Forbidden beta-decay in the field of a strong electromagnetic wave

E. Kh. Akhmedov

I. V. Kurchatov Institute of Atomic Energy, Academy of Sciences of the USSR, Moscow
(Submitted 17 April 1984)
Zh. Eksp. Teor. Fiz. **87**, 1541–1551 (November 1984)

A detailed discussion is given of the effect of a strong monochromatic electromagnetic wave on unique first-forbidden β-transitions. The possibility that the absorption of field quanta from the wave by the nucleus (or the emission of such quanta into the wave) may result in the relaxation of this forbiddeness is examined. It is shown that the predictions that there should be a substantial increase in the probability made by Reiss [Phys. Rev. C **27**, 1199, 1229 (1983)] of forbidden β-transitions in a strong wave are incorrect.

I. INTRODUCTION

The effect of a strong electromagnetic wave on the β-decay of nuclei has been under intensive discussion in recent years because of the advent of powerful sources of electromagnetic radiation.[1–13] It has been suggested[1,2] that the half-life of tritium will decrease by a substantial factor (10^2–10^4) in the electromagnetic wave field produced by powerful modern lasers. However, these results were subsequently shown to be incorrect.[3–10] It has also been reported[4–10] that the relative correction to the β-decay probability of tritium is $\delta W/W_0 \lesssim 10^{-7}$–$10^{-8}$. For nuclei with large energy release ε_0, the effect of the external field on the total β-decay probability turns out to be smaller still.

The principal mechanism responsible for the reduction in half-life in allowed β-transitions in an external field is the increase in the phase space due to the interaction of the emitted electron and the wave field. It has usually been assumed that the interaction of parent and daughter nuclei with the external field could be neglected, and that the matrix elements of the process in the field are practically the same as in the absence of the field.[1–10]

A different mechanism for the increase in the β-decay probability in the field of an electromagnetic wave is possible for forbidden β-transitions and involves the interaction between the nucleus and the external field. The absorption of n dipole quanta from the wave by the nucleus (or the emission of such quanta into the wave) means that the nucleus can receive n units of angular momentum[1)] and may modify the selection rule from n-fold forbidden to allowed. This mechanism of relaxation of forbiddenness was examined in Refs. 11 and 12. The author of these papers concludes that a substantial increase in the probability of the process is possible when the field strength F in the wave and its frequency ω are related by $z \equiv (eFR/\omega)^2 \sim n$ (R is the nuclear radius). These calculations predicted a reduction in the half-life by a factor of 3.8 in the case of the first-forbidden $^{90}Sr \to {}^{90}Y$ transition, by 5 orders of magnitude for the third-forbidden $^{87}Rb \to {}^{87}Sr$ transition, and by 12 orders of magnitude for the fourth-forbidden $^{113}Cd \to {}^{113}In$ transition. Certain corrections to the calculations reported in Refs. 11 and 12 were subsequently published in Ref. 13, but the main results were not reexamined. It will be shown below that these results are, in fact, completely incorrect: the lifting of forbiddenness by external electromagnetic fields will not produce any perceptible reduction in the half-life of nuclei in fields that are currently attainable under laboratory conditions. Nevertheless, the effect of the external field on the created electron is more appreciable for forbidden than for allowed β-decays, which underlines the importance of the study of forbidden transitions.

2. FORBIDDENNESS-LIFTING MECHANISM

The parameter $z^{1/2} = eFR/\omega$ introduced in Refs. 11 and 12 is the ratio of the classical energy studied by a particle of a charge e in the field F at distance R to the energy ω of the quantum. It does not take into account the fact that the nucleus is a quantum-mechanical system and cannot absorb photons in a continuous manner. The actual lifting of forbiddenness occurs as follows. Suppose that the decaying nucleus has an excited state $|1\rangle$ with angular momentum and parity permitting an allowed β-transition from this state to the final state $|f\rangle$. The parent nucleus can then undergo a virtual transition to the state $|1\rangle$ by absorbing one or more quanta from the wave (or by emitting them into the wave) followed by the allowed β-transition. An analogous situation will obtain when the daughter nucleus has a state $|2\rangle$ with quantum numbers permitting an allowed $|i\rangle \to |2\rangle$ β-transition followed by an electromagnetic $|2\rangle \to |f\rangle$ transition under the influence of the external field.

In this paper, we shall examine the simple case of unique first-forbidden β-transitions for which the selection rules have the form $\Delta J^{\Delta\pi} = 2^-$. Higher-order forbidden transitions will be examined in a similar manner.

For unique first-forbidden transitions, the state $|1\rangle$ must be connected with the ground state $|i\rangle$ of the parent nucleus by an electric dipole transition. The $|1\rangle \to |f\rangle$ transition will then be an allowed Gamow-Teller transition with $\Delta J^{\Delta\pi} = 1^+$. An analogous situation will arise for transitions through a virtual state $|2\rangle$ of the daughter nucleus. It is readily seen that any other possibility (for example, electromagnetic $M2$ transition and allowed Fermi transition with $\Delta J^{\Delta\pi} = 0^+$ are less convenient because they result in a much smaller matrix element.

Let us now estimate the matrix element for the process corresponding to the relaxation of forbiddenness in external fields. Let us denote the energy differences between the states $|1\rangle, |i\rangle$ and $|2\rangle, |f\rangle$ by

$$\Delta\varepsilon_1 \equiv \varepsilon_1 - \varepsilon_i, \quad \Delta\varepsilon_2 \equiv \varepsilon_2 - \varepsilon_f.$$

 0038-5646/84/110884-06$04.00

The admixture of the state $|1\rangle$ to the ground state $|i\rangle$ in the external field of frequency $\omega \ll \Delta\varepsilon_1$ will be characterized by the parameter $eFd_{1i}/\Delta\varepsilon_1$, where d_{1i} is the dipole matrix element of the corresponding electromagnetic transition. Similarly, the admixture of the state $|2\rangle$ to the final state $|f\rangle$ of the daughter nucleus will be characterized by the parameter $eFd_{2f}/\Delta\varepsilon_2$. To estimate the matrix element $M_{\rm ind}$, we replace the dipole matrix element d_{1i} and d_{2f} with the nuclear radius R and obtain

$$M_{ind} \sim (eFR/\Delta\varepsilon_{1,2}) \cdot 1. \qquad (1)$$

It is assumed in this that the matrix elements for allowed β-transitions are of the order of unity although, in reality, they are somewhat smaller for intermediate and heavy nuclei.

We note that the "forbiddenness lifting parameter" $eFR/\Delta\varepsilon_{1,2}$ depends on the quantities $\Delta\varepsilon_{1,2}$ and, in contrast to the parameter $z^{1/2}$ of Refs. 11 and 12, cannot be made greater by reducing the external-field frequency ω.

The order of magnitude of the matrix element of the unique first-forbidden transition in the absence of the field is $M_0 \sim k_0 R$, where k_0 is the maximum momentum of the electron, determined by the kinetic-energy release $\varepsilon_0 = M_i - M_f - m$ (M_i and M_f are the masses of the parent and daughter nuclei, and m is the electron mass). It will be shown below that the effect of the external field on the parameters of the forbidden β-decay increases with decreasing energy release. We shall therefore consider the case where $\varepsilon_0 \ll m$. We then have $M_0 \sim (2m\varepsilon_0)^{1/2}R$. The increase in the β-decay probability in the field due to the lifting of forbiddenness is characterized by the factor

$$\delta W/W_0 \sim (M_{ind}/M_0)^2 \sim [eF/(2m\varepsilon_0)^{1/2}\Delta\varepsilon_{1,2}]^2. \qquad (2)$$

This is smaller by the factor $(\omega/\Delta\varepsilon_{1,2})^2$ than the corresponding parameters in Ref. 11 and 12:

$$z/(k_0R)^2 \sim [eF/(2m\varepsilon_0)^{1/2}\omega]^2 \equiv \xi^2 \qquad (3)$$

and, in contrast to the latter, it does not depend on the external-field frequency for $\omega \ll \Delta\varepsilon_{1,2}$. In all known cases, the energies of levels with suitable quantum numbers satisfy the conditions $\Delta\varepsilon_{1,2} \gtrsim 10$ keV. In approximate estimates, if we take $\omega \lesssim 1$, we obtain $(\omega/\Delta\varepsilon_{1,2}) \lesssim 10^{-8}$.

Thus, simple estimates show that the field-induced change in the half-life obtained in Refs. 11 and 12 for first-forbidden transitions is too high by more than eight orders of magnitude. The discrepancy is greater still for higher degrees of forbiddenness (see Sec. 9).

3. INTERACTION OF ELECTRON WITH THE WAVE FIELD

It was shown in Refs. 3–10 that the field-induced increase in the probability of allowed β-transitions was largely due to the increase in the phase space occupied by the created electrons. The corresponding probability is

$$W \approx W_0[1 + (^{35}/_8)\chi^2 + \ldots], \qquad (4)$$

where W_0 is the field-free probability and $\chi = eF/(2m\varepsilon_0)^{1/2}2\varepsilon_0$. The parameter χ has a simple physical interpretation: its order of magnitude is that of the ratio of the energy acquired by the electrons in the field F over the distance equal to the de Broglie wavelength to the energy release ε_0. For tritium ($\varepsilon_0 = 18.6$ keV) at the limit that can be attained under laboratory conditions we have $\chi \lesssim 10^{-5}$–10^{-4}.

This mechanism of field-induced reduction in half-life is also found to occur for forbidden β-transitions. Moreover, the change in the electron wave function in the external field produces a change in the matrix elements for the process in this case. It is readily verified that the corresponding contribution to the total probability for the process should also be of the order of χ^2. In fact, the field-free matrix element is

$$M_0 = \int \varphi_f^* O \varphi_i e^{-i(\mathbf{k}+\mathbf{p})\mathbf{r}}\, d\mathbf{r}, \qquad (5)$$

where φ_i and φ_f are the wave functions of the parent and daughter nuclei, $\mathbf{k}$ and $\mathbf{p}$ are the momenta of the electron and neutrino (we are neglecting the effect of the Coulomb field of the daughter nucleus on the electron wave function), and O the transition operator. Since $k_0R \ll 1$, we can use the expansion $\exp[-i(\mathbf{k}+\mathbf{p})\mathbf{r}] \approx 1 - i(\mathbf{k}+\mathbf{p})\mathbf{r} + \ldots$ in the integral (5). For allowed transitions, the main contribution is provided by the first term of the expansion, whereas for unique first-forbidden transitions, the main contribution is due to the second term. Accordingly, a change in the electron momentum by the amount Δk is accompanied by a change in the square of the matrix element of the forbidden process by the amount $(\Delta k/k_0)^2$. It is readily seen that Δk must be calculated as the change in the electron momentum in the field over a distance of the order of the de Broglie wavelength. Greater distances do not contribute to the total probability of the process.[7-10] Hence, it follows that $(\Delta k/k_0)^2 \sim \chi^2$, which provides us with the required estimate.

The change in the angular momentum of the electron when it absorbs (or emits) field quanta does not lead to lifting of forbiddenness because the degree of forbiddenness is determined by the angular momenta and parities of the nuclear states between which the transition takes place. We have carried out a qualitative analysis of the effect of an intense electromagnetic wave on the probability of a forbidden β-transition. We must now proceed to the calculation of the probability of the process.

4. THE WAVE FUNCTION OF THE ELECTRON IN THE WAVE FIELD

When the β-decay process in an external field was examined in Refs. 1–7, the electron wave function in the field of an intense electromagnetic wave was taken to be the exact solution of the corresponding Dirac equation, i.e., the Volkov wave function.[14] As already noted, the effect of the field on the β-decay parameters of the nuclei is enhanced as the energy release ε_0 is reduced. We shall therefore consider the nonrelativisitic case $\varepsilon_0 \ll m$.

The electron wave function can be written in the following form in the dipole approximation:

$$\psi_e'(\mathbf{r},t) = \exp\left\{ i\mathbf{kr} - \frac{i}{2m}\int^t [\mathbf{k} - e\mathbf{A}(t')]^2\, dt' \right\}. \qquad (6)$$

It constitutes the exact solution of the Schrödinger equation for a particle in a time-dependent uniform electric field described by the vector potential $\mathbf{A}(t)$. This wave function has been used in the analysis of different problems in atomic physics in external fields[15,16] and β-decays in wave fields.[9] It is the nonrelativistic analog of the Volkov solution, and was obtained in the Coulomb gauge: $A'^{\mu} = \{0,\mathbf{A}(t)\}$ the scalar gauge $A^{\mu} = \{\varphi,0\} = \{-\mathbf{F}(t)\mathbf{r},0\}$, where $\mathbf{F}(t)$ is the electric field, has also been frequently empolyed to describe the electric dipole interaction. The electron wave function in the scalar gauge is

$$\psi_e(\mathbf{r},t)=\exp\left\{i[k-e\mathbf{A}(t)]\mathbf{r}-\frac{i}{2m}\int^t[\mathbf{k}-e\mathbf{A}(t')]^2\,dt'\right\} \quad (7)$$

where $\mathbf{A}(t)$ is no longer the vector potential which is equal to zero in the scalar gauge. By definition,

$$\mathbf{A}(t)\equiv-\int^t\mathbf{F}(t')\,dt'.$$

The wave function (7) was used in Ref. 17 to examine multiphoton ionization.

Under the gauge transformation $A^{\mu}\to A'^{\mu} = A^{\mu} - \partial^{\mu}\eta$, the wave function transforms in accordance with the law $\psi\to\psi' = \exp(ie\eta)\psi$. The transformation from the scalar to the Coulomb gauge is accomplished with the aid of the gauge function $\eta(\mathbf{r},t) = \mathbf{A}(t)\mathbf{r}$. It is readily seen that the the wave functions given by (6) and (7) are actually related by

$$\psi_e'(\mathbf{r},\ t)=e^{ie\mathbf{A}(t)\mathbf{r}}\psi_e(\mathbf{r},\ t). \quad (8)$$

As usual, the probability of the process does not, of course, depend on the particular gauge employed, but a successful choice of gauge can simplify calculations. We have found the scalar gauge to be more convenient in the case of the forbidden β-decay in the wave field. The electron wave function (7) is therefore used in the present paper.

We shall suppose that the electromagnetic wave is circularly polarized, i.e.,

$$\mathbf{A}(t)=A_0\{\cos\omega t,\ \sin\omega t,\ 0\}, \quad (9)$$

or, equivalently,

$$\mathbf{F}(t)=F\{\sin\omega t,\ -\cos\omega t,\ 0\},\quad F=\omega A_0. \quad (10)$$

5. NUCLEAR WAVE FUNCTIONS IN THE WAVE FIELD

The wave functions of the initial and final states of the nuclei in the field of an intense electromagnetic wave were taken in Refs. 11 and 12 in the form

$$\psi_{i,f}=\exp[ie_{i,f}\mathbf{A}(t)\mathbf{r}]\varphi_{i,f}, \quad (11)$$

where φ_i and φ_f are the corresponding field-free wave functions and e_i and e_f are the charges on the parent and daughter nuclei. The wave functions (11) were obtained in the Coulomb gauge in the approximation

$$\omega R\ll1,\quad e\omega A_0R/\Delta\varepsilon\ll1, \quad (12)$$

where $\Delta\varepsilon$ are the characteristic nuclear energy differences. The first of the two conditions in (12) is the condition for the validity of the dipole approximation and the second indicates that the external field produces slight mixing of the nuclear states. For fields that can be attained in practice, the two conditions in (12) are satisfied with a considerable margin. Nevertheless, the wave functions (11) are too approximate and, as we shall presently show, cannot describe the removal of forbiddennes by any external electromagnetic field. Direct substitution of (11) in the Schrödinger equation will show that these functions were obtained in zero order in the parameter $e\omega A_0R/\Delta\varepsilon$. This can also be seen from the fact that the factors $\exp[ie_{i,j}\mathbf{A}(t)\mathbf{r}]$ describe the transition from the scalar to the Coulomb gauge [see (11) and (8)]. This means that the functions $\varphi_{i,f}$ should be the nuclear wave functions in the wave field in the scalar gauge. However, as already noted, the functions $\varphi_{i,f}$ in (11) are understood to be the field-free nuclear wave functions. Hence, it follows that the wave functions given by (11) are equivalent from the gauge point of view to the complete neglect of the interaction between the nucleus and the external field. It follows from (6), (7), (11), and charge conservation $e_i - e_f = e$ that the use of nuclear wave functions (11) and the electron wave function (6) in the Coulomb gauge is equivalent to the use of the electron wave function (7) and the nuclear wave functions $\varphi_{i,f}$, that do not take into account the interaction between the nuclei and the external field in the scalar gauge.

As noted in Sec. 2, the mixing of the nuclear wave functions in the field (and this means the lifting of forbiddenness) occurs only in the first order in the parameter $e\omega A_0R/\Delta\varepsilon_{1,2} = eFR/\Delta\varepsilon_{1,2}$. Thus, the wave functions (11) corresponding to the zero-order approximation cannot describe the lifting of forbiddenness in the field. The nonzero results reported in Refs. 11 and 12 will be shown below to be the consequence of certain subsequent and unjustifiable assumptions that produce, in particular, a breakdown in gauge invariance.

When nuclear wave functions in the wave field are calculated, it is convenient to use the scalar gauge because the mixing of the nuclear levels by the external field then appears even in first-order perturbation theory. Let us elucidate this proposition. In the scalar gauge, the condition for the validity of perturbation theory is $eFR/\Delta\varepsilon_{1,2}\ll1$, whereas, in Coulomb gauge, the condition is

$$A_0\sum_i(e_ip_i/M\Delta\varepsilon_{1,2})\ll1,$$

where e_i, M, and p_i are, respectively, the charge, mass, and momentum of the ith nucleon. Using the expressions $F = \omega A_0$ and $i\mathbf{p}_i/M = [\mathbf{r}_i,H_0]$ (H_0 is the field-free nuclear Hamiltonian), we find that the perturbation-theory parameter in the nuclear gauge is of the order of $eFR/\omega\gg eFR/\Delta\varepsilon_{1,2}$ This means that, when the Coulomb gauge is imposed for $eFR/\omega\gtrsim1$, the determination of the nuclear wave functions in the wave field necessarily involves the summation of the entire perturbation theory series. This summation can be performed in a general form in the limit as $eFR/\Delta\varepsilon_{1,2}\to0$ and the final result has the form of (11). However, as already noted, the wave functions given (11) do not describe the lifting of forbiddenness. The determination of the nuclear wave functions $\psi'_{i,f}$ in the Coulomb gauge in the first order in the parameter $eFR/\Delta\varepsilon_{1,2}$ involves laborious calculations. On the other hand, in the scalar gauge, corrections of the order of $eFR/\Delta\varepsilon_{1,2}$ arise even in first-order perturbation theory. If the parameter $eFR/\Delta\varepsilon_{1,2}$ is small, we can confine our attention to the first order:

$$\psi_i(\mathbf{r},t)\approx\varphi_i(\mathbf{r},t)-ie\frac{F}{\sqrt{2}}\left\{(d_{-1})_{1i}\frac{\exp[i(-\varepsilon_i+\omega)t]}{\varepsilon_1-\varepsilon_i+\omega}\right.$$

$$+(d_{+1})_{1i}\frac{\exp[i(-\varepsilon_i-\omega)t]}{\varepsilon_1-\varepsilon_i-\omega}\Big\}\varphi_1(\mathbf{r}),$$

$$\psi_f(\mathbf{r},t)\approx\varphi_f(\mathbf{r},t)-ie\frac{F}{\sqrt{2}}\Big\{(d_{-1})_{2f}\frac{\exp[i(-\varepsilon_f+\omega)t]}{\varepsilon_2-\varepsilon_f+\omega} \tag{13}$$

$$+(d_{+1})_{2f}\frac{\exp[i(-\varepsilon_f-\omega)t]}{\varepsilon_2-\varepsilon_f-\omega}\Big\}\varphi_2(\mathbf{r}),$$

where the unperturbed wave function of the initial state $|i\rangle$ is denoted by $\varphi_i(\mathbf{r},t)\equiv\varphi_i(\mathbf{r}_1,...,\mathbf{r}_A,t)\equiv\varphi_i(\mathbf{r})\exp(-i\varepsilon_i t)$ and similarly for the states $|1\rangle$, $|f\rangle$, and $|2\rangle$; $(d_{\pm 1})_{1i}$ and $(d_{\pm 1})_{2f}$ are the matrix elements of the dipole operator

$$\mathbf{d}=\frac{N}{A}\sum_{i=1}^{Z}(\mathbf{r}_p)_i-\frac{Z}{A}\sum_{i=1}^{N}(\mathbf{r}_n)_i, \tag{14}$$

evaluated in the spherical basis. The quantities Z and N in (14) are, respectively, the numbers of protons and neutrons in the nucleus, $Z+N=A$, and $(\mathbf{r}_p)_i$ and $(\mathbf{r}_n)_i$ are the position vectors of the ith proton and neutron. When the matrix elements of the operator (14) are evaluated, it is important to remember that the numbers of protons in the daughter and parent nuclei differ by unity. This also applies to neutrons because the mass numbers A of the two nuclei are equal.

The normalizing factors of the functions ψ_i and ψ_f are omitted from (13). We shall neglect the difference between these factors and unity (see Sec. 7).

6. EVALUATION OF THE PROBABILITY OF THE PROCESS

The wave function ψ_i in (13) takes into account the admixture to the state φ_i of only the nearest-energy state φ_1 with suitable quantum numbers. We assume that other levels with the same angular momentum and parity lie much higher. This also applies to the state ψ_f. We shall further suppose that either $\Delta\varepsilon_1\gg\Delta\varepsilon_2$ or $\Delta\varepsilon_2\gg\Delta\varepsilon_1$, which is always satisfied in practice.

We begin by considering the case $\Delta\varepsilon_2\gg\Delta\varepsilon_1$ for which the field-induced lifting of forbiddenness proceeds through the excited state $|1\rangle$ of the parent nucleus. To determine the forbidden β-decay probability in the electromagnetic wave field, we use the well-known results of β-decay theory,[18] replacing the usual nuclear and electron wave functions with the expressions given by (13) and (7), respectively. The matrix element for the process can be written in the form

$$M=\int_{-\infty}^{\infty}dt\mathcal{M}(t),$$

where

$$\mathcal{M}(t)=\mathcal{M}_1(t)+\mathcal{M}_2(t)=\frac{G_\beta}{\sqrt{2}}\Big\{\frac{1}{2}g_A B_{ij}[p+k-eA(t)]_j$$

$$-ie\frac{F}{\sqrt{2}}\Big[\frac{(d_{-1})_{1i}e^{i\omega t}}{\Delta\varepsilon_1+\omega}+\frac{(d_{+1})_{1i}e^{-i\omega t}}{\Delta\varepsilon_1-\omega}\Big]g_A\langle f|\sigma_i\tau_\pm|1\rangle\Big\}$$

$$\times\bar{u}_e\gamma_i(1+\gamma_5)u_\nu\exp\Big[i\Big(\frac{k^2}{2m}+\varepsilon_k+p-\varepsilon_0\Big)$$

$$\times t-i\frac{eF}{m\omega^2}k_\perp\sin(\omega t-\varphi)\Big], \tag{15}$$

where $G_\beta=G\cos\theta_c$, G is the Fermi weak interaction constant, θ_c is the Cabibbo angle, $g_A=1.25$ is the axial-vector coupling constant, $\mathbf{k}$ and $\mathbf{p}$ are the electron and neutrino momenta, u_e and u_ν are the electron and neutrino 4-spinors, γ_5 and γ_i $(i=1,2,3)$ are the Dirac matrices,

$$B_{ij}\equiv\langle f|[\sigma_i x_j+\sigma_j x_i-{}^2/_3(\sigma\mathbf{r})\delta_{ij}]\tau_\pm|i\rangle,\quad \langle f|\sigma_i\tau_\pm|1\rangle$$

are, respectively, the nuclear matrix elements of the unique first-forbidden β-transition and allowed Gamow-Teller β-transition,[18] $\varepsilon_k=e^2F^2/2m\omega^2$ is the mean vibrational energy of the electron in the wave field, and

$$k_\perp=(k_x^2+k_y^2)^{1/2},\quad \varphi=\mathrm{arctg}\,(k_y/k_x).$$

The first term in the braces in (15) describes the direct $|i\rangle\to|f\rangle$ β-transition with corrections for the effect of the external field on the electron wave function, and the second term describes the lifting of forbiddenness, i.e., the transition

$$|i\rangle\xrightarrow{\gamma}|1\rangle\xrightarrow{\beta}|f\rangle.$$

We draw attention to the prescence of the term $-eA_j(t)$ in the factor following B_{ij} in (15). The reason for the appearance of this term is the characteristic coordinate dependence of the electron wave function in the wave field, given by (7). The absence of this term would lead to breakdown of gauge invariance of the amplitude for the process.

The parameters of the process are determined by the square of the modulus of the matrix element, summed over the components of the spin of the electron and neutrino and the angular momentum of the daughter nucleus, averaged over the components of the angular momentum of the parent nucleus:

$$\frac{1}{2J_i+1}\sum_{\substack{\sigma_e,\sigma_\nu,\\ M_i,M_f}}|M|^2=\frac{1}{2J_i+1}\int dt_1\,dt_2\sum_{\substack{\sigma_e,\sigma_\nu,\\ M_i,M_f}}\mathcal{M}^*(t_1)\mathcal{M}(t_2). \tag{16}$$

The integrand on the right-hand side of (16) contains a term proportional to

$$|\langle f\|B^{(2)}\|i\rangle|^2\{\mathbf{k}^2+e^2\mathbf{A}(t_1)\cdot\mathbf{A}(t_2)-e[\mathbf{A}(t_1)+\mathbf{A}(t_2)]\cdot\mathbf{k}\}, \tag{17}$$

where $\langle\|.B^{(2)}.\|\rangle$ is the reduced matrix element for B_{ij}. The term p^2 is omitted from the braces in (17) because, in the case that we are considering, $\varepsilon_0\ll m$ so that this term provides a small contribution ($\sim\varepsilon_0/m$) to the probability of the process. We have also omitted terms that are linear in p because they cancel out on integration in the direction of emission of the neutrino.

If we use the simple method developed in Ref. 7 for evaluating the probabilities of quantum-mechanical processes in the field of a strong electro-magnetic wave (see also Refs. 8 and 10), we can readily obtain the expression for the total probability of forbidden β-decay in the wave field in the form of a single integral:

$$W=W_1+W_2,$$

$$W_1=\frac{G_\beta^2 g_A^2}{2}\frac{1}{2J_i+1}|\langle J_f\|B^{(2)}\|J_i\rangle|^2\frac{i(\pi i)^{1/2}}{32\pi^4}m^{5/2}\omega^{9/2}$$

$$\times\int_{-\infty}^{\infty}\frac{dx}{(x+i0)^{11/2}}\Big\{1-\frac{2}{3}i\gamma\Big[\frac{\sin^2 x}{x}+x\cos 2x-\sin 2x\Big]\Big\}$$

$$\times\exp\Big\{-i\Big[\delta x+\gamma\Big(\frac{\sin^2 x}{x}-x\Big)\Big]\Big\}, \tag{18}$$

$$W_2=\frac{G_\beta^2 g_A^2}{2}\left(\chi\frac{\varepsilon_0}{\Delta\varepsilon_1}\right)^2\frac{1}{(2J_i+1)(2J_1+1)}\frac{4}{3}|\langle J_f\|\sigma^1\|J_1\rangle|^2$$
$$\times|\langle J_1\|d^1\|J_i\rangle|^2\frac{(\pi i)^{1/2}}{32\pi^4}(2m\varepsilon_0)m^{3/2}\omega^{7/2}\int_{-\infty}^{\infty}\frac{dx}{(x+i0)^{9/2}}$$
$$\times\left\{\frac{e^{2ix}}{(1+\omega/\Delta\varepsilon_1)^2}+\frac{e^{-2ix}}{(1-\omega/\Delta\varepsilon_1)^2}\right\}$$
$$\times\exp\left\{-i\left[\delta x+\gamma\left(\frac{\sin^2 x}{x}-x\right)\right]\right\} \qquad (19)$$

where

$$x=\omega(t_1-t_2)/2,\quad \gamma=e^2F^2/m\omega^3,\quad \delta=2\varepsilon_0/\omega,$$
$$\chi=(\gamma/\delta^3)^{1/2}=eF/(2m\varepsilon_0)^{1/2}2\varepsilon_0;$$

are the reduced matrix elements for B_{ij}, the dipole operator **d**, and the spin operator $\boldsymbol{\sigma}$, determined in accordance with Ref. 19.

The term W_1 in the total probability corresponds to the direct $|i\rangle\rightarrow|f\rangle$ transition, including corrections for the effect of the external field on the electron wave function. The term W_2 corresponds to the transition through the virtual state $|1\rangle$, i.e., it describes the lifting of forbiddenness. In principle, interference can take place between the direct and the $|i\rangle\rightarrow|1\rangle\rightarrow|f\rangle$ transitions, but the corresponding contribution

$$W_3\infty\mathrm{Im}\,\{\langle J_f\|B^{(2)}\|J_i\rangle\langle J_i\|d^1\|J_1\rangle\langle J_1\|\sigma^1\|J_f\rangle\}$$

vanishes because the reduced matrix elements are real.

The integrals in (18) and (19) can be evaluated numerically. However, they can readily be obtained analytically in all the special cases in which we are interested. Analysis of these integrals can be performed in complete analogy with the allowed β-transitions that have been examined in detail in the literature.[9] In particular, for the realistic case $\chi\ll 1$, we find that, in the first order in the parameters $\chi^2, (\chi\varepsilon_0/\Delta\varepsilon_1)^2$, and including frequency-dependent terms of order up to ω^2, inclusive, we obtain:

$$W_1\approx W_0\left\{1+\frac{315}{8}\chi^2\left[1+\frac{23}{90}\left(\frac{\omega}{2\varepsilon_0}\right)^2\right]\right\}, \qquad (20)$$

$$W_2\approx\left(\chi\frac{\varepsilon_0}{\Delta\varepsilon_1}\right)^2\frac{G_\beta^2g_A^2}{2}\frac{1}{2J_i+1}\frac{1}{2J_1+1}\frac{4}{3}$$
$$\times|\langle J_f\|\sigma^1\|J_1\rangle|^2|\langle J_1\|d^1\|J_i\rangle|^2\frac{8}{105\pi^3}(2m)^{5/2}\varepsilon_0^{9/2}$$
$$\times\left\{1+\frac{35}{2}\left(\frac{\omega}{2\varepsilon_0}\right)^2+14\left(\frac{\omega}{2\varepsilon_0}\right)\left(\frac{\omega}{\Delta\varepsilon_1}\right)+3\left(\frac{\omega}{\Delta\varepsilon_1}\right)^2\right\} \qquad (21)$$

where

$$W_0=\frac{G_\beta^2g_A^2}{2}\frac{1}{2J_i+1}|\langle f\|B^{(2)}\|i\rangle|^2\frac{8}{945\pi^3}(2m)^{5/2}\varepsilon_0^{9/2} \qquad (22)$$

is the probability of unique first-forbidden β-transition in the absence of the field. It is readily seen that the factor following $(\chi\varepsilon_0/\Delta\varepsilon_1)^2$ in the expression for W_2 is of the same order as W_0. The effect of the field on the electron wave function is thus seen to produce corrections to the total probability W_0 of the order of $(\chi\varepsilon_0/\Delta\epsilon_1)^2$ whereas the lifting of forbiddenness by the external field produces corrections of the order of $(\chi\varepsilon_0/\Delta\varepsilon_1)^2$ in accordance with the estimates given in Sections 2 and 3. We have examined the case $\Delta\varepsilon_2\gg\Delta\varepsilon_1$. The case $\Delta\varepsilon_2\ll\Delta\varepsilon_1$, when the lifting of forbiddenness by the external field occurs through the excited state $|2\rangle$ of the daughter nucleus, can be examined in an analogous manner. The result is that W_1 is, as before, given by (18) and (20), and the expression for W_2 is obtained from (19) and (21) by substituting

$$|\langle J_f\|\sigma^1\|J_1\rangle|^2|\langle J_1\|d^1\|J_i\rangle|^2\rightarrow|\langle J_f\|d^1\|J_2\rangle|^2|\langle J_2\|\sigma^1\|J_i\rangle|^2,$$
$$(2J_1+1)^{-1}\rightarrow(2J_2+1)^{-1}\quad\text{and}\quad\Delta\varepsilon_1\rightarrow(-\Delta\varepsilon_2).$$

The contribution W_2 was not taken into account when the probability W of the forbidden β-decay in the wave field was calculated in Refs. 11 and 12. Moreover, the determination of W_1 was based on an approximation corresponding to the neglect of the electron and neutrino momenta in the matrix element of the process. This approximation is commonly used for allowed β-transitions, but is not valid for forbidden transitions. In particular, the field-free matrix element of the unique first-forbidden transition, $B_{ij}(p+k)_j$, is then found to vanish. In the wave field, the matrix element in this approximation is $M\propto(-eA_j)B_{ij}\neq 0$ but, when the field is turned off, the probability W does not tend to W_0 but to zero.

The neglect of the resultant momentum $\mathbf{q}\equiv\mathbf{k}+\mathbf{p}$ of the electron and neutrino in the matrix element for the process is equivalent to the replacement of the matrix element $M_1(\mathbf{q}-e\mathbf{A})$ with $M_1(-e\mathbf{A})$, which leads to the violation of gauge invariance and, as a consequence, to a major error in the probability for the process. In the limit as $\mathbf{q}\rightarrow 0$, the first and third terms in braces in (17) are found to vanish. The three times in (17) correspond to the three terms in square brackets in the expression for W_1 in (18). Thus, as $\mathbf{q}\rightarrow 0$ only the second of these three terms survives. This produces as essential change in the nature of the integrand in (18). As $\mathbf{q}\rightarrow 0$, the expansion for the expression in square brackets for small x begins with terms $\sim x$. The corrections to W_0 calculated in this approximation turn out to be of the order of $\gamma/\delta=\xi^2$ in accordance with Refs. 11 and 12 [see (3)]. When $\mathbf{q}\neq 0$, the corresponding expansion begins with terms $\sim x^3$, and terms $\sim x$ cancel out. As a result, we have $|\langle J_f\|\sigma^1\|J_1\rangle|^2|\langle J_1\|d^1\|J_i\rangle|^2\rightarrow|\langle J_f\|d^1\|J_2\rangle|^2|\langle J_2\|\sigma^1\|J_i\rangle|^2$, $(2J_1+1)^{-1}\rightarrow(2J_2+1)^{-1}$ and $\Delta\varepsilon_1\rightarrow(-\Delta\varepsilon_2)$ [see (20)]. Thus, the neglect of the resultant momentum $\mathbf{q}$ of the electron and neutrino in the matrix element M_1, introduced in Refs. 11 and 12, leads to a result is much too high: $\xi^2\gg\chi^2$, since $\delta\equiv 2\varepsilon_0/\omega\gg 1$. The reason for this is that the three large contributions in the expression for W_1 in (18) no longer mutually cancel out. This cancelation is a consequence of gauge invariance which breaks down when $M_1(\mathbf{q}-e\mathbf{A})$ is replaced with $M_1(-e\mathbf{A})$.

7. DISCUSSION

We begin by considering the range of validity of our results. We have taken into account the interaction of the parent and daughter nuclei with the electromagnetic wave field in first-order perturbation theory [see (13)]. The condition for the validity of perturbation theory is that the paremeters $eFd_{1i}/\Delta\varepsilon_1$, $eFd_{2f}/\Delta\varepsilon_2$ be small. For the purposes of approximate estimates, we may replace the dipole matrix elements d_{1i}, d_{2f} with the nuclear radius R. This yields $\beta\equiv eFR/\Delta\varepsilon_{1,2}\ll 1$. Since $R\lesssim(5\text{–}7)\cdot 10^{-13}$ cm, $\Delta\varepsilon_{1,2}\gtrsim 10$ keV, this is satisfied with considerable margin under the conditions attainable in the laboratory: $\beta\lesssim 10^{-6}$.

In the formulas given by (13), we have neglected the difference between the normalizing factors $N_{i,f}$ of the wave functions $(\Delta\varepsilon_{1,2}/\Delta\varepsilon'_{1\,2})^2$ and unity. It is readily seen that $N_{i,f}-1\sim\beta^2\lesssim 10^{-12}$. Inclusion of corrections $\sim\beta^2$ in the expression for W_2 in (21) would result in the appearance of the terms of the order $(\chi\varepsilon_0/\Delta\varepsilon_1)^4$, which we discard. It follows from (20) that inclusion of terms of the order of β^2 in the wave functions ψ_i and ψ_f in W_1 could become important for $\beta^2\gtrsim\chi^2$, which is equivalent to $R^2>(8m\varepsilon_0)^{-1}$. However, $\varepsilon_0\lesssim 15$ MeV for all known β-transitions. Hence, it follows that $R^2\ll(8m\varepsilon_0)^{-1}$, i.e., $\beta^2\ll\chi^2$, which validates the assumption made above.

We have assumed in our analysis of the field-induced lifting of forbiddenness that only the single highest-lying state of the parent or daughter nucleus with suitable quantum numbers contributes to this process. When there are several such states, the expression for W_2 in (21) must contain the sum over all these states. The correction to W_2 for states with energy $\Delta\varepsilon'_{1,2}$ is then $(\Delta\varepsilon_{1,2}/\Delta\varepsilon'_{1\,2})^2$ if the corresponding dipole matrix elements are of the same order. We note, however, that the states of a given nucleus with equal quantum numbers are rarely found to have similar energies, i.e., the parameter $(\Delta\varepsilon_{1,2}/\Delta\varepsilon'_{1\,2})^2$ is usually small. On the other hand, in all cases in which the states are close to one another, they have an essentially different structure. As a rule, only one of them has a large transition matrix element to the ground state of the nucleus. It is precisely this state that must be taken into accout in (19) and (21).

The electron wave function in the field of the electromagnetic wave that we have used does not allow for the interaction between the electron and the Coulomb field of the daughter nucleus. The validity of this approximation is examined in detail in Ref. 9.

Our calculations of the probability of unique first-forbidden β-transitions in the field of an intense electromagnetic wave have shown that the corrections to W_0 are exceedingly small [see (20) and (21)]. The value of χ^2 for the maximum laser field intensities attainable at present is $\chi^2\sim 10^{-9}$–10^{-8} for $\varepsilon_0\approx 20$ keV, and falls rapidly with increasing ε_0. According to the most optimistic estimates, the parameter $\varepsilon_0/\Delta\varepsilon_{1,2}$ can be of the order of unity (for optimum ε_0 and $\Delta\varepsilon_{1,2}\sim 10$ keV), and hence $(\chi\varepsilon_0/\Delta\varepsilon_{1,2})^2\sim\chi^2$. In practice, the quantities $\Delta\varepsilon_{1,2}$ are at best of the order of a few tens or hundreds of keV. As an example, we may quote unique first-forbidden transitions between the ground states of the nuclei ^{112}Ag$(2^-)\to{}^{112}$Cd(O^+) (here, $\varepsilon_0=3960$ keV; ^{112}Ag has a 1^+ state with $\Delta\varepsilon_1=18.5$ keV) and ^{79}Se$(7/2^+)\to{}^{79}$Br$(3/2^-)$ ($\varepsilon_0=159$ keV; ^{79}Se has a 5/2 level with $\Delta\varepsilon_1=364.5$ keV). Thus, the intensity produced by modern sources of electromagnetic radiation is insufficient for an appreciable change in the total probabilities of forbidden β-transistions.

Our analysis assumes that we are dealing with a nonresonant field: $\omega\ll\Delta\varepsilon_{1,2}$. When $\omega=\Delta\varepsilon_1$ or $\omega=\Delta\varepsilon_2$, the probability of the process should increase substantially because of the increase in W_2. Instead of the estimates $(W_2\sim W_0(\chi\varepsilon_0/\Delta\varepsilon_{1,2})^2$ that follow from (21), we then have $W_2\sim W_0(\chi\varepsilon_0/\Gamma_{1,2})^2$ where $\Gamma_{1,2}$ are the widths of the states $|1\rangle$ and $|2\rangle$. However, the resonance situation would require the use of gamma-ray laser which unfortunately, have not yet been developed.

Our discussion was concerned with unique first-forbidden transitions. It is readilty seen that, for higher degrees of forbiddenness, the corrections to the total probability for the process, due to the lifting of forbiddenness by the external field, can only be lower. In fact, for an n-fold forbidden transition, the absorption (emission) of n dipole quanta gives $\delta W/W_0\sim(\chi\varepsilon_0/\Delta\varepsilon_{1,2})^{2n}$ whereas, for a process involving the participation of one quantum of multipolarity n, we obtain $\delta W/W_0\sim(\chi\varepsilon_0/\Delta\varepsilon_{1,2})^2(\omega^2/2m\varepsilon_0)^{n-1}$. Since $(\chi\varepsilon_0/\Delta\varepsilon_{1,2})$, $(\omega^2/2m\varepsilon_0)\ll 1$, the situation can only deteriorate in the case of transitions with higher degrees of forbiddenness.

It is interesting to note that the coefficient of χ^2 in the expression for W_1 given by (20) is greater by an order of magnitude than the corresponding coefficient for allowed β-decay [see (4)]. This means that unique first-forbidden transitions are more promising than allowed transitions when the effect of the intense electromagnetic wave on the total β-decay probabilities is investigated. In particular, it is interesting to consider the transition ^{187}Re$(5/2^+)\to{}^{187}($Os$(1/2^-)$ which has the very low energy release $\varepsilon_0=2.64(4)$ keV. If the maximum laser radiation intensity were to be increased by two orders of magnitude, the field-induced increase in the probability of the process should reach about 10% according to (20). However, there are serious difficulties with this experiment, including, inter alia, the very long half-life of ^{187}Re (5×10^{10} years).

The author is indebted to V. A. Khodel's for useful discussions.

1) Here and henceforth, we are using the system of units in which $\hbar=c=1$.

[1]W. Becker, W. H. Louisell, J. D. McCullen, and M. O. Scully, Phys. Rev. Lett. **47**, 1262 (1981).
[2]W. Becker, W. H. Louisell, J. D. McCullen, and M. O. Scully, Phys. Rev. Lett. **48**, 653 (1982).
[3]I. M. Ternov, O. F. Dorofeev, and V. N. Rodionov, Preprint No. 08/1982, Moscow State University.
[4]I. M. Ternov, V. N. Rodionov, A. E. Lobanov, and O. F. Dorofeev, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 288 (1983). [JETP Lett. **37**, 342 (1983)].
[5]I. M. Ternov, V. N. Rodionov, and O. F. Dorofeev, Zh. Eksp. Teor. Fiz. **84**, 1225 (1983) [Sov. Phys. JETP **57**, 710 (1983)].
[6]I. M. Ternov, V. N. Rodionov, and O. F. Dorofeev, Yad. Fiz. **37**, 875 (1983) [Sov. J. Nucl. Phys. **37**, 521 (1983)].
[7]A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **85**, 24 (1983) [Sov. Phys. JETP **58**, 14 (1983)].
[8]M. B. Voloshin, Yad. Fiz. **38**, 814 (1983) [Sov. J. Nucl. Phys. **38**, 484 (1983)].
[9]E. Kh. Akhmedov, Zh. Eksp. Teor. Fiz. **85**, 1521 (1983) [Sov. Phys. JETP **58**, 883 (1983)].
[10]A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **85**, 1544 (1983) [Sov. Phys. JETP **58**, 898 (1983)].
[11]H. Reiss, Phys. Rev. C **27**, 1199 (1983).
[12]H. Reiss, Phys. Rev. C **27**, 1229 (1983).
[13]H. Reiss, Phys. Rev. C **28**, 1402 (1983).
[14]D. Volkov, Z. Phys. **94**, 250 (1935).
[15]A. M. Dykhne and G. J. Yudin, Usp. Fiz. Nauk **121**, 157 (1977) [Sov. Phys. Usp. **20**, 80 (1977)].
[16]M. Jain and N. Tzoar, Phys. Rev. A **15**, 147 (1977).
[17]L. V. Keldysh, Zh. Eksp. Teor. Phys. **47**, 1945 (1964) [Sov. Phys. JETP **20**, 1307 (1965)].
[18]Ts. S. Vu and S. A. Moshkovskiĭ, Beta-raspad (Beta Decay), Atomizdat, Moscow, 1970.
[19]A. Edmonds, in: Deformation of Atomic Nuclei [Russian transl. IIL, Moscow, 1958, p. 305].

Translated by S. Chomet