Quantum creation of an inflationary Universe

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It is shown that the process of quantum creation of the Universe must lead with high probability in a large class of elementary-particle theories to the creation of an exponentially expanding (inflationary) Universe whose radius after the end of the expansion exceeds $l \sim 10^{28}$ cm.

There has been much recent interest in attempts to solve a whose series of problems at the frontier of cosmology and elementary-particle theory by means of the scenario of a so-called inflationary Universe,[1–3] and also by means of the related scenario of Starobinskiĭ's model.[4] For a review of the present status of these scenarios, see, for example, Refs. 5 and 6. The main feature of these scenarios is the presence of a long exponential (or quasiexponential) expansion (inflation) with $a(t) \sim a_0 e^{Ht}$ during a time $\Delta t \gtrsim 70 H^{-1}$ in the earliest stages in the evolution of the Universe. Here, $a(t)$ is the scale factor of the Universe, and H is the Hubble "constant," which itself can vary slowly during the time of the inflation, so that $\dot{H} \ll H^2$.

In the inflationary Universe scenario, it is generally assumed that the inflation begins as a result of supercooling in a Universe that expands from a singularity up to a certain time at which the potential energy $V(\varphi)$ of a classical field φ becomes greater than the energy of relativistic particles, which is $\sim T^4$.[1–3] However, there is a further very interesting possibility of realizing this scenario. It is based on the fact that because of strong quantum-gravitational fluctuations of the metric at densities greater than or of the order of the Planck density $\rho_P = M_P^4 \sim 10^{94}$ g/cm^3 the classical description of the evolution of the Universe becomes invalid, and the possibility is therefore opened up that the Universe was never in a singular state but because of quantum-gravitational fluctuations either arose as a whole "from nothing," or "sprouted" out of a different Universe.[7–11] The theory of the corresponding processes is not yet fully developed, and the very conception of creation "from nothing" or from a "different Universe" requires a more detailed development (in this connection, see Ref. 8). Nevertheless, some qualitative features of the possible processes of this kind can already be understood. For example, the process of creation of the Universe can be effectively realized only at an energy density $\rho \gtrsim \rho_P$ of the created Universe, at which quantum-gravitational effects can be important. Moreover, since the quantum creation of the Universe should be regarded as an effect of the type of below-barrier tunneling, it is to expected that the probability of creation when $\rho < \rho_P$ is exponentially suppressed, $P \sim \exp(-C\rho_P/\rho)$, where C is a constant. For the same reason, the probability of creation of a Universe with a large radius must also be exponentially suppressed, so that only a closed Universe with characteristic scale $l \lesssim M_P^{-1} \sim 10^{-33}$ cm should be created.[8] We then face the problem of how a Universe of such small dimensions could give rise to our Universe with $l \gtrsim 10^{28}$ cm, an excess of baryons over antibaryons, its observed spectrum of density inhomogeneities, and so forth. A new stage in the development of ideas about the quantum creation of the Universe began with the papers of Zel'dovich and Grishchuk,[8] who showed that all these problems can be resolved if the created Universe is described by the scenario of an inflating Universe[1–3] or Starobinskiĭ's model.[4] This opened up the basic possibility of constructing a complete scenario of the evolution of the Universe with the Universe never having passed through an initial singularity.[8]

These suggestions made in Refs. 8 referred initially mainly to Starobinskiĭ's model, since the main possibility known until recently[12] of realizing this model was based on the idea of quantum creation of the Universe. At the same time, the possibility of quantum creation of an inflationary Universe appeared relatively improbable, since the vacuum energy density $V(\varphi)$ during the inflation in the first variants of this scenario[1,2] was many orders of magnitude less than the Planck density, and therefore the quantum-gravitational effects that could lead to creation of the Universe were negligibly small in these scenarios. Therefore, the scenario of an inflationary Universe was usually considered in the framework of the more customary approach based on the assumption that the Universe was from the very beginning hot and the exponential expansion began through supercooling in an unstable quasivacuum state.[1–3,5]

An attempt at a quantitative treatment of quantum creation of an inflationary Universe was recently made by Vilenkin.[9] However, it appears to the present author that he obtained a qualitatively incorrect result for the probability of creation of an inflationary Universe, $P \sim \exp(3\rho_P/8\rho)$, from which it would follow that the quantum-gravitational effects are stronger, the smaller ρ. One further attempt to study this question was made by Moss and Wright.[11] However, they investigated only the theory of a conformal scalar field φ, in which, as was anticipated earlier,[5] there is no inflation of the Universe.

In the view of the present author, the most reasonable estimate for the probability of creation of an inflationary Universe is $P \sim \exp(-3\rho_P/8\rho)$. This question is discussed in the Appendix. Here, the only thing important for us is that, as said above, the probability of creation of a Universe with $\rho \ll \rho_P$ must be strongly suppressed. The main aim of the present paper is to propose a scenario of quantum creation of the Universe in which suppression of the probability of creation of an inflationary Universe is absent.

As is clear from what has been said above, the exponential suppression of the creation probability must be absent if the energy density in the created Universe satisfies

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$\rho \gtrsim \rho_P = M_P^4$. However, if such an energy density is achieved by virtue of large gradients of the field φ or by virtue of its kinetic energy $\sim \frac{1}{2}\dot{\varphi}^2$, the corresponding Universe would not inflate, and its characteristic lifetime would be of order M_P^{-1}, i.e., such Universes are not, in a certain sense, created as classical formations but remain quantum fluctuations of the metric and the field φ. We consider now a different case, in which an energy density $\rho \gtrsim \rho_P$ is achieved by a large value of $V(\varphi)$ of a sufficiently uniform (on the scale $l \gtrsim 2H^{-1}$, see Ref. 3) and slowly varying $[(\dot{\varphi})^2 \ll V(\varphi)]$ field φ. In this case, it can be shown that under fairly natural assumptions about the form of the function $V(\varphi)$ the corresponding Universes (or parts of a Universes begin to expand exponentially and grow to dimensions $l \gtrsim 10^{28}$ cm.

Indeed, let us consider as an example the simplest theory of a nonconformal field φ with effective potential $V(\varphi) = \frac{1}{4}\lambda\varphi^4$ for $\lambda \ll 10^{-2}$. Then in accordance with what we said above, exponential suppression of the creation probability is absent if the Universe is created filled with a field $\varphi = \varphi_0$ such that $\frac{1}{4}\lambda\varphi_0^4 \gtrsim M_P^4$, so that $\varphi_0 \gtrsim M_P(4/\lambda)^{1/4} \gg 5M_P$. The further evolution of the homogeneous field φ in the closed Universe is determined by the equation

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} = -\frac{dV}{d\varphi}, \qquad (1)$$

and the scale factor $a(t)$ satisfies the equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{a^2} = \frac{8\pi}{3M_P^2}\left(V(\varphi) + \frac{1}{2}\dot{\varphi}^2\right), \qquad (2)$$

where $V(\varphi) = \frac{1}{4}\lambda\varphi^4$. We also assume that at the beginning $\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi)$. It will be seen that for $\varphi \gtrsim 5M_P$ in this case the field φ evolves very slowly, $\dot{\varphi}/\varphi \ll H$, where $H = (8\pi V(\varphi)/3(M_P^2))^{1/2}$. Therefore, the closed Universe described by Eqs. (1) and (2) moves in accordance with the quasi de Sitter law

$$a(t) = H^{-1}\cosh Ht. \qquad (3)$$

The most probable case is creation of the Universe at the time $t = 0$ when its spatial size is minimal, $a_0 = H^{-1}$. Analysis of Eqs. (1) and (2) shows that for $\varphi = \varphi_0 \gtrsim 5M_P$ the field φ does not significantly decrease during the time $\Delta t \sim H^{-1}$. On the other hand, during this time the scale factor $a(t)$ increases strongly. After this, $\dot{a}/a$ in (1) and (2) becomes equal to H, and the terms $\ddot{\varphi}$ in (1) and $4\pi\dot{\varphi}^2/3M_P^2$ in (2) can be ignored until the field φ becomes less than $M_P/3$. Therefore, the evolution of the field φ for $\varphi \gtrsim M_P/3$ is determined by the equation

$$\varphi = \varphi_0 \exp\left(-\frac{\lambda^{1/2} M_P}{(6\pi)^{1/2}}t\right), \qquad (4)$$

from which it follows that for $\varphi_0 \gtrsim 5M_P$ the Universe succeeds in expanding by more than e^{70} times during the period of exponential expansion (3). This observation was the basis of the scenario of random inflation proposed recently by the author.[3] It can now be seen that this observation greatly facilitates the realization of the idea of quantum creation of the Universe, since the creation process can proceed effectively only with a sufficiently large field φ (in our case $\varphi \gg 5M_P$), and this is simultaneously the condition necessary for realization of the inflationary Universe scenario.[3]

We note that, as follows from Ref. 3, this result is valid not only for the $\frac{1}{4}\lambda\varphi^4$ theory for $\lambda \ll 10^{-2}$ but also for a large class of theories with potential $V(\varphi)$ sufficiently flat for $\varphi \gtrsim M_P$. Thus, if the process of quantum creation of the Universe is possible, then in theories of the indicated type there is a high probability that the created Universe expands exponentially in accordance with the inflationary scenario,[3] and after the expansion has a linear scale exceeding the scale $l \sim 10^{28}$ cm of the observable part of the Universe.

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APPENDIX

Let me now attempt, as promised, to estimate the probability of creation of an inflationary Universe of the considered type. This can be done by means of the formalism for calculating the wave function $\Psi(a,\varphi)$ of the Universe proposed by Hartle and Hawking.[10] They argued that the wave function of a Universe with scale factor $a(t)$ filled with a uniform scalar field φ must have the form

$$\Psi_0(a,\varphi) = \int da(\tau)\, d\varphi(\tau) \exp[-S_E(a,\varphi)], \qquad (A1)$$

where $S_E(a,\varphi)$ is the Euclidean action on the paths $a(\tau)$ and $\varphi(\tau)$ with end points a and φ. Let us recall the motivation of this expression, since it seems to me that it needs to be made more precise in an important respect. To this end, following Ref. 10, we consider the Green's function in quantum mechanics of a particle moving from the point with coordinates $(0, t')$ to the point $(\bar{x}, 0)$:

$$\langle \bar{x}, 0 | 0, t' \rangle = \sum_n \Psi_n(\bar{x}) \bar{\Psi}_n(0) \exp(iE_n t')$$
$$= \int d\bar{x}(t) \exp[iS(\bar{x}(t))]. \qquad (A2)$$

Here, $\Psi_n(x^-)$ are the time-independent eigenfunctions of the energy operator with eigenvalues E_n. We now make the rotation $t' \to -i\tau'$ and go to the limit $\tau' \to -\infty$. In this case, there survives in the sum (A2) only the term corresponding to the lowest energy E_n (normalized to zero), i.e.,

$$\Psi_0(\bar{x}) \sim \int d\bar{x}(\tau) \exp[-S_E(\bar{x}(\tau))]. \qquad (A3)$$

The generalization of this expression to the case we have considered is the expression (A1) of Ref. 10.

In the quasiclassical approximation, it would follow from (A1) that

$$\Psi_0(a, \varphi) \sim \exp[-S_E(a, \varphi)], \qquad (A4)$$

where S_E is the Euclidean action on the solution $a(\tau)$, $\varphi(\tau)$ of the Euclidean equations of motion. In our case, with $\varphi \gg M_P$, an approximate solution of the Euclidean equations of motion is a Euclidean section of the de Sitter Universe with vacuum energy $V(\varphi)$:

$$\varphi \approx \varphi_0, \quad a = H^{-1}\cos H\tau, \quad H = \left(\frac{8\pi}{3M_P^2}V(\varphi)\right)^{1/2}, \qquad (A5)$$

where $0 < \tau < \pi/2H$. Since $\varphi \simeq \varphi_0 = $ const in the considered solution $[\dot\varphi/\varphi \ll H,\ \frac{1}{2}\dot\varphi^2 \ll V(\varphi)]$, the contribution $\sim\dot\varphi^2$ to $S_E(a,\varphi)$ can be ignored, and the action $S_E(a,\varphi)$ for $\varphi \gg M_P$ is with high accuracy equal to the gravitational action in the Euclidean section of the de Sitter Universe with vacuum energy $V(\varphi)$,[13]

$$S_E = -3M_P^4/16V(\varphi).$$

Hence, from (A4) it would follow that $\Psi(a,\varphi) \sim \exp(3M_P^4/16V(\varphi))$, i.e., we would arrive at a result close to Vilenkin's[9] somewhat modified because of the different form of $V(\varphi)$ in the two scenarios. It is however clear that such a result is physically incorrect, since it would mean that the quantum-gravitational effects are larger the lower the energy density. The reason for the mistake is that the effective Lagrangian of the scale factor has the incorrect sign.[14,10] For example, after Euclidization the action $S_E(a,\varphi)$ is

$$S_E(a,\varphi) = -\frac{1}{2}\int d\eta \left[\left(\frac{da}{d\eta}\right)^2 + a^2 - \frac{8\pi V(\varphi)}{3M_P^2}a^4\right]\frac{3\pi M_P^2}{2}, \quad \eta = \int \frac{d\tau}{a(\tau)}. \tag{A6}$$

This means that the energy of "excitations" of the scale factor a is not positive, as is usually the case, but negative, $E_n \leqslant 0$. The physical reason for this is simply that the total energy of a closed Universe, whose creation we consider,[8] is always equal to zero and, therefore, the gravitational energy is negative. But in this case, it would be necessary in the expression analogous to (A2) to make the rotation $t' \to i\tau'$ in the calculation of $\Psi_0(a,\varphi)$, and not $t' \to -i\tau'$, as we did. This leads to a corrected quasiclassical expression for $\Psi_0(a,\varphi)$:

$$\Psi_0(a,\varphi) \sim \exp[+S_E(a,\varphi)]. \tag{A7}$$

Here, to avoid misunderstanding, we must make two important comments. First, in the theory of gravitation there are no real particles with negative energy. The point is that because of the gauge invariance of the theory of gravitation with respect to the choice of the time "scale" there are no physical degrees of freedom in reality corresponding to fluctuations of the scale factor a (see, for example, Refs. 10 and 15). Nevertheless, it is still necessary to quantize the scale factor, and this is done in all studies on quantum cosmology. The introduction of this "redundant" degree of freedom corresponding to the scale factor a is compensated by the introduction of the well-known Faddeev-Popov ghosts, as is always done in the theory of gauge fields.[16,17] However, it is well known that the introduction of ghosts on quantization has an effect only at the level of the single-loop corrections to the quasiclassical expressions, and the quasiclassical expressions themselves must be obtained as if the corresponding variable (in our case, the scale factor a) possessed its own physical degree of freedom. It is for this reason that in deriving (A7) we regarded the scale factor a as a physical field with "incorrect" sign of the energy

The second remark is that the expression (A7) is valid only when it is possible to ignore the evolution of the physical fields with positive energy in the considered process. This in fact is so in the case we consider because the field φ varies slowly for $\varphi \gg M_P$: $\dot\varphi/\varphi \ll \dot a/a$. In this case, it follows from our results that

$$\Psi_0(a,\varphi) \sim \exp(-3M_P^4/16V(\varphi)), \tag{A8}$$

from which we obtain for $V(\varphi) = \frac{1}{4}\lambda\varphi^4$

$$\Psi_0(a,\varphi) \sim \exp(-3M_P^4/4\lambda\varphi^4). \tag{A9}$$

But when the Universe becomes large and the evolution of the scale factor slow, the opposite limiting case holds—at the present time, the evolution of the scale factor can be ignored, and in the construction of the quantum theory of all the remaining fields (with positive energy) the Euclidean rotation is to be done in the standard manner: $t' \to -i\tau'$. As yet, there is no general prescription for quantizing interacting systems with positive and negative energies because of the vacuum instability in the corresponding theory, leading, in particular, to particle creation in an expanding Universe[18] (in this connection, see also Ref. 6).

According to the interpretation of Hartle and Hawking, the square of the wave function $\Psi(a,\varphi)$ determines the probability of quantum creation of the Universe or, more precisely, the probability of finding the Universe in the state with scale factor a filled with the uniform field φ.[10] It can be seen from (A9) that this probability becomes appreciable when $\varphi \gtrsim \lambda^{-1/4}M_P$. In accordance with what was said in the main text, a Universe filled with such a field is almost automatically inflationary,[3] and for $\lambda \lesssim 10^{-2}$ such a Universe acquires a length scale $l \gtrsim 10^{28}$ cm as a result of its expansion.

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