Theory of low-frequency magnetosonic solitons

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We analyze the problem of low-frequency magnetosonic solitons (MSS) propagating in a plasma across the magnetic field. We note that such an analysis is made necessary by the contradictory results of earlier papers, in some of which these solitons are treated as rarefaction solitons and in others as compression solitons. We show that in a plasma with a Maxwellian particle velocity distribution these solitons are compression solitons. We explain that the opposite point of view is an error connected with the use of insufficiently accurate hydrodynamic equations. We obtain a set of hydrodynamic equations which adequately describe low-frequency MSS. We work out a combined kinetic-hydrodynamic approach to the problem of nonlinear low-frequency magnetosonic waves. Using this approach we study low-frequency MSS in a plasma with a non-Maxwellian particle velocity distribution (in that case rarefaction solitons are possible) and also in a plasma with cold and hot impurity ions (the impurity ions can significantly affect the characteristic spatial scale of the solitons). We study the gyro-relaxation damping of low-frequency MSS in a weakly collisional plasma. We show that such a damping is a more important effect than the transverse collisional viscosity which was considered earlier in connection with the shock wave problem.

1. INTRODUCTION

Low-frequency magnetosonic solitons (MSS) propagating across a magnetic field are one of the basic kinds of solitons in a high-temperature magnetized plasma. The first study of these solitons was presented in Refs. 1, 2 in which it was stated that they are rarefaction solitons. The results of Ref. 2 were subjected to criticism in Ref. 3 in which it was noted that magnetosonic solitons in a plasma with a Maxwellian ion distribution must be compression solitons. The validity of the criticism in Ref. 3 was acknowledged in Refs. 4, 5. In accordance with Ref. 3 a solution of the nonlinear equations for compression solitons was given in Ref. 6. However, the discussion started in Ref. 3 bypassed the authors of later papers[7–9] who again considered magnetosonic rarefaction solitons. The incorrect dispersion relation for magnetosonic waves which is the primary cause for the incorrect analysis of magnetosonic solitons in Berezin's paper[1] is reproduced (with an additional error!) also in his recently published book[10] which contains no mention of this discussion (see the equations at the bottom of p. 17 and the top of p. 18 of this book[10]).

In order to warn future researchers against yet another repetition of old errors it is advisable to elucidate the cause for these errors and to show what a valid theory of low-frequency MSS should look like. This is one of the main aims of the present paper. Moreover, we extend the development of the theory of MSS in a direction which we shall indicate later.

The authors of the original papers on the theory of low-frequency MSS[1,2] as well as the contemporary authors of Refs. 7, 8 started from the idea that the dispersion of magnetosonic waves caused by the finite ion Larmor radius can be described using the hydrodynamic equations with the tensor (magnetic) viscosity tensor in Braginskiĭ's form.[11] They were stimulated to use these equations in the MSS problem by their successful application to the problem of the flute instability of a plasma with a finite ion Larmor radius.[12,13] However, this kind of hydrodynamics is suitable only for describing effects which are linear in the Larmor radius ρ. This is sufficient for the flute instability problem. As to the dispersion of magneto-sonic waves, that effect is quadratic in ρ (of order $k_\perp^2\rho^2$, where $k_\perp$ is the transverse wavenumber). The use of this kind of hydrodynamics to study the dispersion of magnetosonic waves is thus unjustified. The formal application of this kind of hydrodynamics in the problem studied here led the authors of Refs. 1, 2, 8, 9 to results which are in error in two respects. Firstly, the dispersive correction to the wave frequency for small β turns out not to be of order $k_\perp^2\rho^2$, but of order $\beta k_\perp^2\rho^2$, where β is the ratio of the plasma pressure to the magnetic field pressure. Secondly, the wave dispersion turns out not to be negative, but positive (in the present case, that of Maxwellian ions).

The problem of low-frequency MSS obtained a correct treatment[6,14,15] thanks to the derivation of a set of hydrodynamic equations which takes into account terms of order $k_\perp^2\rho^2$. This was caused by the inclusion of corrections of order $k_\perp\rho$ to Braginskiĭ's expressions[11] for the viscosity tensor $\hat{\pi}$ and the heat flux $\mathbf{q}$. Since the dispersion emerges as an effect which is linear in the wave amplitude it can also be described by means of the plasma dielectric tensor calculated using the linearized Vlasov kinetic equation. This was done in Ref. 5 and this confirmed the hydrodynamic conclusion[14] about the negative dispersion of magnetosonic waves.

It is clear that the main stumbling block for the development of a theory of low-frequency MSS was the problem of how to take dispersion into account. It is thus advisable to start an analysis of MSS with an analysis of the dispersion of linear magnetosonic waves. We shall proceed in this way in the present paper.

It is clear from what has been said above that an analysis

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of the dispersion may be performed in the framework of two approaches: a hydrodynamic and a kinetic one. At first sight it may turn out that in a hydrodynamic approach which operates with macroscopic plasma parameters we may obtain a more intuitive picture of the dispersion than in the kinetic approach. However, a hydrodynamic formulation which is suitable for describing dispersion, in accordance with what has been said above, is considerably more complicated than the usual (ideal) hydrodynamics and is thus not so intuitive. In a number of cases the kinetic approach possesses greater intuitiveness and turns out to be more effective for studying the effect of various factors on the dispersion of low-frequency magnetosonic waves (for more details see section 5). In this connection we first expound the kinetic theory of the dispersion (section 2) and afterwards the hydrodynamic theory (section 3). This order of exposition is also useful because once we have the kinetic result we can more easily solve the problem of which set of hydrodynamic equations to choose. In other words, we use kinetics not only to obtain physical results, but also to control the hydrodynamics. Moreover, in accordance with what has been said above it turns out to be useful for the study of MSS to use both hydrodynamics and kinetics and to combine these two approaches.

One must bear in mind that the dispersion of magnetosonic waves depends on the nature of the equilibrium state of the plasma. We pay most attention to the case of a Maxwellian plasma with a finite value of β which contains one kind of ions, i.e., the same plasma to which the chain of errors discussed above applied. This case, to be called the standard case, is the subject of the analysis of sections 2, 3 and also of section 4 where we discuss solitons corresponding to this case. Moreover, in section 5 we take into account the non-Maxwellian nature of the particle velocity distribution and consider the case of infinitesimally small β and elucidate the role played by a multi-component ion composition of the plasma. The case of a non-Maxwellian plasma (subsection 5.1) is interesting because the wave dispersion in such a plasma may be positive[3,6] and the corresponding solitons may be rarefaction solitons. The discussion of the case of a plasma with infinitesimally small β (subsection 5.2) is connected with the necessity to reconsider the erroneous criterion for the applicability of the cold-plasma approximation applied in Ref. 2. An interesting feature of a cold plasma with several kinds of ions having different charge-to-mass ratios is an appreciable increase in the wave dispersion even when there is only a small fraction of impurity ions present (for details see subsection 5.3). In the case of a hot plasma the dispersion of low-frequency magnetosonic waves can be appreciably affected by the admixture of high-energy ions if the temperature of those ions is much higher than the temperature of the main ion component (subsection 5.4).

In the present paper we consider mainly the case of a collisionless plasma. A conclusion was reached in Ref. 2 about the possibility of the formation of magnetosonic shock waves in a plasma with weak collisions. We show in that connection in section 6 that weak ion-ion collisions lead not to the formation of shock waves, but to the damping of low-frequency magnetosonic solitons. This damping is caused by the gyro-relaxation effect; we call it gryo-relaxation damping.

The results of the paper are discussed in section 7.

2. KINETIC DESCRIPTION OF LOW-FREQUENCY MAGNETOSONIC WAVES

We assume that the plasma is spatially uniform and is in a uniform magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$. We assume the equilibrium particle velocity distribution to be Maxwellian. We assume that the ion component of the plasma consists of one kind of particle. We consider linear waves propagating at right angles to $\mathbf{B}_0$; to fix our ideas we put their wavevector $\mathbf{k}$ along the x-axis, i.e., $\mathbf{k} = (k,0,0)$. The magnetic field of the waves is characterized by the quantity $\widehat{B}_z$ and the electric field has components E_x, E_y. We assume the wave frequency ω to be small compared to the ion cyclotron frequency ω_{Bi}, $\omega \ll \omega_{Bi}$, and the wavelength large compared to the ion Larmor radius ρ, $k\rho \ll 1$, where

$$\omega_{Bi}=e_iB_0/m_ic, \quad \rho=(T_i/m_i)^{1/2}/\omega_{Bi},$$

e_i, m_i are the ion charge and mass, T_i is the ion equilibrium temperature, and c the velocity of light. We assume the equilibrium plasma density n_0 to be sufficiently large that $c_A^2 < c^2$, where $c_A^2 = B_0^2/4\pi n_0 m_i$ is the square of the Alfvén velocity. In that case the waves may be assumed to be quasineutral.

Under the assumptions made the Maxwell equations reduce to the following

$$\partial \tilde{B}_z/\partial x = -4\pi j_y/c, \tag{2.1}$$

$$\partial \tilde{B}_z/\partial t = -c\partial E_y/\partial x, \tag{2.2}$$

$$j_x = 0, \tag{2.3}$$

where j_x, j_y are the components of the electric current density. Bearing in mind a later generalization of the results to the case of nonlinear waves have written Eqs. (2.1), (2.2) in a coordinate-time representation. In the present section we shall be dealing with the Fourier components of these equations. In that case we must be put $\partial/\partial x \to ik$, $\partial/\partial t \to -i\omega$.

We use the fact that in the case of quasi-neutral waves

$$j_\alpha = -(i\omega/4\pi)\varepsilon_{\alpha\beta}E_\beta, \quad (\alpha, \beta) = (x, y), \tag{2.4}$$

where $\varepsilon_{\alpha\beta}$ is the dielectric permittivity tensor. It then follows from (2.3) that

$$E_x = -\varepsilon_{xy}E_y/\varepsilon_{xx}. \tag{2.5}$$

Using (2.5) we get from (2.4)

$$j_y = -\frac{i\omega}{4\pi}\left(\varepsilon_{yy} - \frac{\varepsilon_{yx}\varepsilon_{xy}}{\varepsilon_{xx}}\right)E_y. \tag{2.6}$$

Substituting (2.6) into (2.5) we are led to the dispersion relation

$$\varepsilon_{yy} - \frac{c^2k^2}{\omega^2} - \frac{\varepsilon_{yx}\varepsilon_{xy}}{\varepsilon_{xx}} = 0. \tag{2.7}$$

Expanding the general kinetic expressions for $\varepsilon_{\alpha\beta}$ given, e.g., in Ref. 16, in series in $k\rho$ and ω/ω_{Bi} to an accuracy sufficient for what follows we get

$$\varepsilon_{yy} = \frac{c^2}{c_A^2}\left(1 + \frac{\omega^2}{\omega_{Bi}^2} - \frac{11}{4}z\right) - \frac{c^2k^2}{\omega^2}\left(\beta - \frac{3}{2}z\beta_i\right),$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = i\frac{c^2}{c_A^2}\frac{\omega}{\omega_{Bi}}\left(1 - \frac{\omega_{Bi}^2}{\omega^2}z\right), \qquad \varepsilon_{xx} = \frac{c^2}{c_A^2}. \tag{2.8}$$

Here

$z=k^2\rho^2$, $\beta=\beta_i+\beta_e$, $\beta_\alpha=8\pi p_\alpha/B_0^2$, $p_\alpha=n_0T_\alpha$ $(\alpha=e,\ i)$.

Substitution of (2.8) into (2.7) gives

$$\omega^2(1+{}^1/_4 z)-c_A^2k^2(1+\beta-{}^3/_8 z\beta_i)=0. \qquad (2.9)$$

Hence it follows, if we use the fact that z is small, that

$$\omega^2=k^2V^2(1-k^2a_D^2), \qquad (2.10)$$

where $V^2=c_A^2(1+\beta)$, a_D is the so-called dispersion length given by the relation

$$a_D^2=\frac{\rho^2}{4}\frac{1+\beta_e+{}^5/_2\beta_i}{1+\beta}. \qquad (2.11)$$

The dispersion length a_D turns, according to (2.11), out to be, of the order of the ion Larmor radius, $a_D\sim\rho$, both for small and for large β.

It is clear from (2.10), (2.11) that with increasing wave-number k the phase velocity ω/k decreases, i.e., the waves have a negative dispersion.

Turning to (2.6) and using (2.8) we note that the electric current of the wave type considered consists of three physically different parts: the inertial part j_y^I, the so-called "β" part j_y^β, and the dispersive part j_y^D,

$$j_y=j_y^I+j_y^\beta+j_y^D. \qquad (2.12)$$

The corresponding parts of the current are given by the equations

$$j_y^I=-\frac{i\omega}{4\pi}\frac{c^2}{c_A^2}E_y, \qquad (2.13)$$

$$j_y^\beta=\frac{ic^2k^2}{4\pi\omega}\beta E_y, \qquad (2.14)$$

$$j_y^D=-\frac{ic^2\omega^2n_0m_i\rho^2k^2}{4B_0^2}\left(1+3\frac{\omega_{Bi}^2}{\omega^2}z\right)E_y. \qquad (2.15)$$

This division of the current into separate structural parts turns out to be useful for comparing the results of the kinetic and the hydrodynamic approaches (see section 3), for constructing equations for nonlinear waves (section 4), and for various generalizations of the magnetosonic wave problem.

HYDROELECTRODYNAMIC APPROACH TO THE MAGNETOSONIC WAVE PROBLEM

In contrast to section 2 we supplement Eqs. (2.1) to (2.3) with the hydrodynamic expression for the electric current (cf. (2.6)):

$$j_y=\frac{c}{B}\left[m_in\left(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}\right)+\frac{\partial p}{\partial x}+\frac{\partial\pi_{xx}}{\partial x}\right]. \qquad (3.1)$$

Here $B=B_0+\tilde{B}_z$ is the total magnetic field, n is the total density of each kind of particle (electron or ion), V_x is the x-component of the velocity of the plasma as a whole, p is the total plasma density, and π_{xx} is the appropriate component of the voscosity tensor $\hat{\pi}$.

To use (3.1) we need to have equations for V_x, n, p, and π_{xx}. As the equation for V_x we use the freezing-in condition

$$\frac{\partial B}{\partial t}+\frac{\partial}{\partial x}(V_xB)=0. \qquad (3.2)$$

We determine the plasma pressure from the heat balance equation:

$$\frac{\partial p}{\partial t}+V_x\frac{\partial p}{\partial x}+2p\frac{\partial V_x}{\partial x}+\frac{\partial q_x}{\partial x}=0, \qquad (3.3)$$

where q_x is the x-component of the heat flux. We can use Eq. (3.3) also to evaluate the pressure of any one of the components (ions or electrons) of the plasma by putting appropriate indexes for the kind of particle on the corresponding quantities.

We note that if we neglect the heat flux we get from (3.2) and (3.3) the equation for the two-dimensional adiabat

$$p^A/B^2=\text{const}, \qquad (3.4)$$

where p^A is the pressure evaluated in the adiabatic approximation. This approximation was used in Ref. 7. It is clear, however, from what follows that when we take into account the dispersion of the magnetosonic waves in a plasma with $\beta\sim1$ this approximation is inapplicable. We need therefore also an expression for q_x (and, indeed, for the ion heat flux q_{xi}).

According to the Appendix the expressions for π_{xx} and q_x differ from the corresponding expressions of Braginskiĭ.[11] They have the form

$$\pi_{xx}=-\frac{1}{2\omega_{Bi}}\left(p_i\frac{\partial V_y}{\partial x}+\frac{\partial\pi_{xy}}{\partial t}+\frac{1}{2}\frac{\partial q_y}{\partial x}\right), \qquad (3.5)$$

$$q_x=-\frac{1}{\omega_{Bi}}\left(\frac{\partial q_y}{\partial t}+\frac{T_i}{m_i}\frac{\partial\pi_{xy}}{\partial x}\right). \qquad (3.6)$$

Here p_i, T_i are the total ion pressure and the ion temperature, V_y is the y-component of the plasma velocity given by the standard equation of motion

$$m_in\partial V_y/\partial t=-\partial\pi_{xy}/\partial x, \qquad (3.7)$$

and the expressions for π_{xx} and q_y have a form similar to the one in Ref. 11:

$$\pi_{xy}=\frac{p_i}{2\omega_{Bi}}\frac{\partial V_x}{\partial x}, \qquad (3.8)$$

$$q_y=\frac{2p_i}{m_i\omega_{Bi}}\frac{\partial T_i}{\partial x}. \qquad (3.9)$$

We note that in Ref. 7 in the expression for π_{xx} only the term with $\partial V_y/\partial x$ was taken into account. This is insufficient for the problem considered.

The basic equations of the present section are closed by the continuity equation in standard form

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nV_x)=0. \qquad (3.10)$$

As in section 2 we consider linear waves. We then write the electric current in the form (2.12) and the perturbed pressure $\tilde{p}$ in the form

$$\tilde{p}=\tilde{p}^A+\tilde{p}^D, \qquad (3.11)$$

where $\tilde{p}^A$, $\tilde{p}^D$ are the adiabatic and dispersive parts of the perturbed pressure. It then follows from (3.1) that

$$j_y^I=\frac{cm_in_0}{B_0}\frac{\partial V_x}{\partial t}, \qquad (3.12)$$

$$j_y{}^\beta=\frac{c}{B_0}\frac{\partial\tilde{p}^A}{\partial x},\qquad(3.13)$$

$$j_y{}^D=\frac{c}{B_0}\left(\frac{\partial\tilde{p}^D}{\partial x}+\frac{\partial\pi_{xx}}{\partial x}\right).\qquad(3.14)$$

We find from (3.2) that

$$V_x=cE_y/B_0,\qquad(3.15)$$

so that, according to (3.12)

$$j_y{}^I=\frac{c^2m_in_0}{B_0^2}\frac{\partial E_y}{\partial t}.\qquad(3.16)$$

It follows from (3.4) that

$$\tilde{p}^A=2p_0\tilde{B}/B_0.\qquad(3.17)$$

Here $p_0=(T_{0e}+T_{0i})n_0$ while T_{0e}, T_{0i} are the equilibrium values of the electron and ion temperatures, i.e., the same as T_e, T_i in section 2. Substituting (3.17) into (3.13) we get

$$j_y{}^\beta=\frac{2cp_0}{B_0^2}\frac{\partial\tilde{B}_z}{\partial x}.\qquad(3.18)$$

To evaluate the right-hand side of (3.14) it is necessary to find solutions of Eqs. (3.7)–(3.10). From (3.10) we find that the density perturbation $\tilde{n}$ has the form

$$\tilde{n}=n_0\tilde{B}_z/B_0.\qquad(3.19)$$

Using (3.17), (3.19) we find from (3.7)–(3.9)

$$(V_y,\pi_{xy},q_y)=\frac{T_{0i}}{B_0\omega_{Bi}}\left(\frac{1}{2m_i}\frac{\partial\tilde{B}_z}{\partial x},-\frac{n_0}{2}\frac{\partial\tilde{B}_z}{\partial t},\frac{2p_{0i}}{m_i}\frac{\partial\tilde{B}_z}{\partial x}\right).\qquad(3.20)$$

It then follows from (3.3), (3.5), (3.6) that

$$\pi_{xx}=\frac{n_0m_i\rho^2}{4B_0}\left(\frac{\partial^2\tilde{B}_z}{\partial t^2}-\frac{3}{2}v_{Ti}{}^2\frac{\partial^2\tilde{B}_z}{\partial x^2}\right),\qquad(3.21)$$

$$q_x=-\frac{3}{2}\frac{p_{0i}\rho^2}{B_0}\frac{\partial^2\tilde{B}_z}{\partial x\,\partial t},\qquad(3.22)$$

$$\tilde{p}^D=\frac{3}{2}\frac{p_{0i}\rho^2}{B_0}\frac{\partial^2\tilde{B}_z}{\partial x^2},\qquad(3.23)$$

where $v_{Ti}^2=2T_{0i}/m_i$. Substituting (3.21), (3.23) into (3.14) we find that

$$j_y{}^D=\frac{cn_0m_i\rho^2}{4B_0^2}\frac{\partial}{\partial x}\left(\frac{\partial^2\tilde{B}_z}{\partial t^2}+\frac{3}{2}v_{Ti}{}^2\frac{\partial^2\tilde{B}_z}{\partial x^2}\right).\qquad(3.24)$$

Comparing (3.16), (3.18), (3.24) with (2.13) to (2.15) and using (2.12) we conclude that when using a hydrodynamic description of the plasma we get exactly the same expression for the electric current as in the kinetic description. It is thus clear that the dispersion Eq. (2.9) and the expression (2.10) for the square of the frequency are the consequence not only of kinetics, but also of hydrodynamics.

4. STANDARD CASE OF MAGNETOSONIC SOLITONS

In contrast to section 3 we now take into account the nonlinear part of the electric current, i.e., we write j_y in the form

$$j_y=j_y{}^L+j_y{}^{NL},\qquad(4.1)$$

where j_y^L, j_y^{NL} are the linear and the nonlinear parts of the current. The linear part of the current is characterized by Eqs. (2.12), (3.16), (3.18), (3.24). To evaluate j_y^{NL} we use Eq. (3.1). The contribution from π_{xx} to j_y^{NL} is unimportant. When evaluating the contribution from the pressure to j_y^{NL} we can use the equation of the adiabat (3.4). It then turns out that the combination $B^{-1}\partial p/\partial x$ does not contain terms quadratic in $\tilde{B}_z$, i.e., the term with the pressure in (3.1) does not contribute to j_y^{NL}. Using (3.2) and (3.10) we find that

$$n/B=\text{const}.\qquad(4.2)$$

Therefore

$$j_y{}^{NL}=\frac{cm_in_0}{B_0}\left(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}\right)^{NL}.\qquad(4.3)$$

We find the nonlinear relation between V_x and $\tilde{B}_z$ which we need by expanding the right-hand side of (3.2) in a series in $\tilde{B}_z/B_0$. We then get [cf. (3.15)]

$$V_x=\frac{cE_y}{B_0}\left(1-\frac{\tilde{B}_z}{B_0}\right).\qquad(4.4)$$

Substituting (4.4) into (4.3) we are led to the required expression

$$j_y{}^{NL}=\frac{m_in_0c^2}{B_0^3}\left[\frac{c}{2}\frac{\partial E_y{}^2}{\partial x}-\frac{\partial}{\partial t}(E_y\tilde{B}_z)\right].\qquad(4.5)$$

Using (2.1), (4.1) and the expressions we have found for the linear and the non-linear parts of the current we get a nonlinear equation for the electromagnetic field of the waves considered [cf. the dispersion Eq. (2.9)]:

$$c_A{}^2(1+\beta)\frac{\partial\tilde{B}_z}{\partial x}+c\frac{\partial E_y}{\partial t}+\frac{\rho^2}{4}\frac{\partial}{\partial x}\left(\frac{\partial^2\tilde{B}_z}{\partial t^2}+\frac{3}{2}v_{Ti}{}^2\frac{\partial^2\tilde{B}_z}{\partial x^2}\right)+\frac{c}{B_0}\left[\frac{c}{2}\frac{\partial E_y{}^2}{\partial x}-\frac{\partial}{\partial t}(E_y\tilde{B}_z)\right]=0.\qquad(4.6)$$

We assume that $\tilde{B}_z$ and E_y depend on $\xi\equiv x-ut$ and $\tau=t$, where τ is the so-called slow time. It then follows from (4.6) that

$$\frac{2}{u}\frac{\partial h}{\partial\tau}-\left(1-\frac{V^2}{u^2}\right)\frac{\partial h}{\partial\xi}+\frac{3}{2}\frac{\partial h^2}{\partial\xi}+a_D{}^2\frac{\partial^3h}{\partial\xi^3}=0,\qquad(4.7)$$

where $h=\tilde{B}_z/B_0$.

In the stationary case $\partial/\partial\tau=0$ and Eq. (4.7) has a soliton solution of the form

$$h=h_0/\text{ch}^2(\varkappa\xi/2),\qquad(4.8)$$

where

$$h_0=1-\frac{V^2}{u^2},\quad\varkappa=\frac{1}{a_D}\left(1-\frac{V^2}{u^2}\right)^{1/2}.\qquad(4.9)$$

The quantity h_0 characterizes the maximum amplitude of the soliton and $\varkappa$ is the reciprocal of the characteristic size of the soliton. For $V/u\ll1$ (as follows from the fact that $\varkappa$ is real), we have $h_0>0$, i.e., in accordance with (4.2) we are dealing with a compression soliton.

Our initial assumption that the dispersion is weak is justified provided $\varkappa a_D\ll1$, i.e., when $h_0\ll1$. In terms of h_0 the characteristic size of the soliton is

$$l_s\sim\rho/h_0^{1/2}.\qquad(4.10)$$

5. SPECIAL CASES OF MAGNETOSONIC SOLITONS

5.1. Plasma with a non-Maxwellian ion distribution

It is clear that the original dispersion relation Eq. (2.7) is independent of the nature of the particle velocity distribution and therefore remains valid. The same is true for Eq. (2.8) for ε_{xx}. We now elucidate how the expressions for ε_{yy} and ε_{xy} are modified when the particle velocity distribution is non-Maxwellian.

We introduce the notation

$$\bar{\varepsilon}_\perp \equiv \frac{1}{n_0}\int \frac{v_\perp^2}{2} f_0\, d\mathbf{v}, \quad \overline{\varepsilon_\perp^2} \equiv \frac{1}{n_0}\int \frac{v_\perp^4}{4} f_0\, d\mathbf{v}, \tag{5.1}$$

where f_0 is the equilibrium distribution function; the integration is over the particle velocities $\mathbf{v}$. We shall also use the following notation, which is analogous to those introduced in section 2:

$$\beta_i = 2\bar{\varepsilon}_\perp/c_A^2, \quad \rho^2 = \bar{\varepsilon}_\perp/\omega_{Bi}^2, \quad z = k^2\bar{\varepsilon}_\perp/\omega_{Bi}^2. \tag{5.2}$$

We then conclude, turning to the general formula[16] for $\varepsilon_{\alpha\beta}$ that Eq. (2.8) for ε_{xy} remains valid if we take for z the quantity (5.2). As to the expression for ε_{yy} it is modified as follows:

$$\varepsilon_{yy} = \frac{c^2}{c_A^2}\left(1 + \frac{\omega^2}{\omega_{Bi}^2} - \frac{11}{4}z\right) - \frac{c^2k^2}{\omega^2}\left(\beta - \frac{3}{2}\lambda\beta_i z\right), \tag{5.3}$$

where

$$\lambda = \overline{\varepsilon_\perp^2}/2(\bar{\varepsilon}_\perp)^2. \tag{5.4}$$

One gets then a formula for the square of the frequency of the form (2.10) with the following expression for the square of the dispersion length a_D^2 replacing Eq. (2.11):

$$a_D^2 = \frac{\rho^2}{4(1+\beta)}\left[1+\beta_e+\beta_i\left(6\lambda - \frac{7}{2}\right)\right]. \tag{5.5}$$

For a Maxwellian particle velocity distribution $\lambda = 1$. In that case (5.5) goes over into (2.11). However, if all particles, for instance, have a single velocity, $f_0 \propto \delta(v_\perp - v_\perp^0)$ where $v_\perp^0$ is a constant, then $\lambda = \frac{1}{2}$. In that case

$$a_D^2 = \frac{\rho^2}{4(1+\beta)}\left(1+\beta_e - \frac{\beta_i}{2}\right). \tag{5.6}$$

It is clear that $a_D^2 < 0$, if

$$\beta_i > 2(1+\beta_e). \tag{5.7}$$

Then the wave dispersion turns out to be positive, $\omega/k > V$.

Using (5.3) we find that the linear part of the current j_y is characterized by the old formulae (2.12) to (2.14) for j_y, j_y^I, j_y^β and the following expression for j_y^D (cf. (2.15)):

$$j_y^D = -\frac{i\omega c^2 n_0 m_i z}{4B_0^2}\left[1+3(4\lambda-3)\frac{\omega_{Bi}^2}{\omega^2}z\right]. \tag{5.8}$$

As the nonlinear part j_y^{NL} of the current does not depend on the nature of the particle velocity distribution, it is clear that for a modification of the nonlinear Eq. (4.7) it is suffient merely to take into account what is new, which results from the difference between (5.8) and (2.15). This difference consists in a new expression for the square of the dispersion length a_D^2 which is now given by Eq. (5.6).

Of considerable interest is the case $a_D^2 < 0$ corresponding to waves with a positive dispersion. In that case we get from (4.7) a soliton solution of the form (4.8) with h_0 of the form (4.9) and with

$$\varkappa = \frac{1}{\tilde{a}_D^2}\left(\frac{V^2}{u^2} - 1\right)^{1/2}, \quad \tilde{a}_D^2 = -a_D^2. \tag{5.9}$$

Such a solution corresponds to a rarefaction soliton, $n < n_0$. We note that Refs. 3, 6 were the first to discuss the problem of positive dispersion of magnetosonic waves and rarefaction solitons in a plasma with non-Maxwellian ions. Recently this problem was studied by Zhdanov.

5.2. Cold plasma with one ion species

According to (2.11) the dispersion length a_D tends to zero when the ion temperature decreases. However, for sufficiently low ion temperatures Eq. (2.11) becomes inapplicable because in its derivation we neglected the electron motion. We consider how (2.11) is modified when we take the electron inertia into account. We shall then assume that $\beta_i \ll 1$, because the ion temperature is low. Moreover, for the sake of simplicity we assume that $\beta_e \ll 1$.

Under those assumptions we have as before Eq. (2.8) for ε_{xy} and the following expressions for ε_{yy}, ε_{xx}, which replace (2.8):

$$\varepsilon_{yy} = \frac{c^2}{c_A^2}\left(1 + \frac{\omega^2}{\omega_{Bi}^2} - \frac{11}{4}z\right), \quad \varepsilon_{xx} = \frac{c^2}{c_A^2}\left(1+\frac{m_e}{m_i}\right), \tag{5.10}$$

where m_e is the electron mass. The term with m_e/m_i in ε_{xx} is caused by the electron inertia.

Accordingly we get instead of (2.9) the dispersion relation of the linear approximation

$$\omega^2\left(1 + \frac{z}{4} + \frac{\omega^2}{\omega_{Bi}^2}\frac{m_e}{m_i}\right) - c_A^2k^2 = 0. \tag{5.11}$$

Hence it follows that the square of the wave frequency is given by Eq. (2.10) with $V^2 = c_A^2$ and an expression for a_D^2 of the form

$$a_D^2 = \rho^2/4 + c^2/\omega_{pe}^2, \tag{5.12}$$

where $\omega_{pe}^2 = 4\pi n_0 e^2/m_e$ is the square of the electron plasma frequency and e the electron charge.

It is clear that one can neglect electron inertia when

$$\beta_i \gg 8m_e/m_i. \tag{5.13}$$

This is the region where the results of sections 2 to 4 are applicable, corresponding to the case of "hot" dispersion. When (5.13) does not hold, $a_D = c/\omega_{pe}$. In that case we are dealing with "cold" dispersion[17] and the solitons are characterized as before by Eqs. (4.8), (4.9) with $a_D = c/\omega_{pe}$. We note also that in Ref. 2 one has instead of (5.13) the incorrect criterion $\beta > (m_e/m_i)^{1/2}$ obtained through using an incomplete expression for the viscosity tensor (see section 3). The correct criterion (5.13) was obtained in Ref. 5. We note this in connection with the fact that the incorrect criterion was again repeated in Ref. 10.

5.3. Cold plasma with two kinds of ions

From Eq. (5.10) for ε_{xx} it is clear that in a cold plasma the dispersion of low-frequency magnetosonic waves is determined by small corrections to the "standard" value of the dielectric tensor. In the case considered in subsection 5.2 we obtained a term with the cold dispersion in Eq. (5.12) "shifting" the standard expression for ε_{xx} by an amount of the order of m_e/m_i, i.e., taking into account the electron inertia. We can obtain a similar result in the case of a plasma containing an admixture of a different kind of ions (having a different particle charge-to-mass ratio) with a density of the order of m_e/m_i times the density of the main component of the plasma. If, on the other hand, the density of the impurity ions substantially exceeds $(m_e/m_i)n_0$ the electron dispersion will be small compared to that caused by the impurity. We give expressions which confirm this statement.

Let the plasma contain two kinds of ions with densities n_1,n_2, charges e_1,e_2 and masses m_1,m_2. For the sake of simplicity we assume both kinds of ions to be cold, $T\to 0$. Instead of (2.8) we then get the following expressions for the components of the dielectric tensor:

$$\begin{aligned}\varepsilon_{yy}&=\frac{c^2}{c_A^2}+\frac{4\pi\omega^4c^4}{B_0^4}\sum\frac{n_\alpha m_\alpha^3}{e_\alpha^2},\\ \varepsilon_{xy}&=-\varepsilon_{yx}=i\frac{4\pi\omega c^3}{B_0^3}\sum\frac{n_\alpha m_\alpha^2}{e_\alpha},\\ \varepsilon_{xx}&=\frac{4\pi c^2}{B_0^2}\sum n_\alpha m_\alpha,\\ c_A^{-2}&=\frac{4\pi}{B_0^2}\sum n_\alpha m_\alpha.\end{aligned} \tag{5.14}$$

The summation is here over the kinds of particles $\alpha = 1,2$. Similarly to (5.12) the square of the dispersion length is now given by the relation

$$a_D^2=\frac{c^2n_1n_2m_1m_2}{4\pi\left(\sum n_\alpha m_\alpha\right)^3}\left(\frac{m_1}{e_1}-\frac{m_2}{e_2}\right). \tag{5.15}$$

For small impurity ion density, $n_2 \ll n_1$, it follows from this that

$$a_D^2=\frac{c^2}{\omega_{pi}^2}\frac{n_2}{n_1}\frac{m_2}{m_1}\left(1-\frac{e_1}{m_1}\frac{m_2}{e_2}\right), \tag{5.16}$$

where $\omega_{pi}^2 = 4\pi n_1 e_1^2/m_1$ is the square of the plasma frequency of the main ion component. In particular, in the case of an electron-proton plasma containing a small fraction of α-particles ($m_2 = 4m_1$, $e_2 = 2e_1$), Eq. (5.5) means that

$$a_D^2=4\frac{n_2}{n_1}\frac{c^2}{\omega_{pi}^2}. \tag{5.17}$$

From a comparison of (5.16) and (5.12) it is clear that in agreement with the discussions given above the impurity dispersion is more important than that caused by the electrons, if

$$\left(1-\frac{e_1}{m_1}\frac{m_2}{e_2}\right)^2\frac{n_2}{n_1}>\frac{m_e}{m_i}. \tag{5.18}$$

Comparing (5.16) with (5.12) we note that the impurity dispersion is more important than the hot dispersion (caused by the term with ρ^2) if

$$n_2/n_1 \gg \beta_i. \tag{5.19}$$

For thermonuclear reactors there is interest in a plasma containing deuterium and tritium. In that case $n_2 = n_1 = n_0/2$, $e_1 = e_2$, $m_2 = 3m_1/2$ so that Eq. (5.15) takes the form

$$a_D^2=\frac{1}{25}\frac{c^2}{\omega_{pi}^2},\quad \omega_{pi}^2=\sum\frac{4\pi e_\alpha^2 n_\alpha}{m_\alpha}=20\pi\frac{e_1^2n_0}{3m_1}. \tag{5.20}$$

The presence of a small numerical factor in Eq. (5.20) for a_D is explained, firstly, by the relatively small difference in the deuterium and tritium masses and, secondly, by the strong dependence of the dispersion on the magnitude of the Alfvén speed.

According to (5.15) $a_D^2 > 0$, i.e., the wave dispersion caused by the presence of two kinds of ions is negative (see section 2) and the corresponding solitons must be compression solitons (see section 4).

5.4. Hot plasma with a component of fast particles

According to the general expressions (2.8) for the components of the dielectric permittivity tensor the components ε_{xy}, ε_{yx} and some of the terms in ε_{yy} are responsible for the dispersion of the low-frequency magnetosonic oscillations. The terms in ε_{yy} responsible for the hot dispersion depend on the pressure and the temperature derivative of the pressure of the ion component of the plasma (the third and fifth terms in the expression for ε_{yy}, respectively). The analogous term in ε_{xy} corresponds to the ion pressure. As the dielectric permittivity tensor for a plasma with several kinds of ions is an additive quantity one may expect that the presence in the plasma of another kind of ions with a pressure comparable to the pressure of the main ions in the plasma will give rise to an appreciable increase in the dispersion.

For impurity particles with a temperature considerably higher than the temperature of the main particles in the plasma this effect may be important even for a low impurity density. Such high-energy particles may arise when neutrals are injected into the plasma, in hf heating or as the result of thermonuclear reactions.

We consider a plasma containing an admixture of hot particles with a temperature $T=\gamma T_i$, a density $n=\alpha n_i$, particle mass $m=\sigma m_i$, where T_i, n_i, m_i are the temperature, density, and ion mass of the main plasma component; γ, α, σ are some dimensionless coefficients. Bearing in mind that in actual cases we shall be interested in the case of low impurity densities we shall assume that $\alpha \ll 1$. For the sake of simplicity we assume that the impurity ions are singly charged and that their distribution function is Maxwellian. The correction to the ε_{yy}-component of the dielectric permittivity tensor caused by the impurity particles will then have the form

$$\delta\varepsilon_{yy}=-\alpha\gamma\frac{c^2k^2}{\omega^2}\left(\beta_i-\frac{3}{2}\sigma\gamma z\beta_i\right), \tag{5.21}$$

where z,β_i are defined in section 2 and refer to the main ions in the plasma. The corrections to the other components of the dielectric permittivity tensor caused by the admixture are unimportant. We then find that the square of the frequency of the low-frequency magnetosonic oscillations is given by an expression similar to (2.10) with an Alfvén speed

$$V^2=c_A^2(1+\beta+\alpha\gamma\beta_i) \tag{5.22}$$

and a dispersion length

$$a_D^2=(\rho^2/4)[1+\beta_e+(5\beta_i/2)(1+2\alpha\gamma/5+3\alpha\gamma^2\sigma/5)]\times(1+\beta+\alpha\gamma\beta_i)^{-1}. \quad (5.23)$$

When

$$2\alpha\gamma/5+3\alpha\gamma^2\sigma/5>1 \quad (5.24)$$

according to these equations the dispersion caused by the fast impurity ions will be larger than the dispersion determined by the main component.

6. GYRORELAXATION DAMPING OF LOW-FREQUENCY MAGNETOSONIC SOLITONS

We neglected so far collisions between particles in the study of MSS. We now show that weak ion-ion collisions, $\nu<\omega$ (ν is the collision frequency and ω a characteristic frequency of the magnetosonic waves) leads to a damping of the MSS. We neglect here the ion-electron collisions.

We shall work in the framework of the two-dimensional equations. We introduce the perturbed part $\tilde{p}^\nu$ of the transverse pressure caused by the collisions. We obtain an equation for $\tilde{p}^\nu$ by integrating the kinetic equation with a weight corresponding to the transverse pressure, taking into account the ion-ion collisions on the right-hand side. The resulting equation will differ from (3.3) by the presence of a dissipative term caused by the collisions. The equation for $\tilde{p}^\nu$ will then have the form

$$\frac{\partial \tilde{p}^\nu}{\partial t}+\frac{1}{5}\tilde{p}^A\nu=0. \quad (6.1)$$

We note that $\tilde{p}^\nu$ is the diagonal part of the three-dimensional viscosity tensor and one can obtain Eq. (6.1) by using general equations of the type (A.1) for the three-dimensional viscosity tensor, taking from them the diagonal part which is independent of the magnetic field. Then solving these equations by expanding in the small parameter ν/ω find in lowest order the adiabatic part of the perturbed pressure, and in the next approximation $\tilde{p}^\nu$.

Using Eq. (6.1), which is analogous to (3.13), we get an expression for the dissipative current (i.e., that part of the current which is caused by $\tilde{p}^\nu$):

$$j^\nu=(c/B_0)\partial\tilde{p}^\nu/\partial x. \quad (6.2)$$

Taking this part of the current into account in the Maxwell equations (see section 4) we get the following nonlinear equation for the low-frequency magnetosonic solitons (cf. (4.7)):

$$\frac{2}{u^2}\frac{\partial h}{\partial\tau}-\left(1-\frac{V^2}{u^2}\right)\frac{\partial h}{\partial\xi}+\frac{3}{2}\frac{\partial h^2}{\partial\xi}+a_D^2\frac{\partial^3 h}{\partial\xi^3}+\frac{2}{5}\frac{c_A^2}{u^2}\frac{\beta_i\nu}{u}h=0. \quad (6.3)$$

Assuming the dissipative term in this equation to be small compared to the nonlinear and dispersive terms we solve it by the method of successive approximations. This is equivalent to an adiabatic treatment of the soliton damping[17] which is valid when $\nu<(h_0/\beta_i)^{3/2}\omega_{Bi}$. We write the solution of Eq. (6.3) in the form $h=h^{(0)}+h^{(1)}$ where $h^{(1)}$ is caused by the collisions. We then have in lowest order for $h^{(0)}$ an equation which is similar to (4.7) in the stationary case:

$$\left(1-\frac{V^2}{u^2}\right)\frac{\partial h^{(0)}}{\partial\xi}+\frac{3}{2}\frac{\partial(h^{(0)})^2}{\partial\xi}+a_D^2\frac{\partial^3h^{(0)}}{\partial\xi}=0. \quad (6.4)$$

This equation has a solution which is similar to (4.8):

$$h^{(0)}=h_0^{(0)}/\mathrm{ch}(\varkappa\xi/2), \quad (6.5)$$

where $h_0^{(0)}$ and $\varkappa$ are connected through Eq. (4.9) and are slowly varying functions of the time. In the next approximation we then get for $h^{(1)}$ the equation

$$\frac{2}{u}\frac{\partial h^{(0)}}{\partial\tau}+\frac{2}{5}\frac{c_A^2}{u^2}\frac{\beta_i\nu}{u}h^{(0)}=\left(1-\frac{V^2}{u^2}\right)\frac{\partial h^{(1)}}{\partial\xi}-3\frac{\partial}{\partial\xi}h^{(0)}h^{(1)}-a_D^2\frac{\partial^3h^{(1)}}{\partial\xi^3}. \quad (6.6)$$

Using the condition that the first and the zeroth approximations must be orthogonal we get the required equation for the amplitude of the damped soliton:

$$\frac{\partial h_0^{(0)}}{\partial\tau}+\frac{1}{5}\frac{c_A^2}{u^2}\beta_i\nu h_0^{(0)}=0. \quad (6.7)$$

Hence

$$h_0^{(0)}(\tau)=h_0^{(0)}(0)\exp(-\tau/\tau_0), \quad \tau_0=5u^2/c_A^2\beta_i\nu. \quad (6.8)$$

It is interesting to note that when $\beta_i\gg1$ the characteristic damping time for the soliton is $\tau_0\sim1/\nu$.

The effect described by the last term in the nonlinear Eq. (6.3) is analogous to the gyrorelaxation effect in linear oscillations. For transverse perturbations of the plasma the longitudinal magnetic field is perturbed and an anisotropy in the particle distribution function then occurs. The collisions tend to equalize the energies of the transverse and the longitudinal motion and the original perturbation then weakens. This mechanism leads to damping of the soliton. We therefore call this effect the gyrorelaxation damping of MSS.

The off-diagonal parts of the collisional viscosity tensor also contribute to the dissipation. In the nonlinear equation for the MSS the off-diagonal parts could correspond to terms with a second derivative of the soliton amplitude with respect to the coordinate. However, in the low-frequency case the off-diagonal parts of the collisional viscosity tensor are corrections, being of order $(\omega/\omega_{Bi})^2$ relative to the diagonal ones.

Only the off-diagonal parts of the collisional viscosity tensor were taken into account in Ref. 2. This is the reason for the incorrect conclusion reached by the authors of Ref. 2 that weak collisions lead to the formation of a magnetosonic shock wave.

7. DISCUSSION OF THE RESULTS

From the analysis given here it is clear that the key to constructing a theory of solitons of low-frequency magnetosonic waves is an adequate description of the dispersion of such waves. Because the dispersion is a small effect of order $k^2\rho^2$ it is necessary to use for their description equations which take corresponding small terms into account. Such equations can be obtained both through kinetic and through hydrodynamic approaches. We elucidated the structure of the equations which are the basis of these two approaches and showed that both lead to the same results.

In contrast to the dispersion the nonlinearity of low-frequency magnetosonic waves is a rather coarse effect. The simplest and most natural way to describe the nonlinearity is thus in the framework of hydrodynamics. On the other hand, due to its specific structure (particularly in a plasma with impurities) dispersion is more fully described in the framework of the kinetic approach. The most effective approach therefore turns out to be the combined kinetic-hydrodynamic one suggested in the present paper in which the dispersion part of the current is determined from kinetics and the nonlinear part from hydrodynamics. Such an approach is also fruitful in studying how dissipative processes affect soliton dynamics.

For a description of the dispersion of low-frequency magnetosonic waves in the framework of the hydrodynamic approach we obtained new expressions for the viscosity tensor and the heat flux which are, respectively the sum of the transverse (magnetic) viscosity and the oblique heat flux in Braginskiĭ's form and terms of order $k^2\rho^2$. When using the kinetic approach we obtained with the necessary accuracy expressions for the components of the dielectric tensor which describe the dispersion of low-frequency magnetosonic waves.

Moreover, we have studied in the present paper special cases of MSS, among them some not previously discussed. We have showed that the dispersion of low-frequency magnetosonic waves is sensitive to details of the plasma ion distribution function. Under certain conditions the dispersion of a given kind of ions may be positive and, hence, in such a plasma the existence of rarefaction MSS is possible (for details see subsection 5.1). We have shown that the dispersion of a cold plasma is determined by the finite ratio of the electron to the ion mass in the case of one kind of ions, whereas in a plasma with an admixture of ions with a different charge-to-mass ratio the dispersion may be appreciably larger. We note that an admixture of ions with a pressure comparable to the pressure of the main ion component may considerably affect the hot dispersion. This may, for instance, be important for the case of an admixture of thermonuclear α-particles.

In studying the effect of ion-ion collisions on the dynamics of MSS we showed that the structure of the dissipative terms depends on the ratio of the collision frequency ν and the characteristic frequency ω of the magnetosonic waves. Taking the dissipation into account in the case of weak collisions, $\nu < \omega$, leads to a gyrorelaxation damping of the MSS, whereas taking dissipation into account in the form of a transverse collisional viscosity for the case of strong collisions, $\nu > \omega$, leads to the formation of a magnetosonic shock wave.

APPENDIX

Derivation of expressions for the viscosity tensor and for the heat flux in the case of a "two-dimensional" collisionless plasma.

To describe magnetosonic waves propagating in a collisionless plasma across the magnetic field one can use the set of two-dimensional hydrodynamic equations obtained by Grad's method[19] in Ref. 18. For us the equations from that set for the viscosity tensor $\hat{\pi}\equiv\pi_{\alpha\beta}$, where $(\alpha,\beta) = (x,y)$ and for the heat flux $\mathbf{q}\equiv(q_x,q_y,0)$ are the important ones. With the accuracy which we need these equations are

$$\frac{\partial\pi_{xx}}{\partial t}+p\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_y}{\partial y}\right)+\frac{1}{2}\left(\frac{\partial q_x}{\partial x}-\frac{\partial q_y}{\partial y}\right)-2\omega_B\pi_{xy}=0,$$

$$\frac{\partial\pi_{xy}}{\partial t}+p\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}\right)+\frac{1}{2}\left(\frac{\partial q_y}{\partial x}+\frac{\partial q_x}{\partial y}\right)+2\omega_B\pi_{xx}=0, \quad (A.1)$$

$$\frac{\partial\mathbf{q}}{\partial t}+\frac{2p}{m}\nabla T+\frac{T}{m}\nabla\hat{\pi}-[\mathbf{q}\omega_B]=0. \quad (A.2)$$

Here $(\nabla\hat{\pi})_\alpha\equiv\partial\pi_{\alpha\beta}/\partial x_\beta$, $\boldsymbol{\omega}_B=\mathbf{e}_z\omega_B$, $\mathbf{e}_z$ is a unit vector along z. We use the fact that $\pi_{yy}=-\pi_{xx}$, $\pi_{xy}=\pi_{yx}$.

We solve Eqs. (A.1), (A.2) by expanding in series in $1/\omega_B$. We then get

$$\hat{\pi}=\hat{\pi}^{(0)}+\hat{\pi}^{(1)},\quad \mathbf{q}=\mathbf{q}^{(0)}+\mathbf{q}^{(1)}. \quad (A.3)$$

The quantities with index zero have a form analogous to that in Ref. 11:

$$\pi_{xx}^{(0)}=-\frac{p}{2\omega_B}\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}\right),$$

$$\pi_{xy}^{(0)}=\frac{p}{2\omega_B}\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_y}{\partial y}\right), \quad (A.4)$$

$$\mathbf{q}^{(0)}=\frac{2p}{m\omega_B}[\mathbf{e}_z\nabla T]. \quad (A.5)$$

The correction terms indicated by the index one are given by the expressions

$$\pi_{xx}^{(1)}=-\frac{1}{2\omega_B}\left[\frac{\partial\pi_{xy}^{(0)}}{\partial t}+\frac{1}{2}\left(\frac{\partial q_y^{(0)}}{\partial x}+\frac{\partial q_x^{(0)}}{\partial y}\right)\right],$$

$$\pi_{xy}^{(1)}=\frac{1}{2\omega_B}\left[\frac{\partial\pi_{xx}^{(0)}}{\partial t}+\frac{1}{2}\left(\frac{\partial q_x^{(0)}}{\partial x}-\frac{\partial q_y^{(0)}}{\partial y}\right)\right] \quad (A.6)$$

$$\mathbf{q}^{(1)}=\frac{1}{\omega_B}\left[\mathbf{e}_z,\frac{\partial}{\partial t}\mathbf{q}^{(0)}+\frac{T}{m}\nabla\hat{\pi}^{(0)}\right]. \quad (A.7)$$

Equations (3.5), (3.6), (3.8), (3.9) follow from (A.3) to (A.7).

Equations (A.6) describe the so-called "inertial" viscosity[18] and the viscosity caused by the heat flux. These equations were first obtained in Ref. 18. Similarly (A.7) describes the "inertial" heat flux and the heat flux caused by the viscosity. In Ref. 18 only the second of these parts of the heat flux was taken into account (see Eq. (1.10) of Ref. 18). In the problem of magnetosonic waves which we have studied the role of both terms on the right-hand side of (A.7) turns out to be the same (see the change from (3.6) to (3.22)).

We note that taking the heat flux into account is important only when $\beta\sim 1$. When $\beta\ll 1$ Eqs. (A.5),(A.7) turn out to be unnecessary and in the right-hand side of Eq. (A.6) it is sufficient to retain solely the terms with $\partial\pi_{\alpha\beta}/\partial t$.

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