of a center that emits near a conducting mirror depends on the distance to the latter. A theory based on a model of a classical dipole emitting near a conducting plane confirms this result.[16–21]

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A correction factor that accounts for the difference between the emission (absorption) spectrum of a thin layer and the spectrum of an infinite homogeneous medium is determined in the present paper. The latter spectrum is assumed known and undistorted by absorption and reradiation of photons on the path to the surface. Such a spectrum can be obtained not only by recalculation, but also, if the absorption coefficient is small enough, by direct experimental measurement. The correcting factor is obtained in quite a general case, without specifying the mechanism and characteristics of the excitation, nor the model of the radiating center.

It is assumed, however, that the layer thickness exceeds considerably the size of the center, so that the electron-vibrational states of the latter and their populations as a result of the excitation are the same as in a homogeneous infinite medium.

The situation considered for thermal excitation is one in which thermal equilibrium is established in the electron-vibrational subsystem, but not in the photon subsystem. The outflowing modes (their normalization intergrals) are localized mainly in the vacuum and their density is infinite even at a finite area of the layer. Finite Planck population of these modes calls for infinite energy (time). Their population is therefore infinitesimally small regardless of the temperature of the electron-vibrational system, i.e., no Planck equilibrium is established in them.[1)]

In the waveguide modes, if the longitudinal dimensions of the layer are large enough, a Planck distribution can be established. We shall consider below, however, also the case when the layer contains optical inhomogeneities that cause intermode scattering of the photon and outflowing modes. If the scattering mean free path of a photon in the waveguide mode is considerably shorter than the absorption path, the population of this mode is likewise in disequilibrium and very small. The photons emitted into the waveguide modes emerge to the vacuum through the layer surface as a result of scattering.

The theory of thermal radiation from small bodies was developed in many studies (see the book by Levin and Rytov[22] and, for example, Ref. 23). It was assumed there, however, in contrast to the present paper, that the electromagnetic waves (the photon subsystem) are also in thermal equilibrium, i.e., the wave absorption length is much shorter than the size of the body (this is realized for radio waves and metallic bodies). The results of the cited studies differ therefore from those that follow.

The solution of the Maxwell equations for an isotropic one-dimensionally inhomogeneous medium, particularly a mutilayered one, when the dielectric constant ε depends only on x, is made complicated both by the Lorentz gauge of the vector potential **A** and by the Coulomb gauge ($\mathrm{div}\mathbf{A} = 0$). When seeking solutions proportional to $\exp(-i\omega t)$ it is convenient to use instead the substitution

$$\mathbf{E}=-\dot{\mathbf{A}}/c, \tag{2}$$

which defines completely the vector **A**. We then obtain from the Maxwell equations

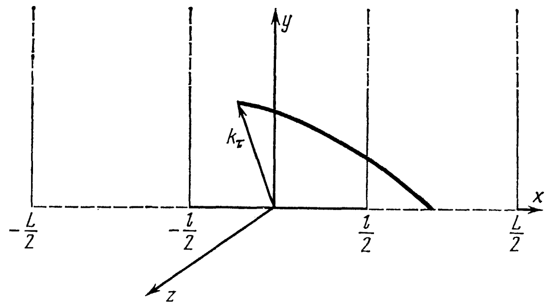

FIG. 1.

$$\mathbf{H}=\mathrm{rot}\,\mathbf{A}, \tag{3}$$

$$\Delta\mathbf{A}-\nabla(\nabla\mathbf{A})+\varepsilon(x)\,\omega^2\mathbf{A}/c^2=0. \tag{4}$$

The coefficients in Eq. (4) are independent of y and z, so that its particular solutions are

$$\mathbf{A}_\nu(\mathbf{r})=\mathbf{f}_\nu(x)\exp(i\mathbf{k}_\tau\mathbf{r}), \tag{5}$$

$$\mathbf{H}_\nu(\mathbf{r})=\mathbf{g}_\nu(x)\exp(i\mathbf{k}_\tau\mathbf{r}), \tag{6}$$

where $\mathbf{k}_\tau$ is a two-dimensional vector with projections k_y and k_z. The subscript ν of the electromagnetic mode includes ω, k_τ, and the polarization index p. The value $p = 1$ corresponds to a direction of $\mathbf{f}_\nu$ perpendicular to the incidence plane that pases through OX and $\mathbf{k}_\tau$. This direction is designated below by the unit vector **s**; $p = 2$ means that $\mathbf{f}_\nu$ lies in the incidence plane.

We consider a homogeneous dielectric layer of thickness l with a real dielectric constant ε. Located in vacuum on both sides of the layer, at very large distance $L/2$ from it, are flat absolutely reflecting mirrors that constitute the superresonator in which the $\mathbf{A}_\nu$ are orthonormalized.

For a wave with polarization $p = 2$, f_ν is a two-component vector with projections f_x and f_τ (f_τ is the projection of **f** on the $\mathbf{k}_\tau$ direction). Inside the layer, **f** is defined by the following equations: From the equation $\mathrm{div}\mathbf{D} = 0$ we get

$$ik_\tau f_\tau+df_x/dx=0. \tag{7}$$

It follows from (4) and (7) that

$$d^2f_\tau/dx^2+\left(\varepsilon\frac{\omega^2}{c^2}-k_\tau^2\right)f_\tau=0. \tag{8}$$

The field in the vacuum is determined by equations that differ from (7) and (8) only in that $\varepsilon = 1$.

A wave with polarization $p = 1$ has a single component f_s and is determined by an equation similar to (8).

It is convenient to place the origin at the center of the layer, so that the plane $x = 0$ is a mirror-symmetry plane. The solutions f_τ and f_s can then be sought in the form of even or odd functions of x and only the region $x > 0$ need be considered.

We denote the parity of the solutions by an index e, which is included in ν ($e = 2$ and $e = 1$ stand respectively for even and odd f_τ and f_s). The solution obtained in the layer and in the vacuum are matched in the $x = l/2$ plane such that the tangential components of the electric and magnetic fields are continuous. Cyclicity conditions with large period L are imposed in the y and z directions. The electromagnetic modes are normalized in a volume bounded in the y and z by the fundamental cyclicity region, and in the x direction by the superresonator mirrors.

It is convenient to choose the normalization constant in (1) equal to[6,7]

$$C=2\pi c^2\hbar/\omega_\nu.$$

We write down the solutions (7) and (8) in the region $x>0$, using the following notation:

$$k=(\varepsilon\omega^2/c^2-k_\tau^2)^{1/2},\quad q\equiv(\omega^2/c^2-k_\tau^2)^{1/2},$$

$$u_p\equiv\begin{cases}-k/|q|, & p=1\\ \varepsilon|q|/k, & p=2\end{cases}.$$

Outflowing modes ($\omega/c>k_\tau$)

Polarization $p=2$

$$\text{at}\quad 0<x<l/2\quad f_\tau=B_\tau\sin[kx+(e-1)\pi/2]$$
$$\text{at}\quad l/2<x<L/2\quad f_\tau=D_\tau\sin[qx+(e-1)\pi/2],$$

where

$$D_\tau/B_\tau=\sin[kl/2+(e-1)\pi/2]\{1+u_p^2\operatorname{ctg}^2[kl/2+(e-1)\pi/2]\}^{1/2}. \tag{9}$$

For $p=1$ it is necessary to replace the subscript τ in these equations by s. From (1) we obtain

$$|D_s|^2=\frac{4\pi c^2\hbar}{\omega L^3},\quad |D_\tau|^2=\frac{4\pi c^2\hbar}{\omega L^3}\left(1+\frac{k_\tau^2}{q^2}\right)^{-1}.$$

Waveguide modes ($\omega/c<k_\tau<\varepsilon^{1/2}\omega/c$).

For these modes, k_τ and ω are not independent but are connected by the dispersion equation

$$\operatorname{tg}\left[\frac{kl}{2}+(e-1)\frac{\pi}{2}\right]=u_p.$$

Polarization $p=$ **2**

$$\text{at}\quad 0<x<l/2\quad f_\tau=B_\tau^0\sin[kx+(e-1)\pi/2]$$
$$\text{at}\quad l/2<x<L/2\quad f_\tau=D_\tau^0\exp(-|q|x),$$

where

$$D_\tau^0/B_\tau^0=\exp(|q|l/2)\sin[kl/2+(e-1)\pi/2]. \tag{10}$$

Solutions with polarization $p=1$ are also obtained by replacing the subscript τ by s. The normalization (1) yields

$$|B_s^0|^2=\frac{\pi c^2\hbar}{\omega L^2}\left[\frac{\varepsilon l}{4}+\frac{\varepsilon|q|^2+k^2}{2|q|(k^2+|q|^2)}\right]^{-1}.$$

In the expressions for the phototransition probabilities, as will be shown below [see Eq. (19)], the mode parameters enter in the form of the following factors:

$$|B_\tau^0|^2=\frac{\pi c^2\hbar}{\omega L^2}\left[\left(1+\frac{k_\tau^2}{k^2}\right)\frac{\varepsilon l}{4}+\frac{k_\tau^2/k^2+k_\tau^2/|q|^2}{2|q|(1+k^2/\varepsilon^2|q|^2)}\right]^{-1}.$$

$$p=1,\quad \int_0^{l/2}|f_s|^2dx=|B_s^0|^2\left\{\frac{l}{4}+(-1)^e\frac{\sin kl}{4k}\right\},$$

$$p=2,\quad \int_0^{l/2}(|f_x|^2+|f_\tau|^2)dx \tag{11}$$

$$=|B_\tau^0|^2\left\{\left(1+\frac{k_\tau^2}{k^2}\right)\frac{l}{4}+(-1)^e\left(1-\frac{k_\tau^2}{k^2}\right)\frac{\sin kl}{4k}\right\}.$$

For the outflowing modes we obtain equations of the same form, but with B_s^0 and B_τ^0 replaced by B_s and B_τ.

We consider now phototransitions in an a radiating center (impurity atom, cluster, or volume element of the medium in the case of intrinsic radiation).

The energy operator for the interaction of radiation with the charges of a radiating complex consisting of an impurity center and the medium molecules surrounding it and strongly interacting with it is of the form

$$\mathscr{H}_{int}=-\sum_i\frac{e_i}{m_ic}\mathbf{A}(\mathbf{R})\mathbf{p}_i.$$

Here e_i, m_i, and $\mathbf{p}_i$ are respectively the charge, mass, and momentum of the ith particle (electron or ion); $\mathbf{R}$ is the coordinate of the complex. The complex is much smaller than the wavelength of the light.

Let $|a\rangle$ and $|b\rangle$ be stationary electron-vibrational states of the complex in the absence of an electromagnetic field. The matrix element of the phototransition $|b\rangle\to|a\rangle$ with emission of a photon into the mode ν is equal to

$$\langle\mathscr{H}_{int}\rangle\equiv\langle a,n_\nu+1|\mathscr{H}_{int}|b,n_\nu\rangle=(n_\nu+1)^{1/2}\vec{\mathscr{L}}\mathbf{A}_\nu(\mathbf{R}),$$
$$\vec{\mathscr{L}}=-\sum_i\frac{e_i}{m_ic}\langle a|\mathbf{p}_i|b\rangle. \tag{12}$$

It is assumed that the states $|a\rangle$ and $|b\rangle$ of the complex are the same as in an infinite homogeneous medium, so that only $\mathbf{A}_\nu(\mathbf{R})$ depends explicitly on l in Eq. (12).

The per-second probability of the phototransition $|b\rangle\to|a\rangle$ with emission of a photon into the mode ν is equal to

$$P_\nu=\frac{2\pi}{\hbar}\overline{\rho_{E_a}|\langle\mathscr{H}_{int}\rangle|^2}, \tag{13}$$

where ρ_{E_a} is the number of finite electron-vibrational states per unit energy interval. Emission of a given photon $\hbar\omega_\nu$ is possible in phototransitions from different initial states $|b\rangle$ into different states $|a\rangle$ for which the energy difference is $E_b-E_a=\hbar\omega_\nu$. The superior bar in (13) denotes averaging over the initial states $|b\rangle$ and summation over the final states $|a\rangle$. The excitation conditions are assumed such that the populations of the different states prior to the phototransition are the same as in an infinite medium. In this case we have[7]

$$P_\nu=(n_\nu+1)D(\omega)|\mathbf{A}_\nu(\mathbf{R})|^2, \tag{14}$$

where $D(\omega)$ is independent of l or $\mathbf{R}$, while $\mathbf{A}_\nu$ does depend on l and $\mathbf{R}$.

Equation (14) is valid also for an infinitely thick layer when the entire volume of the superresonator is filled with matter. The modes are then determined by the same formulas (9), but with $l \to L$. The number of modes of given polarization and parity in an inerval $d\omega$ with any direction of the wave vector in the half-space $k>0$ is

$$\rho_\infty(\omega)\,d\omega=(L^3\varepsilon^{3/2}\omega^2/4\pi^2c^3)\,d\omega. \tag{15}$$

Using (14) and (15) we can express the per-second probability dP of spontaneous emission, in the interval $d\omega$, of a photon of any polarization, parity, and direction:

$$dP=4P_\nu\rho_\infty(\omega)\,d\omega=\frac{2\hbar D(\omega)}{\pi c}\,\varepsilon^{1/2}\omega\,d\omega. \tag{16}$$

On the other hand, dP can be written in the form

$$dP=\varphi(\omega)\,\frac{d\omega}{\tau},\qquad \int_0^\infty \varphi(\omega)\,d\omega=1, \tag{17}$$

where τ is the average radiative lifetime of the center in a thick layer. Identifying (16) with (17) we obtain[7]

$$D(\omega)=\pi c\varphi(\omega)/2\hbar\omega\tau\varepsilon^{1/2}. \tag{18}$$

The quantities $\varphi(\omega)$ and τ can be determined from experiments with a thick layer. Equation (18) determines $D(\omega)$ in this case.

We return now to consideration of radiation of thick layers. If the centers are uniformly distributed in the layer and have a density n_0, the number per second of spontaneous emissions of a photon into the mode ν by the layer is obtained from Eq. (14):

$$n_0\int P_\nu\,d^3\mathbf{R}=n_0D(\omega)\int|\mathbf{A}_\nu(\mathbf{R})|^2\,d^3\mathbf{R}$$
$$=n_0L^2D(\omega)\int_{-l/2}^{l/2}|\mathbf{f}_\nu(x)|^2\,dx. \tag{19}$$

The values of the integral in this equation are given for all mode types in Eq. (11).

To determine the number of photons radiated per second into the frequency interval $d\omega$, we write down the numbers of the modes in the interval $d\omega$. For outflowing modes with fixed polarization and parity we have

$$\rho_\omega d\omega d\Omega=(L^3\omega^2/8\pi^3c^3)\,d\omega d\Omega. \tag{20}$$

Here $d\Omega$ is the solid angle subtending the three-dimensional vector $\mathbf{k}(q, k_\tau)$.

In the case of waveguide modes, as can be seen from (10), we separate, by specifying the polarization p and the parity e, one of four dispersion laws—the equation that connects ω with $|\mathbf{k}_\tau|$. Solving it for $|\mathbf{k}_\tau|$ we obtain a number of roots, which we number by the integer subscript j:

$$|\mathbf{k}_\tau|_j=\varphi_{pej}(\omega). \tag{21}$$

For the dispersion branch defined by the indices p, e, and j, the density $\rho^0_{pej}(\omega)$ of modes with arbitrary direction $\mathbf{k}_\tau\perp X$ is given by

$$\overset{0}{\rho}_{pej}(\omega)\,d\omega=\frac{L^2}{2\pi}|\mathbf{k}_\tau|_j\,d|\mathbf{k}_\tau|_j=\frac{L^2}{2\pi}\varphi_{pej}(\omega)\,\frac{d\varphi_{pej}}{d\omega}\,d\omega. \tag{22}$$

The energy radiated per second from a unit layer area through its two lateral surfaces into all the outlowing modes in the interval $d\omega$ is

$$dW=2\pi\hbar\omega n_0L^{-2}\rho_\infty\,d\omega\sum_{p=1}^{2}\sum_{i=1}^{2}\int\int_0^{\pi/2}P_\nu\,d^3\mathbf{R}\sin\vartheta\,d\vartheta. \tag{23}$$

The spectral density of the energy radiated per second from a unit area into a waveguide mode is given by

$$dW_0=\hbar\omega n_0L^{-2}\sum_{p=1}^{2}\sum_{e=1}^{2}\sum_j\overset{0}{\rho}_{pej}(\omega)\,d\omega\int P_{\nu_0}\,d^3R, \tag{24}$$

where ν_0 is a multidimensional index that includes p, e, j, and k_τ.

The frequency dependences of (23) and (24) were calculated with a computer. The results can be represented in the form

$$dW=Q(a)\,dW_\infty,\qquad a=(\varepsilon-1)^{1/2}\frac{\omega l}{2c}, \tag{25}$$

$$dW_0=Q_0(a)\,dW_\infty,\qquad dW_\infty=n_0l\hbar\omega\varphi(\omega)\,d\omega/\tau. \tag{26}$$

The factor dW_∞ expresses the power radiated in the interval $d\omega$ by a thick-layer volume 1 cm^2 in area and l long. The same power, minus the absorption along the path to the surface, is radiated from 1 cm^2 through both surfaces of the thick layer. The factors $Q(a)$ and $Q_0(a)$ correct dW_∞ for the case of small thicknesses and determine the singularities of the spectrum in this case.

Plots of $Q(a)$ and $Q_0(a)$ are shown in Fig. 2 for $\varepsilon=8$ and in Fig. 3 for $\varepsilon=16$. The tabulation was carried out in the interval $0\leqslant a\leqslant\pi$, where the layer thickness is comparable with or smaller than the wavelength in the medium, and the effects considered are significant. All four dispersion equations for the waveguide modes [see (10)] have one root (21)

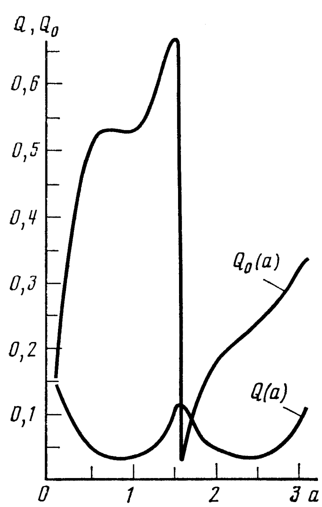

FIG. 2

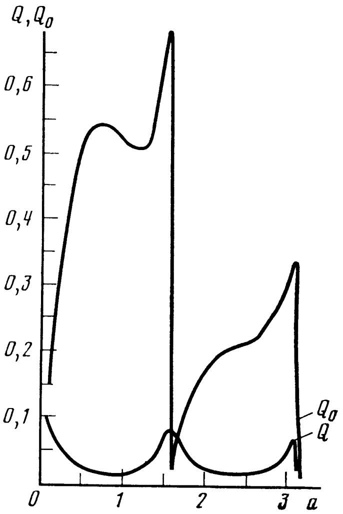

FIG. 3

each in this interval. The $Q_0(a)$ dependence was tabulated for arbitrary $\varepsilon \gg 1$. The approximation $\varepsilon \gg 1$ was used in the tabulation in the expression for $Q(a)$. The case of large ε is of greatest interest, since the depth of the modulation of the spectrum by the factors $Q(a)$ and $Q_0(a)$ decreases with decreasing ε. It can be easily seen that at $\varepsilon = 1$ we have $Q_0(a) = 0$ and $Q(a) = 1$.

Expanding the expression for $Q(a)$ in the limiting case when $\varepsilon \gg 1$ and $\varepsilon^{1/2}a \ll 1$, we obtain $Q(0) = 2/3\varepsilon^{1/2}$.

We consider now some consequences of the results.

1. The radiative lifetime τ_l of an excited center in a thin layer exceeds the radiative lifetime τ in an infinite medium:

$$\frac{1}{\tau_l} = \frac{1}{\tau} \int [Q(a) + Q_0(a)] \varphi(\omega)\, d\omega, \tag{27}$$

where the integral is less than unity, since $Q(a) + Q_0(a) < 1$ and $\varphi(\omega)$ is normalized [see (17)].[2)]

For example, for a narrow radiation line, when we can put $\varphi(\omega) = \delta(\omega - \omega_1)$ in (27), we have $\tau/\tau_l = Q(a_1) + Q_0(a_1)$, where $a_1 \equiv (\varepsilon - 1)^{1/2}\omega_1 l/2c$. It is seen from Fig. 3 that $\tau/\tau_l = 0.77$ at $a_1 = 1.55$, $\tau/\tau_l = 0.1$ at $a_1 = 1.6$, and $\tau/\tau_l = 0.2$ at $a_1 = 2$. The assumption that $\tau_l = \tau$, usually made in elementary interference theory, is thus incorrect for thin layers with $l \lesssim \lambda$, where λ is the light wavelength in the medium. At $l \gg \lambda$ we have $Q + Q_0 \to 1$ and $\tau_l \to \tau$, i.e., the phototransition probability ceases to depend on l.

The probability of phototransition of the center into the interval $d\omega$ in thin layers differs from the case of an infinite medium by the factor $Q(a) + Q_0(a)$, which determines the thickness dependence of the probability and alters substantially its frequency dependence. The frequency ω and the thickness l enter in the factor Q_0 only as the product ωl. At $\varepsilon \gg 1$ this applies also to Q.

2. In the presence of scatterers in the layers, if the per-second probability of scattering a photon from a waveguide mode into outflowing ones exceeds its absorption probability, all the photons radiated into the waveguide modes emerge to the vacuum through the lateral surface. The spectral density of the power radiated through both sides per square centimeter is given by the sum of (25) and (2), i.e., introduction of the scatterers increases the radiation power. The correcting factor in this case is not Q but $Q + Q_0$.

3. As seen from Figs. 2 and 3, for most frequencies we have $Q_0(a) > Q(a)$, i.e., radiation via waveguide modes predominates. According to Kirchoff's law generalized to include the case of thin layers, the light incident on the lateral surface of the layer will be predominantly absorbed via the waveguide modes.

4. The factor $Q + Q_0$ affects strongly the form of the emission and absorption spectra compared with the case of a thick layer, and leads to paradoxical phenomena. Let, for example, a monchromatic wave of frequency ω_0 be incident on the layer, and let l be such that $a = (\varepsilon - 1)^{1/2}\omega_0 l/2c = 1.55$, corresponding to the peak of $Q_0 + Q$. If now at fixed ω_0 we increase l somewhat, then the factor $Q_0 + Q$, and with it also the light-absorption coefficient, decreases sharply, by 6–8 times (see Figs. 2 and 3). Equally anomalous will be also the dependence of the radiation of a narrow line on l. In the absence of scattering, when the light is modulated by the factor Q in lieu of the factor $Q + Q_0$, the absorption decreases by one-half when l and a are increased 1.5 times (see Eq. (25) and Figs. 2 and 3). If, however, the thickness l is fixed, analogous anomalies appear in the emission and absorption when small changes are made in ω_0.

5. By varying l we can stretch and compress the frequency dependence of th factor $Q + Q_0$. This changes the ratio of the radiation brightnesses in the short- and long-wave parts of the spectrum. For example, it becomes possible to suppress the long-wave part of the emission spectrum of a thermal light source, and shift the spectrum into the region where the eye is most sensitive.

If $\varphi(\omega)$ is bell-shaped and the maximum lands on the rising section of the factor $Q + Q_0$, the maximum of the emission band of a thin layer is shifted towards shorter wavelengths than in the case of a thick layer. The maximum of $\varphi(\omega)$, which lands on the descending section of $Q + Q_0$, shifts towards longer wavelengths.

6. In the absence of scattering, the photons of the long-wave modes do not emerge to the vacuum, since the radiation through the end faces of the layer can be neglected. In this case, in the stationary regime, the number of photons emitted per second is equal to the number of the absorbed ones. If the excited thermal and electron-vibrational subsystems are in thermal equilibrium, an equilibrium Planck population of "trapped" waveguide modes sets in. Radiation into vacuum is only from the outflowing modes and is determined by Eq. (25).

If weak scattering of photons from the waveguide modes into the outflowing is introduced, additional radiation to the vacuum, appears, with an intensity proportional to the scattering probability. The spectrum of this scattered radiation equals the Planck spectrum multiplied by the frequency dependence of the scattering probability. When the scattering probability exceeds the probability for absorption of a waveguide-mode photon, the Planck equilibrium of the waveguide modes is upset. The radiation scattered from the waveguide modes is determined by Eq. (26) and ceases to depend on the scattering probability and on its frequency dependence.

The cooling of the layer in vacuum is determined by the thermal radiation power $W(T)$ integrated over ω. At the maximum of the radiation spectrum, $\hbar\omega_{max}$ is usually of the order of several times kT. At $T \approx 1$ K, λ_{max} is of the order of several millimeters, and starting with these values of l the layer becomes optically thin. Its cooling rate becomes equal to

$$\dot{T} = -W(T)/lC(T), \tag{28}$$

where C is the specific heat. So long as l is large enough to make the dielectric layer opaque to millimeter radio waves, the radiation is equilibrium Planckian, and $W(T)$ is determined by the Stefan-Boltzmann law and does not depend on l. The cooling rate is then proportional to l^{-1}. At lower thicknesses the layer becomes transparent:

$$W(T) = \int_0^\infty dW_\infty \propto l,$$

and $-\dot{T}$, having reached the maximum value, ceases to depend on l. With further decrease of l, the layer becomes optically thin, $-\dot{T}$ first decreases, and then becomes an oscillating function of l, since

$$W(T) = \int_0^\infty (dW + dW_0).$$

Removal the scatterers from the layer leads to a considerable decrease of $\dot{T}$.

The author is deeply grateful to S. I. Pekar and V. I. Pipa for a discussion of the work and for important remarks.

1)Establishment of Planck equilibrium in the known example of "a dust speck in a mirror box" requires an infinite time.
2)For an anisotropic center, $\tau_l < \tau$ is also possible.

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