Suppression of interference effects in multiple scattering of light

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The interference effect of amplification of backscattering in propagation of waves in randomly inhomogeneous media is considered. The connection between this effect and the symmetry relative to time reversal is discussed. It is shown that actions that violate this symmetry (random motion of the inhomogeneities and gyrotropy of the medium due to an external magnetic field) suppress the effect.

1. INTRODUCTION

The character of the propagation of waves of any kind in media with random refractive-index inhomogeneities is determined by the ratio of the wavelength λ to the radiation mean free path l. As a rule, in light scattering $\lambda/l \ll 1$. In the zeroth approximation in this parameter, the transport of the wave-field energy is determined by an intensity-transport equation in which interference effects are neglected.[1–3] In this approximation one can calculate the correlation function of the field at any point of space; in particular, the cross section can be obtained for the scattering by an arbitrary volume that contains inhomogeneities.

The form of the principal interference correction to the correlation function of a field in the case of scattering by stationary inhomogeneities is also known.[4–8] The corresponding correction to the cross section is highly anisotropic and differs noticeably from zero only for directions close to backscattering. Although the interference correction to the total cross section is indeed small in the parameter λ/l, the correction for the backscattering direction is of the order of the differential cross section itself.

The cause of so strong an interference effect is the following. The derivation of the transport equation includes incoherent addition of the intensities of waves subject to various multiple scattering processes. The phase relations between them are not taken into account because the inhomogeneities are randomly distributed. If, however, symmetry with respect to time reversal obtains in the medium, a wave successively scattered by certain inhomogeneities, and a wave scattered by the same inhomogeneities but passing through them in a reverse sequence, have the same phase.[4–8] An important role in backscattering is played by the interference between these waves. The presence of amplification of the backscattering is not connected with the statistics of the inhomogeneities, but is determined only by the symmetry properties of the medium with respect to time reversal. It is clear therefore that all the actions that violate this symmetry will suppress this effect.

In scattering by nonstationary inhomogeneities, the phases of these waves are no longer equal, inasmuch as the forward and backward waves pass through the same inhomogeneities at different instants of time. The result is total or partial suppression of the effect. In Refs. 9 and 10 were considered coherent effects in backscattering of sound by bodies located near a choppy sea surface under conditions when single scattering by the surface and the scatterer is substantial.

The present paper deals with the effect of random motion of refractive-index inhomogeneities of a medium, and of the gyrotropy produced in the medium by an external magnetic field via the Faraday effect, on interference effects in multiple scattering. The scattering cross section as a function of these factors is calculated.

2. EFFECT OF MOTION OF THE INHOMOGENEITIES ON THE BACKSCATTERING AMPLIFICATION

We assume that the refractive-index inhomogeneities are produced by pointlike (with characteristic dimensions $a \ll \lambda$) impurities that move perfectly independently, since backscattering is stronger in the case of small-scale inhomogeneities. (In the case of large-scale refractive-index inhomogeneities the analysis is similar to that used in Ref. 11 to solve the physically related problem of the effectiveness of wavefront reversal.)

When light propagates in such a medium, the radiation becomes depolarized rapidly (over distances on the order of the mean free path). We consider therefore the influence of the impurity motion on interference effects in the scattering of scalar waves.

The wave field u satisfies in this model the equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\varepsilon u)-\nabla^2 u=4\pi j,$$

$$\varepsilon(\mathbf{r}t)=\tilde{\varepsilon}(\mathbf{r}t)+1=1+\varepsilon_{\rm imp}\sum_{a=1}^{N}\delta(\mathbf{r}-\mathbf{r}_a(t)), \tag{1}$$

where $j(\mathbf{r}_t)$ is the density of the radiation sources, $\mathbf{r}_a(t)$ are the impurity coordinates, and $\varepsilon_{\rm imp}$ is the dielectric constant of the impurities. It is possible to average the values of the retarded Green function $\langle G^R\rangle$ of Eq. (1) over the realizations $\{\mathbf{r}_a(t)\}$ of the random motion of the impurities by using the usual impurity diagram technique.[1,3] To this end we calculate first the correlator

$$K(\mathbf{r}-\mathbf{r}',\, t-t')=\langle\tilde{\varepsilon}(\mathbf{r}t)\tilde{\varepsilon}(\mathbf{r}'t')\rangle,$$

which corresponds to the dashed lines on the diagrams for $\langle G^R\rangle$. In the principal approximation in the scatterer density we have for the Fourier component of the correlator

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$$K(\mathbf{q},t)=|\varepsilon_{\rm imp}|^2\left\langle \int \exp\{(i\mathbf{q}(\mathbf{r}-\mathbf{r}')\}\right.$$
$$\left.\times\frac{d\mathbf{r}\,d\mathbf{r}'}{V}\sum_a \delta(\mathbf{r}-\mathbf{r}_a(t))\sum_{a'}\delta(\mathbf{r}'-\mathbf{r}_{a'}(0))\right\rangle_r$$
$$\approx\frac{|\varepsilon_{\rm imp}|^2}{V}\sum_a\langle\exp\{i\mathbf{q}(\mathbf{r}_a(t)-\mathbf{r}_a(0))\}\rangle_{\mathbf{r}_a}, \qquad (2)$$

where $\langle\ldots\rangle_r$ denotes averaging over the realizations $\{\mathbf{r}_a(t)\}$, and V is the volume of the system.

Assume that the impurities are Brownian particles moved by random impacts with the molecules of the medium. The intensity of the collisions of the impurities with the molecules of the medium is characterized by the impurity free path time $\tau_{\rm imp}$. If the time t in (2) is much shorter than $\tau_{\rm imp}$, the impurities can be regarded as moving uniformly with velocities $\mathbf{v}_a$ and the averaging in (2) reduces to averaging over the impurity velocities. In a thermodynamic equilibrium state, the distribution of the impurity velocities is Maxwellian and we obtain ultimately

$$K(\mathbf{q},t)=\frac{|\varepsilon_{\rm imp}|^2}{V}\sum_a\langle\exp(i\mathbf{q}\mathbf{v}_at)\rangle_{\mathbf{r}}=n|\varepsilon_{\rm imp}|^2\langle\exp(i\mathbf{q}\mathbf{v}_at)\rangle_{\mathbf{v}}$$
$$=n|\varepsilon_{\rm imp}|^2\exp\left(-\frac{\mathbf{q}^2}{6}\langle\mathbf{v}^2\rangle t^2\right),\qquad |t|\ll\tau_{\rm imp},$$

where $n=N/V$ is the scatterer density and $\langle\ldots\rangle_v$ denotes averaging over the velocities $\mathbf{v}_a$.

In the opposite case $t\gg\tau_{\rm imp}$ each impurity is subjected within the time t to a large number of collisions, and the quantity $\mathbf{r}_a(t)-\mathbf{r}_a(0)$ is a sum of a large number of independent random quantities. Its distribution is thus Gaussian and averaging in (2) yields

$$K(\mathbf{q},t)=n|\varepsilon_{\rm imp}|^2\langle\exp\{i\mathbf{q}(\mathbf{r}_1(t)-\mathbf{r}_1(0))\}\rangle$$
$$=n|\varepsilon_{\rm imp}|^2\exp\left\{-\frac{\mathbf{q}^2}{2}D_{\rm imp}|t|\right\},\qquad |t|\gg\tau_{\rm imp},$$

where $D_{\rm imp}=(1/3)\langle\mathbf{v}^2\rangle\tau_{\rm imp}$ is the impurity diffusion coefficient.

Thus,

$$K(\mathbf{q},t)=n|\varepsilon_{\rm imp}|^2f(\mathbf{q},t)=n|\varepsilon_{\rm imp}|^2\exp\left(-\frac{q^2}{6}\langle\mathbf{v}^2\rangle t^2\right),$$
$$|t|\ll\tau_{\rm imp},$$
$$K(\mathbf{q},t)=n|\varepsilon_{\rm imp}|^2f(\mathbf{q},t)=n|\varepsilon_{\rm imp}|^2\exp\left(-\frac{\mathbf{q}^2}{6}\langle\mathbf{v}^2\rangle|t|\,\tau_{\rm imp}\right),$$
$$|t|\gg\tau_{\rm imp}. \qquad (3)$$

Just as in the case of scattering by stationary inhomogeneities, diagrams with intersecting dashed lines in the expansion of $\langle G^R\rangle$ are small, in the parameter λ/l, compared with diagrams without intersections.[3] Therefore $\langle G^R\rangle$ satisfies in the principal approximation the equation of Fig. 1, whose solution takes at $\omega l(\omega)\ll c$ the form

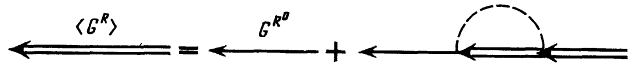

FIG. 1

$$\langle G_\omega{}^R(\mathbf{r})\rangle=(c\mathbf{r})^{-1}\exp\left[-\frac{\mathbf{r}}{2l(\omega)}+i\frac{\omega}{c}\mathbf{r}\right],$$
$$[l(\omega)]^{-1}=n|\varepsilon_{\rm imp}|^2\frac{\omega^4}{4\pi c^4}. \qquad (4)$$

In Eq. (4) the mean free path $l(\omega)$ of a wave of frequency ω does not depend on the characteristics of the motion and is determined only by the density of the scatterers and by the cross section for scattering of a single impurity, as is obvious from geometric considerations. We assume hereafter that the impurities move with an average velocity $\langle\mathbf{v}^2\rangle^{1/2}\ll c$. Then the change of frequency on scattering by moving impurities will be small:

$$\Delta\omega/\omega\sim\langle\mathbf{v}^2\rangle^{1/2}/c\ll 1$$

and we can consider the propagation of almost monochromatic wave packets with a frequency close to some value $\omega_0=ck_0$ and with $l(\omega)\approx l(ck_0)=l$.

To study the transport of the wave-field energy we must average the field correlation function

$$I(\mathbf{R}t,\,\rho\tau)=\langle u(\mathbf{R}+\rho/2,\,t+\tau/2)\,u(\mathbf{R}-\rho/2,\,t-\tau/2)\rangle$$

over the realizations of the random process $\{\mathbf{r}_a(t)\}$. The Fourier component of this correlation function with respect to the difference variable $I(\mathbf{R}t,\mathbf{k}\omega)$ has the meaning of the energy density of a field with frequency ω and wave vector $\mathbf{k}$ at a point $(\mathbf{R},t)$. To find this value we must average, over the realizations $\{\mathbf{r}_a(t)\}$, the product of two Green's functions, retarded G^R and advanced G^A, with $G^A(XX')=G^R(X'X)$. In the principal approximation in λ/l this mean value is represented by the sum of the ladder diagrams of Fig. 2. Transforming in the function

$$F^0(Xx,X_0x_0)=\left\langle G^R\left(X+\frac{x}{2},X_0+\frac{x_0}{2}\right)\right\rangle$$
$$\times\left\langle G^A\left(X_0-\frac{x_0}{2},X-\frac{x}{2}\right)\right\rangle,$$

which corresponds to one link of the diagrams of Fig. 2, to the Fraunhofer approximation[12] in the usual manner and taking the Fourier transform with respect to the difference variable, we obtain an equation for the sum of the ladder diagrams (without the four-point Green functions) in the form

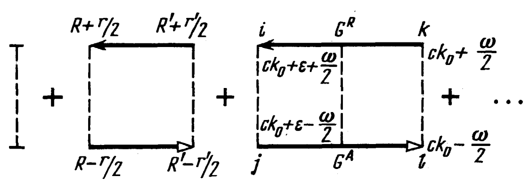

FIG. 2

$$L_{\varepsilon\omega}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')=\frac{1}{4\pi l}\delta(\mathbf{R}-\mathbf{R}')f_{\varepsilon}(k_0(\mathbf{s}-\mathbf{s}'))+\frac{1}{4\pi l}\int f_{\varepsilon_1}(k_0(\mathbf{s}-\mathbf{s}_1))$$
$$\times F_{\omega}^{0}(\mathbf{Rs}_1,\mathbf{R}_1\mathbf{s}_1')L_{\varepsilon-\varepsilon_1,\omega}(\mathbf{R}_1\mathbf{s}_1',\mathbf{R}'\mathbf{s}')\frac{d\varepsilon_1}{2\pi}d\mathbf{R}_1\,d\mathbf{s}_1\,d\mathbf{s}_1', \tag{5}$$

where

$$F_{\omega}^{0}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')=\delta(\mathbf{s}-\mathbf{n})\delta(\mathbf{s}'-\mathbf{n})F_{\omega}^{0}(\mathbf{R}-\mathbf{R}'),\quad \mathbf{n}=\frac{\mathbf{R}-\mathbf{R}'}{|\mathbf{R}-\mathbf{R}'|}$$
$$F_{\omega}^{0}(\mathbf{R})=R^{-2}\exp\left\{\left(-\frac{1}{l}+i\frac{\omega}{c}\right)R\right\}.$$

The scheme for writing the arguments of the four-point diagrams is explained in Fig. 2.

Equation (5) is the transport equation in integral form and has a simple physical meaning: $F_{\omega}^{0}(\mathbf{R}\cdot\mathbf{s},\ \mathbf{R}'\cdot\mathbf{s}')$ is the Green function of the transport operator without allowance for scattering, and the quantity

$$f_{\varepsilon}(k_0(\mathbf{s}-\mathbf{s}'))/4\pi l$$

is equal to the cross section for unit-volume scattering without change of the frequency ε. The function

$$L_{\omega}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')=\int\frac{d\varepsilon}{2\pi}L_{\varepsilon\omega}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')$$

describes the transport of the radiation intensity of all the frequencies and satisfies the equation

$$L_{\omega}(\mathbf{RR}')=\frac{1}{4\pi l}\delta(\mathbf{R}-\mathbf{R}')+\frac{1}{4\pi l}\int F_{\omega}^{0}(\mathbf{R}-\mathbf{R}_1)L_{\omega}(\mathbf{R}_1,\mathbf{R}')\,d\mathbf{R}_1, \tag{6}$$

i.e., the transport equation for isotropic immobile scatterers; in this case $L_{\omega}(\mathbf{R}\cdot\mathbf{s},\ \mathbf{R}'\cdot\mathbf{s}')$ does not depend on the directions of $\mathbf{s}$ and $\mathbf{s}'$.

A solution of (6) can be easily obtained in the diffusion regime $|\mathbf{R}-\mathbf{R}'|\gg l$, $\omega c\ll l$ for the case of an infinite scattering medium. In this case $L_{\omega}(\mathbf{R}\cdot\mathbf{R}')$ depends only on $\mathbf{R}-\mathbf{R}'$ and the Fourier component of the solution (after subtracting $1/4\pi l$, i.e., the values of $L_{\omega}(\mathbf{q})$ in the single-scattering approximation) takes the form

$$L_{\omega}(\mathbf{q})=\frac{1}{4\pi l\tau}\frac{1}{-i\omega+D\mathbf{q}^2}, \tag{7}$$

where $D=l^2/3c$ is the radiation diffusion coefficient and $\tau=l/c$ is the free path time. The solution is of this form because in a medium without true absorption the intensities of all the particles are preserved upon scattering.

To determine the principal interference correction to the correlation function of the field it is necessary to sum the diagrams of first order of smallness in the parameter λ/l. These are the so-called fan diagrams, which contain a maximum number of dashed-line intersections[5] (Fig. 3). The sum of such diagrams

$$V_{\varepsilon\omega}(\mathbf{Rr},\ \mathbf{R}'\mathbf{r}')$$

can be expressed in terms of a new unknown function C (Ref. 13)

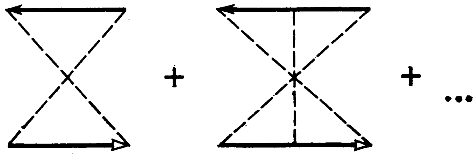

FIG. 3

$$V_{\varepsilon\omega}(\mathbf{Rr},\mathbf{R}'\mathbf{r}')=C_{\varepsilon\omega}\left(\frac{\mathbf{R}+\mathbf{R}'}{2}+\frac{\mathbf{r}-\mathbf{r}'}{4},\mathbf{R}-\mathbf{R}'+\frac{\mathbf{r}+\mathbf{r}'}{2};\frac{\mathbf{R}+\mathbf{R}'}{2}-\frac{\mathbf{r}-\mathbf{r}'}{4},\mathbf{R}'-\mathbf{R}+\frac{\mathbf{r}+\mathbf{r}'}{2}\right), \tag{8}$$

which satisfies an equation of the ladder type

$$C_{\varepsilon\omega}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')=\frac{1}{4\pi l}\int f_{\varepsilon_1}(k_0(\mathbf{s}-\mathbf{s}_1))\times F^{0}_{\varepsilon+\omega-2\varepsilon_1}(\mathbf{Rs}_1,\mathbf{R}_1\mathbf{s}_1')\left\{C_{\varepsilon-\varepsilon_1,\omega-\varepsilon_1}(\mathbf{R}_1\mathbf{s}_1',\mathbf{R}'\mathbf{s}')+\frac{1}{4\pi l}\delta(\mathbf{R}_1-\mathbf{R}')f_{\varepsilon-\varepsilon_1}(k_0(\mathbf{s}_1'-\mathbf{s}'))\right\}\frac{d\varepsilon_1}{2\pi}d\mathbf{R}_1\,d\mathbf{s}_1\,d\mathbf{s}_1'. \tag{9}$$

[The free term in (9) has a different form because it is now necessary to sum the diagrams starting with those having two dashed lines.] Let us examine the large-scale behavior of the solution (9) in the diffusion approximation. Introducing the new variables

$$\alpha={}^1/_2(\varepsilon+\omega),\qquad \beta=\varepsilon-\omega$$

and putting

$$C_{\alpha\beta}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')=\int f_{\varepsilon}(k_0(\mathbf{s}-\mathbf{s}_1))\tilde{C}_{\alpha-\varepsilon,\beta}(\mathbf{Rs}_1,\mathbf{R}'\mathbf{s}')\frac{d\varepsilon\,d\mathbf{s}_1}{2\pi} \tag{10}$$

we reduce Eq. (9), by applying to it the operator

$$[1-2i\tau\alpha+l(\mathbf{s}\nabla_R)],$$

to the form

$$[1-2i\tau\alpha+l(\mathbf{s}\nabla_R)]\tilde{C}_{\alpha\beta}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')=\frac{\delta(\mathbf{R}-\mathbf{R}')}{4\pi l}f_{\alpha+\beta/2}(k_0(\mathbf{s}-\mathbf{s}'))+\int f_{\varepsilon_1}(k_0(\mathbf{s}-\mathbf{s}_1))\tilde{C}_{\alpha-\varepsilon_1,\beta}(\mathbf{Rs}_1,\mathbf{R}'\mathbf{s}')\frac{d\varepsilon_1\,d\mathbf{s}_1}{8\pi^2}. \tag{11}$$

In the diffusion regime $\tilde{C}_{\alpha\beta}(\mathbf{R}\cdot\mathbf{s},\ \mathbf{R}'\cdot\mathbf{s}')$ depends little on the directions of $\mathbf{s}$ and $\mathbf{s}'$. To obtain an equation for the direction-independent part of this function we integrate (11) with respect to $d\mathbf{s}'/4\pi$ and substitute in it the expansion

$$\int\tilde{C}_{\alpha\beta}(\mathbf{Rs},\mathbf{R}'\mathbf{s}')\frac{d\mathbf{s}'}{4\pi}\approx\tilde{C}_{\alpha\beta}(\mathbf{RR}')+3(\mathbf{s}\mathbf{J}_{\alpha\beta}(\mathbf{RR}')).$$

Separating the zeroth and first spherical harmonic and eliminating $\mathbf{J}_{\alpha\beta}(\mathbf{R}\cdot\mathbf{R}')$, we obtain

$$\left(2\tau\frac{d}{dt}-D\tau\nabla_R^2+1-f_t\right)\tilde{C}_{t\beta}(\mathbf{RR}')=\frac{1}{4\pi l}f_t\delta(\mathbf{R}-\mathbf{R}')\exp\left(i\frac{\beta t}{2}\right), \tag{12}$$

where $\widetilde{C}_{t,\beta}$ is the Fourier transform of $\widetilde{C}_{\alpha\beta}$ with respect to the variable α, and

$$f_t=\int\frac{ds'}{4\pi}f(k_0(\mathbf{s}-\mathbf{s}'),t)$$

$$=\begin{cases} y\left(\dfrac{t^2}{\tau_\lambda^2}\right), & |t|\ll\tau_{\text{imp}} \\ y\left(\dfrac{|t|\tau_{\text{imp}}}{\tau_\lambda^2}\right), & |t|\gg\tau_{\text{imp}} \end{cases} \quad (13)$$

where

$$y(x)=x^{-1}(1-e^{-x}),\qquad \tau_\lambda=({}^2/{}_3k_0^2\langle v^2\rangle)^{-1/2}.$$

The parameter τ_λ characterizes the intensity of the impurity motion and has the meaning of the time during which the impurity is displaced a distance $\sim\lambda$ at $\tau_\lambda\ll\tau_{\text{imp}}$ (in the opposite case, at $\tau_\lambda\gg\tau_{\text{imp}}$, the impurity is displaced a distance $\sim\lambda$ after a time $\tau_\lambda\gg\tau_{\text{imp}}$).

In the case of an infinite scattering medium, Eq. (12) is transformed into an ordinary differential equation for the Fourier component of the function $\widetilde{C}$. Solving this equation for the quantity of greatest interest to us

$$C_\omega(\mathbf{q})=\int\frac{d\varepsilon}{2\pi}C_{\varepsilon\omega}(\mathbf{q}),$$

we obtain, taking (10) into account,

$$C_\omega(\mathbf{q})=\frac{1}{4\pi l\tau}\int_0^\infty[f_t]^2\exp\left\{(i\omega-D\mathbf{q}^2)t-\frac{1}{\tau}\int_0^t(1-f_{t'})\,dt'\right\}dt. \quad (14)$$

It can be seen from (14) that as $\tau_\lambda\to\infty$ we have $C_\omega(\mathbf{q})=L_\omega(\mathbf{q})$ and the equivalence of fan and ladder diagrams, which takes place in the stationary case,[5] is restored. The impurity motion does not affect $L_\omega(\mathbf{q})$, but decreases $C_\omega(\mathbf{q})$. At sufficiently large τ_λ, such that the impurities are displaced a distance $\sim\lambda$ within a time much longer than τ, we obtain

$$C_\omega(\mathbf{q})=\int_0^\infty\exp\left[(i\omega-D\mathbf{q}^2)t-\frac{t^3}{\tau_\varphi^3}\right]dt,\quad \tau_\lambda^2\tau\ll\tau_{\text{imp}}^3,\quad \tau\ll\tau_\lambda,$$

$$C_\omega(\mathbf{q})=\int_0^\infty\exp\left[(i\omega-D\mathbf{q}^2)t-\frac{t^2}{\tau_\varphi^2}\right]dt,\ \tau_\lambda^2\tau\gg\tau_{\text{imp}}^3,\quad \tau\tau_{\text{imp}}\ll\tau_\lambda^2, \quad (15)$$

where

$$\tau_\varphi=(3\tau\tau_\lambda^2)^{1/3}\quad\text{at}\quad \tau_\lambda^2\tau\ll\tau_{\text{imp}}^3,$$

$$\tau_\varphi=(2\tau\tau_\lambda^2\tau_{\text{imp}}^{-1})^{1/2}\quad\text{at}\quad \tau_\lambda^2\tau\gg\tau_{\text{imp}}^3.$$

Thus, the contribution to C from scattering processes of longer duration than the time of phase-coherence loss of the forward and backward wave, $\tau_\varphi\gg\tau$, is suppressed.

The meaning of the result is the following: the functions L and C are solutions of Eqs. (5) and (9), which are of the same type, but they have a different large-scale behavior. Let us examine typical diagrams for L and C, which we draw in **R** space (Fig. 4). The time arguments of the four-point diagrams are shown in Fig. 4 with allowance for the fact that we are considering an integral with respect to the change of frequency ε. It is seen immediately that on the diagrams for L the functions G^R and G^A "pass through" each impurity at identical instants of time, while on the diagrams for C at inverse times. This gives rise in the expression for C to an additional phase factor

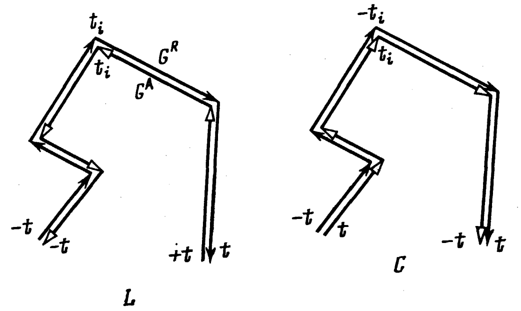

FIG. 4

$$\prod_i\exp\{ik_0\Delta\mathbf{s}_i(\mathbf{r}(t_i)-\mathbf{r}(-t_i))\},$$

where $\Delta\mathbf{s}_i$ is the change of the direction of propagation of the wave when scattered by the ith impurity. Averaging over the motion of the impurities and noting that after a time $2t$ the wave undergoes $\sim 2t/\tau$ scatterings by the impurities, we obtain an estimate accurate to a numerical factor in the argument of the exponential:

$$\exp\left\{-\frac{k_0^2}{\tau}\int_{-t}^{t}\langle(\mathbf{r}(t')-\mathbf{r}(-t'))^2\rangle dt'\right\},$$

which agrees with the exact answer obtained above.

The results can be used to calculate the scattering cross sections of any volume surrounding the impurities. We consider the case when the scattering medium occupies the half-space $z>0$ (the albedo problem), since the effects of multiple scattering are stronger the larger the volume. We confine ourselves here to the case $\omega=0$, i.e., we consider the scattering of a coherent light beam.

The albedo $\alpha(\mathbf{s},\mathbf{s}_0)$ is given by[5]

$$\alpha(\mathbf{s},\mathbf{s}_0)=\mu\mu_0\int F(\mathbf{R}=0,\mathbf{s};\mathbf{R}'=\boldsymbol{\rho},\mathbf{s}_0)\,d\boldsymbol{\rho},$$

$$\boldsymbol{\rho}=(\rho_x,\rho_y,0),\quad \mu=|s_z|,\quad \mu_0=|s_{0z}|. \quad (16)$$

[The function $F(\mathbf{R}\cdot\mathbf{s},\mathbf{R}'\cdot\mathbf{s}')$ is obtained from $\langle G^R G^A\rangle$ by going over to the Fraunhofer approximation and by Fourier transformation with respect to the difference variable.] It is convenient to express the albedo in terms of the total four-point diagram $D(\mathbf{R}\cdot\mathbf{s},\mathbf{R}'\cdot\mathbf{s}')$. For the half-space geometry we have

$$D(\mathbf{R}\mathbf{s},\mathbf{R}'\mathbf{s}')=D((\mathbf{R}-\mathbf{R}')_\perp;\,z,z';\,\mathbf{s}\mathbf{s}')$$

and

$$\alpha(\mathbf{s}\mathbf{s}_0)=\mu\mu_0\iint_0^\infty F^0(R_1\mathbf{s})\left[\int d\boldsymbol{\rho}D(\boldsymbol{\rho};\mu R_1,\mu_0R_2;\mathbf{s}\mathbf{s}_0)\right]\times F^0(R_2\mathbf{s}_0)R_1^2\,dR_1R_2^2\,dR_2. \quad (17)$$

Taking the interference correction into account,

$$D = L + V,$$

with the sum of the fan diagrams expressed in the case of a finite scattering volume Ω in terms of the function C as follows:

$$V(\mathbf{R}, -\mathbf{s}+\mathbf{s}_1; \mathbf{R}', \mathbf{s}) = \delta(\mathbf{R}-\mathbf{R}') \left\{ \int_{\mathbf{R}\pm(\mathbf{r}/2)\in\Omega} C(\mathbf{R}+\mathbf{r}/2, -\mathbf{s}; \mathbf{R}-\mathbf{r}/2, \mathbf{s}) \times \exp(-ik_0\mathbf{s}_1\mathbf{r})\, d\mathbf{r} \right\}, \quad |\mathbf{s}_1| \ll 1, \tag{18}$$

while the functions L and C satisfy Eqs. (5) and (9) supplemented by the condition $\mathbf{R}_1 \in \Omega$.

If the impurity motion is slow enough ($\tau_\lambda \gg \tau$ at $\tau_\lambda^2 \ll \tau_{\text{imp}}^3$ or $\tau_\lambda^2 \gg \tau\tau_{\text{imp}}$ at $\tau_\lambda^2\tau \gg \tau_{\text{imp}}^3$), the dependence of the albedo on the intensity of the impurity motion can be obtained in the diffusion approximation. This approximation is not quite right for the solution of the albedo problem, because it describes correctly on the contribution from scattering processes of high multiplicity. The diffusion approximation, however, is perfectly acceptable for the calculation of the albedo as a function of the intensity of the impurity motion at $\tau_\varphi \gg \tau$, when the impurity motion itself changes only the contribution from scattering processes of long-duration (on the order of τ_φ).

In the diffusion approximation[14] it is assumed that the solutions of the integral equations (5) and (9) satisfy diffusion equations whose forms are determined by the solutions (7) and (14) obtained for the case of an infinite medium. These equations should be supplemented by boundary conditions that are chosen as a rule in the form

$$L(\mathbf{R}\mathbf{R}'),\ C(\mathbf{R}\mathbf{R}') = 0 \text{ where } \mathbf{R}_z = -z_0, \tag{19}$$

where $R_z = -z_0$ is the effective boundary of the scattering volume. The numerical value of z_0 is chosen such as to obtain the correct asymptotic behavior at infinity in the problem with a constant total energy flux for a semi-infinite medium. Comparison with the exact solution of this problem for pointlike isotropic scatterers yields $z_0 = 0.7104l$.[14]

The diffusion equation with the boundary condition (19) is easily solved by the image method, which yields

$$L(\mathbf{R}\mathbf{R}') = \frac{1}{4\pi l}\delta(\mathbf{R}-\mathbf{R}') + \int \frac{d\mathbf{q}}{(2\pi)^3} L(\mathbf{q}) \{e^{i\mathbf{q}(\mathbf{R}-\mathbf{R}')} - e^{i\mathbf{q}(\mathbf{R}-\mathbf{R}'^*)}\},$$
$$C(\mathbf{R}\mathbf{R}') = \int \frac{d\mathbf{q}}{(2\pi)^3} C(\mathbf{q}) \{e^{i\mathbf{q}(\mathbf{R}-\mathbf{R}')} - e^{i\mathbf{q}(\mathbf{R}-\mathbf{R}'^*)}\}, \tag{20}$$

where $L(\mathbf{q})$ and $C(\mathbf{q})$ are the corresponding solutions of (7) and (14) for the case of an infinite medium, while $\mathbf{R}'^*$ is the image of the point $\mathbf{R}'$ in the plane $R_z = -z_0$:

$$\mathbf{R}' = (x', y', z'), \quad \mathbf{R}'^* = (x', y', -z'-z_0).$$

The oscillating factor in the integrand of (18) makes the interference correction to the albedo different from zero only in the narrow direction cone $|\mathbf{s}_1| \sim \lambda/l$. The angular distribution of the correction is similar to that obtained in Ref. 5 for the case of scattering by stationary inhomogeneities; we consider therefore the correction to the albedo for exact backscattering. Using (17), (18), and (20) we obtain for the albedo an expression in the form of three terms corresponding to singly scattered radiation $\alpha^{(1)}$, to the multiple part α^{mult} in the transport equation approximation, and to the interference correction α^{int}:

$$\alpha(-\mathbf{s}, \mathbf{s}) = \alpha^{(1)}(-\mathbf{s}, \mathbf{s}) + \alpha^{\text{mult}}(-\mathbf{s}, \mathbf{s}) + \alpha^{\text{int}}(-\mathbf{s}, \mathbf{s}),$$
$$\alpha^{(1)}(-\mathbf{s}, \mathbf{s}) = \frac{\mu}{8\pi}, \quad \alpha^{\text{mult}}(-\mathbf{s}, \mathbf{s}) = \frac{3}{\pi}\left(\frac{z_0}{l}\mu^2 + \frac{\mu^3}{2}\right),$$
$$\alpha^{\text{int}}(-\mathbf{s}, \mathbf{s}) \approx \alpha^{\text{mult}}(-\mathbf{s}, \mathbf{s}) - \left(\mu^4 + 2\mu^3\frac{z_0}{l} + \mu^2\frac{z_0^2}{l^2}\right) \times \begin{cases} \dfrac{3}{(\pi/6)^{1/2}\Gamma(1/4)}\left(\dfrac{\tau}{\tau_\varphi}\right)^{1/2}, & \tau_\lambda^2\tau \ll \tau_{\text{imp}}^3 \\ \dfrac{6}{(\pi/3)^{1/2}\Gamma(1/6)}\left(\dfrac{\tau}{\tau_\varphi}\right)^{1/2}, & \tau_\lambda^2\tau \gg \tau_{\text{imp}}^3. \end{cases} \tag{21}$$

In the opposite case of fast impurity motion, when a shift by a distance on the order of the radiation wavelength λ takes place in a time much shorter than the free path time τ, the contribution of the high-multiplicity scattering processes to C is small and the interference correction is determined by the first of the fan diagrams of Fig. 3. In this case the calculation of α^{int} yields

$$\alpha^{\text{int}}(-\mathbf{s}, \mathbf{s}) = \frac{\mu}{32\pi^2}\frac{\sqrt{\pi}\,\tau_\lambda}{2\tau}, \quad \tau_\lambda \ll \tau, \tau_{\text{imp}},$$
$$\alpha^{\text{int}}(-\mathbf{s}, \mathbf{s}) = \frac{\mu}{32\pi^2}\frac{\tau_\lambda^2}{\tau\tau_{\text{imp}}}, \quad \tau \gg \tau_\lambda^2\tau_{\text{imp}}^{-1} \gg \tau_{\text{imp}}. \tag{22}$$

It must be noted that in this case the characteristic dimension of the function

$$C(\mathbf{R}+\mathbf{r}/2, -\mathbf{s}; \mathbf{R}-\mathbf{r}/2, \mathbf{s})$$

in terms of the variable $\mathbf{r}$ is $l(\tau_\lambda/\tau)$ at $\tau_\lambda \ll \tau, \tau_{\text{imp}}$ and $l(\tau^2/\tau\tau_{\text{imp}})$ at $\tau \gg \tau_\lambda^2\tau_{\text{imp}}^{-1} \gg \tau_{\text{imp}}$.

Therefore, as seen from (18), the angular dimension of the interference peak is in this case of the order of

$$|\mathbf{s}_1| \sim \frac{\tau\tau_{\text{imp}}}{\tau_\lambda^2}\frac{\lambda}{l}, \quad \tau \gg \frac{\tau_\lambda^2}{\tau_{\text{imp}}} \gg \tau_{\text{imp}},$$
$$|\mathbf{s}_1| \sim \frac{\tau}{\tau_\lambda}\frac{\lambda}{l}, \quad \tau_\lambda \ll \tau, \tau_{\text{imp}}. \tag{23}$$

Thus, the motion of the impurities does not affect the albedo in the transport-equation approximation, but decreases the interference correction, thus violating the equality

$$\alpha^{\text{int}}(-\mathbf{s}, \mathbf{s}) = \alpha^{\text{mult}}(-\mathbf{s}, \mathbf{s})$$

that holds in the stationary case.[5]

In the case of slow impurity motion we have

$$\alpha^{\text{int}}(-\mathbf{s}, \mathbf{s}) - \alpha^{\text{mult}}(-\mathbf{s}, \mathbf{s}) \propto -(\tau/\tau_\varphi)^{1/2},$$

where the time τ_φ of loss of phase coherence of the forward and backward waves is determined by relation (15). In the opposite case of fast motion of the impurities the interference effects are suppressed and the values of the interference peak and of the albedo are small to the extent that the ratio of the time of the impurity shift over a distance $\sim\lambda$ to the free path time of the radiation is small.

3. INFLUENCE OF THE GYROTROPY OF THE MEDIUM ON INTERFERENCE EFFECTS IN MULTIPLE SCATTERING

Consider the propagation of light in a randomly inhomogeneous medium placed in an external constant and uniform magnetic field **H**. Owing to the Faraday effect,[15] the right- and left-polarized electromagnetic waves in such a medium will have different phase velocities. Let us ascertain how this circumstance alters the interference effects in multiple scattering.

Let the wave field E satisfy the equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\hat{\varepsilon}\mathbf{E})-\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}=\frac{4\pi}{c}\mathbf{j}, \tag{24}$$

where

$$\varepsilon_{ij}=\delta_{ij}-ie_{ijk}h_k+\varepsilon_{\mathrm{imp}}\delta_{ij}\sum_a\delta(\mathbf{r}-\mathbf{r}_a)$$

is the dielectric tensor of the randomly inhomogeneous gyrotropic medium and h_k is the gyration vector.

The procedure for investigating the transport of the wave-field energy is similar to that described in §2, and we report here briefly only the results for this case.

The retarded Green function of Eq. (24), averaged over the impurity locations, is equal to

$$G_{ij}^R(\mathbf{p},\omega)=\sum_{\alpha=\pm1}4\pi P_{ij}^\alpha(\mathbf{n})\left[(1+\alpha\mathbf{nh})\frac{\omega^2}{c^2}-\mathbf{p}^2+i\delta\right]^{-1}, \tag{25}$$

where

$$\mathbf{n}=\frac{\mathbf{p}}{|\mathbf{p}|},\quad P_{ij}^\alpha(\mathbf{n})=\frac{1}{2}[\delta_{ij}-n_in_j+i\alpha e_{ijk}n_k],$$
$$\delta=k_0l^{-1}=\frac{k_0^5}{6\pi}n|\varepsilon_{\mathrm{imp}}|^2.$$

The sum of the ladder diagrams satisfies the equation

$$L_{ijkl}(\mathbf{R},\mathbf{R}')=\frac{3}{8\pi l}\delta_{ik}\delta_{jl}\delta(\mathbf{R}-\mathbf{R}')$$
$$+\frac{3}{8\pi l}\int F^0_{ijmn}(\mathbf{R}-\mathbf{R}_1)L_{mnkl}(\mathbf{R}_1,\mathbf{R}')\,d\mathbf{R}_1, \tag{26}$$

$$F^0_{ijkl}(\mathbf{R})$$
$$=\frac{e^{-R/l}}{R^2}\sum_{\alpha,\beta}P_{ik}^\alpha(\mathbf{m})P_{lj}^\beta(\mathbf{m})\exp\left\{i\frac{\omega}{c}R+ik_0\left(\frac{\alpha-\beta}{2}\right)\mathbf{hR}\right\} \tag{27}$$

$$\mathbf{m}=\mathbf{R}/R.$$

In the diffusion regime, in the case of an infinite scattering medium, the Fourier component of the solution of (17) (after subtracting the value in the single-scattering approximation) is of the form

$$L_{ijkl}(\mathbf{q})=\frac{1}{8\pi l\tau}\frac{\delta_{ij}\delta_{kl}}{-i\omega+D\mathbf{q}^2};\quad \omega\tau,\quad D\tau\mathbf{q}^2\ll1, \tag{28}$$

where $D=c^2\tau/3$ is the radiation diffusion coefficient. This form of the solution reflects the fact that at large distances the radiation is completely depolarized and the radiant intensity satisfies the diffusion equation whose form does not depend on the gyration vector **h**.

To calculate the sum of the fan diagrams, we again introduce the function C:

$$V_{ijkl}(\mathbf{Rr},\mathbf{R}'\mathbf{r}')=\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{r}+\mathbf{r}')C_{ilkj}(\mathbf{R}+\mathbf{r}/2,\mathbf{R}-\mathbf{r}/2)$$

and obtain for it an equation of the type (27)

$$C_{ijkl}(\mathbf{R},\mathbf{R}')=\frac{3}{8\pi l}\int\tilde{F}^0_{ijmn}(\mathbf{R}-\mathbf{R}_1)\left\{C_{mnkl}(\mathbf{R}_1,\mathbf{R}')+\frac{3}{8\pi l}\delta_{mk}\delta_{nl}\delta(\mathbf{R}_1-\mathbf{R}')\right\}d\mathbf{R}_1 \tag{29}$$

the only difference being that it contains in lieu of F^0_{ijkl} (**R**) the function

$$\tilde{F}^0_{ijkl}(\mathbf{R})=F^0_{ilkj}(\mathbf{R})=\frac{e^{-R/l}}{R^2}\sum_{\alpha,\beta}P_{ik}^\alpha(\mathbf{n})$$
$$\times P_{lj}^\beta(\mathbf{n})\exp\left\{i\frac{\omega}{c}R+ik_0\left(\frac{\alpha+\beta}{c}\right)\mathbf{hR}\right\}. \tag{30}$$

The function $\tilde{F}^0$ differs from F^0 in that in one of the Green functions it contains the arguments are in reversed order; by virtue of the reciprocity theorem this corresponds to a medium with the direction of the vector **h** reversed.

The solution of (29) in the diffusion approximation is

$$C_{ijkl}(\mathbf{q})=\frac{1}{8\pi l\tau}\frac{\delta_{ij}\delta_{kl}}{-i\omega+D\mathbf{q}^2+2Dk_0^2\mathbf{h}^2};$$
$$D\mathbf{q}^2,\ \omega,\ Dk_0^2\mathbf{h}^2\ll\frac{1}{\tau}. \tag{31}$$

Thus, the gyrotropy of the medium alters the large-scale behavior of the function $C_{ijkl}(\mathbf{R})$ and does not influence $L_{ijkl}(\mathbf{R})$. This can be explained in the following manner: consider again the typical diagrams for L and C (Fig. 4). To each link of these diagrams corresponds the function F^0_{ijkl} or $\tilde{F}_{ijkl}$. At $\mathbf{h}=0$ the relation $F^0_{ijkl}(\mathbf{R})=\tilde{F}^0_{ijkl}(\mathbf{R})$ holds. An important role in the determination of the large-scale behavior of L and C is played by high-order diagrams that describe multiple scattering. Inasmuch as integration is carried out with respect to the vectors $\mathbf{n}_i$ corresponding to different links, what is actually performed in the calculation of L and C at $\mathbf{h}=0$ is multiplication of a large number of matrices of the type

$$\langle P_{ik}^\alpha(\mathbf{n})P_{lj}^\beta(\mathbf{n})\rangle_{\mathbf{n}}=M_{ijkl}^{\alpha\beta}$$
$$=\frac{1}{4}\left[\frac{2}{5}\delta_{ik}\delta_{jl}+\left(\frac{1}{15}+\frac{\alpha\beta}{3}\right)\delta_{ij}\delta_{kl}+\left(\frac{1}{15}-\frac{\alpha\beta}{3}\right)\delta_{il}\delta_{kj}\right].$$

The maximum eigenvalue of M is 1/3 at $\alpha=\beta$ and 1/5 at $\alpha\neq\beta$. It is therefore clear that when calculating the contribution of diagrams of high order account must be taken of only terms with $\alpha=\beta$ on each link, i.e., with equal polarization states of G^R and G^A. It is then clear from the form of the function F^0 (27) that the terms of the sum over α and β with $\alpha=\beta$ do not depend on **h**, but in expression (30) for the function $\tilde{F}^0$ the terms of the sum with $\alpha=\beta$ are found to depend on **h**. Therefore the field **h** does not influence the form of $L_{ijkl}(\mathbf{q})$ at small q, but changes $C_{ijkl}(\mathbf{q})$.

At $h \ll \lambda/l$ the dependence of the interference correction to the albedo on **h** can be obtained in the diffusion approximation. The numerical value of the parameter z_0 is determined as before from the asymptotic form of the solution of the problem with a constant total flux[16] and is equal to $z_0 = 0.7059l$.

The calculation of the albedo is similar to that in §2 and yields

$$\alpha_{ijkl}(-\mathbf{s},\mathbf{s}) = \alpha^{(1)}_{ijkl}(-\mathbf{s},\mathbf{s}) + \alpha^{\text{mult}}_{ijkl}(-\mathbf{s},\mathbf{s}) + \alpha^{\text{int}}_{ijkl}(-\mathbf{s},\mathbf{s}),$$

$$\alpha^{(1)}_{ijkl}(-\mathbf{s},\mathbf{s}) = \frac{3\mu}{16\pi}\sum_{\alpha\beta} P_{ik}{}^{\alpha}(\mathbf{s})P_{jl}{}^{\beta}(\mathbf{s})\,[1+i\mathbf{h}\mathbf{s}k_0l(\alpha+\beta)]^{-1},$$

$$\alpha^{\text{mult}}_{ijkl}(-\mathbf{s},\mathbf{s}) = \frac{3}{8\pi}(\delta_{ij}-s_is_j)(\delta_{kl}-s_ks_l)\left(\frac{z_0}{l}\mu^2+\frac{1}{2}\mu^3\right),$$

$$\alpha^{\text{int}}_{ijkl}(-\mathbf{s},\mathbf{s}) = \alpha^{\text{mult}}_{ilkj}(-\mathbf{s},\mathbf{s}) - (\sqrt{2}\,k_0lh)\frac{3}{8\pi}\left(\mu^4+2\mu^2\frac{z_0^2}{l^2}+\mu^3\frac{z_0}{l}\right)$$
$$\times(\delta_{il}-s_is_l)(\delta_{jk}-s_js_k) - \mathbf{h}\mathbf{s}k_0l\frac{3}{16\pi}\left(2\frac{z_0}{l}\mu^2+\frac{3}{2}\mu^3\right)$$
$$\times[(\delta_{il}-s_is_l)e_{jkn}s_n+(\delta_{jk}-s_js_k)e_{ilr}s_r]. \qquad (32)$$

Particular interest attaches to the intensity of the scattered light when natural (unpolarized) radiation is incident. The albedo is determined in this case by the convolution

$$\alpha(\mathbf{s},-\mathbf{s}) = {}^1/_2\alpha_{iikk}(\mathbf{s},-\mathbf{s})$$

and is equal to

$$\alpha^{(1)} = \frac{3\mu}{16\pi}, \quad \alpha^{\text{mult}}(-\mathbf{s},\mathbf{s}) = \frac{3}{4\pi}\left(\frac{z_0}{l}\mu^2+\frac{1}{2}\mu^3\right), \qquad (33)$$

$$\alpha^{\text{int}}(-\mathbf{s},\mathbf{s}) = \frac{1}{2}\alpha^{\text{mult}}(-\mathbf{s},\mathbf{s}) - \sqrt{2}\,hk_0l\left(2\frac{z_0}{l}\mu^3+\frac{z_0^2}{l^2}\mu^2+\mu^4\right),$$

$$h \ll \lambda/l.$$

Even in the absence of gyrotropy, the interference correction to the intensity of the light backscattering is not equal to its multiple part in the transport-equation approximation, as was the case in the scalar theory.[5] In the diffusion approximation the correction is half as large.[1)] The gyrotropy does not affect the scattered-light intensity in the transport-equation approximation, but decreases the value of the correction.

At $h \gg \lambda/l$ the **h**-dependent terms with $\alpha = \beta$ in expression (30) for the function $\widetilde{F}^0$ (their contribution to C is decisive at $h \ll \lambda/1$) turn out to be rapidly oscillating functions of **R** and their contribution to C can be neglected. Since, however, F_0 contains terms with $\alpha \neq \beta$, which do not depend on **h**, it follows that C does not tend to zero as $h \to \infty$. Therefore even as $h \to \infty$ the interference peak in the scattering cross section has a certain finite value. The reason for this phenomenon is the following. The magnetic field violates the time-reversal symmetry, but the symmetry with respect to a transformation consisting of a simultaneous change of the sign of t and of spatial inversion is preserved also in a magnetic field. The inversion transformation reverses the direction of the circular polarization of the wave. Therefore the phases of the scattering processes connected with time reversal and simultaneous reversal of the polarization state coincide also in a magnetic field.

To calculate the interference correction to the albedo at $h \gg \lambda/l$ we must solve Eq. (29) with the kernel

$$F^0_{ijkl}(\mathbf{R}) = \frac{e^{-R/l}}{R^2}\sum_{\alpha} P_{ik}{}^{\alpha}(\mathbf{n})P_{lj}{}^{-\alpha}(\mathbf{n}).$$

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[1)]The interference correction at $\mathbf{h} = 0$ is expressed in terms of the multiple part of the albedo in the transport-equation approximation by means of the relation

$$\alpha^{\text{int}}_{ijkl}(\mathbf{s},-\mathbf{s}) = \alpha^{\text{mult}}_{ilkj}(\mathbf{s},-\mathbf{s})$$

and can be determined exactly, since there is a known[16] exact solution of the albedo problem in the transport-equation approximation at $\mathbf{h} = 0$. At normal incidence of unpolarized light the ratio obtained from the exact solution is $\alpha^{\text{int}}/\alpha^{\text{mult}}/\approx 0.71$.

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