Influence of phonon interaction on the linewidth and line profile in Brillouin scattering of light

S. V. Gantsevich, V. D. Kagan, R. Katilyus, and E. A. Rzaev

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences
(Submitted 19 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 1734–1744 (November 1984)

The scattering of light by interacting long-wave low-frequency phonons is investigated theoretically under conditions when their intensity exceeds considerably the equilibrium value. Non-Peierls interaction via electron-density waves is considered as the actual mechanism of the phonon-phonon interaction. It was found possible to obtain an expression for the intensity of the scattered light also under advanced turbulence conditions, and the parameters that describe the turbulence serve as the constants of the theory. The equations are analyzed for a typical experimental situation in which the phonon intensity increases along the sample.

1. INTRODUCTION

The long-wave low-frequency part of the phonon distribution in (piezo) semiconductors deviates noticeably from equilibrium near the threshold of acoustic instability. This makes Brillouin scattering (BS) of light by such phonons a highly useful tool for the investigation of the changes that occur in a phonon system. Beyond the instability threshold, the intensity (or number) of the phonons increases even more in this part of the spectrum,[1] and this leads in turn to a steep increase of the BS and improves the observation conditions. A theory of BS under condition of a growing phonon intensity in the linear regime (i.e., when the phonons interact only with the medium, referred to arbitrarily as the thermostat) was developed in Refs. 2 and 3. As the phonon intensity increases (in time or along the sample), nonlinear effects connected with the interaction of the phonons with one another come into play.[4,5] These nonlinear effects alter the frequency-integrated intensity of the scattered light as well as the profile and width of the scattering line. By observing these characteristics we can assess the changes in the properties of the phonon system as the phonons are enhanced. We develop in this paper a theory of BS under conditions when the phonons are not linear in intensity. We can take the nonlinear effects into account either as corrections to the linear theory or else rigorously; the latter permits in principle the light scattering to be described under conditions of advanced phonon turbulence. We derive the BS equations by a kinetic diagram techniques,[6,7] and for the description of the spatial inhomogeneity of the phonon distribution we base the technique on wave packets rather than on plane waves as in Refs. 6 and 7. The technique developed makes it relatively easy to obtain expressions for the BS line shape at various forms of nonlinearity. We consider in this paper a case when the phonon interact via a dissipative electron system. It is known[5] that in this case only departure processes are significant in phonon-phonon collisions, while the collision term does not have the usual Peierls form. The nonlinear interaction of the phonons has here the character of an interaction via a self-consistent field, and differs little in essence from their interaction with the medium. Accordingly, the results will also be somewhat reminiscent of the equations of the linear theory. Whereas, however, in the linear theory the width of the BS line was independent of the occupation numbers of the phonon states, here it turns out to be a functional of the phonon distributions in the momentum and coordinate spaces. Depending on the sign of the nonlinearity, the nonlinear interaction can either broaden or narrow the BS line. It may happen that when only departure-type processes are taken into account the linewidth tends to zero with increasing phonon intensity. In this case the residual width is determined by arrival-type processes, e.g., by emission of phonons from the electron system or else by collisional arrival (the Peierls collision term).

We develop first a diagram technique for light scattering. We consider next the nonlinear effects in the phonon system and their influence on the BS line shape. In the conclusion we use the equations derived to analyze BS in a situation, more or less typical of experiment,[8] wherein the phonon intensity increases along the sample.

2. DIAGRAM TECHNIQUE

The purpose of the present section is to adapt the kinetic diagram technique[6,7] to the description of light scattering. We consider first light scattering in an infinite spatially homogeneous system and obtain the well known[9,10] equation for the BS cross section.

The diagrams for the anti-Stokes component of the scattered light are shown in Fig. 1. The dashed lines represent photons and the wavy one phonons. The photon and phonon states are characterized respectively by wave vectors **k** and **q** by frequencies (energies) $\Omega_{\mathbf{k}}$ and $\omega_{\mathbf{q}}$. The triple vertices describe the photon-phonon interaction. The diagrams are time-ordered (time flows from left to right). We distin-

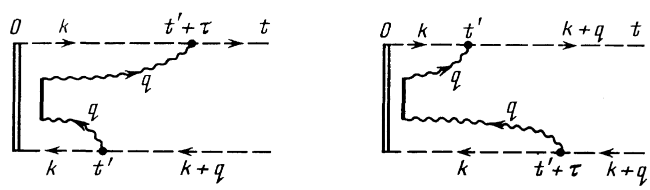

FIG. 1.

 0038-5646/84/110997-06$04.00

guish between retarded Green's functions (propagators) and advanced ones (antipropagators). The propagator direction (indicated by the arrow) coincides with the time direction, while that of the antipropagator is reversed. The propagator is set in correspondence with $\exp(-i\varepsilon t)$, where ε is the energy (frequency) of the corresponding state, and $t \geqslant 0$. Corresponding to the antipropagator is the complex conjugate $[\exp(-i\varepsilon t)]^*$. (This definition is convenient because it can be automatically extended also to the case of complex frequencies, i.e., to allowance for damping.)

The vertical lines joining the propagator and antipropagator on the left are the initial occupation numbers (intensities) specified at the instant of time $t = 0$, viz., $I_{\mathbf{k}}(0)$ for photons and $N_{\mathbf{q}}(0)$ for phonons. The section of the diagram from the instant $t = 0$ to the instant t' when the first interaction act takes place describes the evolution in time of the initial photon and phonon intensities. Neglecting the interaction with the medium, the initial intensities are obviously conserved. We shall neglect photon damping, but phonon damping (enhancement) will be taken into account.

In a semiconductor, the phonons interact with the carriers (electrons). The interaction leads in the linear approximation to the appearance of a complex increment to the phonon frequency:

$$\omega_{\mathbf{q}} \to \omega_{\mathbf{q}} + \delta\omega_{\mathbf{q}} - i\gamma_{\mathbf{q}}/2.$$

The interaction with the medium is taken into account on the diagrams by including in the phonon lines double-vertex points, each of which is set in correspondence with $-i\delta\omega_{\mathbf{q}} + \gamma_{\mathbf{q}}/2 \equiv \beta_{\mathbf{q}}$ (if included in a propagator) or with $i\delta\omega_{\mathbf{q}} + \gamma_{\mathbf{q}}/2 \equiv \beta^*_{\mathbf{q}}$ (for an antipropagator). A complex interaction constant means that the interaction is not instantaneous—the vertex turns out to be a point only to the extent that we are interested in slow variations of the occupation numbers of the phonon states, which are characterized by an absorption coefficient (gain) $\gamma_{\mathbf{q}}$ satisfying the condition

$$\gamma_{\mathbf{q}} \ll \omega_{\mathbf{q}}. \tag{2.1}$$

This condition means that the states of the phonon system at an arbitrary instant of time can be described with sufficient accuracy by a set of occupation number. Summation of diagrams with double vertices yields the obvious result:

$$N_{\mathbf{q}}(t) = N_{\mathbf{q}}(0)\exp(-\gamma_{\mathbf{q}} t). \tag{2.2}$$

To each interaction point on the diagram corresponds its own interaction time t' from the interval $[0, t]$. It is proposed to integrate in succession with respect to all such t', from zero to the subsequent time. These successive integrations can be replaced by drawing vertical cuts through the interaction points and using Laplace transforms. Each section is associated with a frequency (energy) denominator in accordance with the rule

$$s + i\sum_p \omega_p + i\left(\sum_a \omega_a\right)^* = s + i\sum_p \mathrm{Re}\,\omega_p - i\sum_a \mathrm{Re}\,\omega_a + \sum_p \mathrm{Im}\,\omega_p + \sum_a \mathrm{Im}\,\omega_a.$$

Here ω_p are the frequencies (energies) of the cut propagators, ω_a are the frequencies of the cut antipropagators, and s is the Laplace parameter. Application of this rule to the phonon intensity yields

$$N_{\mathbf{q}}(s) = \frac{N_{\mathbf{q}}(0)}{s + \gamma_{\mathbf{q}}} = \int_0^\infty N_{\mathbf{q}}(t)\, e^{-st}\, dt. \tag{2.3}$$

We consider now the right-hand sections of the diagrams of Fig. 1, where the photon propagator and antipropagator emerge at the instant t. This is the measured intensity $I_{\mathbf{k}'}(t)$ of the light scattered by the optical inhomogeneity due to the presence of phonons in the system. The transition $I_{\mathbf{k}} \to I_{\mathbf{k}'}$ requires two $\mathbf{k} \to \mathbf{k}'$ transitions at the level of the amplitudes, which make up the diagrams of the "arrival" type: one interaction point is included in the propagator and the other in the antipropagator. (Diagrams of the "departure" type, when both points are included in one photon line, describe absorption of light by the phonons and are not considered here.) Each transition is due to a perturbation proportional to the phonon amplitude at the instant of the transition. The triple photon-phonon interaction vertices, just as the vertices of the phonon interaction with the medium, have a structure, i.e., they correspond to some complicated process. The duration of this process, however, is very small, of the order of $1/\Omega_{\mathbf{k}}$, much less than all the time scales of interest to us, so that we can regard the vertex as a point and relate it to a matrix element $c^{\mathbf{q}}_{\mathbf{k}\mathbf{k}'}$ of the effective interaction potential. By virtue of the spatial homogeneity we have $\mathbf{k}' = \mathbf{k} + \mathbf{q}$, so that the interaction constants depend only on $\mathbf{k}$ and $\mathbf{q}$. We associate the point to $-ic_{\mathbf{k}}(\mathbf{q})$ in the propagator and to $ic_{\mathbf{k}}(\mathbf{q})$ in the antipropagator, and assume the constants themselves to be real (they are proportional to the usual photoelastic constants).

The vertices of the photon-phonon interaction in Fig. 1 differ in time by τ, so that the produced phonon objects are more complicated than the occupation numbers. These objects are none other than the non-equal-time amplitude correlators that become occupation numbers only at $\tau = 0$. When account is taken of the linear interaction with the medium (with the electrons) these propagators are equal to

$$\begin{aligned} \langle b_{\mathbf{q}}^+(t') b_{\mathbf{q}}(t'+\tau)\rangle &= \exp(-i\tilde{\omega}_{\mathbf{q}}\tau - \gamma_{\mathbf{q}}\tau/2) N_{\mathbf{q}}(t'), \\ \langle b_{\mathbf{q}}^+(t'+\tau) b_{\mathbf{q}}(t')\rangle &= \exp(i\tilde{\omega}_{\mathbf{q}}\tau - \gamma_{\mathbf{q}}\tau/2) N_{\mathbf{q}}(t'), \end{aligned} \tag{2.4}$$

where $\tilde{\omega}_{\mathbf{q}} \equiv \omega_{\mathbf{q}} + \delta\omega_{\mathbf{q}}$ is the phonon frequency renormalized by the interaction with the medium. The factors of $N_{\mathbf{q}}(t')$ in Eqs. (2.4) correspond on the diagram of Fig. 1 to a section, stretched out by τ, of the phonon propagator or antipropagator.

The analytic expressions for the diagrams in Fig. 1, taken in the limit $t \to \infty$, add up to the known formula for the anti-Stokes component of the scattered light:

$$I_{\mathbf{k}+\mathbf{q}} = 2\pi |c_{\mathbf{k}}(\mathbf{q})|^2 \delta(\Omega_{\mathbf{k}} + \tilde{\omega}_{\mathbf{q}} - \Omega_{\mathbf{k}+\mathbf{q}}) N_{\mathbf{q}} I_{\mathbf{k}}, \tag{2.5}$$

where, for greater clarity, we have replaced the Lorentzian obtained in accordance with the correspondence rules by a delta function. The diagrams of Fig. 1 will be used by us for further generalizations. Thus, for example, if the directions of the arrows on the phonon lines are reversed, we find that

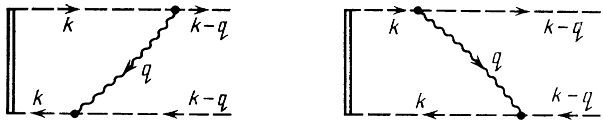

FIG. 2.

part of the scattered-light Stokes component which is proportional to the phonon intensity. For this direction of the phonon lines it is possible to construct also diagrams with only one propagator or antipropagator phonon line—Fig. 2. Such diagrams (which do not contain the phonon intensity) describe light scattering by spontaneously emitted phonons. The sum of four "Stokes" diagrams yields the known expression

$$I_{\mathbf{k}-\mathbf{q}}=2\pi|c_{\mathbf{k}}(-q)|^2\delta(\Omega_{\mathbf{k}}-\tilde{\omega}_{\mathbf{q}}-\Omega_{\mathbf{k}-\mathbf{q}})(N_{\mathbf{q}}+1)I_{\mathbf{k}}. \quad (2.6)$$

We shall neglect hereafter the spontaneous emission of the phonons and put $N_{\mathbf{q}}\gg 1$. The expression for the Stokes component can in this case be obtained by reversing the sign of the frequency and of the wave vector in the corresponding expressions for the anti-Stokes component.

We note in conclusion that a distinguishing feature of the kinetic technique is the use of a special symbol for the initial intensity. It enables us to distinguish between (heretofore equivalent) lines that join two points, viz., lines that carry intensity (amplitude correlators) and pure propagator lines (amplitude commutators). Comparison of the diagrams in Figs. 1 and 2 shows that the intensity appears in a line if it is drawn back to the past. It is thus possible to ascribe intensity to lines of arbitrary particles encountered on diagrams. It must be remembered, however, that all the lines that converge to a certain point cannot carry intensity simultaneously: at least one of them must be a line without intensity. An attempt to ascribe intensity right away to all lines inevitably produces a return point from which not even one line goes into the future. Diagrams with a return point have no physical meaning in kinetics and do not turn up in the technique. On the other hand, since lines with intensity always enter the interaction point from the past, the arrow directions for them are arbitrary. On the contrary, in the case of spontaneous emission (see Fig. 2) the phonon line goes off to the future from the interaction point that is first in time. The arrow direction on the emitted line is then not arbitrary: the propagator emits a propagator line, and the antipropagator an antipropagator line. This is precisely why diagrams with spontaneous emission exist only for the Stokes component of the scattered light.

3. ALLOWANCE FOR SPATIAL INHOMOGENEITY

We consider now light scattering under conditions of spatial inhomogeneity. In this case momentum conservation in the vertices should not hold, so that generally speaking one must forgo the plane-wave representation and change to coordinate-dependent propagators. We, however, are interested in the case of a large-scale inhomogeneity, when the system properties change over distances much larger than the wavelength of the phonons (and photons) that take part in the interaction. The characteristic size of the inhomogeneity is of the order of the damping (or gain) line: the phonon occupation numbers change over distances of this order. To take such a large-scale inhomogeneity into account, it is convenient to use the wave-packet representation. The diagram topology is obviously invariant to the representation, so that a transition to another representation does not change the diagram itself and we can use Fig. 1 for the diagrams in the wave-packet representation. The approach itself consists of introducing a large-scale coordinates $\mathbf{R}$ on which the propagators depend. In contrast to plane waves, wave packets are not exact eigenfunctions of the phonon Hamiltonian. Therefore the large-scale coordinate (as the usual coordinate in the coordinate representation) varies along the propagator. Since the interaction takes place over distances of the order of the wavelength, one can assume the vertices to depend only on one large-scale coordinate that is common for the end points of all the propagators that converge to the vertex. On the contrary, the phonon wave vector (which also becomes large-scale: $\Delta q \gtrsim 1/\Delta R$) is conserved along the line, but changes jumpwise at the vertex (without violating the momentum conservation law). The conservation law for the large-scale momentum is obtained by integration over the small-scale (difference) coordinate about the large-scale point $\mathbf{R}$.

An analytic expression for the principal element of the diagram technique, namely the propagator of the wave packet, can be obtained, for example, by taking the large-scale Fourier transform of the momentum phonon propagator:

$$\begin{aligned} G_{\mathbf{q}}(\mathbf{R}',t|\mathbf{R},t+\tau) &= \sum_{\varkappa} \exp(-i\omega_{\mathbf{q}+\varkappa}\tau+i\varkappa(\mathbf{R}-\mathbf{R}')) \\ &\approx \exp(-i\omega_{\mathbf{q}}\tau)\sum_{\varkappa}\exp(i\varkappa(\mathbf{R}-\mathbf{R}'-\mathbf{w}_{\mathbf{q}}\tau)) \\ &= \exp(-i\omega_{\mathbf{q}}\tau)\,\delta(\mathbf{R}-\mathbf{R}'-\mathbf{w}_{\mathbf{q}}\tau) \end{aligned} \quad (3.1)$$

($\mathbf{w}_{\mathbf{q}}$ is the group velocity). The small momentum $\varkappa$ should be bounded by the usual inequality

$$\varkappa \ll q. \quad (3.2)$$

Equation (3.1) states a perfectly obvious result: the wave-packet propagator is the product of two factors; one is the usual wave propagator $\exp(i\omega_{\mathbf{q}}\tau)$, and the other is a delta function that describes the motion of the packet center in accordance with the laws of geometric optics.

Linear interaction with the medium can be directly included in the wave-packet propagator; this adds a factor $\exp(-\gamma_{\mathbf{q}}\tau/2)$ and renormalizes the phonon frequency. We assume here that the phonon-phonon interaction constant is independent of the coordinate $\mathbf{R}$. The expression for the antipropagator is the complex conjugate of that for the propagator.

We supplement the propagator by introducing also the number of phonons with a given momentum $\mathbf{q}$ in the (large-scale) point $\mathbf{R}$:

$$N_{\mathbf{q}}(\mathbf{R},t)=\sum_{\varkappa} N_{\mathbf{q}}(\varkappa,t)e^{i\varkappa\mathbf{R}}, \quad (3.3)$$

where

$$N_{\mathbf q}(\varkappa,t)\equiv\langle b^{+}_{\mathbf q+\varkappa/2}(t)\,b_{\mathbf q-\varkappa/2}(t)\rangle. \tag{3.4}$$

The coordinate-dependent phonon distribution function in momentum is represented by a corresponding section on the diagrams of Fig. 1, where the propagator lines must now be regarded as dependent on the large-scale coordinate. The first glance on this section may suggest that the distribution function must depend on the two coordinates $\mathbf R$ and $\mathbf R'$ corresponding to the end points of the propagator and antipropagator, which generally speaking need not be identical. This, however, is not so (as is the case also for clear notions concerning the phonon intensity at a point). Indeed, regarding the vertical stroke (the initial coordinate) as diagonal in the large-scale coordinate and integrating with respect to it, the result calls for $\mathbf R = \mathbf R'$. Quantities that are nondiagonal in the large-scale coordinate describe interference effects; they should depend either on the three spatial coordinates $\mathbf R$, $\mathbf R'$ and a certain initial $\mathbf R_0$, or else the corresponding quantity must be a non-equal-time correlator, i.e., must depend not only on t but also on τ. The last quantities, as we shall see presently, indeed appear when light scattering is described.

The phonon distribution function satisfies the kinetic equation. In the linear approximation it takes the form 11

$$(\partial_t+\mathbf w_{\mathbf q}\nabla+\gamma_{\mathbf q})N_{\mathbf q}(\mathbf R,\ t)=\mathfrak{A}_{\mathbf q}(\mathbf R,\ t). \tag{3.5}$$

Here $\mathfrak{U}_{\mathbf q}(\mathbf R,t)$ is the source that describes the creation of phonons by electrons. Its diagrammatic representation in the spatially homogeneous case is given in Ref. 6. Under amplification conditions, when the phonon intensity increases strongly, the source becomes inessential.

We obtain now an expression for the intensity of the scattered light in the spatially inhomogeneous space. It suffices for this purpose to regard the phonon propagator lines in Fig. 1 as lines of phonon wave packets. The phonon-photon interaction vertices are then found to depend explicitly on the large-scale coordinates (say, $\mathbf R$ and $\mathbf R'$ for the upper and lower points). It is necessary to integrate with respect to these coordinates over the entire light-scattering volume V; we integrate also with respect to τ from 0 to ∞. As a result we obtain the known[2,3] formula of the linear theory

$$I_{\mathbf k+\mathbf q}=|c_{\mathbf k}(\mathbf q)|^2\int_0^\infty d\tau\iint_V d\mathbf R\,d\mathbf R'\,\delta(\mathbf R-\mathbf R'-\mathbf w_{\mathbf q}\tau)$$
$$\times\exp[i(\Omega_{\mathbf k+\mathbf q}-\Omega_{\mathbf k}-\tilde\omega_{\mathbf q})\tau-\gamma_{\mathbf q}\tau/2]N_{\mathbf q}(\mathbf R')I_{\mathbf k}. \tag{3.6}$$

In contrast to the spatially-inhomogeneous case, where the scattered light line shape is Lorentzian and its width is determined by the temporal instability of the phonon state on account of absorption or amplification of phonons, we have included here an additional broadening mechanism, since the phonon-state lifetime depends now also on the size of the scattering region, and departure of phonons from this region can compete with absorption (amplification).

4. NONLINEAR EFFECTS IN THE PHONON SYSTEM

The interaction between phonons becomes substantial when their intensity is high. We can take this interaction into account by including in the phonon propagator lines appro-

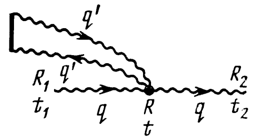

FIG. 3.

priate points—vertices. Just as in the case of interaction with a medium, the internal structure of the new vertices (see Ref. 6) is of no interest to us. We assign to each vertex a certain complex interaction constant.[7]

We consider first weak nonlinearity. It is described by departure-type diagrams with 4 "two by two" wave vertices.[6,7] Figure 3 shows the propagator correction that depends on the phonon intensity. This diagram shows that the nonlinear interaction can be likened to a linear one, but with an effective interaction constant that depends on the phonon occupation numbers. Figure 3 shows the first corrections. To take complete account of a nonlinearity of this type it is necessary to saturate all the encountered propagators with nonlinear-interaction points. We emphasize that in this case both intensity-forming phonon lines should emerge from one vertex—in other words, the topology of the nonlinear diagrams should be the topology of a growing tree. Problems of this type are well enough known—these are problems concerning a self-consistent field. In this case we can formulate the problem as follows: the interaction of an isolated phonon with a "phonon" medium is characterized by a certain $\Gamma_{\mathbf q}$ which is itself a functional of the phonon occupation numbers, and these in turn must be determined from equations with this constant. For weak nonlinearity we have

$$\Gamma_{\mathbf q}(\mathbf R,t)=\sum_{\mathbf q'}\Gamma_{\mathbf q\mathbf q'}N_{\mathbf q'}(\mathbf R,t). \tag{4.1}$$

The constant $\Gamma_{\mathbf q\mathbf q'}$, just as the linear constant $\beta_{\mathbf q}$, is determined by the properties of the dissipative electron system.

The topology of the nonlinear diagram in Fig. 3 can be directly generalized to include strong nonlinearity. In this case the effective interaction constant $\Gamma_{\mathbf q}$ is given by a sum of diagrams of the type of Fig. 3, but with an arbitrary number of phonon lines (that carry intensity). An example of a diagram quadratic in the intensity is shown in Fig. 4.

The analogy between the interaction with a medium and the diagrams of phonon-phonon interaction via a dissipative medium leads to the following conclusion: Under conditions of advanced turbulence, the evolution of the occupation numbers $N_{\mathbf q}(\mathbf R,t)$ is described by an equation that coincides in form with (3.5), but with a coefficient $\tilde\gamma_{\mathbf q}\{N\}$

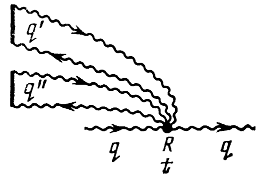

FIG. 4.

that depends on all the phonon occupation numbers:

$$(\partial_t + \mathbf{w}_\mathbf{q}\nabla + \tilde{\gamma}_\mathbf{q}\{N\})N_\mathbf{q}(\mathbf{R}, t) = 0. \tag{4.2}$$

Here and below we have in mind the case of phonon amplification, and neglect the source. The nonlinear amplification coefficient is the sum of the linear coefficient and a nonlinear contribution:

$$\tilde{\gamma}_\mathbf{q}\{N\} \equiv \gamma_\mathbf{q} + 2\,\mathrm{Re}\,\Gamma_\mathbf{q}\{N\}. \tag{4.3}$$

The imaginary part of the nonlinear constant $\Gamma_\mathbf{q}$ makes the following contribution to the renormalization of the phonon frequency:

$$\delta\tilde{\omega}_\mathbf{q} = \mathrm{Im}\,\Gamma_\mathbf{q}\{N\} + \mathrm{Im}\,\beta_\mathbf{q}. \tag{4.4}$$

In contrast to the spatially homogeneous case (see Refs. 6 and 7), the coefficient $\tilde{\gamma}_\mathbf{q}$ and the correction $\delta\tilde{\omega}_\mathbf{q}$ to the frequency depends on the large-scale coordinate **R** via the occupation numbers. The dependence on the time t in the nonstationary case also enters via these numbers.

Equation (4.2) demonstrates the "linearization" typical of problems of the self-consistent-field type. In this case the linearization is natural: as the phonon intensity increases relatively uniformly over the spectrum, the properties of the medium are altered by the very large number of phonon modes. The influence of an individual specific mode is small compared with the total influence of the remaining modes—the self-action is small compared with the interaction.

Thus, the condition for the applicability of Eq. (4.2) is the presence of a large number of independent (noncoherent) modes. This number can be estimated as the ratio of the spectrum width of the spectrum in the gain band to the natural line width determined by gain γ. The criterion $\Delta\omega \gg \gamma$ for the applicability of Eq. (4.2) can be obtained also from the diagrams of Refs. 6 and 7 by comparing, in order of magnitude, the accounted-for diagrams of the self-consistent-field type with the remaining diagrams that describe the correlation between the phonon modes. The topology of the correlation diagrams is such (see Fig. 23 of Ref. 6 and Fig. 7 of Ref. 7) that the ends of the phonon lines are separated in time; when the corresponding cut is made this yields a factor of the order of $1/\Delta\omega$, whereas for diagrams of the self-consistent-field type (which have the same number of intensities and interaction constants) the sections always give $1/\gamma$.

Equation (4.2) for the case of weak nonlinearity was obtained in Ref. 5. Under conditions of advanced turbulence (in the spatially homogeneous case) its derivation is given in Ref. 6. A case when $\Delta\omega \lesssim \gamma$ and (4.2) is not valid was investigated in detail in Ref. 7.

It must be stated that the very existence of an interaction of the self-consistent-field type is due to the dissipative character of the electron system through which the interaction is effected between the phonons. In the case of ordinary anharmonicity (triple vertices) we would have an interaction of the Peierls type with departure and arrival terms in the kinetic equation, in contrast to the pure departure equation (4.2). The Peierls interaction via an electron system is weak compared with a non-Peierls one in a ratio γ/ω (see Ref. 5 and 6).

We note incidentally that the simplicity of Eq. (4.2) is only illusory. The nonlinear gain $\gamma_\mathbf{q}\{N\}$ is very difficult to calculate (as also in any problem with weak interaction). This coefficient, however, can in principle be easily measured, for example by observing the gain or absorption of a weak acoustic signal introduced into the system from the outside. The same coefficient (averaged over the spectrum) enters also in the expression for the acoustoelectric current. Finally, as we shall presently show, it is this coefficient which determines the BS line shape under conditions of advanced turbulence.

5. BS UNDER CONDITIONS OF ADVANCED TURBULENCE

To obtain an expression for the intensity of scattered light under conditions of advanced turbulence, we use again the diagrams of Fig. 1. We now regard the phonon lines on these diagrams not only as dependent on the large-scale coordinates and saturated with linear vertices, but also as fully saturated with all possible vertices of nonlinear interaction, the aggregate of which yields the constant $\Gamma_\mathbf{q}\{N\}$. As a result we obtain

$$I_{\mathbf{k}+\mathbf{q}} = |c_\mathbf{k}(\mathbf{q})|^2\, 2\,\mathrm{Re}\int_0^\infty d\tau \iint_V d\mathbf{R}\, d\mathbf{R}'\, \tilde{G}_\tau(\mathbf{R}'/\mathbf{R}) \times \exp[i(\Omega_{\mathbf{k}+\mathbf{q}} - \Omega_\mathbf{k})\tau] N_\mathbf{q}(\mathbf{R}') I_\mathbf{k}. \tag{5.1}$$

In contrast to the linear theory, the propagator $\tilde{G}_\tau(\mathbf{R}'/\mathbf{R})$ that enters in this formula has no simple form. It can be found from the equation

$$[\partial_\tau + i\omega_\mathbf{q} + \mathbf{w}_\mathbf{q}\nabla + \beta_\mathbf{q} + \Gamma_\mathbf{q}\{N(\mathbf{R})\}]\tilde{G} = 0 \tag{5.2}$$

with the usual initial condition for the coordinate propagator:

$$\tilde{G}|_{\tau=0} = \delta(\mathbf{R} - \mathbf{R}'). \tag{5.2a}$$

Equations (5.2) and (4.2) for the propagator and intensity are interrelated. From Eq. (5.2) and its conjugate (for the antipropagator) we can obtain Eq. (4.2) by defining N as $\tilde{G}\tilde{G}^*$.

BS permits in principle the determination of both N and $\tilde{G}$, and the latter yields in fact the line shape of the resonant light.

We consider now in greater detail a typical experimental situation in which the phonon intensity increases along the sample, so that it suffices to consider only the dependence on the longitudinal coordinate x. The stationary intensity $N_\mathbf{q}(x)$ satisfies then the equation

$$v_\mathbf{q}\frac{\partial N_\mathbf{q}}{\partial x} + \tilde{\gamma}_\mathbf{q}\{N\}N_\mathbf{q} = 0; \tag{5.3}$$

where $v_\mathbf{q}$ is the projection of the group velocity $\mathbf{w}_\mathbf{q}$ on the x axis

$$v_\mathbf{q} = \partial\omega_\mathbf{q}/\partial q_x = w_\mathbf{q}\cos\alpha, \tag{5.4}$$

where α is the angle between the vectors **q** and **x**. For the propagator of the one-dimensional wave packet we have

$$G_\tau(x'/x) = \delta(x - x' - v_\mathbf{q}\tau) \times \exp\left[-i\omega_\mathbf{q}\tau - \beta_\mathbf{q}\tau - \int_{x-v_\mathbf{q}\tau}^{x} \Gamma_\mathbf{q}\{N(x'')\}\, dx''/v_\mathbf{q}\right]. \tag{5.5}$$

Thus for the intensity of the scattered light we have in this case the following formula:

$$I_{\mathbf{k+q}} = \sigma^2 |c_{\mathbf{k}}(\mathbf{q})|^2 2\mathrm{Re}\int_0^\infty d\tau \iint_0^L dx\,dx'\, N_{\mathbf{q}}(x')\,\delta(x-x'-v_p\tau)$$

$$\times \exp\Big[i(\Omega_{\mathbf{k+q}} - \Omega_{\mathbf{k}}^* - \omega_{\mathbf{q}})\tau + i$$

$$\times \int_{x-v_{\mathbf{q}}\tau}^{x} \delta\tilde{\omega}_{\mathbf{q}}(x'')\frac{dx''}{v_{\mathbf{q}}} - \frac{1}{2}\int_{x-v_{\mathbf{q}}\tau}^{x} \tilde{\gamma}_{\mathbf{q}}(x'')\frac{dx''}{v_{\mathbf{q}}}\Big]. \quad (5.6)$$

Here σ is the cross section of the scattering region and L is its length. The total gain $\tilde{\gamma}_{\mathbf{q}}$ and the frequency shift $\delta\tilde{\omega}_{\mathbf{q}}$ depend on the coordinate via the dependence on the occupation numbers $N_{\mathbf{q}}$. Using Eq. (5.3), we can transformed (5.6) into

$$I_{\mathbf{k+q}} = \sigma^2 |c_{\mathbf{k}}(\mathbf{q})|^2 2\,\mathrm{Re}\int_0^\infty d\tau \iint_0^L dx\,dx'\,\delta(x-x'-v_{\mathbf{q}}\tau)$$

$$\times (N_{\mathbf{q}}(x)N_{\mathbf{q}}(x'))^{1/2}\exp\{i[\Delta\omega(\mathbf{k},\mathbf{q}) + \widetilde{\Delta}\omega_{\mathbf{q}}(x,\tau)]\tau\}, \quad (5.7)$$

where we have put

$$\Delta\omega(\mathbf{k},\mathbf{q}) = \Omega_{\mathbf{k+q}} - \Omega_{\mathbf{k}} - \omega_{\mathbf{q}}, \quad (5.8)$$

$$\widetilde{\Delta\omega}_{\mathbf{q}}(x,\tau) = \frac{1}{\tau}\int_{x-v_{\mathbf{q}}\tau}^{x} \delta\tilde{\omega}_{\mathbf{q}}(x'')\frac{dx''}{v_{\mathbf{q}}}. \quad (5.9)$$

We see that the scattered-light line shape is determined by superposition of three factors: the departure of the phonons from the light-scattering region, the nonlinear amplification (damping) of the phonons, and the inhomogeneous broadening due to the coordinate dependence of the phonon-dispersion law. If the scattering region is small enough, the phonon lifetime is equal to the time of their motion in this region. The linewidth is in this case, as in the linear theory, 2 of the order of w/L.

At $w/L \gg 1$ the departure ceases to play any role and the scattered-light linewidth depends on two other mechanisms whose contributions are difficult to separate in the general case. Effects connected with inhomogeneous broadening may turn out to be the principal ones at low supercriticalities in the case of density nonlinearity, since both the linear gain and the nonlinear additions are in this case proportional to the difference between the velocities of the sound and of the electron drift, and vanish when the two are equal, whereas $\delta\tilde{\omega}_{\mathbf{q}}$ does not have this property.

On the other hand, the inhomogeneous broadening may turn out to be insignificant when phonons are amplified from a relatively narrow region of the spectrum, where the integral phonon sensitivity does not change greatly, but the beam spectrum is noticeably broadened because the gain has a maximum at $\mathbf{q} = \mathbf{q}_0$. The explicit form of the distribution of the phonon occupation numbers over the spectrum is given for a model problem of this type in Ref. 7 [(Eq. (4.1)]. In our case we must put in this equation $t = x/v_{\mathbf{q}}$, so that it takes the form

$$N_{\mathbf{q}}(x) = \Big\{1 + W\sum_{\mathbf{q}'} N_{\mathbf{q}'}(0)\frac{\exp\{\gamma_{\mathbf{q}'}x/v_{\mathbf{q}}\}-1}{\gamma_{\mathbf{q}'}}\Big\}^{-1}$$

$$\times N_{\mathbf{q}}(0)\exp\{\gamma_{\mathbf{q}}x/v_{\mathbf{q}}\}. \quad (5.10)$$

Here $N_{\mathbf{q}}(0)$ are the occupation numbers on the boundary. $\gamma_{\mathbf{q}}$ is the linear gain, and W is the nonlinearity constant.

For solutions of this type at sufficiently large x, the integral intensity saturates and it can therefore be assumed that $\delta\tilde{\omega}_{\mathbf{q}}$ ceases to depend on x, so that the inhomogeneous broadening vanishes. In this case the line width is determined by nonlinear amplification and damping effects and depends substantially on how far the light-scattering phonons are from the maximum of the spectrum.

Far from the amplification maximum, the phonons are damped and the line width is determined by their nonlinear decrement. For the model (5.10), the line shape will be Lorentzian with a width equal to the gain difference $\gamma_{\mathbf{q}0} - \gamma_{\mathbf{q}}$. Near the maximum where the weak growth continues (because of the continuing narrowing of the spectrum, we would see a very narrow line whose width is determined either by the small residual increment or by other factors, both accounted for in the present theory (departure and inhomogeneous broadening) and unaccounted for (e.g., phonon-phonon collisions of the Peierls type or else coherence-causing effects (see Ref. 7).

We note in conclusion an interesting possibility of observing the change of the linewidth in our situation by shifting the scattering region from that end of the sample where the linear effects are still small, to the other end, where they are decisive. For example, for phonons not located in the region of the maximum gain, the linewidth, initially equal to $\gamma_{\mathbf{q}}$, decreases (to zero if departure and inhomogeneous broadening are neglected) in the region where the nonlinear damping offsets the gain, and then increases to a value of the order of $\gamma_{\mathbf{q}0} - \gamma_{\mathbf{q}}$. A similar behavior of the scattered-light linewidth can be observed also in one region of the sample (far enough from its low-linearity end) when the supercritically is increased and the linear gain regime saturates. Experimental observation of such a behavior of the linewidth would be of extraordinary interest and would contribute to a better understanding of the nature of phonon turbulence.

[1] A. R. Hutson, J. H. McFee, and D. L. White, Phys. Rev. Lett. **7**, 237 (1961).
[2] V. L. Gurevich and V. D. Kagan, Zh. Eksp. Teor. Fiz. **47**, 1782 (1964) [Sov. Phys. JETP **20**, 1201 (1965)].
[3] Yu. M. Gal'perin, V. L. Gurevich and E. A. Rzaev, Fiz. Tverd. Tela (Leningrad) **25**, 2386 (1983) [Sov. Phys. Solid State **25**, 1369 (1983)].
[4] A. R. Hutson, Phys. Rev. Lett. **9**, 296 (1962).
[5] V. L. Gurevich, V. D. Kagan, and B. D. Laĭkhtman, Zh. Eksp. Teor. Fiz. **54**, 188 (1968) [Sov. Phys. JETP **27**, 102 (1968)].
[6] S. V. Gantsevich, V. D. Kagan, and R. Katilyus, Zh. Eksp. Teor. Fiz. **67**, 1765 (1974) [Sov. Phys. JETP **40**, 878 (1975)].
[7] S. V. Gantsevich, V. L. Gurevich, V. D. Kagan, and R. Katilius, Phys. Stat. Sol. (b) **75**, 407 (1976).
[8] T. S. Velichkina, A. M. D'yakonov, O. N. Vasil'eva, V. V. Aleksandrov, and I. A. Yakovlev, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 438 (1982) [Sov. Phys. JETP **35**, 542 (1982)].
[9] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Pergamon, in press (Nauka, 1982).
[10] I. L. Fabelinskii, Molecular Scattering of Light, Plenum, 1968.
[11] V. L. Gurevich, Zh. Eksp. Teor. Fiz. **47**, 1291 (1964) [Sov. Phys. JETP **20**, 873 (1965)].

Translated by J. G. Adashko