Anomalies in the magnetotransport properties of a two-dimensional electron gas in silicon metal–oxide–semiconductor structures in high magnetic fields

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The electron concentration dependences of the magnetoresistance tensor components ρ_{xx} and ρ_{xy} and also of the magnetoconductivity have been studied in silicon metal–oxide–semiconductor (MOS) structures in a magnetic field ($H \leqslant 20$ T) perpendicular to the two-dimensional electron layers at low temperatures ($T = 1.5$ K). Some features were found which correspond to fractional values of the filling factor, $\nu = 1/3, 2/3, 4/3, 5/3, 7/3, 8/3, 4/5, 6/5$. It is shown that for observation of these features, besides strong magnetic fields and low temperatures, a sufficiently high mobility, μ, of the two-dimensional electrons is required; it is also important that the measuring current or drain-source bias field under the experimental conditions do not exceed 10^{-7} A and 10^{-3} V·cm^{-1} respectively. Anomalies in the transport properties of a two-dimensional electron gas were observed both in MOS structures with a rectangular geometry as well as in structures with a Corbino geometry. It was found that in Si MOS structures at $T = 1.5$ K, fractions with a denominator 3 are observed at $\mu H > 36$ and those with a denominator 5 at $\mu H > 55$.

§1. INTRODUCTION

There is much interest in studying the properties of a two-dimensional electron gas, which has recently grown appreciably with the discovery of the quantum Hall effect (QHE) by von Klitzing *et al.*[1] In essence, this effect consists in the fact that at a sufficiently low temperature in a high magnetic field perpendicular to the two-dimensional electron layer ($H \parallel z$), the components of the magnetoresistance tensor take the values

$$\rho_{xx}=0, \tag{1}$$

$$\rho_{xy}=h/e^2\nu, \tag{2}$$

where $\nu = n_s h/eH$ is the filling factor of the Landau sublevels, n_s is the concentration of two-dimensional electrons, h is Planck's constant and e is the electronic charge. It is remarkable that these relations are not only valid at a single point in electron concentration or magnetic field, but in a certain fairly wide range of these values near the place where the condition $n_s = ieH/h$ (i is an integer) is satisfied, i.e., for complete filling of the highest occupied Landau sublevel. In reality, of course, Eqs. (1) and (2) are not satisfied exactly in an experiment. Instead of this, deep minima are observed in the $\rho_{xx}(n_s)|_H$ [or $\rho_{xx}(H)|_{n_s}$] relation, at which the values of ρ_{xx} decrease by 4–6 orders of magnitude on decreasing the temperature. At the same time, the $\rho_{xy}(n_s)|_H$ [or $\rho_{xy}(H)|_{n_s}$] plot has something like a plateau, on which the deviation of ρ_{xy} from the constant value $h/e^2\nu$ can be 10^{-6}–10^{-7} and decreases as the temperature is lowered.

Until now the main features of the integral QHE have been satisfactorily explained within the framework of the theory of strong localization (of the Anderson–Mott type) in the wings of the Landau levels.[2,3] The QHE has been observed in electron and hole channels both in metal–oxide–semiconductor (MOS) structures and in heterojunctions.[4–6]

Two years after the discovery of the QHE, Tsui *et al.*[7] observed fractional QHE in AsGa–AsGaAl heterojunctions, differing from the normal in that the ρ_{xy} plateau and the ρ_{xx} minimum were not only observed for integral values of the filling factor ν, but also for fractional values ($\nu = 1/3$, 2/3 were found in the first publication.[7]). It is characteristic that fractional QHE is only observed in heterojunctions having an enhanced carrier mobility (μ). For example, fractional QHE in the electron channel in heterojunctions is observed for $\mu_e > 10^5$ cm^2·V^{-1}·s^{-1}, and in the case of the hole channel for $\mu_h > 3\times10^4$ cm^2·V^{-1}·s^{-1} (Ref. 8). A large number of fractional values of ν have been found so far in heterojunctions, at which anomalies in the magnetotransport properties are observed, namely $\nu = 1/3, 2/3, 4/3, 5/3, 2/5, 3/5, 4/5, 2/7, 3/7, 4/7$ (Ref. 8). These anomalies, as distinct from the integral QHE, are undoudtedly related to the effect of interelectron interaction. It was thus originally suggested that they are a result of ordering in the electron system—the formation of a Wigner crystal or of a charge density wave.[7] However, within the framework of these ideas, the observation of fractional values of ν only with odd numerators remains incomprehensible. The experimental fact, on the contrary, finds an explanation in the Laughlin theory,[9] which explains the experimental features in terms of an incompressible Fermi liquid. In this theory the odd integer values of the numerator are a direct consequence of the antisymmetry of the wave function of the electron system. However, in spite of the appearance of the Laughlin theory, fractional QHE remains an incompletely explained phenomenon, and for its understanding investigations on different systems with a different nature and scale of interelectron interaction are quite essential. Studies of the fractional QHE in silicon MOS structures, recently found,[10,11] are thus urgent. One of the important merits of silicon MOS structures, in particular, compared with heterojunctions is the possibility of controlling the two-dimensional electron (hole) gas density within fairly wide limits by means of the gate voltage, and

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TABLE I.

No.	Geometry	d (Å)	V_T, V	L, mm	l, mm	$\tilde{\mu}$(4,2 K), 10^3 cm²/V⁻¹·s⁻¹	$\tilde{\mu}$(1,5 K), 10^3 cm²/V⁻¹·s⁻¹	n_s^0 (1,5 K) 10^{11} cm⁻²
1	Rectangular	1600	−0,2	1,2	0,4	40	52	2
2	Rectangular	1800	−0,05	2,5	0,28	30	36	3
3	Corbino	1300	0,05	0,22	1,41	23	28	4,5
4	Rectangular	1800	−0,1	2,5	0,28	13	13	6

also the large size of the effective Rydberg, which characterizes the Coulomb interaction.

In the present work, which is an extension of the work of Gavrilov *et al.*[11] on several silicon MOS structures, in which the mobility of the two-dimensional electrons exceeded 2.8×10^4 cm²·V⁻¹·s⁻¹, in very high magnetic fields, the dependences of the components ρ_{xx} and ρ_{xy} of the magnetoresistance tensor (specimens with rectangular geometry), and also of the conductivity σ_{xx} (specimen with Corbino geometry), on the density of the two-dimensional electrons have been studied, and anomalies have been found at fractional values of the filling factor: 1/3, 2/3, 4/3, 5/3, 7/3, 8/3, 4/5, 6/5.

§2. EXPERIMENTAL METHOD AND MOS STRUCTURES

Starting from the fact that the fractional QHE in heterojunctions was only observed on structures with high carrier mobility, we chose specimens according to just this parameter. In characterizing a large number of MOS structures prepared on the (100) surfaces of p type Si, it can be noted that the majority of them had a maximum mobility $\tilde{\mu}$ in the range $(13\text{–}20)10^3$ cm²·V⁻¹·s⁻¹ with a maximum for $n_s^0=(6\text{–}8)\times10^{11}$ cm⁻², with the position of the maximum and the magnitude of $\tilde{\mu}$ depending very weakly on temperature for $T<4.2$ K. A fractional QHE was not observed with such structures for any realizable experimental conditions.

We were able to select three specimens with unusually high (for MOS structures) electron mobility, in which we observed anomalies in magnetrotransport properties for fractional values of the filling factor. The chief parameters of these structures are given in Table I and the $\mu(n_s)$ plots for

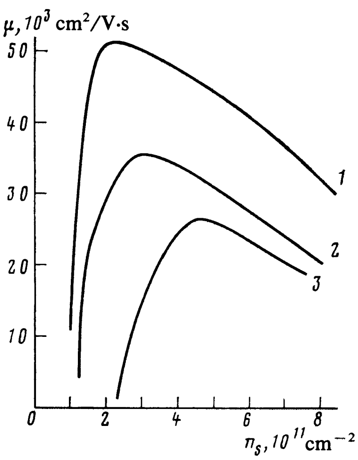

FIG. 1. The concentration (n_s) dependence of the mobility of two-dimensional electrons μ in different silicon MOS structures (Nos. 1, 2 and 3) at $T=1.5$ K.

$T=1.5$ K are shown in Fig. 1. [The following symbols are used in Table I: d is the SiO_2 thickness, V_T the threshold voltage, L the distance between drain and source, l the gate width (in the Corbino geometry $l=\pi(R_1+R_2)$, where R_1 and R_2 are the external and internal radii of the ring gate), $\tilde{\mu}$ is the maximum value of the mobility, realized for $n_s=n_s^0$]. It is a characteristic feature that the values of the Hall mobility and of the mobility determined from the magnitude of the conductivity, coincided. The distinguishing features of MOS structures with high electron mobility are: first, a shift in the mobility maximum in the direction of smaller concentrations, down to $n_s^0=2.6\times10^{11}$ cm⁻² (Fig. 1); second, an appreciably stronger dependence of the mobility on the lattice temperature for $T<4.2$ K (see Table I); third, an unusually high sensitivity to heating by the drain-source bias electric field E. The last feature is illustrated in Fig. 2 in which the dependences of the maximum mobility on bias field E at $T=1.5$ K are shown. It can be seen that unlike the usual situation, in which noticeable heating occurs for $E\geqslant10^{-1}$ V·cm⁻¹ (Ref. 2), heating already had an effect for $E\approx10^{-2}$ V·cm⁻¹ in specimens with high electron mobility. For example, in a magnetic field $H=6$ T, noticeable heating of the electron system in specimen No. 3, estimated from the amplitude of the Shubnikov oscillations ($\Delta T\approx0.2$ K), occurred for $E=10^{-2}$ V·cm⁻¹. We estimated the electron energy relaxation time in our structures and found that at $T=1.5$ K and $n_s=(3\text{–}5)\times10^{11}$ cm⁻², the energy relaxation time $\tau_\varepsilon\sim3\times10^{-7}$ s. This value is almost an order of magnitude greater than values known in the literature.[12,13]

Because in our MOS structures heating up of the electron system occurred at a relatively small power introduced by the bias field, the measuring current for measurements in a magnetic field on structures with rectangular geometry (Nos. 1 and 2) was $I_0\leqslant10^{-8}$ A, and the bias voltage for the Corbino geometry (No. 3) was $E\leqslant10^{-3}$ V·cm⁻¹. It was only under these conditions that the current-voltage (I-V) charac-

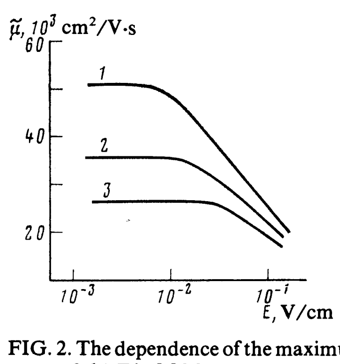

FIG. 2. The dependence of the maximum electron mobility $\tilde{\mu}$ on the drain-source field E in MOS structures (Nos. 1, 2 and 3) at $T=1.5$ K.

teristics became linear and heating of the electron system, consequently, was absent.

The measurements were made in the standard way[14] both for a steady and an alternating current at frequencies from 10 to 150 Hz and always gave identical results. It was more convenient to work under alternating current conditions for small values of $E\,(<10^{-3}\ \mathrm{V{\cdot}cm^{-2}})$ and $I_0(<10^{-9}\ \mathrm{A})$.

In order to check on the equilibrium state of the two-dimensional electron system, we studied the ρ_{xx} and ρ_{xy} dependences on different sections of the MOS structure, using five potential contacts positioned on both sides of the transistor. We may note that we did not observe any appreciable difference in the dependences taken from different sections of the structure. The experiments were carried out at fixed magnetic field. The gate voltage sweep was carried out slowly enough for possible hysteresis in the $\rho_{xx}(n_s)$, $\rho_{xy}(n_s)$, $\sigma_{xx}(n_s)$ relations, associated with charging up processes, to be totally eliminated.

§3. ANOMALIES IN THE VARIATIONS OF THE COMPONENTS OF THE MAGNETORESISTANCE TENSOR $\rho_{xy}(n_s)$ and $\rho_{xx}(n_s)$

MOS structures with rectangular geometry, Nos. 1, 2 and 4 were studied to determine the values of the components ρ_{xx} and ρ_{xy} of the magnetoresistance tensor. The dependences of the voltage between the Hall (u_{xy}) and potential (u_{xx}) contacts on the gate voltage V_g, measured from the threshold voltage V_T, were studied at a given measuring current $I_0<10^{-8}$ A, and from this the $\rho_{xx}(n_s)$, $\rho_{xy}(n_s)$ dependences can be obtained:

$$\rho_{xy}=R_H=u_{xy}/I_0, \quad (3)$$

$$R_x=u_{xx}/I_0, \quad (4)$$

$$\rho_{xx}=R_x l/a, \quad (5)$$

$$n_s=(V_g-V_T)\,\varepsilon\varepsilon_0/de, \quad (6)$$

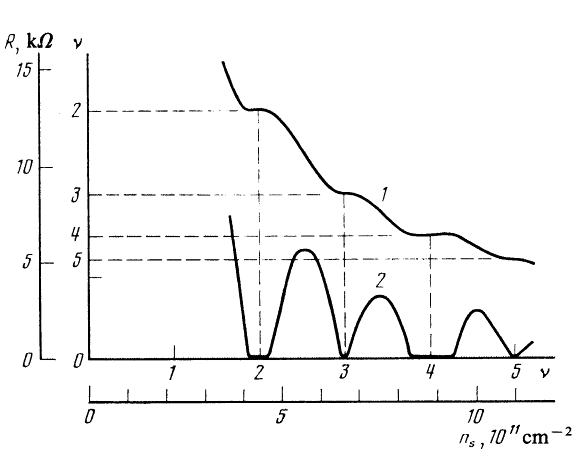

FIG. 3. The dependence of Hall resistivity ρ_{xy} (curve 1) and resistivity ρ_{xx} (curve 2) on the concentration of two-dimensional electrons, obtained on MOS structure No. 4 at $T=1.7$ K, $H=9$ T, $I_0=100$ nA. The values of the filling factor ν are shown both on the abscissa axis and also on the ordinate axis. The absolute values of the resistivity ρ_{xx}, shown in the figure, are multiplied by 10.

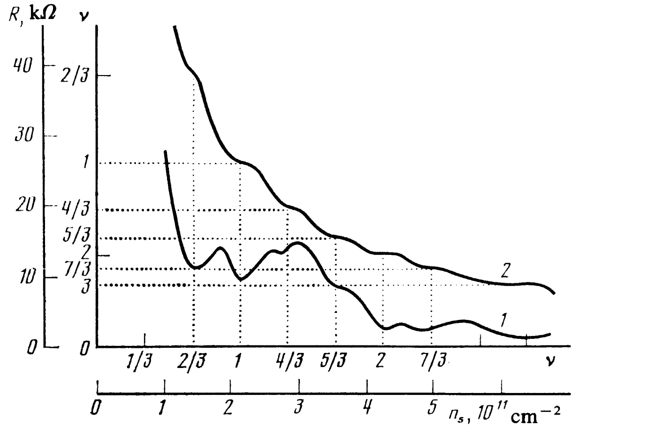

FIG. 4. The dependence of Hall resistivity ρ_{xy} (curve 2) and resitivity ρ_{xx} (curve 1) on the concentration of two-dimensional electrons for MOS structure No. 1 at $T=1.7$ K, $H=8.8$ T, $I_0=10$ nA. The positions along the abscissa and ordinate axes of some fractional and integral values of the filling factor ν are shown dotted. The values of ρ_{xx} are multiplied by 15.

where ε and ε_0 are the dielectric permittivities of the SiO_2 and vacuum, d is the thickness of the SiO_2 layer, l is the gate width and a is the distance between the potential contacts.

The clearest illustration of the integral QHE is provided by the $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ relations obtained on specimens with $\tilde{\mu}=(0.7\text{–}1.5)\times10^4\ \mathrm{cm^2{\cdot}V^{-1}{\cdot}s^{-1}}$. The broadest plateaus in the ρ_{xy} dependences are observed in just this region of values of μ, with $\rho_{xy}=h/e^2\nu$, and also the widest and deepest minima in the $\rho_{xx}(n_s)$ dependence for integral values of the filling factor. Typical $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ plots for $T=1.7$ K, $H=9$ T on MOS structure No. 4 in which $\tilde{\mu}=13\times10^3\ \mathrm{cm^2{\cdot}V^{-1}{\cdot}s^{-1}}$ are shown in Fig. 3. It is characteristic that the most clearly expressed features are observed for filling factors which are multiples of 4 ($\nu=4$, 8, 12, etc.) for complete filling of a Landau level, which contains four sublevels as a result of there being a degenerate electron spectrum in Si(100)—doubly for the valleys and doubly in electron spin. Since the cyclotron energy is appreciably more than the paramagnetic and intervalley splitting energies, the energy gap in the electron density of states is largest for $\nu=4$. The smallest gap in the density of states corresponds to the intervalley splitting and is realized for odd ν. As a result, all the features at $\nu=1, 3, 5,\ldots$ are least pronounced. Features for $\nu=2, 6, 10,\ldots$ correspond to paramagnetic splitting.

Figure 4 shows $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ plots at 8.8 T and $T=1.7$ K, measured on MOS structure No. 1, in which the largest electron mobility was achieved (see Table I and Fig. 1). It can be seen, above all, that all the integral QHE features are, naturally, present in this case, but do not appear as clearly as for specimen No. 4. In addition, because of the relatively narrow electron concentration region of strong localization in structure No. 1 even for $H=8.8$ T, the $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ dependences could be studied up to $\nu=0.5$. It can be seen from Fig. 4 that besides the $\rho_{xx}(n_s)$ minima and $\rho_{xy}(n_s)$ plateaus at integral ν, additional anomalies are observed—minima in $\rho_{xx}(n_s)$ and inflections in $\rho_{xx}(n_s)$ for non-integer ν. It is characteristic that the additional features for $H=8.8$ T

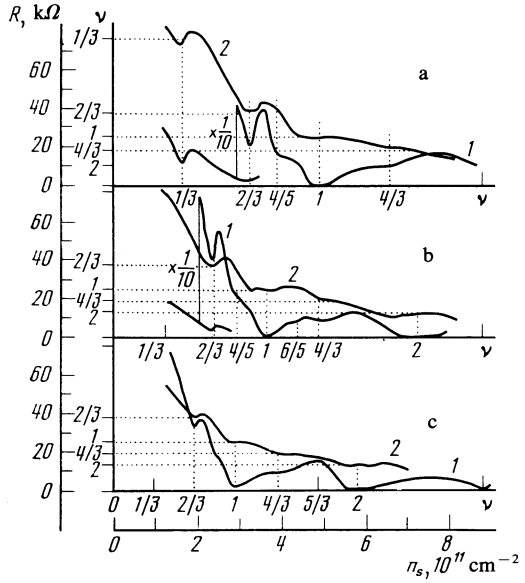

FIG. 5. Plots of $\rho_{xy}(n_s)$ (curve 2) and $\rho_{xx}(n_s)$ (curve 1) obtained on the electron channel of MOS structure No. 2 at $T = 1.5$ K, $I_0 = 10$ nA in different magnetic fields: a) $H = 20$ T, b) $H = 15$ T, c) $H = 12$ T. The absolute values of ρ_{xx} are multiplied by 15. The positions of some integral and fractional values of the filling factor ν are shown along the abscissa and ordinate axes.

and $T = 1.7$ K are observed exclusively in the regon of the anomalously high electron mobility—for 1.5×10^{11} cm$^{-2} < n_s < 5 \times 10^{11}$ cm^{-2}, i.e., for $\mu_e > 4 \times 10^4$ cm^2·V^{-1}·s^{-1} (see Figs. 1 and 4).

There are three independent means of determining the filling factor at points where the anomalies in magnetotransport properties are observed. The values of ν can be found, first, from the absolute value of R_H, using Eq. (2), and also from the positions, on the concentration scale, of the singularities on the $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ plots, since $\nu = n_s h/eH$. All three methods gave practically the same value of ν for each singularity, which is shown by the dotted lines in Fig. 4 (and 5).

The following fractional values of the filling factor ν were thus found for MOS structure No. 1 for $T = 1.7$ K and $H = 8.8$ T: 2/3, 4/3, 5/3 and 7/3, observed in such a region of concentration of two-dimensional electrons at which $\tilde{\mu} > 40 \times 10^3$ cm^2·V^{-1}·s^{-1}. The fractions 2/3, 4/3, 5/3 and 6/5 were found[11] for the same specimen in a magnetic field of 11 T. Unfortunately, structure No. 1 went out of order before it became possible to study the magnetotransport properties in magnetic fields up to 20 T.

As can be seen from Fig. 1, the electron mobility in MOS structure No. 2 did not exceed 36×10^3 cm^2·V^{-1}·s^{-1}, so that it is not surprising that no features were observed for $H = 10$ T for fractional values of ν. Plots of $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ in magnetic fields of 12, 15 and 20 T are shown in Fig. 5. It can be seen that singularities at fractional values of the filling factor ($\nu = 2/3$, 4/3) only appear for $H \geqslant 12$ T, and in the region of n_s for which the electron mobility is a maximum and exceeds $\mu = 30 \times 10^3$ cm^2·V^{-1}·s^{-1}. The characteristics of the features observed in the $\rho_{xx}(n_s)$ plot for fractional and integral values of the filling factor then differ appreciably. While the $\rho_{xy}(n_s)$ dependence is very weak and is a monotonic function of n_s in the region of integral ν, a nonmonotonic form of $\rho_{xy}(n_s)$ behavior is clearly observed for fractional ν. It is not impossible that this last fact is associated with additions from ρ_{xx} and ρ_{xy} (Refs. 15, 16), but it is possible that the observed differences are a manifestation of the fact that different physical causes lead to anomalies for integral and fractional ν.

As can be seen from Fig. 5, with an increase in magnetic field in the region of maximum electron mobility, the singularities for $\nu = 2/3$ and 4/3 become more significant and, in addition, new anomalies appear for $\nu = 4/5$, 6/5 ($H = 15$ T) and for $\nu = 1/3$ ($H = 20$ T). It is of interest to trace the value of the minimum electron mobility μ^*, starting from which anomalies at fractional ν are observed, as a function of magnetic field at a fixed temperature. Unlike the case of heterojunctions, this can be done comparatively easily for MOS structures, since at fixed H the two-dimensional electron concentration can be changed within wide limits, and the values of n_s^* and $\mu^* \equiv \mu(n_s^*)$ (see Fig. 1) at which anomalous magnetotransport properties corresponding to fractional ν arises and disappear can be determined. It is possible to find the values of μ^* separately for fractions with denominator 3 (μ_3^*) and with denominators 5 (μ_5^*). For example, at $T = 1.5$ K, $H = 20$ T, we obtain from Figs. 1 and 5: $\mu_3^* = (19 \mp 3) \times 10^3$ cm^2·V^{-1}·s^{-1}, $\mu_5^* = (27 \mp 2) \times 10^3$ cm^2·V^{-1}·s^{-1}. The composite graph on which the results obtained on all three structures (Nos. 1, 2 and 3) are illustrated, is shown in Fig. 6 and will be discussed in §5.

§4. ANOMALIES IN THE VARIATION OF THE DIAGONAL COMPONENT OF THE MAGNETOCONDUCTIVITY TENSOR

The influence of contact phenomena can be eliminated in studies of magnetotransport properties in structures of rectangular geometry, since in them measurements are made according to a four-contact system. However, there are oth-

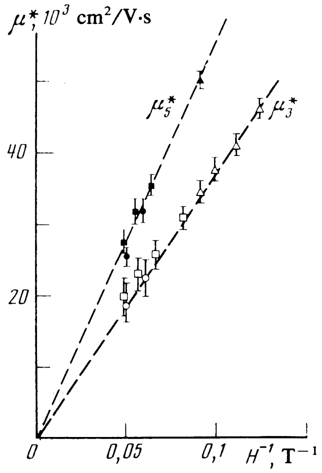

FIG. 6. The magnetic field dependence at $T = 1.5$ K of the minimum mobility μ^*, starting from which fractional anomalies are observed in the magnetotransport properties of a two-dimensional electron gas. The triangles, squares and circles show results obtained on structures Nos. 1, 2 and 3 respectively. The dark symbols refer to fractions with denominators 5, the light circles to denominators 3.

er reasons which can affect the $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ dependences studied. One of them consists in the fact that the low resistance regions of drain and source in structures of rectangular geometry, short circuit the edges of the electron layer and shunt the Hall voltage which arises in a magnetic field. As a result of this the current lines in such structures are greatly disturbed by the presence of drain and source.[17] Another reason is that reflection of electrons takes place over the perimeter at the boundaries of the two-dimensional electron layer, and in a strong magnetic field perpendicular to the layer, discontinuous electron trajectories can arise (static skin effect).[18] The trajectories can, under certain conditions, end at the potential and Hall contacts, and it is not impossible that additional anomalies in the magnetotransport properties can then arise. These and other causes are due to the absence of the proper symmetry in the rectangular geometry of the structure and are nonexistent in structures with Corbino geometry, in which the drain, source and gate are in the shape of concentric rings.[15] The observation of anomalies in magnetotransport properties in such structures is especially basic, since all the anomalies at fractional values of the filling factor were only found in structures with rectangular geometry.

The maximum mobility of the two-dimensional electrons in the best structure with Corbino geometry (specimen No. 3) was $\tilde{\mu} = 28\times10^3$ cm^2·V^{-1}·s^{-1} at $T = 1.5$ K, so that it is not surprising that the first anomaly in $\rho_{xx}(n_s)$ at $\nu = 4/3$ only appears for $H \geqslant 15$ T. It is shown in Fig. 7 how the fractional anomalies in magnetotransport properties in MOS structure No. 3 at $T = 1.5$ K develop as the magnetic field increases. It can be seen that they appear in the region of the maximum electron mobility for $n_s = (3.5–7.5)\times10^{11}$ cm^{-2}, where $\mu_e > 20\times10^3$ cm^2·V^{-1}·s^{-1}. More precisely, at $T = 1.5$ K and $H = 20$ T the minimum value of the mobility, starting from which, fractions with denominator 3 are found, is $\mu_3^* = (19 + 3)\times10^3$ cm^2·V^{-1}·s^{-1}, and for fractions with denominator 5 it is $\mu_5^* = (25 + 2)\times10^3$ cm^2·V^{-1}·s^{-1}, which agrees with the values obtained for structure No. 2.

Figure 7 shows the effect of the drain-source bias field on the form of the $\rho_{xx}(n_s)$ relation for $T = 1.5$ K and $H = 20$

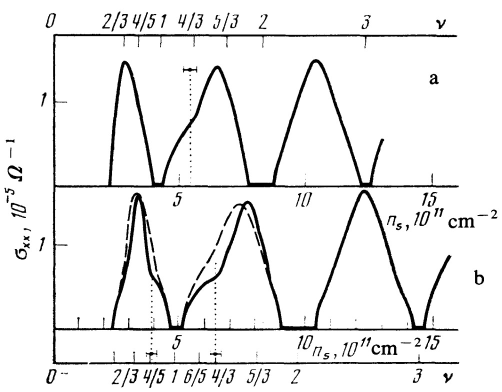

FIG. 7. $\sigma_{xx}(n_s)$ plots obtained on a MOS structure with Corbino geometry at $T = 1.5$ K, $E = 10^{-3}$ V·cm^{-1} and a) $H = 17$ T, b) $H = 20$ T. The change in the $\sigma_{xx}(n_s)$ relation on increasing the drain-source bias field up to the value $E = 4\times10^{-2}$ V·cm^{-1} is shown dashed. Values of the filling factor ν are shown on the abscissa axis.

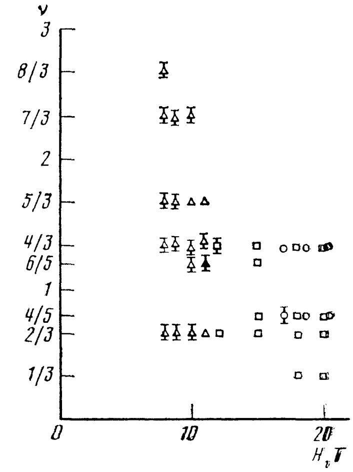

FIG. 8. Summary of the values of fractional filling factors found experimentally in structures Nos. 1, 2 and 3 for different magnetic fields at $T = 1.5$ K: △) 1, □) 2, O) 3.

T. It can be seen that it is essential that $E \ll 10^{-2}$ V·cm^{-1} for observing anomalies in the magnetotransport properties, since the electron subsystem already warms up appreciably for $E = 4\times10^{-2}$ V·cm^{-1} (according to estimates based on analysis of Shubnikov oscillations, the electron temperature increases by more than 2 K) and singularities at fractional ν disappear from the $\rho_{xx}(n_s)$ relation.

Anomalies in the magnetotransport properties of a two-dimensional electron gas in silicon MOS structures for fractional values of the filling factor have thus been observed both in specimens with rectangular geometry and in specimens with Corbino geometry. The essential conditions for observing these singularities are a high electron mobility and a sufficiently small value of the drain-source bias voltage.

§5. CONCLUSIONS

The investigations carried out show that in silicon MOS structures, differing in dielectric thickness, channel width, geometry and prepared on different substrates, but with very high electron mobility, anomalies are observed in the $\rho_{xx}(n_s)$, $\rho_{xy}(n_s)$ and $\sigma_{xx}(n_s)$ relation at sufficiently low temperatures and high magnetic fields for fractonal values of the filling factor $\nu = 1/2, 2/3, 4/3, 5/3, 7/3, 8/3, 4/5$ and $6/5$.

All the fractional values of the filling factors in structures Nos. 1, 2, and 3 found experimentally at different magnetic fields are shown in Fig. 8. By using the results for all three structures, we tried to construct, in different coordinates, the magnetic field dependence of the value of the minimum electron mobility (at $T = 1.5$ K) starting from which anomalies at fractional ν with numerators 3 and 5 are observed. These results can be represented in $\mu^*–H^{-1}$ coordinates (see Fig. 6). It can be seen that the results obtained on different structures agree well among themselves, and that for observing fractions with denominator 3 in silicon MOS structures at $T = 1.5$ K, it is necessary that $\mu^*H > 36$, while for denominator 5 it is $\mu^*H > 55$.

It also follows from this that at $T = 1.5$ K, anomalies of the magnetotransport properties at fractional ν with denominator 7 could only appear in magnetic fields $H > 25$ T in

MOS structures Nos. 2 and 3 which we studied. As distinct from the "normal" quantum Hall effect with integral filling of the corresponding quantum states, which are satisfactorily explained in the noninteracting particle approximation, the anomalies in the magnetotransport properties observed in two-dimensional electron systems at fractional values of the filling factors are a direct consequence of interelectron interaction. As a result of this interaction, new gaps appear in the spectrum of single particle excitations for fractional filling of the corresponding quantum states by the two-dimensional electrons. It follows from the experiment that the scale of the gaps in the energy spectrum under the conditions of fractional filling, decreases as the odd denominator increases.

According to Laughlin's hypothesis,[9] the fractional QHE is produced by condensation of the two-dimensional electron gas in a strong magnetic field into a new type of quantum Fermi liquid, in which the elementary excitations are essentially fermions with fractional charge $1/m$ (where m is an odd integer). The stability of such a liquid is due to its incompressibility and the absence of gapless single particle excitations. In Laughlin's theory the odd values of the denominator of the fractional filling factors are a direct consequence of the antisymmetry of the wave functions describing the ground state of the interacting electron system. However, one must bear in mind that the proposed wave function is not an exact solution of the problem of interacting two-dimensional electrons, but has the nature of well chosen variational function. Laughlin's hypothesis explains the main features of the phenomenon, the odd denominators of the fractional filling factors; however, the theory of this problem is on the whole still far from complete, and the appearance itself of the fractional QHE is not fully examined.[19–22]

In particular, the essential quenstion remains; for which denominators does a Fermi liquid cease to be the ground state of a system of interacting particles and, as an alternative, does its crystallization (the formation of a Wigner crystal or of a charge density wave) become energetically favorable. Anomalies of magnetotransport properties at fractional ν with large numerators are not clear within the framework of existing theory. In particular, the appearance of fractions with $\nu > 2$ in the case of silicon MOS structures is evidence that spin orientation of electrons, or their belonging to a single valley, is not required for observation of fractional QHE.

It is also of interest to know how the nature of the interparticle interaction influences the wave function of the ground state of the zero-dimensional Fermi liquid. It must be borne in mind that interparticle interaction in space-charge layers in heterojunctions and MOS structures can differ not only in character but also in its scale. The characteristic magnitude of the Coulomb interaction of two-dimensional electrons in MOS structures, calculated from the formula $V = e^2 n_s^{1/2}/\varepsilon = e^2 \nu^{1/2}/\varepsilon a_H$ (a_H is the magnetic length, $\varepsilon = (\varepsilon_{Si} + \varepsilon_{SiO_2})/2 = 7.7$ is the mean dielectric permittivity of Si and SiO_2), is double the corresponding value for GaAs–GaAlAs heterojunctions. For just this reason the anomalies in magnetotransport properties at fractional values of the filling factor in silicon MOS structures are observed at noticeably higher temperatures compared with heterojunctions. We also point out that in the experiments carried out ($H \approx 20$ T) on MOS structures, the Coulomb interaction energy exceeds the cyclotron energy, while in heterojunctions for the same magnetic fields the situation is the opposite: the Coulomb energy is much less than the cyclotron energy.

In discussing the experimental possibilities of further investigation of the effects of Coulomb interaction in a two-dimensional electron system, one should first select spectroscopic methods, for example studies of recombination radiation of two-dimensional electrons with photoexcited holes. Values of the Coulomb energy and the energy of the ground state[23] as functions of the density of two-dimensional electrons can be determined in such experiments. As regards traditional magnetotransport measurements, the most important are studies of the dependence of the fractional anomalies on temperature and on measuring current frequency.

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