Anomalous magnetoresistance of amorphous metals

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(Submitted 1 April 1983)
Zh. Eksp. Teor. Fiz. **86**, 180–186 (January 1984)

The magnetoresistance of amorphous films of Bi, Ga, Yb, and V has been studied at low temperatures in magnetic fields up to 4 T. For all these metals, the magnetoresistance is positive, falls off sharply with increasing temperature, and depends on the magnetic field in an anomalous way. The results for amorphous superconductors agree satisfactorily with the theory of an anomalous magnetoresistance which incorporates a scattering of electrons by superconducting fluctuations.

1. INTRODUCTION

In amorphous metals and alloys in which the electron mean free path is comparable to interatomic distances, the ordinary magnetoresistance should be negligibly small. In fact, experiments have shown that the magnetoresistance of certain nonmagnetic amorphous metals and alloys lies within the measurement error[1,2]: $\Delta\rho/\rho < 10^{-5}$ in a field $\sim$4T. (Here $\Delta\rho = \rho_B - \rho$, where ρ_B and ρ are the electrical resistivities in a magnetic field with induction B and in the absence of a magnetic field, respectively.) It has also been found, however, that amorphous films of ytterbium,[3] vanadium,[4] and certain amorphous metal alloys[5] exhibit a significant magnetoresistance ($\Delta\rho/\rho \sim 10^{-4}$–$10^{-3}$). The B dependence of $\Delta\rho/\rho$ is anomalous: There is a pronounced deviation from the quadratic law $\Delta\rho/\rho \sim B^2$ which is characteristic of crystalline materials in a weak field.

A theory has been derived for the anomalous magnetoresistance of disordered metals and semiconductors.[6] It has been shown that the quantum corrections to the conductivity with or without allowance for the interaction between electrons lead to a magnetoresistance with an anomalous field dependence. The magnetic field is influential even in classically weak magnetic fields, under the condition $\omega_c\tau \ll 1$ (ω_c is the cyclotron frequency, and τ is the electron momentum relaxation time). The effect of the magnetic field on the conductivity correction for the localization of noninteracting electrons gives rise to a negative magnetoresistance. In the quasi-2D case we have

$$G(B)-G(0)=\frac{e^2}{2\pi^2\hbar}f_2\left(\frac{4DeB}{\hbar}\tau_\varphi\right), \tag{1}$$

where $G(B)$ and $G(0)$ are the surface conductivities of the film in a magnetic field B and without a field, respectively; D is the electron diffusion coefficient; τ_φ is the relaxation time of the phase of the electron wave function for the relaxation due to inelastic collisions (for metals at low temperatures, τ_φ is equal to the energy relaxation time of an electron); e is the charge of the electron; and $\hbar$ is Planck's constant.

Asymptotic expressions for the behavior of the function $f_2(x)$ in Eq. (1) in weak and strong fields are

$$\begin{aligned} f_2(x)&=x^2/24, \quad x\ll 1,\\ f_2(x)&=\ln x, \quad x\gg 1. \end{aligned} \tag{2}$$

In the 3D case we have

$$\sigma(B)-\sigma(0)=\frac{e^2}{2\pi^2\hbar}f_3\left(\frac{4DeB}{\hbar}\tau_\varphi\right)\left(\frac{eB}{\hbar}\right)^{1/2}, \tag{3}$$

where $\sigma(B)$ and $\sigma(0)$ are the conductivities in a field B and without a field. The asymptotic expression for $f_3(x)$ are

$$\begin{aligned} f_3(x)&=x^{3/2}/48, \quad x\ll 1,\\ f_3(x)&=0.605, \quad x\gg 1. \end{aligned} \tag{4}$$

It is apparently the localization of noninteracting electrons in the system which gives rise to the negative magnetoresistance of certain crystalline[7] and amorphous[8] semiconductors.

The interaction between electrons gives rise to the same field dependence of the magnetoresistance as for the noninteracting electrons.[6] One type of electron-electron interaction is a scattering of electrons by superconducting fluctuations. We know that in amorphous superconductors, even at temperatures twice the critical temperature T_{cr}, there is a significant additional conductivity due to Cooper pairs of fluctuational origin.[9] The Maki-Thompson corrections known from the theory of fluctuational superconductivity (at $T > T_{cr}$) lead to the following expressions for the magnetoconductivity[6]:

$$\Delta G^{MT}(B)=-\beta(T)\Delta G(B), \tag{5}$$

$$\Delta\sigma^{MT}(B)=-\beta(T)\Delta\sigma(B). \tag{6}$$

Here $\Delta G(B)$ and $\Delta\sigma(B)$ are given by (1) and (3). The parameter $\beta(T)$ does not depend on the dimensionality of the sample. The function $\beta(T)$ is related to the electron interaction constant $g(T)$ and has been calculated and tabulated by Larkin.[10] Regardless of the sign of the interaction constant, the anomalous magnetoresistance due to the scattering of electrons by superconducting fluctuations is always positive.

Summing the effect of the noninteracting electrons and the effect due to the scattering of electrons by superconducting fluctuations, we find the following expressions for the magnetoconductivity in the quasi-1D and 3D cases[6]:

$$G_s(B)-G_s(0)=[C_2-C_2'\beta(T)]\Delta G(B), \tag{7}$$

$$\sigma_s(B)-\sigma_s(0)=[C_3-C_3'\beta(T)]\Delta\sigma(B). \tag{8}$$

If the spin-orbit interaction of the electrons with impurities is of minor importance, then we could have $C_2 = C_2' = C_3 = C_3' = 1$; in the opposite limit we would have $C_2 = C_3 = -1/2$ and $C_2' = C_3' = 1/4$.

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In the present experiments we studied the magnetoresistance of amorphous films of four metals—bismuth, gallium, vanadium, and ytterbium—in a magnetic field (up to 4T) normal to the plane of the film. In the cases of the ytterbium and vanadium we also studied the magnetoresistance in a field up to 1.6 T parallel to the plane of the film (and to the current). We will compare the experimental results with the predictions of the theoretical expressions discussed above.

2. EXPERIMENTAL PROCEDURE

The procedure for producing and studying the amorphous metal films has been described in detail elsewhere.[11] The Bi, Ga, V, and Yb films were produced by condensing vapor on a glass surface cooled with liquid helium. Before the evaporation of the metal was begun, the pressure in the working cell was $\sim 10^{12}$ Torr. The purity levels of the initial metals were 99.9998% for Ga, 99.999% for Bi, 99.99% for V, and 99.7% for Yb. We produced and studied amorphous films 10–90 nm thick in the case of Ga, 10–80 nm thick in the case of Yb, 20–30 nm thick in the case of Bi, and 20–40 nm thick in the case of V. The crystallization temperatures $T_{a\to c}$ of these films ranged from 15 to 30 K, depending on their thickness,[11] so we studied the magnetoresistance at $T_{cr} < T < T_{a\to c}$. The ratio of the resistivity at room temperature, $\rho(300\text{ K})$, to the residual resistivity ρ_{res} (at $T = 4.2$–10 K), lay in the following ranges for the thickest of these films in their crystalline state: 2.5–3 for Yb, 3–4 for V, and 7–10 for Ga. The values of $\rho(300\text{ K})$ of the annealed films of these metals corresponded to the values of the bulk crystalline state. The Bi films produced in the amorphous state and then crystallized during a warming are known to have a large resistivity and negative temperature coefficient of the resistance (see, for example, the review by Kuz'menko *et al.*[11]). Typical resistivities of these amorphous films are 140 ± 20 $\mu\Omega\cdot$cm for Bi, 220 ± 30 for V, 120 ± 20 for Yb, and 26 ± 3 for Ga. A magnetic field normal to the plane of the film was produced with a superconducting solenoid. A field parallel to the plane of the film was produced with an electromagnet with a superconducting winding. The electromagnet was positioned inside the same cryostat, so that the same film samples could be studied in transverse and longitudinal fields.

3. EXPERIMENTAL RESULTS AND DISCUSSION

Let us first look at the general behavior of the magnetoresistance of these amorphous metals. The magnetoresistance is found to be positive, and its field dependence is essentailly the same for fields perpendicular and parallel to the plane of the film if the film thickness $d > 40$ nm. A curve of $\Delta\rho/\rho$ vs B is typically convex upward. We also find that for all the amorphous metals studied the magnetoresistance falls off sharply with increasing temperature. Above 15–20 K, $\Delta\rho/\rho$ becomes smaller than the resolution of our measurement method ($< 10^{-5}$). Figure 1 shows some representative results on the temperature dependence of the magnetoresistance of amorphous Yb and Ga films in a field $B = 1.6$ T. The last general result which we find for these amorphous films was mentioned earlier: The magnetoresistance depends on the thickness. Specifically, the magnetoresistance increases sharply with decreasing thickness of the amorphous films.

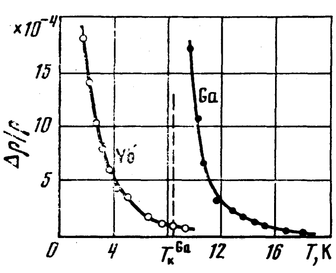

FIG. 1. Change in the magnetoresistance vs the temperature in a magnetic field of 1.6 T. The field is directed normal to the plane of the film. The thickness of the Yb and Ga films is ~ 10 nm.

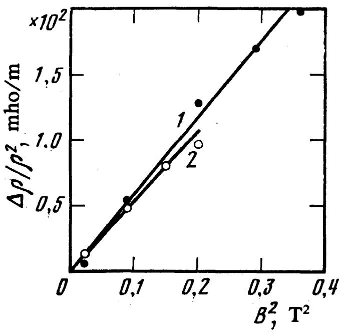

FIG. 2. 1—Dependence of $\Delta\rho/\rho^2$ on B^2 for an amorphous Bi film ~ 25 nm thick at 7.8 K; 2—for an amorphous V film 40 nm thick at 4.2 K (weak-field region).

A. Amorphous superconducting metals[1)] (Bi, Ga, V).

Since the amorphous films of Bi, Ga, and V are superconductors, superconducting fluctuations should contribute substantially to their conductivity at temperatures above T_{cr}. The magnetoconductivity of such films can be described by Eq. (7) or (8), depending on whether we are dealing with a quasi-$1D$ or $2D$ sample. We find that in weak fields (up to 0.2–0.5 T) the magnetoresistance can be described satisfactorily by a quadratic law. Figure 2 shows the magnetoresistance $\sigma_s(0) - \sigma_s(B) \approx \Delta\rho/\rho^2$ vs B^2 for amorphous films of Bi

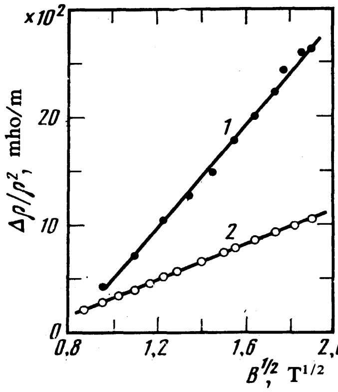

FIG. 3. Behavior of $\Delta\rho/\rho^2$ in the field interval ~ 0.8–3.6 T for films of Bi (1) and V (2). The samples and the experimental conditions are described in the caption of Fig. 2.

TABLE I.

Metal	d, nm	T, K	β	$-g$	T^*_{cr}, K	T_{cr}, K	L_φ, nm
Bi	25	7,8	9,45	4,85	6,34 (6,24)	6,0	24
V	40	4,2	3,89	2,4	2,77 (2,48)	2,46	34
Ga	42	9,9	15,66	7,4	8,65 (8,63)	8,6	50
Ga	10	12,8	5,17	2,95	9,1 (8,56)	8,4	33

and V. As the field is increased to 0.7–1 T (Fig. 3), $\Delta\rho/\rho^2$ becomes proportional to $B^{1/2}$. These films thus behave as $3D$ films and can evidently be described by Eq. (8). The criterion of Ref. 6 for judging the dimensionality of a sample[2)] involves a characteristic length $L_\varphi = (D\tau_\varphi)^{1/2}$. In accordance with Eqs. (8), (3), and (4), the values of the parameters β and L_φ were determined from the slopes of the corresponding curves of $\Delta\rho/\rho^2$ vs $B^{1/2}$ (Fig. 3) and $\Delta\rho/\rho^2$ vs B^2 (Fig. 2). The values of β which were calculated for the amorphous films of Bi, Ga, and V under the assumption of a slight spin-orbit interaction are listed Table I. Also shown here are the values of L_φ and the constants of the effective interaction between electrons, $g(T) = -1/\ln(T/T^*_{cr})$, which were determined from the tables of Ref. 10 and the values which we found here for the parameter β. Let us compare the superconducting transition temperature T^*_{cr} calculated from the interaction constants with the direct measurements of T_{cr} for the same amorphous films. Table I shows some representative results for Bi, Ga, and V films. We see that the values of T_{cr} and T^*_{cr} agree satisfactorily (within 12%), showing that these theoretical agruments (for the $3D$ case) do correspond to the experimental results for the "thick" amorphous films of Bi, V, and Ga. For all the samples studied, the difference between T^*_{cr} and T_{cr} falls below 4% if we ignore noninteracting electrons, i.e., if we use Eq. (6) for the calculations. The values of T^*_{cr} calculated from this equation are shown in parentheses in Table I. The good agreement with experiment here might results from a small localization effect in these amorphous films.

Calculations under the assumption of a strong spin-orbit interaction values of T^*_{cr} very different from the actual values.

It should be noted that the magnetoresistance of these films depends on their thickness. As the thickness of the Ga films increases from 10 to 90 nm, for example, the magnetoresistance in a field of 3.6 T at a temperature of 11.5 K falls from $6\cdot10^{-4}$ to $2\cdot10^{-4}$. The thickness dependence of the magnetoresistance can be studied by working from expression (1), since we would have $\tau_\varphi \sim d$ in the quasi-$2D$ case.[6] In the $3D$ case, τ_φ does not depend on d, so the magnetoresistance should not depend on the thickness of the sample [see expression (3)]. Pursuing this discussion, we might suggest that the observed thickness dependence of the magnetoresistance stems from a manifestation of quasi-$2D$ effects in amorphous Ga films thinner than $d = (D\tau_\varphi)$.$^{1/2}$ The field dependence of the magnetoresistance remains "three-dimensional" [describable by Eq. (8)] even for the thinnest of the Ga films ($d \approx 10$ nm). As can be seen from Table I, the calculated values of T^*_{cr} agree satisfactorily with the measured values. Consequently, the behavior of the amorphous Ga films remains primarily a $3D$ behavior up to $d \approx 10$ nm, despite some evidence of quasi-two-dimensionality. The electron velocity at the Fermi surface ($v_F = 0.2\cdot10^7$ m/s) and the electron mean free path ($l = 1.7$ nm) are known for amorphous gallium.[12] We can thus estimate the diffusion coefficient and the energy relaxation time for the amorphous films of the metal studied in the present experiments. For the film $\sim$42 nm thick at $T = 9.9$ K, for example, we find $D = (1/3)\, v_F\, l = 1.1\cdot10^{-3}$ m²/s and $\tau_\varphi \approx 2\cdot10^{-2}$ s.

B. Amorphous ytterbium.

The behavior of the magnetoresistance of amorphous Yb is similar to that of the amorphous superconducing metals. That this is true can be seen from Fig. 4, which shows $\Delta\rho/\rho^2$ vs B^2 and vs $B^{1/2}$ in the asympotoic regions of weak and strong fields, respectively, for one of the amorphous Yb films. Using Eqs. (8), (3), and (4) to analyze the results of the type in Fig. 4, we can find the values of the parameters $\beta(T)$ and $L_\varphi = (D\tau_\varphi)^{1/2}$ for amorphous Yb. Table II shows some typical values calculated for β for Yb films thicker than 40 nm at $T = 4.2$ K under the assumption of weak and strong spin-orbit interactions. The typical value of L_φ for these films is 36–38 nm. Over the temperature interval 10–1.7 K, the product $D\tau_\varphi$ is described approximately by a T^{-2} law. If we assume that the electron diffusion coefficient D is essentially independent of T in this temperature interval, we may conclude $\tau_\varphi \sim T^{-2}$. This temperature dependence of τ_φ may result from a predominant electron-electron scattering[13] in amorphous Yb at low temperatures.

As Larkin[10] has pointed out, superconducting fluctuations exist even if the electrons repel each other. In this case, however, the interaction constant $g(T)$ and thus $\beta(T)$ should logarithmically approach zero decreasing tempera-

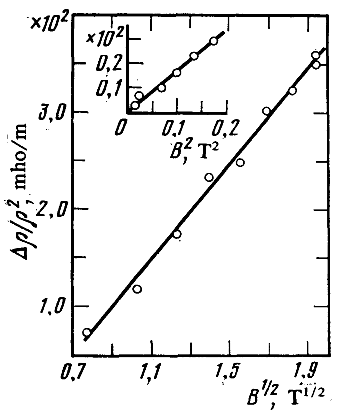

FIG. 4. Behavior of $\Delta\rho/\rho^2$ in the field interval $\sim$0.6–3.7 T for a Yb film 45 nm thick at 4.2 K. The inset shows the B^2 dependence of $\Delta\rho/\rho^2$ for the same film in the low-field region.

TABLE II.

Spin-orbit interaction	β	$-g$	T^*_{cr}, K
Weak	1.8–1.85	1.4–1.45	2–2.1
Strong	1.2–1.4	1.1–1.2	1.7–1.8

ture. For the Yb films studied in the present experiments, β increases by a factor of about four as the temperature is lowered from 9 to 2 K. This $\beta(T)$ behavior is evidence of an attraction between electrons and of a transition of amorphous ytterbium to a superconducting state. Estimating the possible superconductivity temperature of amorphous Yb films from the tabulated[10] dependence of β on $-g$, we find 1.7–2.1 K (Table II). Ignoring localization and the spin-orbit interaction, i.e., working from Eq. (6), we find $T^*_{cr} = 1.1$–1.3 K. These results are at odds with experiment: The amorphous Yb films studied in the present experiments exhibit no evidence at all of superconductivity down to $T = 1.5$ K. Futhermore, according to Ref. 14, amorphous Yb films are not superconducting even at $T = 0.35$ K. We do not see a logical explanation for this discrepancy between theory and experiment in the case of amorphous Yb. All we can suggest is that in addition to the contributions to the anomalous magnetoresistance discussed above there is another important contribution in the case of Yb, from magnetic-field dependent corrections to the state density because of other types of electron-electron interactions.[6]

A behavior of the magnetoresistance similar to that discussed above for thick Yb films (Fig. 4) is also observed in thinner films, with $d < L_\varphi$ ($d \approx 5$–10nm). As in the case of the amorphous Ga films, the magnetoresistance depends on the thickness of the amorphous Yb films (for magnetic fields both normal to and parallel to the plane of the film; Fig. 5). As mentioned above, this behavior is not characteristic of $3D$ samples. Again in the case of Yb at $d < L_\varphi$ we are apparently beginning to see manifestations of a $2D$ behavior of the films. Evidence for this suggestion also comes from the increasing difference between the magnetoresistances of films thinner than 40 nm in a magnetic field either normal or parallel to the plane of the film (Fig. 5).

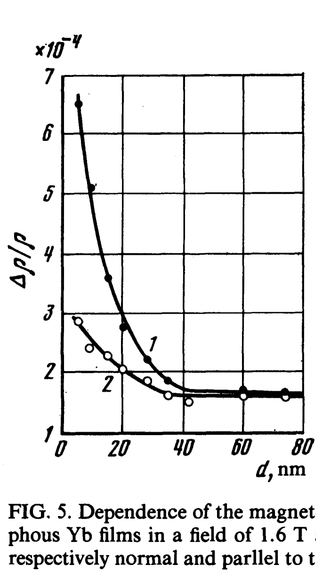

FIG. 5. Dependence of the magnetoresistance on the thickness of amorphous Yb films in a field of 1.6 T at 4.2 K. 1, 2—The magnetic field is respectively normal and parllel to the plane of the film.

Unfortunately, we did not analyze the curves of $\Delta\rho/\rho$ vs B in the case of a magnetic field parallel to the plane of the film because the maximum field ($B \approx 1.6$ T) available from the magnet at our disposal was not strong enough. Regarding this question we can only point out that in modest fields ($\lesssim 0.5$ T) we find $\Delta\rho/\rho \sim B^2$.

CONCLUSION

In summary, the positive magnetoresistance with an anomalous magnetic-field dependence which is observed in amorphous Bi, Ga, and V films can be explained satisfactorily in terms of a predominant effect of the magnetic field on the correction to the conductivity for electron scattering by superconducting fluctuations. The ordinary magnetoresistance of these metals, which is observed at $T > 15$–20 K, is negligibly small ($< 10^{-5}$) because of the short electron mean free path. Amorphous Yb films also exhibit a positive magnetoresistance which decreases sharply with increasing temperature and which has an anomalous field dependence. The B dependence of $\Delta\rho/\rho$ is the same as in the case of amorphous superconducting metals. The $\beta(T)$ dependence found in this study for amorphous Yb, which yields the strength of the effective interaction between electrons, $g(T)$, is evidence that a superconducting transition may occur in amorphous Yb at low temperatures.

1) We will discuss here the behavior of the resistance only in a magnetic field normal to the plane of the film.

2) A sample may be regarded as $3D$ if its thickness satisfies $d > L_\varphi$; if the thickness is instead small in comparison with L_φ, the sample behaves as if it were a $2D$ film.

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Translated by Dave Parsons