Cyclotron resonance in ionic spectra

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A theory is proposed of the line shape of resonant absorption (gain) of monochromatic radiation by ions moving on helical trajectories in a uniform dc magnetic field. It is shown that the ion Larmor rotation can be accompanied by the appearance of an equidistant superposition of cyclotron resonances against the background of the spectrum Zeeman structure. The cyclotron resonances are separated by an interval determined by the ion Larmor frequency ω_L for linear absorption and by $\omega_L/2$ for nonlinear absorption. The dependences of the cyclotron resonances in the spectrum on the magnetic field direction, on the radiation polarization, and on the angular momenta of the particles are analyzed. It is demonstrated that Lorentz cyclotron resonances with widths equal to the homogeneous linewidth can be produced by linear absorption or amplification of the radiation if the magnetic field is perpendicular to the wave vector.

§1. INTRODUCTION

We consider here the spectral manifestations of the action of a magnetic field on the motion of ions in a plasma. The gyromagnetic effects in a magnetoactive plasma[1,2] and in a solid[3] have already been thoroughly investigated. How quasistationary fields that accelerate ions in a gas-discharge plasma influence the ion spectra has also been recently described, as were the magneto-optic resonances due to the Zeeman effect.[5] Spectrum broadening due to ion scattering in a Coulomb field was investigated in Refs. 9 and 10.

Ion motion on helical trajectories along the force line of a constant magnetic field **H** leads to the appearance of a number of new effects in absorption or amplification of resonant radiation and in the velocity distribution of the excited-state populations. The peculiarities of the spectrum formation in this situation are due to the anomalies that appear when a detuning of the radiation frequency ω from the Zeeman-shifted transition frequency ω_{mn} coincides with a harmonic of the ioo-cyclotron frequency

$$\omega_L=ZeH/m_ic, \tag{1.1}$$

where Ze is the charge and m_i is the particle mass. The cyclotron-resonance condition takes in the optical region the form

$$\omega-\omega_{mn}+\alpha\Delta=l\omega_L,\quad l=0,\pm1,\pm2,\dots . \tag{1.2}$$

The parameter $\Delta=\mu gH/\hbar$ characterizes here the Zeeman splitting of the line at different g factors of the Landé states m and n (normal Zeeman effect); μ is the Bohr magneton (or the nuclear magneton); $\alpha=0$ for the π component of the radiation, and $\alpha=\pm1$ for the σ components.

We investigate next on the basis of the kinetic equation for the density matrix the resonant interaction of two-level ions placed in a magnetic field with monochromatic radiation. Particular attention is paid to gyromagnetic effects in the formation of linear and nonlinear spectral structures. The dependences of the cyclotron resonances in the spectrum on the magnetic-field direction, on the radiation polarization, and on the particle angular momenta are analyzed.

§2. QUANTUM KINETIC EQUATION. GENERAL RELATIONS

Consider a gas of ionized particles that interact resonantly with electromagnetic radiation in a constant uniform magnetic field **H** with a vector potential

$$\mathbf{A}=[\mathbf{Hr}]/2. \tag{2.1}$$

The kinetic energy of the ion is determined by the Hamiltonian

$$U=\left(\mathbf{p}-\frac{Ze}{c}\mathbf{A}\right)^2(2m_i)^{-1}, \tag{2.2}$$

where $\mathbf{p}=-i\hbar\nabla_\mathbf{r}$ is the momentum operator corresponding to the coordinates **r** of the mass center. The Hamiltonian operator

$$\mathcal{H}'=\mathcal{H}_0-\mu g\mathbf{JH} \tag{2.3}$$

corresponds to interaction of the constituent parts of an ion having an angular momentum **J** with one another ($\mathcal{H}_0$) and with the magnetic field. The total Hamiltonian $\mathcal{H}$, whose eigenvalues determine the energy levels in the absence of interaction with the perturbing particles, can thus be represented as the sum

$$\mathcal{H}=\mathcal{H}'+U+\hbar V, \tag{2.4}$$

where $V(\mathbf{r},t)$ is the operator of the interaction with the electromagnetic radiation.

A derivation, suitable for any interaction between the particles and the magnetic field, of the kinetic equation for the density matrix is described in Ref. 5. In analogy with this reference, we obtain a kinetic equation for the Wigner density matrix $\rho(\mathbf{r},\mathbf{v},t)$ of the ions in the interaction representation

$$\frac{\partial\rho}{\partial t}+\{U,\rho\}=-i[V,\rho], \tag{2.5}$$

where Poisson brackets for U and ρ are used. The commutator of the operators V and ρ in the right-hand side of (2.5) describe induced transitions with the recoil effects neglected, i.e., at $\hbar k^2m_i\ll\Gamma$, where Γ is the homogeneous line width and **k** is the wave vector of the electromagnetic wave. The

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recoil in photon absorption or emission is accounted for by the derivatives $\nabla_r V$ (Refs. 6 and 11). The left-hand side of (2.5) is specified by an operator similar to that used in the kinetic equation for the classical distribution function of a system of charged particles.[12]

In the two-level approximation with allowance for the relaxation processes and for the excitation of the working levels m and n, Eq. (2.5) reduces to the form

$$\left(\frac{d}{dt}+\Gamma_{jl}\right)\rho_{jl}=q_j\delta_{jl}-i[V,\rho]_{jl},\qquad j,l=m,n,$$
$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{v}\nabla_\mathbf{r}+[\mathbf{v}\boldsymbol{\omega}_L]\nabla_\mathbf{v},\qquad \boldsymbol{\omega}_L=\frac{Ze\mathbf{H}}{m_ic}. \qquad (2.6)$$

Here Γ_{jl} are the decay constants and q_j are the level-excitation Maxwellian functions. We use hereafter the notation $\Gamma=\Gamma_{mn}$ and $\Gamma_j=\Gamma_{jj}$.

Taking the Zeeman splitting into account, we can describe the electrodipole interaction of the ions with a traveling monochromatic wave or arbitrary polarization by the matrix elements of the operator V, described with the aid of Clebsch-Gordan coefficients:

$$V_{mM,nM'}=-\sum_\alpha G_\alpha(-1)^{J_n-M'}\langle J_mMJ_n-M'|1\alpha\rangle$$
$$\times\exp[-i(\Omega-\Delta_{MM'})t+i\mathbf{kr}], \qquad (2.7)$$
$$G_\alpha=E_\alpha d_{mn}/2\hbar,\qquad \Omega=\omega-\omega_{mn},\qquad \alpha=0,\pm1.$$

E_α denote the spherical components of the complex amplitude of the electric vector; d_{mn} is the transition dipole-moment matrix element; M is the projection of the angular momentum **J**. In the normal Zeeman effect $\Delta_{MM'}=\Delta(M-M')$.

The set of equations for the density-matrix elements is made much simpler by converting to the representation of irreducible tensor operators.[13] The coefficients of the expansion of the matrix elements of the operator V in terms of the irreducible tensors satisfy the relations

$$V_{mn,\lambda\alpha}=-\delta_{\lambda1}G_\alpha\exp[-i(\Omega-\alpha\Delta)t+i\mathbf{kr}]. \qquad (2.8)$$

We note that for neutral particles whose motion is not acted upon by the magnetic field, the solution of the system (2.6) was found earlier by D'yakonov and Perel'[14,15] on the basis of an expansion of the matrix ρ in powers of the electromagnetic-wave amplitude. The spectral manifestations of the effect of the magnetic field on the medium reduce in this case to the Zeeman effect.

§3. LINEAR THEORY

We track first the spectral-shape changes due to Larmor rotation of the ions, in the case of weak electromagnetic radiation when the nonlinear effects are negligible. We seek for Eqs. (2.6) a perturbation-theory solution in the form of an expansion in Green's functions. In the irreducible-tensor representation, the zeroth term of the expansion of the density-matrix element ρ_{jj} diagonal in the level index is proportional to the total j-level population density

$$\rho_{jj}^{(0)}=N_jF(\mathbf{v}),\qquad F(\mathbf{v})=\frac{1}{(\sqrt{\pi}\bar{v})^3}\exp\left(-\frac{\mathbf{v}^2}{\bar{v}^2}\right). \qquad (3.1)$$

Here N_j is the total number of the ion excitations per unit volume on the level j in a time Γ_j^{-1}, and $\bar{v}$ is the mean thermal velocity of the ions.

Let, for the sake of argument, the z axis be directed along the magnetic field **H**, and let the x axis be in the plane of the vectors **k** and **H**. In this coordinate frame, the differential operator is

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{v}\nabla_\mathbf{r}-\omega_L\frac{\partial}{\partial\varphi}, \qquad (3.2)$$

where φ denotes the angle between the x axis and the vector $\mathbf{v}_\perp=\mathbf{v}\times\mathbf{H}/H$. In accordance with (2.9), the density matrix elements that are not diagonal in the level index

$$\rho_{mn,1\alpha}=R_\alpha\exp[-i(\Omega-\alpha\Delta)t+i\mathbf{kr}] \qquad (3.3)$$

satisfy in the irreducible-tensor representation the equation

$$[\Gamma-i(\Omega'-\alpha\Delta-k_\perp v_\perp\cos\varphi)-\omega_L\partial/\partial\varphi]R_\alpha=-iNF(\mathbf{v})G_\alpha, \qquad (3.4)$$
$$\Omega'=\Omega-k_zv_z,\qquad N=\frac{N_m}{2J_m+1}-\frac{N_n}{2J_n+1},\qquad \mathbf{k}_\perp=\frac{[\mathbf{kH}]}{H}.$$

The ion motion along the helical trajectories in a uniform magnetic field is described by the equation

$$\dot{\mathbf{v}}=[\mathbf{v}\boldsymbol{\omega}_L]. \qquad (3.5)$$

Its solutions (see, e.g., Ref. 16) correspond to the characteristics of Eq. (2.6). On the characteristics, the expression corresponding to (3.4) for the Green's function $f_\alpha(t|t')$ is of the form

$$f_\alpha(t|t')=\theta(\tau)\exp\{-(\Gamma-i\Omega'+i\alpha\Delta)\tau-i[\sin(\omega_Lt+\varphi)$$
$$-\sin(\omega_Lt'+\varphi)]k_\perp v_\perp/\omega_L\}; \qquad (3.6)$$
$$\tau=t-t',\quad \theta(\tau)=1 \text{ at } \tau\geqslant0 \text{ and } \theta(\tau)=0 \text{ at } \tau<0.$$

The solution for the off-diagonal element of the density matrix R_α, with (3.6) taken into account, can be represented by an expansion in the Bessel functions $J_l(x)$:

$$R_\alpha(\varphi)=-iNF(\mathbf{v})G_\alpha\exp\left\{i\frac{k_\perp v_\perp}{\omega_L}\sin\varphi\right\}$$
$$\times\sum_{l=-\infty}^{\infty}J_l\left(\frac{k_\perp v_\perp}{\omega_L}\right)e^{-il\varphi}\{\Gamma+i(\alpha\Delta+l\omega_L-\Omega')\}^{-1}. \qquad (3.7)$$

For Larmor rotation of ions, the emission-spectrum concept is meaningful only relative to motion along the entire time axis; a reflection of this in (3.7) is an infinite set of resonances at the frequencies

$$\Omega=\alpha\Delta+k_zv_z+l\omega_L. \qquad (3.8)$$

In the absence of rotation ($Ze=0$) or at $\mathbf{k}\|\mathbf{H}$ relation (3.7) reduces to the known result for a particle with constant velocity **v** (Ref. 15):

$$R_\alpha=-iNF(\mathbf{v})G_\alpha[\Gamma-i(\Omega-\alpha\Delta-\mathbf{kv})]^{-1}. \qquad (3.9)$$

In accordance with (3.6) and (3.7), the work of the optical field

$$P=-2\hbar\omega\,\mathrm{Re}\,\langle \mathrm{Sp}_M(iV^+\rho)\rangle_\mathbf{v} \qquad (3.10)$$

is expressed in terms of the Fourier transform F_α of the correlation function $\Phi(t)$:

$$P=2\hbar\omega N\sum_{\alpha}|G_\alpha|^2F_\alpha(\Omega_\alpha),\quad F_\alpha=\mathrm{Re}\int_0^\infty e^{i\Omega_\alpha t}\Phi(t)\,dt,$$

$$\Omega_\alpha=\Omega-\alpha\Delta, \tag{3.11}$$

$$\Phi(t)=\exp\left[-\Gamma t-\left(\frac{k_z\bar{v}t}{2}\right)^2-\left(\frac{k_\perp\bar{v}}{\omega_L}\right)^2\sin^2\frac{\omega_L t}{2}\right]. \tag{3.12}$$

The argument of the exponential in the correlation function $\Phi(t)$ on the background of its damping represents oscillations with Larmor frequency ω_L; this leads to the appearance of cyclotron resonances in the spectrum when the detuning coincides in the convolution (3.11) with harmonics of the frequency ω_L. A more illustrative picture of spectral deformations of this type is provided by expansion of the function $F_\alpha(\Omega_\alpha)$ in terms of the modified Bessel functions $I_l(\mu)$:

$$F_\alpha=\sum_{l=-\infty}^{\infty}e^{-\mu}I_l(\mu)\frac{\sqrt{\pi}}{k_z\bar{v}}\mathrm{Re}\,w\left(\frac{\Omega-l\omega_L+i\Gamma}{k_z\bar{v}}\right),$$

$$\mu=\frac{k_\perp^2\bar{v}^2}{2\omega_L^2},\quad w(t)=e^{-t^2}\left(1+\frac{2i}{\sqrt{\pi}}\int_0^t e^{t'^2}dt'\right). \tag{3.13}$$

It can be seen that in the general case the spectrum $P(\Omega)$ contains three Zeeman components with centers at $\Omega=\alpha\Delta$, each of which is represented in turn by an equidistant superposition of cyclotron resonances. Their centers are at the frequencies $\Omega_\alpha=l\omega_L$, and the amplitudes are related as $I_l(k_\perp^2\bar{v}^2/2\omega_L^2)$. A similar infinite set of harmonics is obtained for phase modulation or for parametric resonance.[17]

The spectrum is spatially anisotropic, since the widths of the individual components and their number are determined by the angle between the wave propagation direction **k** and the magnetic-field intensity vector **H**. For an axial magnetic field (**H**||**k**) only one term with $l=0$ is unequal to zero in the sum (3.13), and this equation yields a well-known expression for the Doppler spectral profile, i.e., the action of the magnetic field on the spectrum reduces to formation of a normal Zeeman triplet. In the case of transverse wave propagation, when **H**⊥**k**, the cyclotron resonances are represented by Lorentz contours $[\Gamma^2+(\Omega_\alpha+l\omega_L)^2]^{-1}$ with homogeneous width Γ. If the wave is linearly polarized and the vector **H** lies in the polarization plane, we have $E_{\pm1}=0$ and the spectrum contains only the π component of the radiation E_0. Consequently, in this specific case there is no Zeeman splitting of the spectrum and the influence of the magnetic field on the line shape manifests itself only via Larmor rotation of the ions.

If $\mu\ll 1$, then $[\exp(-\mu)]I_l(\omega)\propto\mu^l$ and only terms with $l=0$ and ±1 need be retained in (3.13). To estimate the number of resonances l_0 whose contribution to the line contour is substantial at $\mu\gg1$ we use the asymptotic expansion of the modified Bessel function at $\mu\gg l$ (Ref. 18):

$$e^{-\mu}I_l(\mu)\propto\exp(-l^2/2\mu).$$

Since $l^2/2\mu=(l\omega_L/k_\perp\bar{v})^2$, the spectrum envelope $F_\alpha(\Omega_\alpha)$ is a Lorentz contour. The characteristic number of cyclotron resonances is determined by the width of this contour:

$$l_0\sim\mu^{1/2}\sim k_\perp\bar{v}/\omega_L\gg1.$$

In an approximation in which the Doppler broadening is large ($k\bar{v}\gg|\Omega|,\Gamma,\Omega_L$) the expression for the work $P(\Omega)$ of the light field can be greatly simplified by using an asymptotic expansion of the integral in (3.11) by the saddle-point method.[19] Thus, in the case considered of transverse wave propagation (**k**⊥**H**) and at $E_{\pm1}=0$ we have

$$P(\Omega)\propto N|G_0|^2\left\{1+2\,\mathrm{Re}\left[\exp\left(2\pi\frac{\Gamma-i\Omega}{\omega_L}\right)-1\right]^{-1}\right\}. \tag{3.14}$$

The maxima of the periodic function $P(\Omega)$ that describes the contour of the spectral line are located at the points $\Omega=l\omega_L$, and the minima at $\Omega=(l+1/2)\omega_L$. The contrast of the cyclotron resonances in the spectra is determined by the expression $\xi=\mathrm{sech}(\pi\Gamma/\omega_L)$. At $\Gamma\ll\omega_L$, the resonances have high contrast ($\xi\approx1$), whereas at $\Gamma\gtrsim\omega_L$ the contrast is exponentially small: the resonances coalesce to form a Doppler contour.

§4. NONLINEAR EFFECT

We consider now the spectrum of resonant absorption (amplification) of a standing wave

$$\vec{\mathscr{E}}=2\mathrm{Re}(\mathbf{E}e^{-i\omega t}\cos\mathbf{kr}), \tag{4.1}$$

taking into account only the first nonlinear-in-radiation-intensity correction δP to the work of the light field, for Larmor rotation the ions and under conditions of significant Doppler broadening:

$$k\bar{v}\gg\Gamma,\ |\Omega|,\ \Delta,\ \omega_L. \tag{4.2}$$

In addition, to simplify the description of the dependence of δP on the radiation polarization and on the angular momenta, we confine ourselves for the time being to the case

$$k_\perp\bar{v}\lesssim\omega_L, \tag{4.3}$$

when the vector of the magnetic field **H** is almost axial and the contribution of the radiation component E_0 to δP can be neglected. Since the quantization axis is chosen as usual along the vector **H**, we have $|E_0\sim|E|k_\perp/k\ll|E|$.

In the approximation indicated, the nonlinear correction δP to the work of the light field is given by

$$\delta P\approx-\frac{2\sqrt{\pi}\,\hbar\omega N}{k\bar{v}}\mathrm{Re}\sum_{\alpha=\pm1}|G_\alpha|^2\Big\{|G_\alpha|^2A_0\sum_j\Big[Z_j(0,0)$$
$$+Z_j(\Omega_\alpha,0)\Big]+|G_{-\alpha}|^2\sum_j A_{j2}[Z_j(-\alpha\Delta,2\alpha\Delta)+Z_j(\Omega_\alpha,2\alpha\Delta)]$$
$$+|G_{-\alpha}|^2[A_{m2}Z_n(\Omega,0)+A_{n2}Z_m(\Omega,0)+A_{m2}Z_n(-\alpha\Delta,0)$$
$$+A_{n2}Z_m(-\alpha\Delta,0)]\Big]\Big\}, \tag{4.4}$$

$$A_{m\varkappa}=3\begin{Bmatrix}1&1&\varkappa\\J_m&J_n&J_n\end{Bmatrix}^2,\quad A_{n\varkappa}=3\begin{Bmatrix}1&1&\varkappa\\J_n&J_n&J_m\end{Bmatrix}^2, \tag{4.5}$$
$$A_0={}^1/_3A_{m0}+{}^1/_2A_{m1}+{}^1/_6A_{m2};$$

$$Z_j(\Omega_\alpha,q\Delta)=\sum_{l=-\infty}^{\infty}\int_0^\infty\frac{2\,dt}{\Gamma_j+i(q\Delta-l\omega_L)}\exp\Big[-2(\Gamma-i\Omega)t-il\omega_L t$$
$$-4\mu\sin^2\frac{\omega_L t}{2}\Big]I_l\left(4\mu\sin^2\frac{\omega_L t}{2}\right),\quad\mu=\frac{k_\perp^2\bar{v}^2}{2\omega_L^2},\quad j=m,n. \tag{4.6}$$

The dependence of the nonlinear absorption on the magnetic field was calculated by the method developed in §3. In the absence of Larmor rotation ($\omega_L = 0$) when the vector **H** is oriented along the wave vector **k**, the equation for δP reduces to the result obtained by D'yakonov and Perel' for the nonlinear Zeeman effect.[14,15] In the general case the spectral structures described by the functions $Z_j(\Omega,0)$, $Z_j(\Omega_\alpha,0)$, and $Z_j(\Omega_\alpha,2\alpha\Delta)$ correspond to Zeeman splitting of the Lamb dip in the absorption-line profile into three components with coordinates $\Omega = 0$ and $\pm\Delta$. It follows from (4.6) that at $\Gamma \ll \omega_L$ each of these components is represented by an infinite set of cyclotron resonances having a width of order Γ and centers at the frequencies

$$\Omega = l\omega_L/2, \quad \Omega = \pm\Delta + l\omega_L/2; \quad l = 0, \pm 1, \pm 2, \ldots \tag{4.7}$$

The spacing of the cyclotron resonances in nonlinear absorption is equal to half the cyclotron frequency ω_L for each of the three components.

If the wave is circularly polarized, there exists only one set of cyclotron resonances corresponding to a Zeeman shift of the Lamb dip at the point $\Omega = \Delta$ or $\Omega = -\Delta$ (left- or right-hand polarization, respectively). This set is a reflection of the analogous splitting of the Bennett dips in the velocity distribution of excited atoms into an infinite sequence of cyclotron structures.

The contributions of the structures $Z_j(\Omega_\alpha,q\Delta)$ to the nonlinear absorption are determined by the polarization of the radiation and by the coefficients $A_{j\varkappa}$ expressed in terms of Wigner 6*J*-symbols. In particular, for the energy states j with angular momenta $J_j = 0$ and 1/2 the coefficient A_{j2} is zero and δP does not contain the structures $Z_j(-\alpha\Delta,2\alpha\Delta)$ and $Z_j(\Omega_\alpha,2\alpha\Delta)$ that result from the Raman scattering by the split Zeeman sublevels at $J_j \geqslant 1$; nor does it contain the structures $Z_j(\Omega,0)$ and $Z_j(-\alpha\Delta,0)$. The resonant factor $(\Gamma_j + iq\Delta + l\omega_L)^{-1}$ in (4.6) reflects the action of the Larmor rotation on the excited-level populations; its appearance is equivalent to splitting of the levels into an infinite number of equidistant components separated by the frequency interval ω_L. The simplest example of such a splitting of working levels into an infinite number of sublevels is the interaction of a two-level system with a bichromatic electromagnetic field.[6]

§5. REAL OBSERVATION AND UTILIZATION OF CYCLOTRON EFFECTS

The main conclusion of our study is that Larmor rotation of ions in a magnetic field can lead to cyclotron splitting of both linear and nonlinear spectral structures. For cyclotron resonances to manifest themselves in a spectrum it is necessary that the time τ of stay of the ion in a light beam propagating through a partially ionized gas exceed substantially the period of the cyclotron oscillations:

$$\tau\omega_L > 1, \quad \tau = a/\bar{v}, \tag{5.1}$$

where a is the radius of the light beam. The corresponding maximum magnetic field strength is defined by the inequality H [Oe] $> 10^{-4} M\bar{v}/a$, where M is the ion mass in atomic units. In particular, at $\bar{v} \sim 10^5$ cm/sec, $a \sim 1$ cm, and $M \sim 10$ we have $H > 10^2$ Oe. Another natural condition for separating cyclotron resonances in a spectrum is that the homogeneous line width be small compared with the ion cyclotron frequency:

$$\Gamma < \omega_L. \tag{5.2}$$

The magnetic field intensity H_0 starting with which this condition is satisfied is listed in the table for individual transitions of certain elements under the assumption that the width Γ is determined by radiative decay. In this case $H_0 \approx 10^{-4} M\gamma$, where γ is the natural line width.

It is known[20] that the radiative lifetime τ_r of an excited state of a hydrogenlike ion increases steeply with increasing principal quantum number $n(\tau_r \propto n^{9/2})$. The inequality $\gamma < \omega_L$ for the natural line width of the $n-1 \leftrightarrow n$ transition of He II begins therefore to be satisfied at $n \geqslant 6$ if $H \gtrsim 2 \cdot 10^4$ Oe. In the far infrared, the maximum magnetic field intensity H_0 decreases with increasing n. The smallest values of H_0 correspond to the Ca II ion $4s$–$3d$ transitions which are forbidden in the electrodipole approximation. Cyclotron resonances at these transitions of calcium or strontium vapor can be recorded by the methods of two-photon-transition spectroscopy.[6] Particularly convenient for this purpose is a spectral analysis of two-photon absorption in the $4s$–$3d$ transition, or of resonance Raman scattering in interaction of radiation with the $4s$–$3d$–$4p$ three-level system in the relatively weak magnetic field $H \sim 10^2$ Oe. Lasing in which cyclotron effects can likewise manifest themselves was obtained on the electrodipole transitions $3d$–$4p$ in Ca II ions and $4d$–$5p$ in Sr II.

In a low temperature plasma, account must be taken of the role of inelastic collisions with electrons, as well as of the Stark effect; the latter can turn out to be significant for highly excited states. At not too high an electron density n_e in the plasma, the contribution of the Stark broadening to Γ will be acceptable. Estimating, by the method described in Ref. 21, the contribution of collision broadening by electrons and by quasistatic ions in He II transitions, we find that the electron density should not exceed $n_e \sim 10^3$ cm^{-3} in order of magnitude; level quenching by electrons can be neglected in this case. For allowed transitions of nonhydrogenlike ions, the total contribution to the homogeneous widths of inelastic collisions with electrons and of the Stark broadening ν_e at $\gamma \sim 10^7$ sec^{-1} is characterized by the relation

$$\nu_e/\gamma \sim n_e \cdot 10^{-12} \text{ cm}^3, \tag{5.3}$$

from which it follows that these effects are immaterial at $n_e < 10^{12}$ cm^{-3}. The natural widths γ used to calculate H_0 in (5.3) were obtained from the transition-probability tables of Refs. 22 and 23. For forbidden transitions in Ca II and Sr II ions this contribution is smaller by about an order of magnitude:

$$\nu_e \sim n_e \cdot 10^{-6} \text{ cm}^3 \cdot \text{sec}^{-1}. \tag{5.4}$$

Elastic collisions of particles in a plasma can be accompanied by a change of their velocity. When an excited ion moves in a low temperature plasma, it is convenient to distinguish between two types of collisions that change the ion velocity: 1) collision of the ion with neutral atoms of the same species, wherein resonant charge exchange with simulta-

TABLE I

Element	Transition	Terms	Wavelength μm	H_0, Oe
^{4}He II	6–7	–	3,09	$2\cdot 10^4$
^{3}He II	13–14	–	27,95	$6\cdot 10^2$
^{7}Li II	$1s2s-1s2p$	$^3S_1-{}^3P_1{}^0$	0,5485	$8{,}7\cdot 10^3$
^{40}Ca II	$3d-4p$	$^2D_{5/2}-{}^2P^0_{3/2}$	0,8542	$7\cdot 10^4$
	$3d-4p$	$^2D_{3/2}-{}^2P^0_{1/2}$	0,8662	$7{,}5\cdot 10^4$
	$4s-3d$	$^2S_{1/2}-{}^2D_{5/2}$	0,7293	$\sim 10^{-2}$
	$4s-3d$	$^2S_{1/2}-{}^2D_{3/2}$	0,7326	$\sim 10^{-2}$
^{88}Sr II	$4d-5p$	$^2D_{5/2}-{}^2P^0_{3/2}$	1,0326	$\sim 10^5$
	$4d-5p$	$^2D_{3/2}-{}^2P^0_{1/2}$	1,0918	$\sim 10^5$
	$5s-4d$	$^2S_{1/2}-{}^2D_{5/2}$	0,6740	$\sim 0{,}1$
	$5s-4d$	$^2S_{1/2}-{}^2D_{3/2}$	0,6870	$\sim 0{,}1$
^{173}Yb II	$4f^{14}(^1S)6s-$ $-4f^{14}(^1S)6p$	$^2S_{1/2}-{}^2P^0_{3/2}$	0,3694	$2\cdot 10^5$

neous excitation transfer takes place, and 2) Coulomb ion-ion scattering.

The charge-exchange scattering cross section $\sigma_a \sim 10^{-14}$–10^{-15} cm^2 greatly exceeds the gas-kinetic value. The relative contribution of the effective charge-exchange frequency ν_a to the line width at $\sigma_a \sim 10^{-14}$ cm^2 and $\gamma \sim 10^7$ sec^{-1} can be estimated from the formula

$$\nu_a/\gamma \sim n_a \cdot 10^{-16}\ \text{cm}^3, \tag{5.5}$$

where n_a is the neutral-atom density. Collisions with charge exchange can therefore be neglected at $n_a \lesssim 10^{15}$ cm^{-3}.

In a gas-discharge plasma, the electron temperature usually exceeds greatly the ion temperature, so that diffusion in velocity space is due mainly to ion-ion collisions. As shown in Ref. 9, the Coulomb increments $\gamma_{m,n}$ to the Lorentz widths Γ of the nonlinear-absorption (gain) resonances are less dependent on the ion temperature T_i than the ion-collision frequency ν_{ii}, which is determined by the dynamic-friction coefficient and is proportional to $T_i^{-3/2}$. For example, in $3d$–$4p$ transitions of Ca II ions the Coulomb broadening of narrow nonlinear resonances are determined by the constant

$$\gamma_m \sim (10^2-10^3)\, n_i^{1/2} T_i^{-1/4}\ \text{cm}^{3/2}\cdot\text{sec}^{-1}, \tag{5.6}$$

where n_i is the ion density. It follows hence that at $T_i \sim 5000$ K and $n_i \sim 10^{12}$ the mechanism of Coulomb scattering through small angles during the lifetime of the upper level $\Gamma_{4p}^{-1} \sim 10^{-7}$ sec can play a rather noticeable role, alongside the Stark broadening, in the shaping of the nonlinear-resonance profiles; the magnetic field needed for their cyclotron splitting to become manifest increases accordingly.

Cyclotron splitting of the spectrum can be observed also in other systems, primarily in resonant interaction of radiation with vibrational-rotational transitions of molecular ions. Since the scale of the Zeeman splitting of the spectrum is set in this case by the nuclear magneton, the Zeeman-splitting parameter Δ can be of the same order as the frequency ω_L. For atomic ions, on the contrary, $\omega_L \ll \Delta$ since the electron mass $m_e \ll m_i$. Cyclotron effects can be observed in weak magnetic fields $H \lesssim 10^3$ Oe in molecular-ion transitions with small radiative widths.

The problem of obtaining spectral resonances of minimum width is of paramount importance for ultrahigh resolution spectroscopy. We have demonstrated in §3 the feasibility, in linear absorption or amplification of radiation, of cyclotron spectral resonances with widths equal to the homogeneous line width, if the magnetic field is perpendicular to the wave vector. Larmor rotation of the ions eliminates the Doppler broadening of the ion spectrum, so that it becomes much easier to obtain information on the structure of the quantum transition of individual particles, on radiative relaxation, and on collision processes in a plasma. Thus, a principally new ultrahigh-resolution ion-spectroscopy method can be based on recording narrow cyclotron resonances of linear absorption or amplification of radiation. Linear cyclotron resonances can find another important application as the basis for development of laser systems of highly stable frequency. One can use for this purpose, in particular, the effect of frequency locking in lasing, which becomes much more resonant in the presence of cyclotron spectrum and when the Doppler broadening of the spectrum is eliminated.

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