Cyclotron absorption of fast magnetosonic wave in a stellarator

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It is demonstrated theoretically that a fast magnetosonic wave can be strongly absorbed by a stellarator plasma under cyclotron-resonance conditions.

High-frequency heating of plasma by a fast magnetosonic (FMS) wave at frequencies on the order of the ion cyclotron frequency is effectively used at present in various installations.[1,2] For the main plasma ions, however, the cyclotron-frequency heating regime considered in the present article is inefficient.[3,4] This is due not only the relatively small absorption band in which $|\omega - \Omega_i| \leqslant k_\parallel V_i$, but also to the unfavorable polarization of the electric field of the FMS wave in this band (here ω and $k_\parallel$ are the frequency and longitudinal wave number of the FMS wave, and V_i is the thermal velocity of the ions). The question of the peculiarities of FMS wave absorption in a tokamak at $\omega \approx \Omega_i$ with allowance for the nonuniformity of the magnetic field and the finite rotational conversion was discussed in Refs. 5–7. The results of these studies do not agree with one another. From our point of view the conclusions[6,7] based on the value of the dielectric constant averaged over the magnetic surface are not quite correct. In a stellarator, a more important cause is the helical character of the confining magnetic field, and the effects of enhancement of the absorption can be considerably larger than in a tokamak with analogous plasma parameters.

To obtain an expression for the plasma current in a stellarator magnetic field we integrate the linearized Vlasov equation along the characteristics. We choose a coordinate frame (x^1, x^2, x^3) such that $x^1 = \text{const}$ would be the equation of the magnetic surface. We introduce an orthonormal system of vectors $\mathbf{e}_\parallel = \mathbf{B}_0/B_0$, $\mathbf{e}_a = \nabla x^1/|\nabla x^1|$, and $\mathbf{e}_b = \mathbf{e}_\parallel \cdot \mathbf{e}_a$ ($\mathbf{B}_0$ is the stellarator magnetic field. Then any vector, say $\mathbf{E}$, can be specified by the components $E_a = \mathbf{e}_a \mathbf{E}$, $E_b = \mathbf{e}_b \mathbf{E}$, $E_\parallel = \mathbf{e}_\parallel \mathbf{E}$. We transform to rotating components of the velocity and of the field $V^\pm = V_a \pm iV_b$, $E^\pm = E_a \pm iE_b$. Assuming the ion Larmor radius to be small compared with the characteristic dimension of the magnetic-field inhomogeneity and with the transverse wavelength, and considering only untrapped ions,[1] we obtain for the left-polarized component of the ion current the expression

$$j_i^+ = \frac{\omega_{pi}^2}{4\pi^{3/2} V_i} \int_{-\infty}^{\infty} \exp\left(-\frac{V_\parallel^2}{V_i^2}\right) dV_\parallel \int_{-\infty}^{t} E^+(t') \times \exp\left[i \int_{t}^{t'} (k_\parallel V_\parallel - \omega + \Omega_i)\, d\tau \right] dt'. \tag{1}$$

We introduce the dimensionless parameters

$$\eta = \varepsilon_s \Omega_{i0}/\omega_{b0}, \quad \chi = \varepsilon_s \Omega_{i0}/k_\parallel V_i, \quad \omega_{b0} = \alpha V_i. \tag{2}$$

Here ω_{pi} and Ω_i are the plasma and cyclotron ion frequencies, Ω_{i0} and $\varepsilon_s \Omega_{i0}$ are the mean value and amplitude of the alternating part of the cyclotron frequency, $\alpha = 2\pi/L$, and L is the period of the helical winding. We consider conditions when

$$1 \ll \eta \ll \chi^2. \tag{3}$$

The main contribution to f_i^+ is made then by the vicinities of the stationary-phase points, which are simultaneously the points of local cyclotron resonance

$$k_\parallel V_\parallel - \omega + \Omega_i = 0. \tag{4}$$

If the condition (4) is satisfied periodically with period T as the ion moves along the trajectory, the ions with resonant velocities $V_\parallel = V_{\parallel r}$ which satisfy the conditions

$$\Delta\Phi = \int_{t-T}^{t} (k_\parallel V_{\parallel p} - \omega + \Omega_i)\, d\tau = 2\pi p \tag{5}$$

(p is an integer) will pass through analogous successive resonances in one and the same phase relative to the electric field of the wave. In a straight stellarator the magnetic field depends only on the helix angle $\theta = \vartheta - \alpha z$, and the characteristic value is $T \approx 2\pi/\omega_{b0}$. Allowance for the small toroidality $\varepsilon_t \ll \varepsilon_s$ (ε_t is the reciprocal of the aspect ratio) leads to a dependence of Ω_i also on the poloidal angle ϑ. Consequently, condition (5) is satisfied at all angles ϑ only if $\varepsilon_t \Omega_{i0} \ll \omega_{b0}$. This requirement is usually not satisfied. In addition, weak ion Coulomb collisions with $\nu_i \ll \omega_{b0}$ lead to a random small change of the longitudinal velocity, and this causes a random phase change $\Delta\Phi$ amounting to $\sim 2\pi\eta(2\pi\nu_i/\omega_{b0})^{1/2}$, which frequently is not small compared with 2π (e.g., under the condition of the experiment with the L-2 stellarator[8]).

The foregoing remarks lead to the conclusion that the resonances (5) are not realized in practice. For the average absorbed power, recognizing that the main contribution is made by small vicinities $2l$ of the resonance angles θ_s (where $\omega = \Omega_i$), we obtain

$$P = \frac{\omega_{pi}^2}{32\pi} \sum_{s=1}^{2l} \frac{|E^+(\theta_s)|^2}{|(d\Omega_i/d\theta)_{\theta_s}|}. \tag{6}$$

Here l is the multiplicity of the stellarator winding. The wave fields $E^+(\theta_s)$ is conveniently expressed with the aid of Maxwell's equations in terms of the function G, which depends little on the proximity of the angle θ to the resonant one θ_s:

$$E^+(\theta_s) = 4\sqrt{2}\pi G(\theta_s)/\omega\varepsilon^+(\theta_s), \tag{7}$$

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$$\varepsilon^+(\theta_s)=\frac{i\omega_{pi}^2}{\omega\omega_{b0}^{1/2}}\int_{-\infty}^{\infty}\exp(-u^2)\left[-2iu\left(\frac{d\Omega_i}{d\theta}\right)_{\theta_s}\right]^{-1/2}du, \tag{8}$$

$$G=\frac{c}{4\pi}\frac{1}{(2g^{11})^{1/2}}\left[g^{1j}+i\frac{g^{kj}}{g^{1/2}}\frac{B_0^i}{B_0}(g_{2k}g_{3i}-g_{3k}g_{2i})\right]\frac{\partial B_\parallel}{\partial x^j}. \tag{9}$$

Here g_{ij} is the metric tensor, $g=\mathrm{Det}\, g_{ij}$, and $B_\parallel$ is the longitudinal component of the wave's magnetic field. Substituting in (6) we obtain

$$P\approx\frac{\omega_{b0}}{\omega_{pi}^2}\sum_{s=1}^{2l}|G(\theta_s)|^2. \tag{10}$$

The increase of the power P with the frequency ω_{b0} and with l can be treated as proportionality of the absorbed power to the number of ions passing through resonance per unit time.

Averaging over θ the local values of the power, given by the theory for a uniform magnetic field at the local plasma parameters, we obtain for $\bar{P}$ a value $\varkappa\simeq\chi^2/\eta$ times smaller than (10). Under conditions (3) the amplification coefficient $\varkappa\gg1$. Absorption amplification is possible also in a tokamak, but there it is connected with a finite rotational conversion and is considerably smaller. Comparison of the powers absorbed P_s and P_t averaged over the magnetic surface in a stallarator and a tokamak having the same plasma parameters yields the large value $P_s/P_t\approx m_s q_t$, where m_s is the number of periods of the stellarator magnetic field on the large circuit along the torus ($m_s=14$ for the L-2) and q_t is the stability margin factor in the tokamak.

[1)] Trapped ions contribute little to cyclotron absorption.

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