Effect of the relative delay of rays focused by a rotating massive body

I. G. Dymnikova

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences
(Submitted 3 June 1983)
Zh. Eksp. Teor. Fiz. **86**, 385–389 (February 1984)

The existence is shown and the magnitude calculated of a relative delay of rays emanating from a point source of radiation and focused by the gravitational field of a rotating massive body before they arrive at a point of observation situated on the same straight line as the source and the focusing mass in its equatorial plane. It is also shown that the additional gravitational delay of the signals due to the rotation is negative for photons moving parallel to the rotation axis and for photons moving in the equatorial plane in the direction of the rotation. Photons scattered by a rotating black hole with impact parameters near the critical value for capture have a relative delay because their cross section is asymmetric.

In 1936, Einstein predicted[1] the gravitational lens effect on the basis of the deflection of rays in the gravitational field of a massive body. This has now become a very topical effect because of the recent observations of one double quasar and two triple quasars convincingly interpreted as the images of single objects formed by gravitational lenses on the line of sight between the respective quasars and the observer (see Ref. 2 and the bibliography there).

Since the gravitational lens effect leads to the appearance of multiple images of a single source formed by rays that traverse different optical paths, it is necessary to consider the question of simultaneity for these images. In the case when the source of the radiation, a point mass that plays the part of the gravitational lens, and the observer are on a straight line, the resulting images are simultaneous, i.e., the propagation times of signals from a given point of the source along different optical paths corresponding to different images are equal. In the more general case of an extended gravitational lens with spherically symmetric density distribution and an observer situated on the line joining the radiation source to the center of the lens, all rays reaching a given point of observation also arrive at it simultaneously (this can be readily verified by using, for example, Ref. 3, p. 324).

The situation is quite different if a rotating massive body is the gravitational lens. As is shown below, rotation introduces an asymmetry into the gravitational delay of the signals, and the additional gravitational delay due to the rotation has different signs depending on the orientation of the (orbital) angular momentum of the photon relative to the rotation axis. This leads to nonsimultaneity of the resulting images even in the simplest case when the radiation source, the focusing mass, and the observer are on a straight line. The relative delay between rays arriving at the same point of observation along different optical paths is maximal in the equatorial plane of the focusing body. In this case, the signal propagation time obtained by integrating the geodesic equations for the Kerr metric describing the gravitational field of the rotating body (see, for example, Ref. 3, p. 418) in the rest frame of the distant observer and under the condition that the wavelength of the radiation is much shorter than the characteristic scale of variation of the field is given by the expression

$$t_{bd}=\left\{(b^2-d^2)^{1/2}+2\ln\frac{b+(b^2-d^2)^{1/2}}{d}+\left(\frac{b-d}{b+d}\right)^{1/2}+\frac{15\pi-8}{4d}-\frac{4a}{d}\frac{\rho}{|\rho|}\right\}\frac{GM}{c^3}, \tag{1}$$

where $bGMc^{-2}=r_*$ is the coordinate of the point of emission of the signal, $dGMc^{-2}=r_{\min}$ is the distance of its closest approach to the deflecting body, which has mass M and angular momentum $I=aGM^2c^{-1}$, ρGMc^{-2} is the impact parameter in the equatorial plane, G is the gravitational constant, and c is the velocity of light. The signal propagation time is written down here in the approximation $b\gg d$, $d\gg r_+$, where $r_+GMc^{-2}=r_h$ is the radius of the event horizon (see also Ref. 4, in which Kostyukovich and Mitjanock consider the delay of a radar signal in the equatorial plane of a rotating mass by means of a method they developed for analyzing the equatorial motion in Tomimatsu-Sato fields). Here an in what follows, we use Boyer-Lindquist coordinates (see, for example, Ref. 3), which at infinity go over into ordinary spherical coordinates in flat space. The distances are measured from the center of the gravitating body.

As follows from Eq. (1), the additional "rotational" gravitational delay is negative for photons moving in the direction of the rotation and positive for photons moving in the opposite direction to the rotation of the central body. This situation has analogies in scattering theory (for example, the asymmetry of the scattering by a polarized target associated with the phase difference) and can be characterized as a manifestation of the spin-orbit coupling in the gravitational field of the rotating massive body. It is particularly interesting to note that in the given case this coupling has the consequence that the gravitational delay corresponding to the rotation may be a "gravitational acceleration."

The propagation time of a signal moving parallel to the rotation axis,

$$t_{\parallel}=\left\{(b^2-d^2)^{1/2}+2\ln\frac{b+(b^2-d^2)^{1/2}}{d}+\left(\frac{b-d}{b+d}\right)^{1/2}+\frac{15\pi-8}{4d}-\frac{4a^2+23\pi-120}{4d^2}\right\}\frac{GM}{c^3}, \tag{2}$$

includes a dependence on the rotation in a higher order in the small quantity $1/d$ than (1), the additional "rotational" gravitational delay being negative. The quadratic dependence of this delay on $1/d$ is in a certain sense analogous to the transverse Doppler effect in the special theory of relativity and indicates that in the case of propagation of the radiation parallel to the axis the effect of the dragging of the inertial frames in the field of the rotating body is not so important as in the equatorial plane, though the additional rotational gravitational delay is manifested as an acceleration and not a retardation.

For rays propagating in the equatorial plane of the rotating gravitational lens, the signal propagation time from the point r_* of the source to the point of observation $r_0 = fGMc^{-2}$ (see Fig. 1) is $t_{bd} + t_{df}$. In the approximation in which $d \gg 1$ and $b, f \to \infty$, the deflection angle of a ray propagating in the equatorial plane of the rotating body is

$$\Delta\varphi = \frac{4}{d} + \frac{15\pi - 16}{4d^2} - \frac{4a}{d^2}\frac{\rho}{|\rho|}.$$

Rays with negative impact parameters are deflected more strongly than those with positive. Bearing in mind that the distances from the lens to the source and to the observer are finite, we can obtain a condition for the radiation source, the focusing mass, and the observer to be situated on one line. It is

$$\frac{d}{b} + \frac{d}{f} = \frac{4}{d} - \frac{4a}{d^2}\frac{\rho}{|\rho|} + \frac{15\pi - 32}{4d^2} \tag{3}$$

and it shows that rays from the same point of the source but with positive and negative impact parameters arrive at the same point of observation only if they satisfy the condition

$$d_- = d_+ + a. \tag{4}$$

As a result, there is a relative time delay due to the difference between the optical paths for rays that arrive at the same point of observation; it is determined by Eqs. (1) and (4) and is

$$\Delta t = t_- - t_+ = \frac{8a}{d}\frac{GM}{c^3} = \frac{8I}{Mcr_{min}}\frac{GM}{c^3}. \tag{5}$$

The magnitude of this delay depends on the type of focusing object and can be estimated as

$$\begin{aligned} \Delta t &= \left[\frac{M}{5M_\odot}\frac{I}{GM^2c^{-1}}\frac{100}{d}\right] \cdot 2 \cdot 10^{-6}[\mathrm{sec}] \\ &= \left[\frac{M}{5 \cdot 10^8 M_\odot}\frac{I}{GM^2c^{-1}}\frac{100}{d}\right] \cdot 200\ [\mathrm{sec}]. \end{aligned} \tag{6}$$

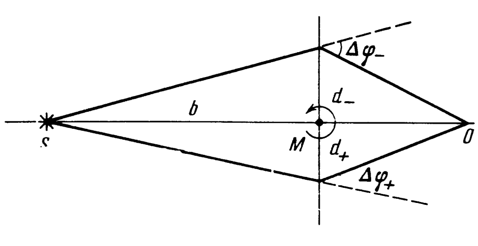

FIG. 1. Deflection of rays propagating from the source point S in the equatorial plane of a rotating point mass M to the point of observation O; $d_-, d_+, \Delta\varphi_-, \Delta\varphi_+$ are, respectively, the distances of closest approach and the deflection angles of rays with negative and positive impact parameters.

It is a somewhat speculative but basically plausible suggestion that the relative delay of signals in the field of rotating bodies could be responsible for the extremely rapid fluctuations in the intensities of some distant objects such as are observed, for example, for the quasar 1525 + 227 with characteristic time $\tau \approx 200$ sec,[5] if it is assumed that there is a massive rotating gravitational lens (for example, a black hole with mass $\sim 5 \cdot 10^8 M_\odot$) on the line of sight between the quasar and the observer.

The effects associated with the spin-orbit interaction in the field of a rotating black hole become particularly important for photons propagating in the immediate proximity of the region of gravitational capture; in particular, they lead to a strong distortion of the shape of the capture cross section. In the case of an extended source of radiation and a rotating black hole between the source and the observer, the black hole will appear literally as a black hole in the image of the source bounded by the curve formed by the photons deflected by the hole through angles $\Delta\varphi = 3k\pi$, where $k = \pm(1,2,3,...)$. This curve is the boundary of the cross section of gravitational capture of the photons by the rotating black hole and appears as a circle symmetric with respect to the center only for photons incident on the hole parallel to the rotation axis. For an extremal ($a = 1$) hole and arbitrary angle of incidence the curve is described by the equation

$$\begin{aligned} &\rho_\parallel^2 + (\rho_\perp - \sin\vartheta_0)^2 - 4[\rho_\parallel^2 + (\rho_\perp - \sin\vartheta_0)^2]^{1/2} \\ &\quad + 4(\rho_\perp \sin\vartheta_0 - 1) = 0, \end{aligned}$$

where $\rho_\parallel$ and $\rho_\perp$, the impact parameters of the photon, are related to its energy $\mathscr{E}$, angular momentum $\mathscr{L}$, the (conserved) projection Φ of $\mathscr{L}$ onto the rotation axis, and the initial value ϑ_0 of the polar angle (the angle of incidence) by

$$\rho_\perp = \Phi/\mathscr{E} \sin\vartheta_0, \quad \rho_\parallel = [\mathscr{L}^2/\mathscr{E}^2 - \rho_\perp^2]^{1/2}.$$

Photons with impact parameters within this curve are captured at the points

$$r_c = \tilde\rho/2 = [\rho_\parallel^2 + (\rho_\perp - \sin\vartheta_0)^2]^{1/2}/2;$$

the remaining photons are deflected at the points

$$d = {}^1/_2\{\tilde\rho + [\tilde\rho^2 - 4\tilde\rho + 4(\rho_\perp \sin\vartheta_0 - 1)]^{1/2}\}.$$

The magnitude of the capture cross section is almost independent of the angle ϑ_0, but the shape of the capture cross section changes strongly when this angle is increased. For angles of incidence in the interval

$$\sin^{-1}(\sqrt{3} - 1) \approx 47° \leqslant \vartheta_0 \leqslant 90°,$$

the capture cross sections have in the region of positive $\rho_\perp$ a straight section due to the fact that all photons with impact parameters

$$\rho_\perp = \frac{2}{\sin\vartheta_0}, \quad \rho_\parallel \leqslant \left[4 - \left(\frac{2}{\sin\vartheta_0} - \sin\vartheta_0\right)^2\right]^{1/2}$$

are captured by the hole at the point $r_c = r_+$. The capture cross section is maximally asymmetric for $\vartheta_0 = 90°$; in this case, $-7 \leqslant \rho_\perp \leqslant 2$.[6]

Because of the asymmetry of the cross section for scattering of photons by a rotating black hole, there must be a relative delay in the propagation time for photons that form the boundary of the "hole" in the image of the radiation source. For photons propagating in the equatorial plane with the maximal and minimal impact parameters, this delay, obtained from the analytic expressions for the propagation time and the photon deflection angle in the immediate proximity of the capture region,[7] is described by

$$\Delta t \approx 3 |\Delta\varphi| GM/c^3. \tag{7}$$

In the case of a variable radiation source, the delay may have the consequence that as the brightness of the source changes the brightness of the boundary curve changes nonuniformly, a "hare" running from the point corresponding to the minimal value of the impact parameter $\rho_\perp$ with velocity $v_3 \approx cR_s/6R_h$ (for an extremal black hole), where R_h is the distance from the observer to the black hole, and R_s is the distance from the observer to the radiation source.

I thank V. L. Ginzburg and the participants of his seminar for discussing the work and making helpful comments, and also I. G. Mitrofanov and D. G. Yakovlev for stimulating discussions at various stages in the work.

[1]A. Einstein, Science **84**, 506 (1936) [Russian translation published in Vol. 2 of the Russian translations of Einstein's Scientific Works (Nauka, Moscow 1966, p. 436)].
[2]D. W. Weedman, R. J. Weymann, R. F. Green, and T. M. Heckman, Astrophys. J. **255**, L5 (1982).
[3]L. D. Landau and E. M. Lifshitz, Teoriya polya, Nauka, Moscow (1973); English translation: The Classical Theory of Fields, 4th ed., Pergamon Press, Oxford (1975).
[4]N. N. Kostyukovich and V. V. Mitjanock, Acta Phys. Pol. **B10**, 4, 279 (1979).
[5]T. Matilsky, C. Shraeder, and H. Tananbaum, Astrophys. J. **258**, L1 (1982).
[6]J. M. Bardeen, in: Black Holes (eds. C. DeWitt and B. S. DeWitt), New York (1973), p. 215.
[7]I. G. Dymnikova, Preprint No. 795 [in Russian], A. F. Ioffe Physicotechnical Institute (1982).

Translated by Julian B. Barbour