Superconductivity of layered metals

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A theory of a mixed state of a new class of layered materials, namely layered metals, is developed. It is shown that Abrikosov vortex lattices localized along the magnetic field can appear in such systems. If the magnetic field is parallel to the layers, the resultant superconducting state takes the form of a sequence of parallel superconducting plates. The energy of such states and the magnetic moment are determined.

Superconductors of a new type, layered metals, are being intensively investigated at present. These substances, obtained by molecular epitaxy, are sequences of layers of different metals, each layer varying widely in thickness from 5 to 10^4 Å. Besides the already obtained substances of this type, it is possible to obtain by a similar method substances with a period that differs from the layer thickness.[1–3]

Clearly, the physical properties of such substances can differ strongly from the properties of ordinary layered materials, which are crystals with microscopic layered structure.[4] The difference should be particularly strong in the superconducting properties of such systems. Indeed, the electrons move freely over the sample in such superconductors, and the principal effect that determines the properties of the systems is the "proximity effect."[5,6] In addition, in such superconductors the boundaries between the layers are apparently not abrupt.

The study of the superconductivity of the described systems was preceded by a number of investigations of the superconducting properties of a single inhomogeneous layer in an infinite superconductor.[7–12] In Ref. 13 were determined the critical magnetic fields and the critical temperatures of superconductors with an electron-electron interaction $g = g(\mathbf{r})$ that varies periodically in space. In this paper we determine the structure of the superconducting state and the magnetic moment of an inhomogeneous type-II superconductor with periodic spatial inhomogeneity.

§1. BASIC EQUATIONS

The problem was solved formally under the following model assumptions: 1) the Fermi velocities and the Debye frequencies of the metallic constituents of the layered metals are equal; 2) the electron-electron interaction $g(\mathbf{r})$ is a periodic function of the coordinates; 3) the impurities are uniformly distributed through the sample.

The microscopic equations for a dirty superconductor with a coordinate-dependent value of $g(\mathbf{r})$ in the field interval $\Delta H/H_0 \ll 1$ ($\Delta H = H_0 - H_e$, H_0 is the field at which the superconductivity sets in and H_e is the external magnetic field) are of the form

$$[\chi(\xi_T^2\hat{P}_1^2)+V(\mathbf{r})-\varepsilon]\psi^+(\mathbf{r}_+)+\frac{1}{8\pi^2T^2}\times\sum_{n=0}^{\infty}\frac{n+{}^1/_2+(v_0^2\tau/48\pi T)[(\hat{P}_1-\hat{P}_3)^2+(\hat{P}_2-\hat{P}_4)^2]}{\prod_{i=1}^{4}[n+{}^1/_2+(v_0^2\tau/12\pi T)]}$$

$$\times\psi^+(\mathbf{r}_1)\psi(\mathbf{r}_2)\psi^+(\mathbf{r}_3)_{\mathbf{r}_1=\mathbf{r}_2=\mathbf{r}_3=\mathbf{r}}=0, \qquad (1)$$

$$\mathbf{j}(\mathbf{r})=-\frac{e\tau_{tr}N}{4\pi mT}\sum_{n=0}\frac{(\hat{P}_1-\hat{P}_2)\psi^+(\mathbf{r}_1)\psi(\mathbf{r}_2)}{\prod_{i=1}^{2}[n+{}^1/_2+(v_0^2\tau_{tr}/12\pi T)P_i^2]}\Bigg|_{\mathbf{r}_1=\mathbf{r}_2=\mathbf{r}} \qquad (2)$$

where

$$\hat{P}_i^2=\left[i\partial_i+(-1)^i\frac{2e}{c}\mathbf{A}\right]^2;\quad \hat{P}_4=\hat{P}_1-\hat{P}_2+\hat{P}_3;\quad V(\mathbf{r})=\frac{1}{g(\mathbf{r})}\cdot$$

$$\chi(z)=\Psi\left(\frac{1}{2}+\frac{z}{2}\right)-\Psi\left(\frac{1}{2}\right);\quad E=\frac{T}{1{,}14\omega_D};$$

$$\xi_T^2=v_0l/6\pi T;\quad l=v_0\tau_{tr};\quad \varepsilon=-\ln E;\quad \hbar=k_B=1.$$

Here Ψ is the digamma function, T the temperature, $\mathbf{A}$ the magnetic-field vector potential, v_0 the Fermi velocity, l the electron mean free path, $\hbar$ the Planck constant, c the speed of light τ_{tr} the transport time between collisions, N the electron density, k_B the Boltzmann constant, $\mathbf{j}$ the current density, and ψ the order parameter. $V(\mathbf{r})$ plays in these formulas the role of the potential energy in the nonlinear Schrödinger equation (1) of a "particle" with a dispersion law $\chi(\xi_T^2P^2)$.

Equation (1), which is linear in $\psi(\mathbf{r})$, determines the critical superconductivity field in the inhomogeneous system. From the formal viewpoint this is the Schrödinger equation of a "particle" with a dispersion χ in a potential $V(\mathbf{r})$:

$$\hat{L}\psi^+\equiv[\chi(\xi_T^2\hat{P}^2)+V(\mathbf{r})-\varepsilon]\psi^+(\mathbf{r})=0. \qquad (3)$$

The eigenvalues and eigenfunctions of (3) depend on the relations between the characteristic lengths of the problem, namely the length of localization of the particle by the potential $V(\mathbf{r})$, the Larmor radius $a_H = (c/2eH)^{1/2}$, and the distance D between the inhomogeneities.

§2. SINGLE INHOMOGENEITY

To obtain the structure of the state and the magnetic moment of a superconductor with a periodic $V(\mathbf{r})$ it is necessary to determine the order parameter, the state structure, and the magnetic moment of a single inhomogeneity in an infinite superconductor. As already indicated in the Introduction, the critical characteristics for this case were considered earlier. It sufficed for this purpose to use Eq. (3) which is linear in ψ. In our case we must solve the nonlinear equation (1).

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Just as in Refs. 14–16, the nonlinear equation (1) has a solution that depends on the spatial coordinates in the same manner as the solution of the linear equation (3). We consider a situation wherein the metallic matrix contains a metallic superconducting layer with a large electron-electron interaction. In this case the potential $V(\mathbf{r})\equiv V(x)$, where x is the coordinate perpendicular to the inhomogeneity. The function $V(x)$ has a minimum, $V(0)\equiv V_m$, and is bounded at infinity, $V(\infty)\equiv V_\infty$.

Both the structure of the resultant state and the magnetic moment depend on the relations between the characteristic lengths of the problem as well as on the angle between the direction of the external magnetic field and the inhomogeneity plane. Let us consider various cases.

a) Magnetic field perpendicular to inhomogeneity plane. In the gauge $A_y = Hz$ (Fig. 1) the variables in Eq. (3) are separable, and the solution of the nonlinear equation (1) can be sought (for weak and strong magnetic fields) in the form

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})+\psi_1(\mathbf{r}),\quad \psi_0(\mathbf{r})\equiv\varphi_0(y,z)\psi_0(x),$$
$$\varphi_0(y,z)=\sum_n C_n e^{iqny}\varphi(z-z_n)=\sum_n \varphi_n,$$
$$\hat{q}=-i\partial_y,\quad \varphi(z-z_n)=\exp[-(z-z_n)^2/2a_H^2],$$
$$z_n=cqn/2eH,\quad r=(x,y,z)\equiv(x,\boldsymbol{\rho}),$$
$$V_m''\equiv V''(x\to 0). \qquad (4)$$

Here $\psi_0(\mathbf{r})$ is an eigenfunction of Eq. (3). The function $\psi_0(\mathbf{x})$ can be obtained with asymptotic accuracy in two cases: for a broad continuous inhomogenity $d\ll l$, where $L^2 = V_m/V_m''$ and d is the characteristic length of variation of the function $\psi_0(x)$ along the magnetic field, and for a narrow inhomogeneity if $d\gg L(V_\infty/V_m)^{1/2}$. These two cases will be referred to hereafter as the "well bottom" and "shallow well."

For the "well bottom" we can expand the potential in (3) in powers of x. As a result we get

$$\psi_0(x)\propto\exp(-x^2/d_{1H}^2),$$
$$d_{1H}^2=2^{3/2}\xi_m L\left[\frac{\chi'(u)}{V_m}\right]^{1/2},\quad \chi'(u)=\frac{d\chi}{du}, \qquad (5)$$
$$u(H)=2\xi_T^2 eH/c,\quad \xi_m\equiv\xi_{Tm},\quad T_m=1{,}14\omega_D\exp(-V_m).$$

The critical magnetic field is then of the form

$$\Delta H=-cV_m^{1/2}/2^{3/2}L\xi_T\chi'^{1/2}(u_1),\quad u_1=u(H_{c2}^m)\quad (a_H\ll d_{1H}),$$
$$H_0=\frac{4}{\pi^2}\frac{c}{e\xi_m^2}\frac{\Delta T}{T_0}\quad (a_H\gg d_{1H}), \qquad (6)$$
$$\Delta T=T_0-T,\quad \Delta H=H_0-H_{c2}^m,\quad T_0=T_m(1-{}^1/{}_2\pi\xi_m V_m^{1/2}/L).$$

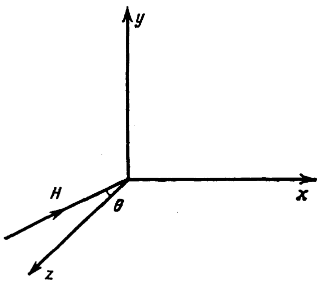

FIG. 1. Coordinate system. yz is the inhomogeneity plane.

Here $H_{c2}{}^m$ is the critical field of the inhomogeneity material.

In the "shallow well" region the corresponding quantities are

$$\psi_0(x)\propto\exp(-|x|/d_{2H}),$$
$$d_{2H}=\xi_T^2\chi'(u_\infty)/I,\quad u_\infty=2\xi_T^2 eH_{c2}^\infty/c, \qquad (7)$$
$$I=\int\tilde{V}(x)\,dx,\quad \tilde{V}(x)=V(x)-V_\infty,\quad T_\infty=1{,}14\omega_D\exp(-V_\infty);$$
$$\Delta H=\frac{c}{8e\xi_T^4}\frac{I^2}{[\chi'(u_\infty)]^2}\quad (a_H\ll d_{2H}),$$
$$H_0=\frac{4}{\pi^2}\frac{c}{e\xi_T^2}\frac{\Delta T}{T_\infty}\quad (a_H\gg d_{2H}), \qquad (8)$$
$$\Delta T=T_0-T,\quad T_0-T_\infty=4I^2/\pi^2\xi_\infty^2,\quad \Delta H=H_0-H_{c2}^\infty.$$

Here $H_{c2}{}^\infty$ is the critical field of the superconductor far from the inhomogenity.

Substituting the order parameter in the form (4) in Eqs. (1) and (2) we obtain in the usual manner

$$H(x)=H_e-\frac{e\tau_{tr}N}{mcT}\rho(\varkappa)$$
$$\times\overline{|\varphi_0(y,z)|^2}\,\psi_0^2(x)\,\frac{e}{c}\,\xi_T^2\rho(\varkappa)\Delta H\overline{|\psi_0(\mathbf{r})|^2}$$
$$+\frac{1}{8\pi^2T^2}\left\{\frac{4\pi N}{3m}[e\tau_{tr}v_0\rho(\varkappa)]^2-f_1(\varkappa)\right\}\overline{|\psi_0(\mathbf{r})|^4}=0. \qquad (9)$$

We have introduced here two functions

$$f_1(\varkappa)=\sum_{n=0}^{\infty}(n+{}^1/{}_2+\varkappa)^{-3},\quad \rho(\varkappa)=\sum^{\infty}(n+{}^1/{}_2+\varkappa)^{-2} \qquad (10)$$

of the parameter $\varkappa=\xi_T^2(eH/c)$, and the superior bar denotes averaging:

$$\bar{u}=\int u\,d^3\mathbf{r}\Big/\int d^3\mathbf{r}.$$

The Gibbs potential for a lattice of superconducting states localized along the magnetic field is given by

$$G=\frac{1}{8\pi}\int(\mathbf{H}-\mathbf{H}_e)^2\,d^3\mathbf{r}-\frac{N_3 f_1(\varkappa)}{16\pi^2T^2}\int|\psi|^4\,d^3\mathbf{r}. \qquad (11)$$

Here N_3 is the density of states on the Fermi surface.

Using (10), we obtain from (11) the magnetic moment and the free energy:

$$M=-\frac{1}{4\pi}\frac{\Delta H\gamma}{(2k^2-1)\beta},\quad G_s=-\frac{1}{8\pi}\frac{(\Delta H)^2\gamma}{(2k^2-1)\beta},$$
$$\gamma=\left(\int\psi_0^2(x)\,dx\right)^2\Big/\int\psi_0^4(x)\,dx\sim d_{iH}\quad (i=1,2). \qquad (12)$$

Here

$$k^2=\frac{3f_1(\varkappa)mc^2}{8\pi N[e\tau_{tr}v_0\rho(\varkappa)]^2},\quad \beta=\frac{\overline{|\psi_0(\rho)|^4}}{(\overline{|\psi_0(\rho)|^2})^2}. \qquad (13)$$

The energy of a lattice of superconducting states localized along the magnetic field, just as a lattice of Abrikosov vortices, depends only on the parameter β. For quadratic and triangular lattices $\beta = 1.18$ and 1.16, respectively. Corresponding to the minimum of the free energy is a triangular lattice.

The length of the characteristic change of the order parameter along the magnetic field is determined by Eqs. (5)

and (7). It depends both on the type of the inhomogeneity and on the temperature. At a temperature $T^* \approx T_0/2$ the quantity d_{iH} has a minimum. In the region of the "well bottom" $d_{1H}^{\min} \approx 0.9 d_{iH}(0)$, and in the "shallow-well" region $d_{2H}^{\min} \approx 0.8 d_{2H}(0)$.

In a magnetic field perpendicular to the inhomogeneity plane, the inhomogeneous superconducting state is thus bounded along the magnetic field by an Abrikosov-structure layer of thickness d_{iH} (Fig. 2a).

b) Magnetic field at an angle to the inhomogeneity plane. If the magnetic field is oriented at an angle θ to the inhomogeneity plane, the situation is different for strong and weak fields. In the region of weak fields ($d_{iH} \ll a_H$) in the angle interval $\theta > \theta_1 \sim d_{iH}/a_H$ one can neglect in (3) the magnetic-field component parallel to the inhomogeneity plane. The structure of the lattice of the superconducting states localized along the magnetic field remains unchanged, and everywhere in the equations for the weak field it is necessary to replace H by $H \sin\theta$. It is clear that the equations for the critical fields are changed in this case:

$$\Delta H(\theta) = \Delta H(\pi/2)/\sin\theta. \tag{14}$$

At an arbitrary angle $\theta > \theta_1$ the lattice generally speaking need not be regular, since a preferred direction in the inhomogeneity plane appears. For a rectangular lattice (in weak fields)

$$\varphi_0(y,z) = \sum_n \exp\left(i\frac{2\pi n}{a}y\right)\varphi_0(z+nb), \tag{15}$$

$$b = q/2eH\sin\theta, \quad q = 2\pi/a, \quad ab\sin\theta = \pi/eH;$$

here a and b are the periods of the rectangular structure. From (15) we obtain the quantity β:

$$\beta = \frac{(2\pi)^{1/2}}{a}\tilde{a}_H(\sin\theta)^{1/2}\sum_n \exp\left(-\frac{n^2b^2\sin\theta}{\tilde{a}_H^2}\right)$$
$$\tilde{a}_H = a_H/(\sin\theta)^{1/2}, \tag{16}$$

the minimum of which is reached at $b^2\sin\theta/a_H^2 = 2\pi$. In this case

$$a = b = a_H(2\pi/\sin\theta)^{1/2}, \tag{17}$$

i.e., the lattice of the localized superconducting states remains quadratic in the principal order in d_{iH}/a_H. For a triangular lattice, the solution, found in the class of isosceles triangles, also brings about in principal order a unit cell in the form of an equilateral triangle.

Thus, in weak magnetic fields (at temperatures close to critical) there exists a wide range of angles between the direction of the magnetic field and the inhomogeneity plane, in which the localized superconducting states are perpendicular to the inhomogeneity plane.

c) Magnetic field at an angle to the inhomogeneity plane. Strong fields. In strong magnetic fields ($d_{iH} \gg a_H$) the magnetic field "localizes" the superconducting state along a force line. The resultant state is in this case one-dimensional. It is convenient to obtain a formal solution of the problem in a reference frame with the x axis along the magnetic field (Fig. 2b). In this case

$$A_{\tilde{y}} = \int^{\tilde{z}} H\,d\tilde{z}, \quad x = \tilde{x}\cos\theta - \tilde{z}\sin\theta, \quad y = \tilde{y}. \tag{18}$$

The order parameter must then be found in the form

$$\psi_0(\mathbf{r}) = \sum_{n,m}\varphi_n(y,z)\psi_0(x-x_m), \quad \psi = \psi_0 + \psi_1. \tag{19}$$

The magnetic moment has as before the form (12) with the localization length d_{iH} along the magnetic field replaced by $d_{2H}\sin\theta$ in the "shallow well" region. The values of the critical magnetic fields are also modified. To obtain the critical fields it is necessary to divide the corresponding equation at $\theta = \pi/2$ by $\sin\theta$ in the region of the "well bottom" and by $\sin^2\theta$ in the region of the "shallow well."

The superconducting state in the region of strong magnetic fields (low temperatures) is thus a periodic lattice of superconducting states localized along the magnetic field. The unit cell of the periodic structure in the plane of the inhomogeneity is deformed along the projection of the magnetic field into a plane. For a quadratic lattice the deformation leads to a rectangular unit cell with sides $a \approx a_H$, $b \approx a_H/\sin\theta$. The triangular lattice is made up of isosceles triangles with vertex angle

$$\alpha = 2\,\mathrm{arctg}(\sin\theta/\sqrt{3}).$$

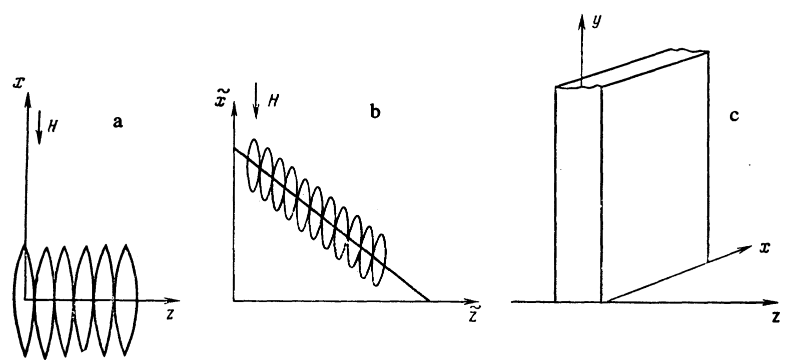

FIG. 2. a) Structure of inhomogeneous superconducting state in a magnetic field perpendicular to the inhomogeneity plane. b) Structure of inhomogeneous superconducting state at low temperatures in the case when the magnetic field makes an angle θ with the inhomogeneity plane. c) Inhomogeneous superconducting state in the case when the magnetic field is parallel to the inhomogeneity plane.

(We note that just as in bulky homogeneous superconductor a triangular structure is realized in this case, but in view of the small energy difference between a quadratic and a triangular lattice the external conditions can lead to a quadratic (rectangular) structure.)

The cited results for strong magnetic fields are valid only in the region of not too small angles $\theta \gg \tilde{\theta}_1 \sim a_H/d_{iH}$, inasmuch as in the angle interval $\theta \sim \tilde{\theta}_1$, owing to the condition $a_H \sim d_{iH}$, a state localized along the magnetic field ceases to be one-dimensional and the results are incorrect.

d) Magnetic field parallel to the inhomogeneity plane. Strong fields. We consider separately the case when the magnetic field is parallel to the inhomogeneity plane ($\theta = 0$). The potential $V(x)$ in Eqs. (1) and (3) does not localize in this case the superconducting state along the magnetic field. On the other hand in a plane perpendicular to the magnetic field, the state is localized both by the magnetic field and by the potential.

In the case of strong magnetic fields [$d_0 \gg a_H$, where d_0 is the characteristic length of localization by the potential $V(x)$], the potential $V(x)$ in Eq. (1) can be taken into account by perturbation theory. From Eq. (3), which is linear in the order parameter, we obtain the critical field $H_0''(T)$ and the "eigenfunction" $\psi_0(x)$ of the ground state. In the "well bottom" region

$$\Delta H_0''(T) = -c^2 V_m / 8e^2 \xi_T^2 \chi'(u_1) H_{c2}^m L^2,$$
$$\Delta H_0'' = H_0'' - H_{c2}^m. \tag{20}$$

In the "shallow well" region

$$\Delta H_0'' = \frac{c}{4e\xi_T^2} \left(\frac{eH_{c2}^\infty}{\pi c} \right)^{1/2} \frac{I}{\chi'(u_\infty)}, \tag{21}$$
$$\Delta H_0'' = H_0'' - H_{c2}^\infty.$$

In both cases the spatial part of the order parameter is given by

$$\psi_0(x) \propto \exp(-x^2/2a_H^2), \quad a_H^2 = c/2eH_0''(T). \tag{22}$$

The solution of the nonlinear equation (1) is sought in the form

$$\psi = C_0 \psi_0(x) + \psi_1, \tag{23}$$

where ψ_1 is due to the nonlinearity of Eq. (1). Substituting (23) in Eq. (2) for the current density we obtain an expression for the vector potential A:

$$A_y = H_e x - \frac{e\tau_{tr} N}{mcT} \rho(\varkappa) \int^x \psi_0^2(x) C_0^2 \, dx. \tag{24}$$

Using (23) and (24) we obtain expressions for the amplitude of the order parameter C_0 and for the magnetic field

$$\frac{e}{c} \xi_T^2 \rho(\varkappa) \Delta H \overline{\psi_0^2(x)} - \frac{(2k^2-1) 8\pi N}{48\pi^2 T^2 m} (e\tau_{tr} v_0 \rho(\varkappa))^2 \overline{\psi_0^4(x)} C_0^2 = 0, \quad \bar{a} = \int a \, dx; \tag{25}$$

$$\frac{M}{S} = -\frac{\Delta H}{4\pi(2k^2-1)\beta^*}, \quad \Delta H = H_0'' - H_e;$$
$$\beta^* = \overline{\psi_0^4(x)} / [\overline{\psi_0^2(x)}]^2 \sim a_H^{-1}, \tag{26}$$

(S is the area of the inhomogeneity plane). The resultant superconducting state takes the form of a superconducting plane placed at the origin (at the center of the inhomogeneity) and having a thickness $\sim a_H$ (Fig. 2c).

With further decrease of the external magnetic field, there are produced in the system vortex chains arranged along the inhomogeneity plane, and the superconducting plate will be amplitude modulated. After the field H_{c2}^∞ is reached, an Abrikosov structure arises in the system.

e) Magnetic field parallel to the inhomogeneity plane. Weak fields. To determine the magnetic moment in weak magnetic fields parallel to the inhomogeneity plane ($a_H \gg d_0$), we must determine the order parameter $\psi_0(x,H)$ as $H \to 0$. In this case the nonlinear equation (1) is transformed into a nonlinear Schrödinger equation with a potential $V(x)$. The solution of this equation must be sought in the form of a series in the eigenfunctions of that part of Eq. (1) which is linear in ψ:

$$\psi = C_0 \psi_0(x) + \sum_{n=1}^{\infty} C_n \psi_n(x), \tag{27}$$

where $\psi_0(x)$ is the eigenfunction of the ground state of Eq. (3). In the "well-bottom" region $\psi_0(x)$ takes the form (5) as $H \to 0$. In the region of the "shallow-well" parameters $\psi_0(x)$ is determined by Eqs. (7) ($H \to 0$).

As shown in Ref. 15, the solution of the nonlinear equation (1) as $H \to 0$ in the temperature region $\delta T = T_{0H} - T \ll T_0 - T_1$ (here T_1 is the temperature corresponding to the "energy of the first excited level") is

$$\psi(x) \approx C_0 \psi_0(x) + O((\delta T)^{3/2}), \tag{28}$$
$$C^2 = \frac{8\pi^2 T^2}{7\zeta(3)} \frac{\overline{\psi_0^2(x)}}{\overline{\psi_0^4(x)}} \frac{T_{0H} - T}{T_{0H}}.$$

The temperature T_{0H} is determined from the solution of Eq. (3) (the ground energy level of this equation). In the "well-bottom" region

$$\frac{T_0 - T_{0H}}{T_0} = \frac{\pi}{2} \frac{e^2 \xi_m^2}{c^2} \frac{L}{L_0} H_\parallel^2, \quad L_0 = \xi_m (2V_m)^{1/4}. \tag{29}$$

In the "shallow-well" region

$$\frac{T_0 - T_{0H}}{T_0} = \frac{\pi^6}{32} \frac{e^2 \xi_\infty^6}{c^2} \frac{H_\parallel^2}{I^2}. \tag{30}$$

To determine the spatial dependence of the magnetic field $H(x)$, we must substitute (28) in (2) and solve the resultant equation

$$\Delta A = \frac{\delta\tau}{2^{1/2}\lambda^2} \exp\left(-\frac{2x^2}{d_0^2}\right) A \tag{31}$$

with the boundary condition $H(\infty) = H_e$. Here λ is the depth of penetration of the magnetic field into the superconductor, and $\delta\tau = (T_{0H} - T)/T_{0H}$.

The nondimensional form of this equation is

$$\frac{\partial^2 A}{\partial \bar{x}^2} = \frac{d_0^2 \delta\tau}{2^{5/2}\lambda^2} \exp(-\bar{x}^2) A, \quad \frac{2^{1/2} x}{d_0} = \bar{x}. \tag{32}$$

For type-II superconductors $d_0^2 \delta\tau / \lambda^2 \ll 1$ and the magnetic field can be found by perturbation theory. In this case

$$H(x)=H_e\left[1-\frac{d_0^2\delta\tau}{2^{5/2}\lambda^2}\exp\left(-\frac{2x^2}{d_0^2}\right)\right], \qquad (33)$$

$$M=\frac{1}{4\pi}\int(H(x)-H_e)\,dx=-H_e\frac{d_0^3}{\lambda^2}\frac{\delta\tau}{32\pi^{1/2}}. \qquad (34)$$

The magnetic moment for the case when the order parameter changes over distances that are large compared with the inhomogeneity dimension ("shallow well") was determined for pure superconductors in Ref. 8. In our case

$$M=-\frac{H_e}{8\pi}\frac{\delta\tau}{\lambda^2}d_1^3. \qquad (35)$$

Thus, in weak magnetic field the resultant superconducting state takes the form of a superconducting plate of thickness $d_0=d_{0i}$. In the case of a type-II superconductor the magnetic field penetrates into the superconducting layer, and this leads to a decrease of the diamagnetic moment by a factor $\delta\tau d_{oi}^2/\lambda^2$.

§3. PERIODIC SPATIAL INHOMOGENEITY

We consider now the superconducting properties of a metal in which $V(x)$ is a periodic function of the coordinate x:

$$V(x)=V(x+D), \qquad (36)$$

where D is the period of the superstructure. Depending on the relations between the parameters a_H, d_{iH}, and D, and also on the angle θ between the magnetic field direction and the inhomogeneity plane, various situations are possible.

We consider first the case when $\theta>\{\theta_1,\tilde{\theta}_1\}$ and $d_{iH}\ll D$. The x-dependent part of the order parameter is then of the form

$$\psi_0(x)=\sum_n\psi_{0n}(x-nD), \qquad (37)$$

here ψ_{0n} is the x-dependent part of the order parameter in the n-th layer. The free energy can be represented in this case in the form

$$G_s=\sum_n G_n+\sum_n G_{n,n+1},$$

$$G_n\approx-\frac{1}{8\pi}\frac{(\Delta H)^2 d_{iH}}{(2k^2-1)\beta}, \quad G_{n,n+1}\approx\frac{G_nJ_n}{d_{iH}}, \qquad (38)$$

$$J_n=\left[\int\psi_{0n}(x-x_n)\psi_{0n+1}(x-x_{n+1})\,dx\right]^2\Big/\int\psi_{0n}^4(x-x_n)\,dx.$$

The minimum of the energy $G(D)$ is reached at the maximum value of the overlap integral between the neighboring layers. Thus, at arbitrarily weak interaction between the layers, the superconducting states localized in the layer are spatially continuations of one another. The magnetic moment takes in this case the form

$$\tilde{M}=-\frac{1}{4\pi}\frac{(H_0-H_e)}{(2k^2-1)\beta_\Delta}d_{iH}\tilde{n}, \qquad (39)$$

where n is the number of layers.

In the opposite limiting case ($d_{iH}\gg D$) the situation is different in the "well-bottom" and "shallow-well" regions. Near the bottom of the well the potential energy $V(x)$ is given by

$$V(x)\approx V_m+D^2f(x/D)V_m'', \qquad (40)$$

where $f(x/D)$ is a periodic function with the period of the structure. Recognizing that D is much less than the other characteristic lengths, we average Eqs. (1)–(3) over small-scale changes of $f(x/D)$ with amplitude $\sim D^2/L^2$. As a result we obtain in the principal order in D/L an equation of the type (1)–(3) with $V(x)$ replaced by V_m. The critical magnetic field determined from (3) has the form usual for a homogneous superconductor ($g=g_m$)

$$\ln(T/T_m)+\chi(u_m)=0. \qquad (41)$$

The structure of the inhomogeneous state and the magnetic moment in such superconductors have the usual Abrikosov form with critical parameters T_m and H_{c2}^m.

For sufficiently thin layers, when the order parameter for an individual inhomogeneity changes over distances that are large compared with the layer dimension ("shallow-well"), the potential $V(x)$ is given by

$$V(x)=\langle V\rangle+\tilde{f}\left(\frac{x}{D}\right), \quad \langle V\rangle=\int_{-D/2}^{D/2}\frac{V(x)\,dx}{D}. \qquad (42)$$

Here $\tilde{f}(x/D)$ is an alternating-sign rapidly oscillating function. Substituting the potential (42) in Eqs. (1)–(3) and averaging these equations over the fast oscillations, we obtain for the critical magnetic field $\langle H\rangle_0$ and for the magnetic moment M the following expressions:

$$\ln(T/\langle T\rangle_0)+\chi(\bar{u})=0, \qquad (43)$$

$$\langle T\rangle_0=1{,}14\omega_De^{-V}, \quad \bar{u}=\frac{2e}{c}\xi_T^2\langle H\rangle_0, \qquad (44)$$

$$M=-\frac{1}{4\pi}\frac{\langle H\rangle_0-H_e}{(2k^2-1)\beta_\Delta}. \qquad (45)$$

The inhomogeneous superconducting field has in this case the Abrikosov form at any orientation of the magnetic field relative to the inhomogeneity plane.

At $\theta=0$ the structure of the inhomogeneous superconducting state depends in a more complicated manner on the relations between the parameters.

In strong magnetic fields at $a_H\ll d_0\ll D$ the inhomogeneous superconducting state is a system of parallel superconducting plates each with center at the middle of the inhomogeneity plane. The magnetic moment is in this case the sum of the magnetic moments of the individual plates. With decreasing distance between the plates, the amplitude of the modulations in each of the plates increases, and at $D\sim\xi$ an Abrikosov state is produced in the system.

At $D\ll\{a_H,d_0\}$ the potential energy $V(x)$ in Eqs. (1)–(3) can be represented in the form (40)–(42). After averaging, these equations are transformed into equations with a constant electron-electron interaction. The superconducting state arises in this case at the first critical field $\langle H\rangle_0$ defined by Eqs. (41)–(43) (for broad and narrow layers) and has the same properties as in a homogeneous superconductor near the upper critical field. At $\tau\ll 1$ we have $a_H\sim\xi(\tau)$, $\tau=1-T/\langle T\rangle_0$; at $\tau\sim 1$ we have $a_H\sim\xi$.

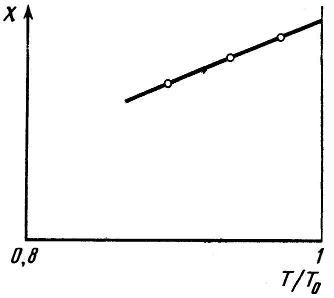

FIG. 3. Dependence of the diamagnetic susceptibility on the temperature of a layered superconductor.

§4. CONCLUSION

Thus, layered metals have different superconducting properties than layered superconductors of the metal-insulator type. The usual layered superconductors are materials that are anisotropic in the normal state, and their superconducting properties are connected in most cases with this anisotropy.

Layered metals, which are isotropic in the normal state, can have considerable anisotropy in the superconducting state. The anisotropy in such substances is connected not only with the distance between layers, but also with the orientation of the magnetic field relative to the layers. The anisotropy of the superconducting properties of superconductors, which is the result of the distance between the layers, depends then on the orientation of the magnetic field. Thus, in strong magnetic fields parallel to the inhomogeneity plane the interlayer distance at which the anisotropy sets in is $D \sim \xi$, where ξ is the coherence length, whereas for fields perpendicular to the inhomogeneity plane the anisotropy sets in at a distance $D \sim \xi^2/a$, where a is the thickness of the layer (for thin layers).

The anisotropic properties of layered metals should manifest themselves in experiments on the measurement of a Josephson current that flows along a magnetic field, and in measurements of the critical magnetic field and of the magnetic moment in such superconductors.

A number of experiments have recently been reported on superconductivity in layered metals. In Ref. 17 were measured the critical magnetic field, the critical temperatures, and the diamagnetic susceptibility of an Nb–Ti system obtained by layer-by-layer evaporation of one metal on the other.

Figure 3 shows the temperature dependence of the diamagnetic susceptibility for a layered Nb–Ti metal with distance $D = 34.5$ Å between the layers. In this case $D \ll a_H$ and the condition $D \ll d_{iH}$, at which Eq. (44) is valid, is satisfied. The magnetic susceptibility takes then the universal form

$$\chi^{-1} = 4\pi [2k^2(T) - 1] \beta_\Delta, \quad k(\langle T \rangle_0) = 0.96\lambda_L/\xi. \tag{46}$$

Here λ_L is the Londen depth of penetration of the magnetic field into the superconductor, and $k(T)$ has the well-known form.[18] The relation defined by Eq. (46) is plotted in Fig. 3 (solid line). The value $k(\langle T \rangle_0 \approx 0.72$ coincides in this case with that obtained in Ref. 17.

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