Spectrum and polarization of the gravitational radiation of pulsars

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A hydrodynamic model of an inclined rotator is constructed in the form of a rotating self-gravitating homogeneous fluid drop with internal magnetic field inclined at an angle to the rotation axis. The spectrum and polarization of the gravitational radiation of the system are calculated. It is shown that the spectrum contains the first and second harmonics of the rotation frequency, the first harmonic being entirely due to the contribution of Newtonian stresses. The appearance of the first harmonic is not acompanied by precession of the magnetic axis of the pulsar. The reaction of a heterodyne detector to gravitational radiation of the described type is calculated. It is shown that use of the polarization properties of the radiation makes it possible to increase effectively the sensitivity of the detecting device.

1. INTRODUCTION

The discovery of the radio pulsar PSR 1937 + 214, which has a millisecond period $T = 1.557807$ msec,[1] has reopened the question of the detection of monochromatic gravitational radiation from neutron stars. For such rapidly rotating pulsars the main factor responsible for gravitational radiation is evidently the combined effect of the rapid rotation and the stresses of the magnetic field inclined at an angle to the rotation axis.[2–4] Although the power of the gravitational radiation estimated for the above pulsar on the basis of the spindown parameter appears to be insufficient for experimental detection, it is not impossible that similar objects with larger spindown parameter could be observed.

It should also be borne in mind that the estimate of the power of the gravitational radiation of pulsars based on the spindown parameter may be an underestimate. For it is based on the relation

$$dE/\omega = dJ/m \tag{1.1}$$

between the loss of energy dE and loss of angular momentum dJ in the case of radiation by a multipole field with frequency ω and azimuthal number m,[5] which for rigid-body rotation with frequency $\Omega = \partial E/\partial J$ leads to a result proportional to the spindown parameter.[1)] However, if one takes into account the possible change in the configuration of the rotating star and the redistribution of the masses within it, the relationship between the energy losses on radiation (including gravitational) and the observed spindown will be more complicated. Energy for the radiation can be drawn from not only the rotational energy but also the Newtonian potential energy of the star. Thus, from the classical theory of figures of equilibrium of a rotating homogeneous gravitating fluid it is known that the Jacobi ellipsoids increase their angular velocity of rotation through loss of angular momentum and energy on gravitational radiation.[6] Because of this, an experiment to look for gravitational radiation from rapidly rotating pulsars does not appear to be hopeless even in the case of a small spindown parameter $\dot{T}/T$, which for PSR 1937 + 214 has the order 10^{-19} sec.

In the present paper, we show that such an experiment would permit not only the detection of gravitational waves but also the verification of the general relativistic prediction of nonlinear relativistic transformation of the quasistationary Newtonian field into gravitational waves (in quantum language, the existence of the three-graviton vertex), which is of interest, in particular, in connection with the existence of alternative classical theories of gravitation.[7]

The effect to which we wish to draw attention consists of the generation of the first harmonic of the gravitational radiation (at the rotation frequency) of a neutron star through the contribution of the Newtonian stresses of the gravitational field. We note that when gravitational radiation is calculated by means of the quadrupole formula information about the contributions to the radiation from the motion of the gravitating masses and the force stresses cannot be obtained separately. However, if one does not use in explicit form the condition of the energy-momentum tensor's being conservative, then it is possible to separate the contributions to the Riemann tensor in the wave zone from the motion of masses and the magnetic and gravitational stresses. It is then found that the kinetic terms and the force stresses contribute equally to the radiation at twice the rotation frequency. But the radiation at the first harmonic is entirely due to the Newtonian stresses. This separation of the contributions of mass and stresses does not occur, for example, in the case of gravitational radiation from a binary system, for which the similarly calculated contributions differ only by a numerical coefficient. Of course, the separation of the mass and stress contributions to the gravitational radiation is not gauge invariant, i.e., it depends on the choice of the frame of reference. In the considered problem however such a choice is natural and leads to the asymptotically Galilean system in which the center of mass of the pulsar is at rest.

The proposed experiment consists of measuring the Riemann tensor of gravitational waves propagating away from the pulsar at two frequencies: the rotation frequency of the pulsar and twice that frequency. The detector can be based, for example, on the heterodyne principle proposed in Ref. 8. The ratio of the contributions to the gravitational radiation of the pulsar at the first and second harmonics depends on the angle of inclination of the magnetic axis of

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the star to the rotation axis, which can be estimated independently in other ways. We shall show below that general relativity predicts absence of radiation at the first harmonic only when the magnetic axis is at right angles to the rotation axis (which appears is improbable).

The possible appearance of the first harmonic in the gravitational radiation of pulsars was noted earlier in Refs. 9 and 10. However, this was associated with the possible effect of precession for a pulsar rotating as an asymmetric rigid body about an axis that does not coincide with any of the principal axes of inertia. Such a model leads to the prediction of modulation of the sequence of electromagnetic pulses from the pulsar at the precession frequency. The available observational data[11] apparently rule out the existence of precession. We wish to emphasize that the first harmonic must appear in the gravitational radiation even if the magnetic axis of the star keeps a constant angle of inclination to the rotation axis (and, therefore, there is no modulation of the electromagnetic pulses). The reason is that in the case of rotation of a figure with a magnetic field only the total angular momentum of the masses and the electromagnetic field is conserved, which makes possible rotation about an axis that does not coincide with any of the principal axes of the body though the angle of inclination of the magnetic moment to the rotation axis remains constant. This will be shown explicitly for the example of the hydrodynamic model of an inclined rotator—a rotating self-gravitating fluid drop with internal magnetic field inclined at an angle to the rotation axis.[3] In principle, such an effect is also possible for a rigid model if allowance is made for the nonconservation of the mechanical angular momentum in the presence of the angular momentum of the electromagnetic and Newtonian fields of the star, which was not done in Refs. 9 and 10.

In the present paper, we calculate the spectrum and polarization of the gravitational radiation of pulsars and consider the possibility of measuring them with a heterodyne detector of gravitational waves. We show that the use of the polarization properties of the gravitational radiation make it possible not only to increase effectively the sensitivity of the detecting device but also to obtain data on the angle of inclination of the rotation axes of pulsars relative to the direction to the Earth.

2. HYDRODYNAMIC MODEL OF AN INCLINED ROTATOR

It is well known that when a self-gravitating fluid drop with an internal magnetic field rotates the equilibrium configuration acquires a quadrupole moment.[12] This is regarded as one of the reasons for the occurrence of quadrupole deformation and, therefore, gravitational radiation of pulsars.[2] However, analytic calculations of equilibrium configurations can be made only for zero angle of inclination of the magnetic axis to the rotation axis[12] or for the rather unrealistic case of mutually perpendicular axes.[2] Below, we construct a simple model in which calculations can be made by the method of successive approximation for any angle of inclination. The accuracy of the approximation is sufficient for subsequent calculation of the quadrupole gravitational radiation.

We consider the rotation of a self-gravitating drop of a homogeneous fluid with internal magnetic field and a nearly spherical shape. We shall assume that in the absence of rotation the drop, which has equilibrium radius R_0, has an internal homogeneous magnetic field

$$\mathbf{B}_{in}=2\boldsymbol{\mu}/R_0^3, \tag{2.1}$$

(μ is the magnetic moment of the pulsar), this being matched to the magnetic dipole field in the exterior region, which in the near zone is

$$\mathbf{B}_{ext}=-[\boldsymbol{\mu}r^2-3(\boldsymbol{\mu}\mathbf{r})\mathbf{r}]r^{-5}. \tag{2.2}$$

If the drop rotates with constant angular velocity ω inclined at angle α to the direction μ of the magnetic moment, the equilibrium configuration takes the form of a triaxial ellipsoid (due to the anisotropy created by the magnetic stresses), and we write the equation of its surface Σ in the form

$$(r^2-R_0^2+a_{ij}x_ix_j)\,|_\Sigma=0. \tag{2.3}$$

We shall consider the case of fairly slow rotation, when the components of the tensor a_{ij} are small compared with unity; the corresponding conditions will be obvious from the obtained approximate solution.

The energy-momentum tensor of this system is a sum of four terms:

$$T_{\mu\nu}=\rho_{\mu\nu}+b_{\mu\nu}+t_{\mu\nu}+\tau_{\mu\nu}, \tag{2.4}$$

corresponding to the contributions of the masses, the magnetic field, the Newtonian field, and the viscous stresses. In what follows, we shall be interested in the spatial components of the energy-momentum tensor, which for the terms on the right-hand side of (2.4) have the form

$$\rho_{ij}=\rho v_iv_j+P\delta_{ij} \tag{2.5}$$

[ρ is the density of the fluid, assumed constant, P is the pressure, and $\mathbf{v}(\mathbf{r},t)$ is the velocity field];

$$b_{ij}=-\frac{1}{4\pi}\left(B_iB_j-\frac{1}{2}\delta_{ij}B^2\right), \tag{2.6}$$

it being necessary to take into account both the internal magnetic field (2.1) and the external field in the near zone (2.2);

$$t_{ij}=\frac{1}{4\pi G}\left(\Phi_{,i}\Phi_{,j}-\frac{1}{2}\delta_{ij}(\nabla\Phi)^2\right), \tag{2.7}$$

where Φ is the Newtonian gravitational potential, and G is the gravitational constant. The viscous stress tensor τ_{ij} (of anisotropic pressure) is calculated by solving the equations of hydrostatic equilibrium in a rotating coordinate system.

To determine the motion of the masses and calculate the equilibrium configurations of the fluid, we shall use the equation

$$\partial T^{\mu\nu}/\partial x^\nu=0, \tag{2.8}$$

which holds in the case of a sufficiently weak gravitational field [Eq. (2.8) contains the ordinary and not the covariant derivative, since the Newtonian stresses are already included in $T^{\mu\nu}$]. The nonlinear general relativistic corrections can be found by the standard method of post-Newtonian approximations.[11,13] From the conservation condition (2.8) we obtain the equations of hydrostatic equilibrium:

$$P+\rho\Phi-\rho v^2/2+B^2/8\pi=\text{const}, \quad (2.9)$$

$$(\partial/\partial x_j)(\tau_{ij}-B_iB_j/4\pi)=0. \quad (2.10)$$

An expression for the Newtonian potential of the deformed drop can be readily derived by using the expressions for the potential of a homogeneous triaxial ellipsoid. Taking into account the terms linear in a_{ij}, we can represent the potential in the interior region in the form

$$\Phi_{in}=\varkappa^2\left(-\frac{1}{2}R_0^2+\frac{1}{6}r^2-\frac{1}{30}ar^2+\frac{1}{10}a_{ij}x_ix_j+\frac{1}{6}aR_0^2\right), \quad (2.11)$$

where $a=\operatorname{tr} a_{ij}$ and $\varkappa^2=4\pi G\rho$ is the square of the Jeans frequency. The corresponding expression for the Newtonian potential outside the star will have the form

$$\Phi_{ext}=\varkappa^2R_0^3\left(-\frac{1}{3r}+\frac{a}{6r}\left(1-\frac{R_0^2}{5r^2}\right)+\frac{R_0^3}{10r^5}a_{ij}x_ix_j\right). \quad (2.12)$$

Of practical interest is the case when the magnetic energy of the drop is less than the Newtonian energy, i.e.,

$$\beta=B^2/4\pi G\rho^2R^2\ll 1, \quad (2.13)$$

and the rotation frequency ω is small compared with the Jeans frequency $\varkappa$. The equilibrium configurations are found by expanding all quantities in powers of the parameter β and the ratio $\omega/\varkappa$. In accordance with the hydrodynamic equations (2.9) and (2.10) and Maxwell's equations, the following boundary conditions must be satisfied on the surface Σ of the drop:

$$8\pi P|_\Sigma=B_{ext}^2-B_{in}^2,$$
$$\tau_{ij}|_\Sigma n_j=\frac{1}{4\pi}(B_{ext\,i}-B_{in\,i})(\mathbf{Bn}), \quad (2.14)$$
$$(\mathbf{B}_{ext}\mathbf{n})=(\mathbf{B}_{in}\mathbf{n}),$$

where $\mathbf{n}$ is the vector of the normal to the surface. It is readily seen that the conditions of matching of the interior magnetic field (2.1) to the exterior (2.2) cease to hold on the surface of the deformed drop. This means that, with allowance for the deformation, the interior and exterior magnetic fields acquire small corrections δB_{in} and δB_{ext}, which ensure fulfillment of the boundary conditions. However, since δB_{in} and δB_{ext} are small quantities of first order, they are to be ignored when the equilibrium configuration is found, i.e., the tensor a_{ij}. Bearing in mind what we have said, we obtain from Eqs. (2.9), (2.10), and (2.14) in the lowest nonvanishing approximation in the parameters β and $\omega/\varkappa$ the following expression for the tensor a_{ij}:

$$a_{ij}=-\frac{15}{2\varkappa^2}\omega_i\omega_j+\frac{45R_0^4}{8\pi\varkappa^2\rho}\mu_i\mu_j. \quad (2.15)$$

In the same approximation, the tensor τ_{ij} has the form

$$\tau_{ij}=\frac{3R_0^4}{2\pi}(\mu_i\mu_jr^2+\delta_{ij}(\boldsymbol{\mu}\mathbf{r})^2). \quad (2.16)$$

It can be seen from the expression (2.14) that in the general case the drop will have the shape of a triaxial ellipsoid whose rotation axis coincides with none of the principal axes of inertia of the mass distribution. However, in contrast to the case of a rigid top,[9] precession does not arise in our case, and the angle of inclination of the interior magnetic field to the rotation axis remains constant.

3. CURVATURE TENSOR IN THE WAVE ZONE

The reaction of a detector to the gravitational waves is determined by the periodically varying components R_{0i0j} of the curvature tensor. These components are readily calculated by standard methods.[13,14] In the region of the detector, the space-time metric is nearly flat, and we can therefore write

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}, \quad (3.1)$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor. In the considered asymptotic region $h_{\mu\nu}\ll 1$, and the components of the curvature tensor in which we are interested have the form

$$R_{0i0j}=\frac{1}{2}(h_{0i,0j}+h_{0j,0i}-h_{ij,00}-h_{00,ij}). \quad (3.2)$$

In this region, the field $h_{\mu\nu}$ satisfies the linearized equations

$$h_{\mu\nu,\lambda\tau}\eta^{\lambda\tau}=0, \quad (3.3)$$

which admit a large degree of freedom in the choice of the gauge. Since under the gauge transformation

$$h_{\mu\nu}\to h_{\mu\nu}+\xi_{\mu,\nu}+\xi_{\nu,\mu} \quad (3.4)$$

the components of the curvature tensor do not change, in the asymptotic region it is possible to use the doubly transverse traceless gauge:

$$h_{0i}=0;\quad \operatorname{tr} h_{ij}=0,\quad \partial h_{ij}/\partial x_j=0, \quad (3.5)$$

the choice of which, as is readily shown, does not conflict with Eqs. (3.3). Then (3.2) takes the simple form

$$R_{0i0j}=-{}^1/{}_2\partial^2h_{ij}/\partial t^2. \quad (3.6)$$

The field h_{ij} is determined by solving the equation

$$\Box h_{ij}=16\pi GT_{ij}, \quad (3.7)$$

whose right-hand side contains the spatial components of the total energy-momentum tensor of the system, including the Newtonian stresses of the gravitational field. We expand the right-hand side in a Fourier series, taking into account the periodic dependence of $T_{\mu\nu}$ on the time with frequency ω:

$$T_{ij}(\mathbf{r},t)=\int\sum_{n=-\infty}^{\infty}T_{ij}(\mathbf{k},n)\exp(-in\omega t+i\mathbf{kr})\,d\mathbf{k}/(2\pi)^3, \quad (3.8)$$

and we introduce the standard polarization tensors

$$e_{ij}^{+}=(e_i^1e_j^1-e_i^2e_j^2)/2^{1/2};\quad e_{ij}^{\times}=(e_i^1e_j^2+e_i^2e_j^1)/2^{1/2}, \quad (3.9)$$

where $\mathbf{e}^1$ and $\mathbf{e}^2$ are unit vectors orthogonal to each other and the vector $\mathbf{k}$ (and form with it a right-handed triplet). Then the solutions of Eq. (3.7) in the wave zone corresponding to the two independent polarization states (3.9) will have the form[14]

$$h_{+,\times}(t)=-(4G/R)\sum_{n=-\infty}^{\infty}T_{+,\times}(n\omega\mathbf{R}/R,n)\exp[-in\omega(t-R)], \quad (3.10)$$

where $\mathbf{R}$ is the radius vector from the pulsar to the point of observation. The intensity of the gravitational radiation can be expressed in terms of the quantities $T_{+,\times}\equiv T_{+,\times}(n\omega\mathbf{R}/$

$R,n)$ as follows:

$$I=(G/\pi)\sum_{n=1}^{\infty}(n\omega)^2(|T_+|^2+|T_\times|^2). \qquad (3.11)$$

It is sufficient to calculate the Fourier components of the stress tensor in the long-wave approximation, replacing $\exp(-i\mathbf{k}\cdot\mathbf{r})$ by unity. Note that if we are not interested in the contributions to T_{ij} of the terms of (2.4) separately, we can, using the conservation condition (2.8), write down a relation that holds for $kR_0\ll 1$:

$$T_{ij}(\mathbf{k},n)=\frac{1}{2}(n\omega)^2\frac{\partial^2 T_{00}(\mathbf{k},n)}{\partial k^i\,\partial k^j}, \qquad (3.12)$$

the use of which enables us to reduce (3.11) readily to the usual quadrupole form. Calculations show that the total intensity (3.11) of the gravitational radiation is the sum of two harmonics:

$$I=I_\omega+I_{2\omega},\quad I_\omega=I_{2\omega}/16\,\mathrm{tg}^2\,\alpha,$$
$$I_{2\omega}=0{,}4\mu^2\omega^6\sin^4\alpha/GM^2, \qquad (3.13)$$

where α is the angle of inclination of the magnetic axis to the rotation axis, and M is the mass of the drop [the gravitational constant is in the denominator of the expression (3.13), since, as is readily seen from (2.15)], the quadrupole moment of the equilibrium configuration is inversely proportional to G).

We show that the radiation at the rotation frequency of the drop is due to the contribution of the Newtonian stresses t_{ij} to (2.4). For this, we calculate $T_{+,\times}$, substituting as T_{ij} the sum (2.4) of the contributions of the masses, ρ_{ij}, the magnetic stresses, b_{ij}, the Newtonian stresses, t_{ij}, and the anisotropic pressure, τ_{ij}. As in the calculation of the equilibrium configuration of the drop, to find the Fourier transforms of the quantities in (2.4) we use perturbation theory, regarding the parameters β and $\omega/\varkappa$ as small. In the lowest order of perturbation theory, the contribution of the mass vanishes, and the remaining terms in (2.4) cancel each other:

$$\tau_{ij}(\mathbf{k},n)=6t_{ij}(\mathbf{k},n)=-\frac{6}{7}b_{ij}(\mathbf{k},n)=\frac{1}{10\pi R_0^3}\int_0^{2\pi}\mu_i\mu_j d(\omega t). \qquad (3.14)$$

This corresponds to the obvious absence of dipole gravitational radiation. In the second order, we take into account the terms that together give the quadrupole radiation. Calculation shows that in this approximation the contributions of the magnetic stresses and the anisotropic pressure are zero, while the contributions of the mass and the Newtonian stresses are

$$\rho_{ij}(\mathbf{k},n)=\frac{\omega^2}{2GM}\int_0^{2\pi}\mu_{\perp i}\mu_{\perp j}\exp(in\omega t)\frac{d(\omega t)}{2\pi}, \qquad (3.15)$$

$$t_{ij}(\mathbf{k},n)=\rho_{ij}(\mathbf{k},n)-\frac{\omega^2}{4GM}\int_0^{2\pi}(\mu_{\parallel i}\mu_{\perp j}+\mu_{\perp i}\mu_{\parallel j})\exp(in\omega t)\frac{d(\omega t)}{2\pi}, \qquad (3.16)$$

where $\mu_{\perp i}$ and $\mu_{\parallel i}$ are the components of the magnetic moment perpendicular to and parallel to the rotation axis.

In a coordinate system with z axis along the rotation axis, the nonvanishing components of the vector of the magnetic moment are

$$\mu_{\perp x}=\mu\sin\alpha\cos\omega t,\quad \mu_{\perp y}=\mu\sin\alpha\sin\omega t;\quad \mu_{\parallel z}=\mu\cos\alpha. \qquad (3.17)$$

It can be seen from Eqs. (3.15)–(3.17) that the mass tensor ρ_{ij} has a nonvanishing Fourier component only at the second harmonic ($n=2$), whereas the tensor of the Newtonian stresses also contains a contribution of the first harmonic $n=1$ [the second term in (3.16)]. Note that the contributions of the masses and the stresses at the second harmonic are equal, as can be seen from (3.15) and (3.16).

We give the final expressions for the projections of the Riemann tensor corresponding to the two independent polarization states $R_{+,\times}=-\frac{1}{2}\partial^2 h_{+,\times}/\partial t^2$. We attach to the pulsar a coordinate system in which the z axis points toward to the observer and makes angle θ with the rotation axis, and the x axis lies in the same plane as they. Substituting the Fourier transforms of (3.15) and (3.16) in (3.10), we find

$$R_+=-\omega^2(h_1\sin 2\theta\cos\omega t/2^{3/2}+2^{1/2}h_2(\cos 2\theta+3)\cos 2\omega t), \qquad (3.18)$$

$$R_\times=\omega^2(h_1\sin\theta\sin\omega t/2^{1/2}+2^{5/2}h_2\cos\theta\sin 2\omega t), \qquad (3.19)$$

the dimensionless amplitudes h_1 and h_2 in these expressions being related to the radiation intensities (3.13) at the first and second harmonics by

$$h_1=(\omega R)^{-1}(20GI_\omega/3)^{1/2};\quad h_2=(\omega R)^{-1}(5GI_{2\omega}/8)^{1/2}, \qquad (3.20)$$

where R is the distance from the pulsar to the Earth.

It can be shown that $R_+ + iR_\times$ determined by means of (3.18) and (3.19) is an expansion of the component ψ_4 of the Weyl tensor in the asymptotic region with respect to the spin spherical harmonics $_{-2}S_{lm}(\theta)$, the first terms (radiation at frequency ω) corresponding to $l=2$, $|m|=1$, and the second (radiation at frequency 2ω) to $l=2$, $|m|=2$.

4. EFFECT OF GRAVITATIONAL WAVES ON A HETERODYNE DETECTOR

We calculate the response of a heterodyne detector to the gravitational waves described by the Riemann tensor (3.18), (3.19). In the simplest case, such a detector is a rotating dumbbell oriented at right angles to the direction of propagation of the wave.[8,15] A more complicated system is an asymmetric rotator,[16] which has the advantage that the working and signal rotations can be separated, since the incident gravitational wave gives rise to a rotation of the rotator about the axes perpendicular to the axis of resonance rotation. The separation of the working and signal motions reduces the requirements on the accuracy of synchronization of the rotation by several orders of magnitude. Under certain conditions, the effect has however the same order as the one considered in Refs. 8 and 15. Therefore, for simplicity we shall make numerical estimates for the simplest model of a heterodyne detector.[8,15]

As a result of calculations, the details of which are given in Ref. 15, it is possible to obtain an equation for determining the angle φ between the x axis and one of the arms of the dumbbell under the influence of the gravitational radiation (without allowance for damping):

$$\ddot{\varphi}_1 = -{}^1/_4\omega^2 h_1 (\sin 2\theta \sin 2\varphi \cos \omega t + 2 \sin \theta \cos 2\varphi \sin \omega t),$$
$$\ddot{\varphi}_2 = -4\omega^2 h_2 (\cos 2\theta \sin 2\varphi \cos 2\omega t + \cos \theta \cos 2\varphi \sin 2\omega t). \quad (4.1)$$

It is convenient to represent the solution of Eq. (4.1) in the form

$$\varphi_{1,2} = \Omega t + \varphi_0 + \Delta\varphi_{1,2}(t), \quad (4.2)$$

where Ω is the angular velocity of the dumbbell rotation without allowance for the effect of the gravitational wave, and $\Delta\varphi_{1,2}$ are small corrections due to the influence of the gravitational radiation of frequencies ω and 2ω, respectively. Assuming that over the time interval in which we are interested $h_{1,2}$, Ω, and ω change negligibly little, we find from (4.1) and (4.2)

$$\Delta\varphi_1 = -\frac{1}{4}\omega^2 h_1 \sin\theta \left[\frac{1+\cos\theta}{(\omega+2\Omega)^2}F_1^+ + \frac{\cos\theta - 1}{(\omega-2\Omega)^2}F_1^-\right],$$
$$\Delta\varphi_2 = -\frac{1}{4}\omega^2 h_2 \left[\frac{(1+\cos\theta)^2}{(\omega+\Omega)^2}F_2^+ + \frac{(1-\cos\theta)^2}{(\omega-\Omega)^2}F_2^-\right], \quad (4.3)$$

where

$$F_1^\pm = \pm\cos 2\varphi_0[(\omega\pm2\Omega)t - \sin(\omega\pm2\Omega)t] + \sin 2\varphi_0[1-\cos(\omega\pm2\Omega)t],$$
$$F_2^\pm = \pm\cos 2\varphi_0[2(\omega\pm\Omega)t - \sin 2(\omega\pm\Omega)t] + \sin 2\varphi_0[1-\cos 2(\omega\pm\Omega)t]. \quad (4.4)$$

As can be seen from (4.3), a change in the direction of rotation of the dumbbell gives rise to a difference in the responses of the heterodyne detector to the gravitational radiation. Thus, the simultaneous use of two dumbbells rotating in different directions can significantly increase the sensitivity of the detector by separating the difference between their responses.

In the case $\Omega = \pm\omega$, $\pm\omega/2$, i.e., when the condition of exact synchronism is satisfied, we obtain from (4.1)

$$\Delta\varphi_1^\pm(t) = -{}^1/_8 h_1\omega^2 t^2 \sin 2\varphi_0(\cos\theta \mp 1)\sin\theta,$$
$$\Delta\varphi_2^\pm(t) = -{}^1/_2 h_2\omega^2 t^2 \sin 2\varphi_0(1\mp\cos\theta)^2. \quad (4.5)$$

In deriving Eqs. (4.5), we have assumed that the amplitude of the gravitational wave is constant, which corresponds to a steady pulsar rotation regime. One can also consider the period in which steady rotation is established after the formation of the star. It was shown in Ref. 4 that if the pulsar rotation frequency is equal to the characteristic frequency (bifurcation point) the intensity of the gravitational radiation at the second harmonic increases strongly, and its time dependence is

$$I_{2\omega}(t) = I_{2\omega}(0)\tau^{2/3}(t+\tau)^{-2/3} \quad (4.6)$$

(the parameter τ has the order 0.03 sec). Taking into account (4.6), we obtain instead of (4.5) in the given case

$$\Delta\varphi_2^\pm(t) = -0{,}1 h_2(0)\sin 2\varphi_0(1\mp\cos\theta)^2\omega^2\tau^{1/3}t^{5/3}. \quad (4.7)$$

In making estimates in what follows, we shall be optimistic and assume that the condition of perfect synchronism is satisfied. Estimates of the noise for a heterodyne detector are given in Refs. 16 and 17.

Choosing $\omega = 4\times10^3$ sec^{-1} (which corresponds to the period of the pulsar PSR 1937 + 214), $R = 10^{22}$ cm, $\sin 2\varphi_0 = 1$, $\cos\theta = 2^{-1/2}$, $t = 10^3$ sec, we find on the basis of our expressions the values for $\Delta\varphi^\pm_{1,2}$ given in Tables I, II, and III. Table I corresponds to the effect on the detector of the gravitational radiation of frequency 2ω at the time of formation of a pulsar whose figure is near the bifurcation point. The quantities $\Delta\varphi_2^\pm$ will have the same order of magnitude at any time if as a result of freezing of the crust the figure of the pulsar remains unchanged from about 10^4 sec after its formation. In Table II we give estimates of $\Delta\varphi_2^\pm$ under the assumption that the figure of the pulsar is far from the bifurcation point and corresponds to the equilibrium configuration of a fluid drop. In Tables I and II, B_0 is the interior magnetic field of the pulsar. In Table III we give the corresponding estimates for $\Delta\varphi_1^\pm$.

It can be seen from the estimates that $\Delta\varphi_2^+$ and $\Delta\varphi_2^-$ differ by about two orders of magnitude (for $\theta = \pi/2$), which creates favorable possibilities for increasing the sensitivity of the detector and using it to measure the polarization of the gravitational waves at frequency 2ω emitted by pulsars with millisecond period. The values of $\Delta\varphi_1^\pm$ differ by somewhat less (by a factor of about six).

5. DISCUSSION OF THE RESULTS

On the basis of a simple hydrodynamic model of an inclined rotator we have shown that the gravitational radiation of a pulsar must contain components with frequencies ω and 2ω (ω is the rotation frequency), which corresponds to quadrupole radiation of multipole fields $l = 2$, $|m| = 1$ and $l = 2 = |m|$. The radiation at frequency ω is due to the ab-

TABLE I.

B_0, G	h_2	$\Delta\varphi_2^+$, rad	$\Delta\varphi_2^-$, rad
$4\cdot10^{14}$	$0.8\cdot10^{-20}$	$0.93\cdot10^{-8}$	$2.7\cdot10^{-10}$
$4\cdot10^{12}$	$1.7\cdot10^{-23}$	$1.9\cdot10^{-11}$	$5.7\cdot10^{-13}$
$2\cdot10^{10}$	$0.5\cdot10^{-24}$	$0.57\cdot10^{-12}$	$1.7\cdot10^{-14}$
$6.3\cdot10^{8}$	$0.5\cdot10^{-25}$	$0.57\cdot10^{-13}$	$1.7\cdot10^{-15}$

TABLE II.

B_0, G	h_2	$\Delta\varphi_2^+$, rad	$\Delta\varphi_2^-$, rad
$5\cdot10^{15}$	$1.7\cdot10^{-23}$	$1.9\cdot10^{-11}$	$5.7\cdot10^{-13}$
10^{15}	$0.69\cdot10^{-24}$	$0.77\cdot10^{-12}$	$2.3\cdot10^{-14}$
10^{14}	$0.69\cdot10^{-26}$	$0.77\cdot10^{-14}$	$2.3\cdot10^{-16}$
10^{12}	$0.69\cdot10^{-30}$	$0.77\cdot10^{-18}$	$2.3\cdot10^{-20}$

TABLE III.

B_0, G	h_1	$\Delta\varphi_1^+$, rad	$\Delta\varphi_1^-$, rad
$5\cdot10^{15}$	$4.3\cdot10^{-24}$	$0.18\cdot10^{-11}$	$-1.03\cdot10^{-11}$
10^{15}	$1.7\cdot10^{-25}$	$0.72\cdot10^{-13}$	$-0.41\cdot10^{-12}$
10^{14}	$1.7\cdot10^{-27}$	$0.72\cdot10^{-15}$	$-0.41\cdot10^{-14}$
10^{12}	$1.7\cdot10^{-31}$	$0.72\cdot10^{-19}$	$-0.41\cdot10^{-18}$

sence of axial symmetry of the rotating magnetized drop, which takes the shape of a triaxial ellipsoid. The drop rotates about an axis that coincides with none of the principal mechanical axes of inertia, but this motion is not accompanied by precession because of the nonconservation of the mechanical angular momentum. This is the fundamental difference between the mechanism considered here for generation of the first harmonic from the precession mechanism proposed in Refs. 9 and 10. In contrast to the model of a precessing top, the present model does not lead to the prediction of modulation of the train of electromagnetic pulses at the precession frequency. Evidently, the nonconservation of the angular momentum of the masses separately must also be taken into account for a rigid model of a pulsar whose magnetic axis and rotation axis are inclined at an angle $\alpha \neq 0, \pi/2$ to each other.

In the framework of the considered model, the gravitational radiation of the pulsar at the first harmonic of the rotation frequency is entirely due to the Newtonian stresses. For a sufficiently small angle of inclination of the magnetic and rotation axes, the radiation at the first harmonic may be predominant.

Using the polarization properties of the radiation, one can effectively increase the sensitivity of a heterodyne detector. It follows from our estimates that the detection of gravitational radiation from pulsars with millisecond period is not technically hopeless.

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