Chaotic inflation of the Universe in supergravity

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(Submitted 26 October 1983)
Zh. Eksp. Teor. Fiz. **86**, 1594–1599 (May 1984)

A new, improved implementation of the scenario for the chaotic inflation of the Universe is proposed within the framework of models based on $N = 1$ supergravity. This scenario ensures both sufficient inflation of the Universe and the required amplitude of density perturbations that appear after inflation and subsequently lead to the formation of galaxies. The superpotential of matter fields in this model can be made as small as desired at the absolute minimum of the effective potential, which is a necessary condition for the solution of the gauge hierarchy problem in such models.

One of the most interesting implementations of the inflating Universe scenario[1–3] is based on the cosmological consequences of $N = 1$ supergravity interacting with matter.[4–7] This invariant of the inflating Universe scenario has become particularly attractive since the resolution[6,7] of its basic difficulty, namely, the problem of primordial monopoles.[4]

On the other hand, the scenario has been incomplete to some extent. It was implemented by introducing into the theory a certain additional chiral superfield Φ and by studying the effective potential $V(z,z^*)$ in terms of the first (scalar) component of the superfield z (Ref. 8):

$$V(z, z^*) = e^{zz^*/2}[2|g_z'(z)+{}^1/_2 z^* g|^2 - 3|g|^2], \tag{1}$$

where $g(z)$ is an arbitrary superpotential, $g_z'(z) \equiv dg/dz$,

$$g(z) = \mu^3 f(z), \tag{2}$$

μ is a factor with dimensions of mass, and $f(z)$ is an arbitrary function of the field z. In all these expressions, we used the system of units in which $M_p/(8\pi)^{1/2} = 1$ (Ref. 8).

Since $f(z)$ is arbitrary, the form of $V(z,z^*)$ has also remained arbitrary to a considerable extent. For example, the superpotential $f(z)$ was chosen in Refs. 4 and 5 in the form of the series

$$f(z) = \sum_{n=0}^{m} \frac{\lambda_n}{n} z^n, \tag{3}$$

where it was assumed that $\lambda_0 \geqslant 0$, $\lambda_1 > 0$. The effective potential in terms of the real part φ of the field z then assumes the form

$$V(\varphi) = \mu^6(\alpha + \beta\varphi^2 - \gamma\varphi^3 + \delta\varphi^4 + \ldots), \tag{4}$$

where α, β, γ, δ are certain combinations of λ_n. The coefficients λ_n are chosen so as to ensure that the function $V(\varphi)$ has a minimum at $\varphi = \varphi_0 = 1$ and, at the same time, $V(\varphi = 1) = 0$. The latter condition is necessary to ensure that the vacuum energy (cosmological term) turns out to be zero after symmetry-breaking.

In our previous papers,[6,7] the effective potential was taken in the form

$$V(\varphi) = 3\mu^6\left(1 - \alpha^2\varphi^2 + \frac{\alpha^4}{4}\varphi^4\right). \tag{5}$$

The minimum of $V(\varphi)$ was then at $\varphi = \varphi_0 = 2^{1/2}/\alpha$, which automatically ensures that $V(\varphi_0) = 0$. The question as to which of these potentials is the more natural is, at present, largely academic since we have practically no idea what the form of the function $f(z)$ should be (see, however, the account given below). We shall therefore confine our attention, for the moment, to the identification of the basic features of the inflation scenario in supergravity, which may not be very dependent on some of the details of the behavior of $f(z)$. Moreover, one would at least like to verify that this scenario can be implemented, if only in principle, for some particular form of $f(z)$.

The most important restriction on the form of $f(z)$ is due to the fact that the gravitino mass m_G is proportional to $g(z)$ in these theories. However, if we try to solve the hierarchy problem and explain the scale of symmetry breaking in weak-interaction theory in terms of supergravity effects, we find that, as shown in Ref. 9, the gravitino mass must be $m_G \sim 100$ GeV or, in units of $M_p/(8\pi)^{1/2}$, $m_G \sim 10^{-16}$. This means that, in this type of theory, the quantity $g(z)$ must be closely equal to zero at the minimum of $V(z)$. In our previous papers,[6,7] it was not our aim to achieve this because the low gravitino mass gives rise to cosmological difficulties[10] that can be overcome[11,5] but lead to considerable restrictions on the structure of the theory.[11] Nevertheless, it was important to elucidate the question as to whether the scenario of primordial inflation can, in fact, be implemented under the condition $g(\varphi_0) = 0$, the satisfaction of which was demanded in, for example, Refs. 4 and 5. This question seems trivial at first sight because of the functional freedom in the choice of $g(z)$ but, in reality, it conceals a certain danger.

To examine this question, it will be convenient to transform from the function $f(z)$ to $\psi(z)$, given by

$$\psi(z) = e^{z^2/4} f(z). \tag{6}$$

The convenience of this form becomes clear if we write

$$V(z, z^*) = \mu^6 e^{-(z-z^*)^2/4}[2|\psi_z'(z) + {}^1/_2(z^*-z)\psi|^2 - 3|\psi|^2], \tag{7}$$

where $\psi_z' \equiv d\psi/dz$. We then have on the real axis ($z = \varphi$)

$$V(\varphi) = \mu^6[2|\psi_\varphi'|^2 - 3|\psi|^2]. \tag{8}$$

We now note that, from the condition $f(\varphi_0) = 0$, it also follows that $\psi(\varphi_0) = 0$, and (6) leads to

$$\psi(0) = f(0), \quad \psi_\varphi'(0) = f_\varphi'(0).$$

Suppose, for example, that the superpotential $f(z)$ is given by

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the series (3) with $\lambda_0 \geqslant 0$, $\lambda_1 > 0$, in accordance with Refs. 4 and 5. Hence, it follows that, in the theories examined in Refs. 4 and 5,

$$\psi(0)=\lambda_0\geqslant 0, \quad \psi'(0)=\lambda_1>0.$$

It is readily seen that, in this case, the condition $\psi(\varphi_0) = 0$ can be satisfied only if $\psi(\varphi_0)$ vanishes at some point φ^* lying between $\varphi = 0$ and $\varphi = \varphi_0$, where $\psi(\varphi^*)\neq 0$. However, this would mean that, according to (8), the quantity $V(\varphi^*)$ is negative, i.e., the absolute minimum of $V(\varphi)$ does not lie at $\varphi = \varphi_0$, as was suggested in Refs. 4 and 5, but somewhere in the range $0 < \varphi < \varphi_0$, and the vacuum energy at this minimum is negative. This produces a considerable complication in the implementation of the primordial inflation scenario with superpotentials $g(z)$ of the above type.

Thus, it is not entirely clear whether it will be possible to construct the theory with the required type of effective potential when $g(\varphi_0) = 0$, and the first attempts in this direction have not been entirely successful. The aim of this paper is to propose a superpotential that would lead to a theory with all the properties necessary for the implementation of the primordial inflation scenario, including the property $g(\varphi_0) = 0$. We emphasize that it is not our aim to propose the "prettiest" superpotential, since we have no criterion for this choice. We merely wish to show that superpotentials of the required type do actually exist, and to exhibit the basic features of our scenario.

As an example of a superpotential with all the necessary properties, let us consider the function

$$\psi(z)=\mathrm{th}\,\zeta\,\mathrm{sh}\,\zeta, \quad \zeta=\xi+i\eta, \tag{9}$$

where

$$\zeta=(^3/_2)^{1/2}(z-\varphi_0), \tag{10}$$

and φ_0 is some real number (see below). It is clear that $\psi(z) = 0$ at $z = \varphi_0$, as required. In terms of the new variables, the effective potential takes the form

$$V(\zeta,\zeta^*)=3\mu^6\exp\left\{\frac{2(\mathrm{Im}\,\zeta)^2}{3}\right\}\left\{\left|\psi_\zeta'-\frac{(\zeta-\zeta^*)}{3}\psi\right|^2-|\psi|\right\}. \tag{11}$$

Using the superpotential (9), we find that, on the real axis

$$V(\xi)=3\mu^6[(\psi_\xi')^2-(\psi)^2]=9\mu^6\left[1-\frac{2}{3\,\mathrm{ch}^2\,\xi}-\frac{1}{3\,\mathrm{ch}^4\,\xi}\right]. \tag{12}$$

It is clear that the effective potential has a minimum at $\xi = 0$ ($\varphi = \varphi_0$) with $V(0) = 0$, and asymptotically reaches a constant as $|\xi|\to\infty$: $V(\xi)\to 9\mu^6$. Analysis of the behavior of $V(\zeta,\zeta^*)$ in the complex plane has shown that the quantity $V(\zeta,\zeta^*)$ is positive semi-definite for all ζ; $V(\zeta,\zeta^*) = 0$ only for $\zeta = i\pi n$, $n = 0, \pm 1, \pm 2,\ldots$.

Because of the presence of the factor $\exp\{2(\mathrm{Im}\,\zeta)^2/3\}$ in (11), the quantity $V(\zeta,\zeta^*)$ is exponentially large (it exceeds the Planck energy by many orders of magnitude) everywhere outside a narrow band of width of the order of 4–5 units of $M_p/(8\pi)^{1/2}$ near the real axis. The only exception to this rule is provided by exponentially narrow (widths $\sim\exp\{-2(\mathrm{Im}\,\zeta)^2/3\}$) wells lying close to the above points $\zeta = i\pi n$, $n = 0, \pm 1, \pm 2,\ldots$.

The above potential is ideally suited to the implementation of the inflating Universe scenario. Actually, it is shown in Ref. 7 that inflation occurs in supergravity not within the framework of the standard scenario, based on the theory of high-temperature phase transitions, but only within the framework of the chaotic inflation scenario.[3] It is assumed in the latter that the Universe was initially filled with some field distribution $z(x)$ such that $V(z,z^*)\lesssim M_p^4$, and all the field values for which $V(z,z^*)\lesssim M_p^4$ (i.e., $V(z,z^*)\lesssim 1$ in our units) were more or less equally probable.

For our potential $V(z,z^*)$, this means that most of the Universe was initially in the state in which the field ζ was somewhere within the band $|\mathrm{Im}\,\zeta|\lesssim 5$, and typical values of $\mathrm{Re}\,\zeta = \xi$ could be as large as desired. The field then rapidly rolled down into the "rut" on the real axis, and continued to roll very slowly and for a long time toward the minimum of $V(\varphi)$ at $\varphi = \varphi_0 (\zeta = 0)$. During this roll-over period, the Universe continued to expand in accordance with the law

$$H=\frac{dZ}{dt}=\frac{1}{\sqrt{3}}\left(V+\frac{\dot\varphi^2}{2}\right)^{1/2}, \tag{13}$$

where Z is the logarithm of the scale factor $a(t)$, i.e., $Z = \ln a$, and the evolution of the field φ was described by

$$\ddot\varphi+3H\dot\varphi=-\frac{dV}{d\varphi}. \tag{14}$$

In the region of high values of the field φ in which we are interested, the evolution of this field occurs exceedingly slowly, and the terms $(\dot\varphi)^2$ and $\ddot\varphi$ in (13) and (14), respectively, can be neglected.[3] For large φ, the function $V(\varphi)$ is given by the asymptotic expression,

$$V=9\mu^6[1-\gamma\exp(-\sqrt{6}|\varphi-\varphi_0|)], \tag{15}$$

where $\gamma = \frac{8}{3}$. We must now determine the factor by which the Universe expands as the field φ falls to the minimum of $V(\varphi)$. We have

$$\ln\frac{a(\varphi=\varphi_0)}{a(\varphi)}=Z(\varphi_0)-Z(\varphi)\equiv\Delta Z_\varphi$$

$$=\int_\varphi^{\varphi_0}\frac{dZ}{d\varphi}d\varphi\approx\int_{\varphi_0}^{\varphi}\left(\frac{d\ln V}{d\varphi}\right)^{-1}d\varphi\approx\frac{3}{2\gamma}\exp(\sqrt{6}\,|\varphi-\varphi_0|). \tag{16}$$

It is clear that expansion by a factor in excess of e^{70}, which is necessary for the inflating Universe scenario, will occur if the rolling process begins for fields $|\varphi-\varphi_0|\gtrsim 2$. These initial conditions are completely natural within the framework of the chaotic inflation scenario with the potential given by (15), and should occur in most parts of the Universe.

In other words, according to the chaotic inflation scenario,[3] the Universe should contain many regions filled with the field φ such that $|\varphi-\varphi_0|\gtrsim 2$. In the course of subsequent expansion, all such regions with initial size $l\gtrsim 2H^{-1}$ assume dimensions exceeding the size of the observable part of the Universe [$l\sim 10^{28}$ cm (Ref. 3)].

The superpotential that leads asymptotically to a slowly-varying potential on the real axis is not unique. All superpotentials that tend asymptotically (as $\xi\to\infty$) to $\mathrm{sh}\,\xi$, lead to the desired result. In point of fact, the asymptotic form of $g(\xi)$ for $\xi\to\infty$ should be

$$g(\xi)\underset{\xi\to\infty}{\longrightarrow}(1-Ae^{-2\xi})\,\mathrm{sh}\,\xi. \qquad (17)$$

This property is exhibited, for example, by the superpotential

$$g(\zeta)=\left[1-\sum_{n=1}^{\infty}a_n(e^{-2\zeta})^n\right]\mathrm{sh}\,\zeta. \qquad (18)$$

To ensure that $g(\zeta)=0$, $\zeta=0$, the coefficients must satisfy the obvious relation

$$\sum_{n=0}^{\infty}a_n=1. \qquad (19)$$

It is readily verified that all superpotentials of the form (17)–(19) are zero at the points $\zeta=i\pi n$ ($n=0, \pm 1, \pm 2...$), and so are their derivatives. This obviously leads to zero potential at these points, just as for the potential generated by the superpotential (9). Computer calculations have shown in addition that the (positive semidefinite) potentials corresponding to (17)–(19) have generally similar form in the complex plane of ζ, which we have described in connection with the superpotential (9). The clear difference as compared with the superpotential (9) is that, as $\xi\to-\infty$, the potential will now grow exponentially, so that the above scenario can be implemented only for a fall on the side of large positive ξ. This is, of course, quite adequate for the implementation of our scenario. The factor $1-Ae^{-2\xi}$ in (17) is important. The superpotential is asymptotically (as $\xi\to+\infty$) equal to

$$(1-Ae^{-\lambda\xi})\,\mathrm{sh}\,\xi,$$

and, for $\lambda<2$ leads to an exponentially growing potential. For $\lambda>2$, the potential again reaches a constant, but from above, in accordance with the expression $1+Ae^{-(\lambda-2)}$. It must therefore pass through a maximum at

$$\xi_{\max}=\frac{1}{2}\left\{\left(1-\frac{\lambda-2}{4}\right)\ln\left(1+\frac{4}{\lambda-2}\right)+\ln 2\right\}.$$

The field can also evolve in this potential in the direction of increasing $|\xi|$, but this does not suit our purposes. For λ approaching $\lambda=2$, and for values of $|\xi|$ that are not too high, the effective potential will be close to that corresponding to $\lambda=2$, depending on $|\lambda-2|$. (In other words, the potential $V_\lambda(\varphi)$ tends to $V_{\lambda=2}(\varphi)$, but it does not do so uniformly.) Since only limited regions of the field ζ are important (such that $V(\zeta,\zeta^*)\lesssim M_p^4$), the above scenario will obtain even for $\lambda\neq 2$, but close to this value. Moreover, all the results will be close to those obtained for $\lambda=2$ precisely because the two potentials are close to one another. We shall therefore suppose that $\lambda=2$.

With the exception of the essential $n=1$ term, expression (18) need not contain any other term. Moreover, we can add terms such as $e^{-3\zeta}$, etc., which fall more rapidly than $e^{-2\xi}$ as $\xi\to+\infty$. The addition of terms of the form of $e^{-3\zeta}$ can, for example, remove some of the zeros of V. All the potentials $V(\varphi)$ obtained from the above superpotentials are given by (15) for large $\varphi-\varphi_0$. For $\varphi-\varphi_0\gg 1$, for which the asymptotic expression (15) is valid, the conditions $|V'|\ll V$ and $|V''|\ll V$, will be satisfied. These conditions are necessary to allow us to neglect the terms $\ddot{\varphi}$ and $\dot{\varphi}^2/2$ in (13) and (14).

It is important to note that, since the entire observable part of the Universe was formed in this scenario whilst the field φ was in the region $|\varphi-\varphi_0|\lesssim 2$ (see above), the entire scenario can actually be implemented for a wide class of theories in which the potential $V(\varphi)$ is close in form to (15) for $|\varphi-\varphi_0|\lesssim 2$, and increases in an arbitrary manner for $|\varphi-\varphi_0|>2$, i.e., this is not confined to theories with superpotentials (9) and (18).

We now turn to the spectrum of inhomogeneities that arise after the inflation of the Universe. According to Ref. 12, these inhomogeneities are produced from quantal fluctuations of the field φ which, at $\varphi=\varphi^*$, have the following momentum at the beginning of the inflation process:

$$k_*^2=|V''(\varphi^*)|\approx V_0\gamma\alpha^2e^{-\alpha\varphi^*}=V_0/\Delta Z_*. \qquad (20)$$

At the end of the inflation process, this momentum is reduced:

$$k^2=k_*^2e^{-2\Delta Z_*}=(V_0/\Delta Z_*)\,e^{-2\Delta Z_*}. \qquad (21)$$

The spectrum of fluctuations with momentum k is given by[12]

$$\frac{\delta\rho(k)}{\rho}=\frac{1}{(2\pi^3)^{1/2}}\frac{H^2}{|\dot{\varphi}|}\bigg|_{\varphi=\varphi^*}$$

$$=\left(\frac{V_0}{6\pi^3}\right)^{1/2}\alpha\Delta Z_*=0{,}3\mu^3\ln\frac{\mu^6k^{-2}}{50}. \qquad (22)$$

Galaxies appear as a result of the amplification of fluctuations with $\Delta Z_*\approx 50$ (Ref. 12), so that, on the galactic scale

$$\delta\rho/\rho\approx 30\mu^3. \qquad (23)$$

Hence, it is clear that $\delta\rho/\rho\sim 10^{-4}$ for $\mu^3\approx 3\times 10^{-6}$, i.e., for $\mu\sim 10^{-2}$, which seems quite reasonable. In contrast to all other theories considered so far, this theory does not demand any additional small parameters to ensure substantial inflation and small $\delta\rho/\rho\sim 10^{-4}$. As far as the symmetry-breaking parameter φ_0 is concerned, the rate of decay of the particles φ into other particles $\Gamma_\varphi\sim\mu^3\varphi_0^3$ depends on this parameter,[5] so that, by varying this parameter, we can vary the temperature T_p of the Universe after the heating-up process, which is important for the gravitino problem. In particular, in our scenario $T_p=10^{10}$–10^{11} GeV for $\varphi_0=1$, and a reduction (increase) in φ_0 leads to a proportional reduction (increase) in T_p. The gravitino problem and the generation of baryon asymmetry in this scenario is examined in Ref. 11.

We have thus succeeded in finding a sufficiently wide class of superpotentials that lead to an effective potential with all the properties necessary for chaotic inflation.

The attractive feature of the above scenario is that its complete implementation within the framework of the above class of superpotentials does not demand the introduction of any small parameters, other than the parameter $\mu\sim 10^{-2}$, into the theory. This distinguishes our scenario from all other variants of the inflating Universe proposed previously.

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Translated by S. Chomet