

# “Antilocalization” of electrons and electron–electron interaction in thin bismuth films

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The dependences of the resistance of bismuth films of thickness 50–1500 Å with resistance per square 40–6500 Ω on temperature  $T = 0.2$ –20 K and magnetic field ( $H$  up to 60 kOe) have been measured. The observed anomalous behavior of the resistance is interpreted in terms of the theories of “weak” localization and electron–electron interaction in two-dimensional disordered systems. Values of characteristic times for scattering mechanisms determining the magnitude and sign of the quantum correction to the resistance are obtained. The observed peculiarities of the effects in low-resistance films are accounted for by a change in the effective dimensionality of the specimens.

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## I. INTRODUCTION

There have recently been theories of the physics of disordered systems which predict an anomalous behavior of the kinetic properties of weakly disordered metals (mean free path of the electrons exceeding the de Broglie wavelength:  $l \gg \lambda$ ).

Due to interference between the electron wave packets, there is an addition to the residual resistivity of the metal for elastic scattering by impurities and statistical structural lattice defects (the theory of weak localization (WEI)<sup>1,2</sup>) The presence of impurities in the system also leads to an increase in electron–electron interaction and to the appearance of a quantum correction of a different kind to the conductivity (the theory of interelectron interaction (EEI)<sup>3,4</sup>).

The magnitude of the correction to the conductivity in WEL theory depends on the characteristic times for various electron scattering mechanisms. The time for loss of phase coherence  $\tau_\varphi$  of the interfering waves is determined by the times for inelastic scattering  $\tau_E$  and for elastic scattering by paramagnetic impurities with spin reversal,  $\tau_s$ . Scattering by impurities for which electron spin reversal takes place due to spin–orbit interaction (time  $\tau_{so}$ ) plays a special part. Not only the magnitude, but also the sign of the correction to the conductivity in WEL theory is connected with this mechanism.<sup>5</sup>

In the absence of spin–orbit interaction, electron localization leads to an increase in resistivity with decreasing temperature. A magnetic field destroys the interference, as a result of which a negative magnetoresistance arises. Strong spin–orbit interaction ( $\tau_{so} \ll \tau_\varphi$ ) changes the sign of the interference correction to the conductivity; the “antilocalization” effect<sup>1</sup> appears; the sign of the magnetoresistance then changes.

Up until now the study of quantum corrections to the conductivity of thin films has been carried out on films of a number of metals (for example, Refs. 7 and 8), on semiconductor<sup>9</sup> and semimetal films.<sup>10–14</sup> In semiconductors and semimetals the feature of the WEL and EEI effects consists of an appreciable change in the resistivity of these films. Interest in semimetallic films is explained, in addition, by the possibility of observing the “antilocalization” phenomenon, be-

cause of the strong spin–orbit interaction, in semimetals of the bismuth type.

Results of studying the WEL and EEI effects in the conductivity of thin bismuth films are presented here. A summary of theoretical results appropriate for discussing the results obtained, is given in the following section.

## II. THE MAIN RESULTS OF THE THEORIES OF WEAK LOCALIZATION AND INTERELECTRON INTERACTION FOR THE TWO-DIMENSIONAL CASE

If the thickness of the film satisfies the condition  $d \ll L_\varphi = (D\tau_\varphi)^{1/2}$  ( $D$  is the diffusion coefficient), then the temperature and magnetic field (perpendicular to the film) dependences of the interference correction to the conductivity have the form:<sup>1–5</sup>

$$\Delta\sigma_\square(T) = -\frac{\Delta R_\square(T)}{R_\square^2} = -\alpha \frac{e^2}{2\pi^2\hbar} \ln \tau_\varphi(T) = \alpha p \frac{e^2}{2\pi^2\hbar} \ln T, \quad (1)$$

$$\Delta\sigma_\square(H) = -\frac{\Delta R_\square(H)}{R_\square^2} = \alpha \frac{e^2}{2\pi^2\hbar} f_2(\gamma\tau_\varphi). \quad (2)$$

Here  $\sigma_\square$  is the conductivity of a square film,  $\gamma = 4DeH/c\hbar$ , and

$$f_2(x) = \ln x + \Psi(1/2 + 1/x) = \begin{cases} \ln x + \Psi(1/2), & x \gg 1 \\ x^2/24, & x \ll 1 \end{cases}$$

where  $\Psi(x)$  is the logarithmic derivative of the gamma function. It is assumed in Eq. (1) that  $\tau_\varphi \sim T^{-p}$ . The relation between  $\Delta\sigma(H)$  and  $\Delta R(H)$  in Eq. (2) is valid for a small Hall conductivity ( $\sigma_{xy} \ll \sigma_{xx}$ ). This condition is realized in bismuth films because of the existence of two types of current carriers. The coefficient  $\alpha$  in Eqs. (1) and (2) characterizes the spin–orbit interaction and takes the values 1 for  $\tau_\varphi \ll \tau_{so}$  and  $-1/2$  for  $\tau_{so} \ll \tau_\varphi$ .

The parallel magnetoresistance is appreciably less than the perpendicular, depends on the film thickness, arises in much stronger fields and is independent of the orientation of the field  $H_\parallel$  relative to the measuring current:<sup>15</sup>

$$\Delta\sigma(H_\parallel) = \frac{e^2}{2\pi^2\hbar} \ln \left( \frac{d^2 L_\varphi^2}{3L_H^4} + 1 \right), \quad L_H = \left( \frac{c\hbar}{eH} \right)^{1/2}; \quad (3)$$

$L_H$  is the magnetic length.

In a sufficiently high perpendicular magnetic field

( $L_H \ll L_\varphi$ ), the minimum length for loss of phase coherence becomes  $L_H$ , and the change in  $L_\varphi$  with temperature does not lead to a change in  $\sigma_\square$ . A strong magnetic field thus suppresses the temperature dependence of the interference correction. This fact will be used later to separate out the WEL and EEI contributions to the  $\Delta\sigma_\square(T)$  dependence of bismuth films. A parallel magnetic field acts on the  $\Delta\sigma_\square(T)$  dependence in a similar way.

The two-dimensional case is realized in the EEI theory for  $d \ll L_I = (D\hbar/kT)^{1/2}$ . The correction to the conductivity due to interelectron interaction then depends on  $T$  in the following way:<sup>16</sup>

$$\Delta\sigma_\square(T) = \frac{e^2}{2\pi^2\hbar} \left\{ \left[ 1 - \frac{3}{4}F \right] \ln T\tau - \ln \frac{\ln T_c \cdot \tau}{\ln(T_c^*/T)} \right\} \approx \frac{e^2}{2\pi^2\hbar} \left[ 1 - \frac{3}{4}F - g(T) \right] \ln T\tau; \quad (4)$$

$F$  is an interaction parameter in the particle-hole channel with total spin  $J = 1$ ;  $\tau$  is the momentum relaxation time;  $kT_c^* = \varepsilon_F \exp(-1/\lambda_0)$  is the effective critical temperature for the normal metal;  $\lambda_0 < 0$  is the bare interaction constant;  $g(T) = [\ln(T_c^*/T)]^{-1} \ll 1$  is the interaction constant in the particle-particle channel.

It can be seen from Eq. (4) that the temperature dependence of the corrections to the conductivity in the WEL and EEI theories have the same logarithmic form. The magnetoresistance in the EEI theory becomes appreciable in the region of fields satisfying the condition  $L_H \ll L_T$ ,  $g_L \mu H / kT \gg 1$  ( $g_L$  is the Landé factor and  $\mu$  the Bohr magneton). The values of the fields concerned exceed the value of a high field for the WEL theory ( $L_H \ll L_\varphi$ ). In the region  $L_H \ll L_T$ ,  $g_L \mu H / kT \gg 1$  the magnetoresistance in the EEI theory depends on  $H$  in the following way:<sup>4,16</sup>

$$\Delta\sigma_\square(H) = \frac{e^2}{2\pi^2\hbar} \left[ -\frac{F}{2} \ln \frac{g_L \mu H \tau}{\hbar} - \ln \frac{\ln T_c \cdot \tau}{\ln(kT_c^* \pi c / DeH)} \right] \approx \frac{e^2}{2\pi^2\hbar} \left[ -\frac{F}{2} \ln \frac{g_L \mu H \tau}{\hbar} - g(H) \ln \frac{DeH}{\pi c kT} \right], \quad (5)$$

with the first term independent of field direction while the second is determined by the component of field normal to the film.

In the EEI theory a strong magnetic field does not completely suppress the temperature dependence of  $\sigma_\square$ :

$$\Delta\sigma_\square(T) = \frac{e^2}{2\pi^2\hbar} \left( 1 - \frac{F}{4} \right) \ln T\tau \quad (6)$$

for  $L_H \ll L_T$ ,  $g_L \mu H / kT \gg 1$ . The Maki-Thompson contribution is absent in Eqs. (4) to (6) as it can be considered small due to the smallness of the parameter  $\beta(T)$  for a normal metal:  $\beta(T) \approx g^2(T) \ll 1$ .<sup>17</sup>

We should note that strong spin-orbit interaction does not change the sign of the correction in EEI theory, but reduces the contribution from terms describing the interaction in the electron-hole channel with total spin 1 (terms proportional to the constant  $F$ ). This influence is appreciable for  $\hbar/\tau_{so} > kT$ ,  $g\mu H$ .<sup>16</sup>

### III. EXPERIMENTAL RESULTS AND DISCUSSION OF THEM

The bismuth films were obtained by thermal deposition of pure (99.9999%) bismuth in a vacuum of  $10^{-5}$ – $10^{-6}$  Torr

onto a mica substrate at  $T = 300$  K. The thickness of the "thick" (500–1500 Å) films was measured with a MII-9 interference microscope, and the thickness of the "thin" (50–500 Å) films was estimated from the known rate of deposition. The Bi films were textured polycrystalline specimens with the  $C_3$  axis along the normal to the surface. The geometry of the films was: width 1 mm, distance between potential leads 5 mm. The resistance was measured by a four-lead balance method with 20 Hz alternating current. The voltage along the specimen was 1 to 10 mV, the sensitivity of the apparatus  $\Delta R/R \sim 10^{-5}$ .

Films with resistance per square in the range  $R_\square = 40$ – $6500 \Omega$  were studied; the value of  $R_\square$  decreases as the film thickness is increased.

#### 1. Results of measuring "high-resistance" films ( $R_\square = 150$ – $6500 \Omega$ )

##### a) Measurement of the temperature dependence of resistance

In all the films an increase in resistivity with decreasing temperature was observed in the temperature range 1.5–6 K. A minimum was observed on the  $R(T)$  curve in the region  $T > 10$  K for films with thickness less than 400 Å. This minimum was not observed for films thicker than 400 Å. The nature of the  $R(T)$  curve for  $T > 10$  K is evidently determined by the complicated form of the temperature dependences of the carrier concentration  $n$  and of the mobility  $\mu$  in Bi films<sup>18</sup> and was not studied in detail by us.

The experimental variation in the temperature range 1.6–6 K is well described by the logarithmic law

$$\Delta R = -(A e^2 R_\square^2 / 2\pi^2 \hbar) \ln T. \quad (7)$$

Most of the measurements were carried out over this temperature interval. Measurements were carried out<sup>2)</sup> over the wider temperature range 0.2–5 K for several specimens to check the logarithmic form of the  $R(T)$  dependence. It also held for  $T < 1.5$  K (Fig. 1).

It was found that the increase in resistivity  $\Delta R_\square$  was proportional to the values of  $R_\square^2$  (independent of the film deposition conditions and their thickness). The slope  $A$  of the  $\Delta R_\square(T)$  plot changes little as  $R_\square$  changes from 150 to 6500  $\Omega$  (Fig. 2) and on the average  $A \approx 0.4$ . The constancy of the coefficient  $A$  means that the anomalous increase in resis-

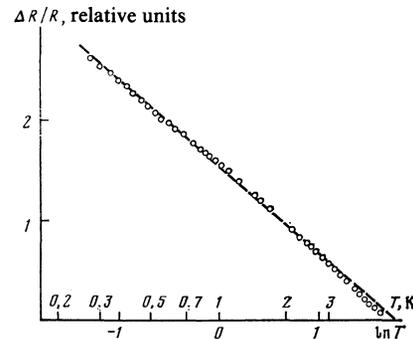


FIG. 1. Temperature dependence of resistivity of a bismuth film ( $R_\square = 100 \Omega$ ). The dashed straight line is produced by the least squares method.

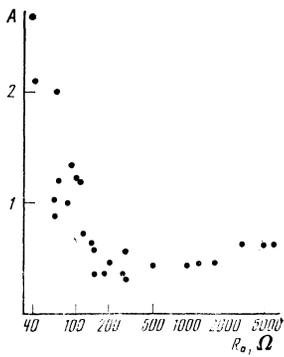


FIG. 2. Dependence of the gradient  $A$  of the logarithmic temperature dependences (Eq. (7)) on the resistance of the film per square.

tance cannot be considered a manifestation of the Kondo effect, which is an exchange effect and for which  $\Delta\rho_K(T) \propto -\ln T$ . The magnitude of  $A$  would then change by about an order of magnitude as  $R_\square$  changed 40-fold.

The effects of WEL and EEI can be considered to show up additively for small corrections to the resistivity:  $A = A_{\text{WEL}} + A_{\text{EEI}}$ . In order to separate their contributions, the temperature dependence of  $R_\square$  was measured in a magnetic field which should suppress the WEL contribution to  $\Delta R_\square(T)$  for  $L_H \ll L_\varphi$ . Under the experimental conditions ( $H \parallel C_3$ ,  $H < 5$  kOe) the strong-field criterion for EEI theory is not satisfied since  $g_T \mu H / kT \lesssim 1$ ,  $L_T / L_H \lesssim 1$  for  $D \sim 10$  cm<sup>2</sup> sec<sup>-1</sup>. A magnetic field should therefore not change the temperature dependence of the EEI contribution appreciably. An enhancement of the  $\Delta R_\square(T)$  dependence was found experimentally<sup>11</sup> on increasing  $H_\perp$ , with the coefficient  $A$  reaching a plateau at  $A^H \approx 1$ . An increase in  $A$  is also observed for a parallel field  $H_\parallel$  (Fig. 3).

It follows from the results that the contribution of the WEL effect at  $H = 0$  is negative, i.e., the "antilocalization" case is achieved ( $\alpha = -1/2$ ,  $\tau_{\text{so}} \ll \tau_\varphi$ ). The value of the parameter  $A$  at saturation, determined by the effect of interaction (Eq. 4), is  $A^H = 1 - 3/4F - g(T)$ . It follows from the value obtained,  $A^H = 1$  that  $g(T) \approx 0$ . It cannot then be concluded that the parameter  $F$  is small, since the term proportional to  $F$  can be absent from Eq. (4) at  $F \neq 0$  in the case of strong spin-orbit interaction ( $\hbar/\tau_{\text{so}} \gg kT$ ). The value of  $\tau_{\text{so}}$  in the films studied was determined from results on magnetoresistance, to be discussed below:  $\tau_{\text{so}} \sim 3 \times 10^{-13}$  sec. This indicates that the last inequality is satisfied in the experiment.

By taking into account that  $A^H - A^{(H=0)} = \alpha p = -0.6$  and  $\alpha = -1/2$ , we find that  $p = 1.2$  and the

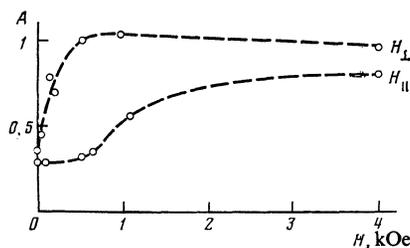


FIG. 3. Saturation of the temperature coefficient  $A$  in perpendicular and parallel magnetic fields ( $R_\square = 200 \Omega$ ).

law for the variation of  $\tau_\varphi$  with temperature:  $\tau_\varphi \propto T^{-1.2}$ . Mechanisms for phase destruction which lead to such a variation will be discussed below.

#### b) Measurement of magnetoresistance

A positive magnetoresistance with an anomalous dependence on  $H$  (close to logarithmic in sufficiently high fields)<sup>10</sup> was observed for all the films studied. The sign of the magnetoresistance corresponds to the WEL effect (Eq. (2)) for  $\alpha = -1/2$  (this value of  $\alpha$  was determined from the temperature dependence of the resistivity). Using Eq. (2) to evaluate the results obtained in a field up to 5 kOe for the case when  $\tau_{\text{so}} \ll \tau_\varphi$ ,  $L_H \ll L_T$ , showed that they are well described by the theory. The experimental value  $\alpha \approx -0.45$  agrees with the theoretical  $\alpha = -1/2$ .

Measurement of the angular dependences showed that the magnetoresistance of the films is of two-dimensional character. It is mainly determined by the component of field normal to the film,  $H_\perp$  (Fig. 4).

The perpendicular magnetoresistance can thus be interpreted as a manifestation of the WEL effect for strong spin-orbit coupling. The contribution of the EEI effect (Eq. (5)) is small because of the small  $g(T)$ , confirming the results of measurements of the temperature dependence of resistivity. The isotropic component of magnetoresistance (the first term in Eq. (5)) in bismuth should be noticeable because of the large value of the  $g_L$  factor: the spin splitting  $g_L \mu H \sim 4$  meV for  $H = 50$  kOe,  $H \parallel C_3$  (the parameters of bulk Bi are used<sup>19</sup>). The condition  $g_L \mu H / kT \gg 1$  could therefore be satisfied under the experimental conditions. However, the strong spin-orbit interaction ( $\hbar/\tau_{\text{so}} \sim 2$  meV) smears out the singularity in the electron density of states<sup>20</sup> and suppresses the correction to the conductivity (Eq. (5)) associated with it. This fact also agrees with the results of our tunneling experiment, in which it was not possible to observe singularities in the electron density of states in Be films at distances  $\pm g_L \mu H$  from the Fermi level.

The parallel magnetoresistance of the films was positive

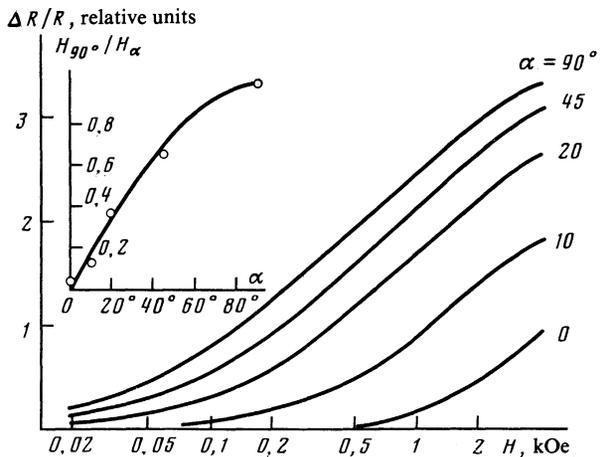


FIG. 4. Dependences of the resistance of a Bi film ( $R_\square = 240 \Omega$ ) on the strength of a magnetic field at an angle  $\alpha$  to the surface of the film. In the inset  $H_\alpha$  is the field strength corresponding to a fixed value of  $\Delta R$  ( $\Delta R(H_\alpha) = \Delta R(H_{90^\circ}) = \text{const}$ ); full line—the dependence  $H_{90^\circ} = H_\alpha \sin \alpha$ , characteristic of the two-dimensional situation.

and appeared in larger (compared with the perpendicular magnetoresistance) magnetic fields. Its value was the same for longitudinal and transverse configurations ( $\mathbf{H} \parallel \mathbf{I}$  and  $\mathbf{H} \perp \mathbf{I}$ ,  $\mathbf{I}$  is the current in the specimen). There is, however, considerable difficulty in comparing experiment with theory (Eq. (6)) since the residual contribution to the magnetoresistance, connected with  $H$  being off-parallel, and also the effect of the classical size effect have to be taken into account.

Equation (2) in the WEL theory is given for the limiting case of strong or weak spin-orbit interaction. In the intermediate region the following expression<sup>5</sup> is valid:

$$\Delta\sigma_{\square}(H) = \frac{e^2}{2\pi^2\hbar} \left[ \frac{3}{2} f_2(\gamma\tau_{\varphi}^*) - \frac{1}{2} f_2(\gamma\tau_{\varphi}) \right], \quad (8)$$

$$\tau_{\varphi}^{-1} = \tau_E^{-1} + 2\tau_s^{-1}, \quad (\tau_{\varphi}^*)^{-1} = \tau_E^{-1} + 2/3\tau_s^{-1} + 1/3\tau_{so}^{-1}.$$

According to Eq. (8) the positive magnetoresistance in large fields should have a maximum and a change of sign in the dependence for  $\gamma\tau_{\varphi}^* > 1$ . The position of the maximum depends on the ratio of the times  $\tau_{\varphi}$  and  $\tau_{so}$ . Measurements of the perpendicular magnetoresistance were carried out in fields  $H$  up to 60 kOe and over a wider temperature range,  $T = 1.5$ –20 K, in order to obtain information on the magnitude of  $\tau_{so}$ . The experimental curves agree much better with the full expression (8) than with the asymptotic Eq. (2). A series of experimental field dependences at various temperatures is shown in Fig. 5 for Bi films. The dashed, theoretical curves from Eq. (8), are obtained by varying two parameters:  $L_{\varphi} = (D\tau_{\varphi})^{1/2}$  and  $L_{\varphi}^* = (D\tau_{\varphi}^*)^{1/2}$ . The difference between the experimental and theoretical curves at high fields is proportional to  $H^2$ . From this, the addition to the WEL effect can be considered to come from the classical magnetoresistance and the value of the carrier mobility can be estimated from the compensated semimetal model. The value of  $\mu$  for the series of curves in Fig. 5 is  $\sim 150$  cm<sup>2</sup>/V sec. The value  $D \sim 3$  cm<sup>2</sup>/sec can be estimated from the Einstein relation and the  $\tau_{\varphi}(T)$  and  $\tau_{so}(T)$  dependences can be found. The value  $3 \times 10^{-13}$  sec is obtained for  $\tau_{so}$ , independent of  $T$ , invariant as the film is degraded (increase in resistivity) and is the same for films of different thickness. These properties are a confirmation of the correctness in determining the value of  $\tau_{so}$ .

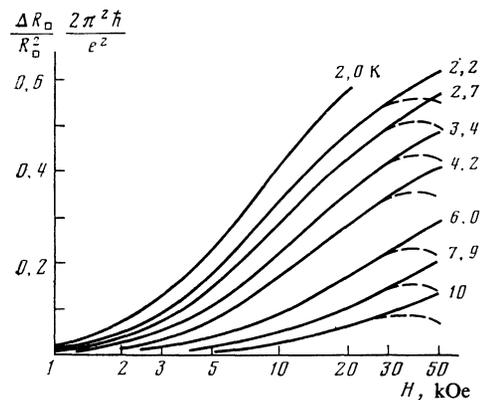


FIG. 5. Dependence of the resistance of a Bi film on the strength of a perpendicular magnetic field at different temperatures. Dashed lines— theoretical dependences (Eq. 7);  $R_{\square} = 5.7$  K  $\Omega$ .

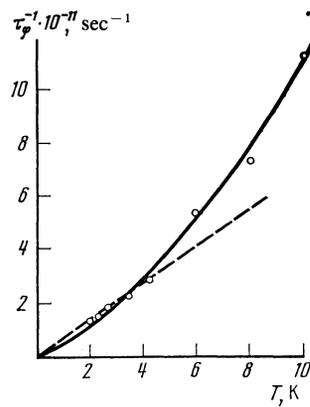


FIG. 6. Dependence of the time of loss of phase coherence  $\tau_{\varphi}$  on temperature for a Bi film ( $R_{\square} = 5.7$  K  $\Omega$ ). The dashed line corresponds to the relation  $\tau_{\varphi} \propto T^{-1}$ , the full line to  $\tau_{\varphi}^{-1} = aT + bT^2$ .

The  $\tau_{\varphi}(T)$  dependence in the temperature range  $T = 1.6$ –4 K agrees with the dependence found earlier<sup>11</sup> by analyzing the results in fields  $H < 10$  kOe, without taking account of  $\tau_{so}$  ( $\tau_{\varphi} \propto T^{-1}$ ) and with the results given above of analyzing the  $\Delta R_{\square}(T)$  curves ( $\tau_{\varphi} \propto T^{-1/2}$ ). In the temperature range  $T > 4.2$  K an increase in the power of the temperature dependence of  $\tau_{\varphi}$  is observed (Fig. 6). It can be deduced from the value  $p \approx 1$  that the main mechanism of phase destruction in the films studied is electron-electron scattering in the “dirty” limit.<sup>21</sup> The  $\tau_{\varphi}(T)$  relation is well described by the expression  $\tau_{\varphi}^{-1} = aT + bT^2$  (Fig. 6). The value of the coefficient  $a$  agrees in order of magnitude with the theoretical value<sup>21</sup>

$$a = \frac{\pi k R_{\square} e^2}{\hbar 2\pi^2 \hbar} \ln \frac{\pi \hbar}{e^2 R_{\square}}.$$

For a variation in  $R_{\square}$  over the range 150–6500  $\Omega$ ,  $T_{\varphi}$  (4 K) varies between the limits  $5 \times 10^{-12}$ – $2 \times 10^{-10}$  sec. The value of  $b$  is much greater than the theoretical value for electron-electron scattering:

$$b_{th} \approx \pi^2 k^2 / 64 e_F \hbar = 4 \cdot 10^7 \text{ K}^{-2} \cdot \text{s}^{-1}, \quad b_{exp} \approx 10^{10} \text{ K}^{-2} \cdot \text{s}^{-1}$$

This evidently indicates that at high temperatures other inelastic mechanisms make a contribution to loss of phase coherence, apart from electron-electron scattering.

The time  $\tau_s$  is independent of  $T$ . Since in the experiments there is no saturation of the temperature dependence of resistivity down to  $T = 0.2$  K, the value of  $\tau_s$  is at least an order of magnitude greater than  $\tau_{\varphi}$  (4 K) in the specimens studied.

The results given were obtained on films with resistivity lying in the range  $R_{\square} = 150$ –6500  $\Omega$ . Their thickness did not exceed 300 Å and satisfied the condition for two-dimensionality,  $L_{\varphi} \gg d, L_T \gg d$  (the experimental values of  $L_{\varphi}$  were  $\sim 800$  Å for films with thickness  $d \sim 50$  Å and  $L_{\varphi} \sim 3000$  Å for  $d \sim 300$  Å; correspondingly  $L_T \approx 200$  Å and  $L_T \approx 800$  Å).

## 2. Results of measuring low-resistance ( $R_{\square} = 40$ –150 $\Omega$ ) films ( $d > 250$ Å)

It was established when measuring the  $\Delta R_{\square}(T)$  relation that the coefficient  $A$  in the logarithmic dependence increases on reducing  $R_{\square}$  below 150  $\Omega$  (see Fig. 2). This in-

crease cannot be explained by the temperature dependence of the current carrier concentration, an effect which can be comparable in magnitude with the WEL and EEI effects in bismuth films with a low value of  $R_{\square}$ . The temperature dependence of concentration calculated according to the results of Belov and Serova<sup>22</sup> for bulk bismuth is clearly not of logarithmic form, which contradicts the experiment.

The increase in the coefficient  $A$  in low-resistance specimens can be associated with the change in effective dimensionality of a film as its thickness is increased. If the experimental temperature range is small, then in the thickness transition region from two-dimensional to three-dimensional, the  $\Delta\sigma_{\square}(T)$  relation will be almost of logarithmic form with an increased coefficient  $A$ . This assertion was confirmed by us for the temperature dependence of the interference correction in the thickness region  $d \sim L_{\varphi}$  plotted according to Volkov<sup>23</sup> in the range 1.5–4.2 K.

Calculations of  $L_{\varphi}$  and  $L_T$  show that low-resistance films are two-dimensional with respect to the WEL effect ( $d < L_{\varphi}$ ) and satisfy the condition for a transition region for the EEI effect ( $d \sim L_T$ ). For example, we have  $L_{\varphi} \sim 3500 \text{ \AA}$  and  $L_T \sim 700 \text{ \AA}$  for films with  $R = 60 \Omega$ , thickness  $1000 \text{ \AA}$  with mobility  $\mu \sim 10^3 \text{ cm}^2 \text{ V}^{-1} \cdot \text{s}^{-1}$  at  $T = 1.6 \text{ K}$ . The experimental value of  $A^H$  (the temperature coefficient in a high field) for low resistance films also exceeded the value  $A^H \approx 1$  obtained for high resistance films. Since  $A^H$  corresponds to the EEI contribution, this fact confirms the suggestion that low resistance films do not satisfy the requirement of two-dimensionality for the EEI theory.

An increase in the slope of the logarithmic part of the magnetoresistance curves in fields  $H < 10 \text{ kOE}$  is observed for low-resistance films as well as the features of the temperature dependence of  $\sigma$ . Estimates of  $L_{\varphi}$  and  $L_H$  show that this result can also be associated with the change in dimensionality of the films: the transition to the three-dimensional case for the WEL effect for satisfaction of the condition  $d \ll L_H \ll L_{\varphi}$ .

#### 4. CONCLUSIONS

The observed anomalous  $T$  and  $H$  dependences of resistivity of thin bismuth films are described well by the theories of weak localization for the case of strong spin–orbit interaction and electron–electron interaction in a disordered system. Both effects occur in the temperature dependence of the conductivity and separate out in a magnetic field; localization dominates in the field dependence of the positive magnetoresistance. The main mechanism for phase destruction in the temperature region  $T = 1.6\text{--}4 \text{ K}$  is then electron–electron interaction, modified for “dirty” systems. Singularities in the manifestation of quantum effects are observed in films with a low value of  $R_{\square}$ , which can be explained by a change

in the effective dimensionality of the specimens: a change from the two to the three-dimensional case. A transition then takes place in the temperature dependence of  $\Delta\sigma_{\square}$  for interelectron interaction; a change in dimensionality for localization occurs in the  $\Delta\sigma_{\square}(H)$  dependence.

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<sup>1</sup>This term was introduced by Bergmann.<sup>6</sup>

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