Nonlinear cyclotron resonance in a normal magnetic field

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This paper reports on the generation of the second harmonic by a bismuth single crystal in a magnetic field that is normal to the crystal's surface. A resonance peak in generation was observed in the magnetic field. This peak is due to the nonlinear cyclotron resonance of conduction electrons in the central cross section of the Fermi surface.

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INTRODUCTION

The surface impedance of a semi-infinite metal in a magnetic field normal to its surface has been studied frequently. As was shown in Ref. 1, the surface impedance in the anomalous skin effect does not depend on the magnetic field in first-order approximation. In the succeeding approximations in the anomaly parameter, components appear in the surface impedance that depend nonanalytically (as $\tau\rightarrow\infty$) on the magnetic field[2] (τ is the time of free flight). These features are connected with the electrons near the elliptic limiting points (if there are such) and also with the electrons of those parts of the Fermi surface where extrema of the quantities $\partial S/\partial p_y$ and $m^*(p_y)$ are reached simultaneously. Here $S(p_y)$ is the area of the intersection of the Fermi surface with the plane p_y = const and $m^*(p_y)$ is the cyclotron mass, while the magnetic field is directed along the y axis.

The following must be noted. Although the linear conductivity in the first-order approximation in the anomaly does not depend on the magnetic field, a significant contribution to it and, it seems, to the surface impedance also, can be made by electrons near the central cross section, which remain for a long time in the skin layer and which interact resonantly with the electromagnetic field. Actually, by using the well-known expression for the conductivity tensor $\sigma_+(k,\omega)$, it is not difficult to establish the fact that at $\omega=\Omega,\omega\tau\gg 1,k\sim\delta^{-1}\gg\omega/V_F$ (ω is the frequency of the wave, k is the wave vector, Ω is the cyclotron frequency, V_F is the Fermi velocity, δ is the skin depth) the basic contribution to $\sigma_+(k,\omega)$, and to the corresponding surface impedance is made by electrons with velocities $V_y\lesssim\delta/\tau$. In the given situation, the resonant growth of the nonequilibrium increment to the distribution function, which is linear in the electric field in the range of velocities $V_y\lesssim\delta/\tau$ is compensated in the calculation of the conductivity $\sigma_+(k,\omega)$, by the smallness of this region. The surface impedance of Bi in a normal magnetic field has been studied in a number of works.[3,4] In Refs. 5 and 6, the generation of the second harmonic in bismuth under the Azbel'-Kaner conditions of cyclotron resonance has been studied. Here the magnetic field lies in the plane of the sample. In the present work, we have investigated the generation of the second harmonic in a geometry in which the magnetic field is perpendicular to the surface of the sample. From symmetry considerations, it is seen that in such a geometry of the experiment, anisotropy of the dispersion law of the carriers in necessary, since in the isotropic case the second harmonic will not be radiated. In an oblique field, the generation of the second harmonic is possible even in the isotropic case. As will be shown below, in the case of an anisotropic dispersion law for the carriers, a strong resonance takes place at $\omega\simeq\Omega$ i.e., the radiation of the second harmonic is many times greater at resonance than far from resonance. The point is that the nonequilibrium contribution to the distribution function, which is quadratic in the amplitude of the electromagnetic field, experiences much stronger resonance growth than the linear at $\omega\approx\Omega$ and in the region of velocities $\overline{V}_y\simeq\delta/\tau$ (where $\overline{V}_y$ is the mean over a single period of the value of the velocity along the magnetic field). Therefore, electrons with velocities $\overline{V}\lesssim\delta/\tau$ can make the principal contribution to the nonlinear conductivity at the frequency of the second harmonic, increasing with increase in τ. Consequently, even in the fundamental approximation in the anomaly, there is resonance with the reflected second harmonic, due to the electrons that are close to the central cross section.

We shall show, using an illustrative example, whence the resonance in the radiation of the second harmonic can come. We consider the Fermi surface in the form of a severly elongated, inclined ellipsoid of revolution. As will be shown below in the section "Theory," the principal contribution to the resonance is made by electrons close to the central cross section $\overline{V}_y\lesssim\delta/\tau$. For simplicity we shall approximate this region of the Fermi surface by a cylinder inclined to the surface of the sample at an angle α (Fig. 1). The metal occupies

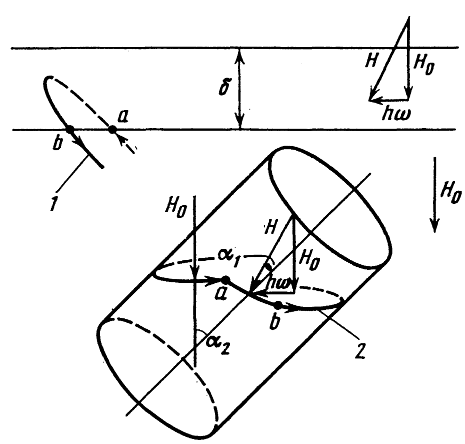

FIG. 1. 1—trajectory of electron in coordinate space; 2—orbit of electron in momentum space; a and b—points of electron entry into and departure from the skin layer.

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the half-space $y > 0$ and is located in a constant magnetic field H_0. We denote the alternating field in the skin layer (depth δ) by $\mathbf{h}_\omega$. We consider an electron moving in the magnetic field. Its orbit in real space is a circle of radius r_H, which depends on the magnitude and direction of the magnetic field. Let α_1 and α_2 be the angles between the directions of the magnetic field $H_0 + h_\omega$ and H_0, respectively, and the cylinder axis. We shall assume that the time of flight in the skin layer τ_{fl} is much smaller than the period of rotation in the magnetic field $2\pi/\Omega$ and the period of the electromagnetic field $2\pi/\omega$. At the moment when the electron flies into the skin layer, it begins to move over the cross section perpendicular to the acting magnetic field in the skin layer $\mathbf{H}$, and its cyclotron mass in the skin layer is

$$m_1^* = m_0^*/\cos\alpha_1,$$

where m_0^* is the cyclotron mass at $\alpha = 0$. Outside the skin layer,

$$m_2^* = m_0^*/\cos\alpha_2.$$

Because of the difference in directions of the magnetic field, the cyclotron mass in and out of the skin layer are different. This leads to a change in the time of flight in the skin layer:

$$\tau_{fl} = \left(\frac{4\delta}{V_F\Omega_0 \sin 2\alpha_2}\right)^{1/2}\left(1 + \mathrm{tg}\,\alpha_2 \frac{h_\omega}{H_0}\right) = \tau_0 + \tau_\omega,$$

where τ_0 is the part of τ_{fl} independent of h_ω, Ω_0 is the cyclotron frequency at $\alpha = 0$, and τ_ω is the part of τ_{fl} that depends linearly on h_ω.

Within a time τ_{fl} (t_1) the electron receives the following increment to its velocity:

$$\Delta\mathbf{V}(t_1) = \frac{e\mathbf{E}(t_1)}{m}\tau_{fl}(t_1), \qquad \mathbf{E}(t) = \mathbf{E}_0 \exp(-i\omega t),$$

where $\mathbf{E}(t)$ is the electric field in the skin layer, τ is the time of free flight, m is the mass in the direction of $\mathbf{E}(t_1)$ and e is the charge of the electron.

In order to find the velocity at the instant of time t, it is necessary to sum these increments over the re-entries into the skin layer, with account of damping:

$$\mathbf{V}(t) = \sum_{p=0}^{\infty} \Delta\mathbf{V}(t_p), \qquad t_p = t - p\frac{2\pi}{\Omega} + \sum_{n=0}^{p} \tau_\omega(t_n).$$

Under the condition $\omega^2\tau\tau_\omega \ll 1$ we have for the velocity at the frequency 2ω,

$$\mathbf{V}^{2\omega}_{(t)} = \frac{e\mathbf{E}_0\tau_0 i\omega\tau_\omega}{m}\,\frac{\exp\{i2\pi\omega_*/\Omega\}\exp(-2i\omega t)}{(1-\exp\{i2\pi\omega_*/\Omega\})(1-\exp\{i4\pi\omega_*/\Omega\})},$$

$$\omega_* = \omega + i\tau^{-1}.$$

It is seen from the formula that under the condition $\omega = n\Omega$, $n = 1,2$, the term $\mathbf{V}^{2\omega}$ increases resonantly by a factor $(\Omega\tau)^2$. Such a strong increase in the nonlinear current of resonant particles is no longer compensated by the smallness of their number: $N_{\rm eff} \propto 1/\tau$ and the total nonlinear current has a resonant character.

In the present model, we have considered the generation mechanism connected with the change in the time of flight in the skin layer. Complete account of all the mechanisms leading to generation of the second harmonic within the framework of Newton's equations of motion of the carriers is very complicated. We shall carry out the analysis and interpretation of the results on the basis of the solution of the Boltzmann kinetic equation for electrons in a magnetic field.

EXPERIMENT

In the experiment, we studied the generation of the doubled frequency in a single Bi crystal located in a magnetic field normal to its surface. The sample, which was placed at the bottom of a rectangular bimodal resonator (modes TE_{101}, TE_{202}), was irradiated by a microwave electromagnetic field at a frequency of $\omega/2\pi = 9.37$ GHz (see Fig. 2). The signal of double frequency was investigated, detected by a superheterodyne receiver. The source of the electromagnetic field at the frequency 9.37 GHz was a magnetron operating in the pulse mode: the pulse duration was $\tau_p = 2\,\mu$sec, the repetition rate 30 Hz. The power in the pulse was of the order of several tens of watts. To obtain a constant magnetic field, we used a Helmholtz system. The resonator with the sample was placed in a long metallic vessel, and these were in turn placed in the liquid helium; in this way it was possible to eliminate helium bubbles in the resonator. The experiment was conducted at $T = 4.2$ K.

Change in the pulse duration and the repetition frequency used in the study did not change the value of the useful signal. This permits us to state that the sample was not heated. The temperature near the sample was monitored by a germanium thermometer. Details of the experimental setup have been reported in Ref. 5.

The experiment was carried out on Bi samples having the shape of disks of diameter 18 mm and thickness 1 mm. The bisector axis C_1 was perpendicular to the plane of the sample. The investigations were carried out at various directions of the external microwave magnetic field $\mathbf{h}_\omega$ and of the magnetic field of the second harmonic $\mathbf{h}_{2\omega}$ relative to the crystallographic axes of the sample. Figure 3 shows the char-

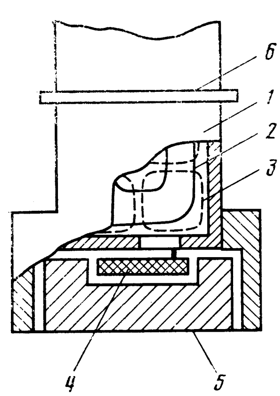

FIG. 2. 1—resonator- 2—lines of force of the magnetic field at the frequency ω (the TE_{101} mode); 3—lines of force of the magnetic field at the frequency 2ω (the TE_{202} mode); 4—sample; 5—sample holder; 6—diaphragm. The sample was irradiated through a rectangular window at the bottom of the resonator. The center of the window was displaced from the center of the bottom of the resonator in order that the radiation at the double frequency to be excited in the TE_{202} mode.

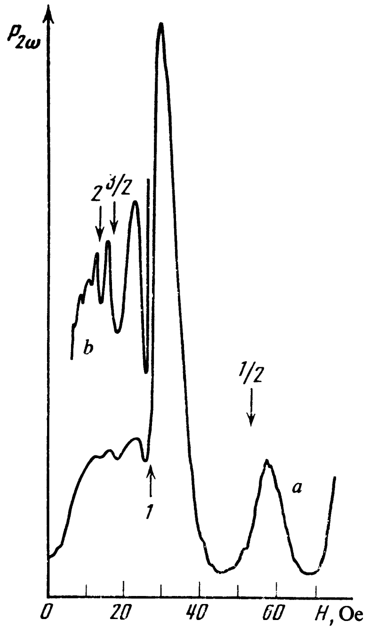

FIG. 3. Dependence of the amplitude of the second harmonic $P_{2\omega}$ on the constant magnetic field H: 1—theoretical position of the cyclotron resonance $\omega = \Omega$ of the electrons of the α ellipsoid; 1/2— resonance $2\omega = \Omega$; 3/2—resonance $2\omega = 3\Omega$; 2—resonance $\omega = 2\Omega$. The polarization is $\mathbf{h}_\omega \| \mathbf{h}_{2\omega} \| C_2$. The curve b is recorded with larger gain than a.

acter of the dependence of the second harmonic signal on the external magnetic field. The most intense generation of the second harmonic was observed in the region of a magnetic field corresponding to cyclotron resonance $\omega = \Omega$ from an almost vertical ellipsoid, which we designate as the α ellipsoid. (The angle between the long axis of the α ellipsoid and the normal to the surface of the sample **n** was about 6°.) To the left of it is observed a series of resonances at multiple fields of significantly lower intensity. The peak to the right corresponds to the resonance $2\omega = \Omega$. It should be noted that at such an orientation of the crystal in an external magnetic field $\mathbf{H}_0 \| \mathbf{n} \| C_1$ the cyclotron mass of the other two ellispoids (we denote these as the β and γ ellipsoids) are the same and equal to double the cyclotron mass of the vertical ellipsoid. Therefore, resonances from these two ellipsoids lie in

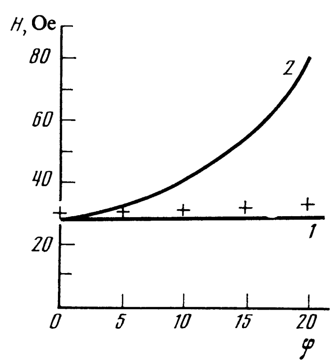

FIG. 4. Dependence of the position of cyclotron resonance $\omega = \Omega$ on the angle φ between C_1 and $\mathbf{H}$ in rotation in the binary-bissector plane: 1—theoretical location of the resonance for the α ellipsoid; 2—for the β ellipsoid; + —experimental position of the vertex of the fundamental peak.

the doubled magnetic field and can be superimposed on the resonances from the vertical ellipsoid. On the other hand, the cyclotron masses of these two ellipsoids are more sensitive to the inclination of the magnetic field than the cyclotron mass of the vertical ellipsoid. In the experiment we could rock the magnetic field direction from -20 to $+20°$ both in the plane formed by the C_1 and C_3 axes, and in the plane formed by the C_1 and C_2 axes. Measurements showed that the singularities move in correspondence with the change in the cyclotron mass of the α ellipsoid; therefore, the observed feature is connected precisely with it (Fig. 4). Upon increase in the temperature of the sample to 10–15 K, the fundamental peak was shifted toward large magnetic fields by 2–3 Oe, became diffuse and then disappeared.

THEORY

We now turn to the theoretical consideration of the generation of the second harmonic in a normal magnetic field. In the solution of the problem of the reflection of an electromagnetic wave at the second harmonic, it is necessary to calculate the nonlinear current $\mathbf{j}^{(2)}(y)e^{-2i\omega t}$, created by the fields of the fundamental frequency in the semi-finite conductor. Since the character of the reflection of the electrons by the surface is not very significant in the given case, we limit ourselves to a simplified approach, assuming that the fields and the currents are excited in an unbounded conductor by an external current sheet.

We now consider an unbounded conductor. Let the electromagnetic field in it have the form

$$\begin{aligned} \mathbf{E}(y,t) &= \mathbf{E}(k_1)e^{ik_1y-i\omega t} + \mathbf{E}(k_2)e^{ik_2y-i\omega t}, \\ \mathbf{h}(y,t) &= \mathbf{h}(k_1)e^{ik_1y-i\omega t} + \mathbf{h}(k_2)e^{ik_2y-i\omega t}. \end{aligned} \tag{1}$$

We now find the nonlinear current $j_\alpha^{(2)}(y,t)$, created by the electromagnetic field (1) and proportional to $e^{i(k_1+k_2)y}$, i.e.,

$$j_\alpha^{(2)}(y,t) = j_\alpha^{(2)}(k, 2\omega)e^{iky-2i\omega t}, \quad k = k_1 + k_2. \tag{2}$$

We introduce the nonlinear conductivity tensor $\sigma_{\alpha\beta\gamma}(k_1, k_2)$ in the following manner:

$$\begin{aligned} j_\alpha^{(2)}(k, 2\omega) = &\hat{P}(k_1, \beta; k_2, \gamma)\sigma_{\alpha\beta\gamma}(k_1, k_2) \\ &\times E_\beta(k_1)E_\gamma(k_2), \end{aligned} \tag{3}$$

where $\hat{P}(k_1, \beta; k_2, \gamma)$ is the symmetrization operator over the wave vectors and the vector indices (summation over the permutations). The tensor $\sigma_{\alpha\beta\gamma}$ is represented in the form

$$\sigma_{\alpha\beta\gamma}(k_1, k_2) = \overset{(h)}{\sigma}_{\alpha\beta\gamma}(k_1, k_2) + \overset{(E)}{\sigma}_{\alpha\beta\gamma}(k_1, k_2), \tag{4}$$

where the first term is the contribution to the nonlinear conductivity due to the Lorentz force, while the second is due to the nonlinearity which is quadratic in the electric field.

For the description of the location of the electron in momentum space we introduce, as usual, the variables ε, p_y and φ where ε is the energy, p_y is the projection of the momentum of the electron of the magnetic field $\mathbf{H}$ and φ is the phase ($\varphi = \Omega t_1$), where t_1 is the time of motion along the trajectory in a constant magnetic field, and Ω is the cyclotron frequency).

Solving the kinetic equation, we can obtain an expression for both terms in (4). Omitting the intermediate calculations, we write out the results at once. The nonlinearity due to the electric field is

$$\sigma_{\alpha\beta\gamma}^{(E)}(k_1,k_2)=-i\frac{e^3}{2\pi^2}\sum_{n,l}\int d\varepsilon\, dp_y m^*$$
$$\times\left\{V_{l+n}^{\alpha*}(k)\left[l\left(\frac{\partial\varphi}{\partial p_\gamma}\right)_n(k_2)-k_1\left(\frac{\tilde V_H}{\Omega}\frac{\partial\varphi}{\partial p_\gamma}\right)_n(k_2)\right]V_l^\beta(k_1)\right.$$
$$\times[k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}]^{-1}[k_1\bar V_H+l\Omega-\omega-i\tau^{-1}]^{-1}$$
$$-i\frac{V_{l+n}^{\alpha*}(k)V_n^\gamma(k_2)}{k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}}\frac{\partial}{\partial\varepsilon}\frac{V_l^\beta(k_1)}{k_1\bar V_H+l\Omega-\omega-i\tau^{-1}}$$
$$-\frac{V_{l+n}^{\alpha*}(k)k_1F_n^\gamma(k_2)V_l^\beta(k_1)}{[k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}][k_1\bar V_H+l\Omega-\omega-i\tau^{-1}]}$$
$$-\frac{\dot V_{l+n}(k)\delta_{\gamma y}I_n(k_2)}{k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}}\frac{\partial}{\partial p_y}\frac{V_l^\beta(k_1)}{k_1\bar V_H+l\Omega-\omega-i\tau^{-1}}$$
$$\left.-\frac{\dot V_{l+n}(k)k_1\Phi_n(k_2)\delta_{\gamma y}V_l^\beta(k_1)}{[k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}][k_1\bar V_H+l\Omega-\omega-i\tau^{-1}]}\right\}\frac{\partial f_0}{\partial\varepsilon},\quad(5)$$

where $\bar V_H$ is the average, over the period, of the projection of the velocity on the y axis along which the field $\mathbf{H}$ is directed, the alternating part of the velocity is $\tilde V_H = V_y - \bar V_H$, m^* is the cyclotron mass, n and l are integers, and f_0 is the equilibrium distribution function.

In addition,

$$V_l^\alpha(k)=\frac{1}{2\pi}\int_0^{2\pi}d\varphi V^\alpha(\varphi)\exp\left\{-il\varphi+\frac{i}{\Omega}\int_0^\varphi d\varphi' k\tilde V_H(\varphi')\right\},$$
$$\left(\frac{\partial\varphi}{\partial p_\gamma}\right)_l(k)=\frac{1}{2\pi}\int_0^{2\pi}d\varphi\frac{\partial\varphi}{\partial p_\gamma}\exp\left\{-il\varphi+\frac{i}{\Omega}\int_0^\varphi d\varphi' k\tilde V_H(\varphi')\right\},\quad(6)$$
$$\left(\frac{V_H}{\Omega}\frac{\partial\varphi}{\partial p_\gamma}\right)_l(k)=\frac{1}{2\pi}\int_0^{2\pi}d\varphi\frac{\tilde V_H}{\Omega}\frac{\partial\varphi}{\partial p_\gamma}\exp\left\{-il\varphi+\frac{i}{\Omega}\int_0^\varphi d\varphi' k\tilde V_H(\varphi')\right\}.$$

The quantities $F_l^\gamma(k)$, $\Phi_l(k)$, $I_l(k)$ are expressed in terms of the functions $F_l(\varphi)$, $\Phi(\varphi)$, $I(\varphi)$, respectively, according to formulas similar to Eqs. (6), namely,

$$F^\gamma(\varphi)=V^\gamma(\varphi)\int_0^\varphi d\varphi'\frac{\partial}{\partial\varepsilon}\left(\frac{\tilde V_H(\varphi')}{\Omega}\right),$$
$$\Phi(\varphi)=\int_0^\varphi d\varphi'\frac{\partial}{\partial p_y}\left(\frac{\tilde V_H(\varphi')}{\Omega}\right),\qquad I(\varphi)=1.\quad(7)$$

The nonlinearity due to the magnetic field is

$$\sigma_{\alpha\beta\gamma}^{(h)}(k_1,k_2)=i\frac{e^3}{2\pi^2}\frac{1}{\omega}\sum_{n,l}\int_{\varepsilon=\varepsilon_F}dp_y m^*$$
$$\times\left\{\frac{V_{l+n}^{\alpha*}(k)[lk_2W_n^\gamma(k_2)-k_1k_2(\tilde V_H W^\gamma/\Omega)_n(k_2)]V_l^\beta(k_1)}{[k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}][k_1\bar V_H+l\Omega-\omega-i\tau^{-1}]}\right.$$
$$-i\frac{V_{l+n}^{\alpha*}(k)k_2(1-\delta_{\gamma y})V_n^\gamma(k_2)}{[k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}]}\frac{\partial}{\partial p_y}\frac{V_l^\beta(k_1)}{[k_1\bar V_H+l\Omega-\omega-i\tau^{-1}]}$$
$$\left.-\frac{V_{l+n}^{\alpha*}(k)k_1k_2(1-\delta_{\gamma y})P_n^\gamma(k_2)V_l^\beta(k_1)}{[k\bar V_H+(l+n)\Omega-2\omega-i\tau^{-1}][k_1\bar V_H+l\Omega-\omega-i\tau^{-1}]}\right\},\quad(8)$$

where

$$W^\gamma=\frac{\partial\varphi}{\partial p_y}V^\gamma-\frac{\partial\varphi}{\partial p_\gamma}V^y,\quad P_{(\varphi)}^\gamma=V_{(\varphi)}^\gamma\int_0 d\varphi'\frac{\partial}{\partial p_y}\left(\frac{\tilde V_H(\varphi')}{\Omega}\right).$$

In obtaining Eqs. (5) and (8), we have assumed the relaxation time to be independent of the momentum.

Let the dispersion law have the form

$$\varepsilon(\mathbf{p})={}^1/_2\alpha_{\mu\nu}p_\mu p_\nu.\quad(9)$$

Solving the equations of motion in a constant magnetic field

$$\Omega\frac{\partial\mathbf{p}}{d\varphi}=\frac{e}{c}[\mathbf{V}\times\mathbf{H}],\quad(10)$$

we find the connection of the variables ε, φ, p_y with the components of the momentum $\mathbf{p}$. We shall use the following parametrization of the trajectory:

$$p_x=\bar p_x+\frac{\tilde p_x}{(A^2+B^2)^{1/2}}[A\cos\varphi-B\sin\varphi\,\mathrm{sign}(eH)],$$
$$p_z=\bar p_z-\frac{\tilde p_x}{(A^2+B^2)^{1/2}}\left[(\alpha_{xz}\alpha_{xy}-\alpha_{xx}\alpha_{yz})\cos\varphi-\frac{\alpha_{xy}}{m^*}\mathrm{sign}(eH)\sin\varphi\right],\quad(11)$$

where

$$\bar p_x=m^{*2}(\alpha_{zy}\alpha_{xz}-\alpha_{xy}\alpha_{zz})p_y,$$
$$\bar p_z=m^{*2}(\alpha_{xz}\alpha_{xy}-\alpha_{xx}\alpha_{zy})p_y,\quad(12)$$
$$\tilde p_x=m^*\alpha_{zz}^{1/2}\{2\varepsilon-[\alpha_{yy}+m^{*2}(2\alpha_{xy}\alpha_{zy}\alpha_{xz}-\alpha_{xy}^2\alpha_{zz}-\alpha_{yz}^2\alpha_{xx})p_y^2]\}^{1/2},$$
$$A=\alpha_{zz}\alpha_{xy}-\alpha_{yz}\alpha_{xz},\quad B=\alpha_{yz}/m^*.$$

Using (11), we can determine V^α, $\bar V_H$, $\tilde V_H$, $\partial\varphi/\partial\mathbf{p}$; substituting these expressions in (5), we find $V_n^\gamma(k)$, $F_n^\gamma(k)$, $W_n^\gamma(k)$, $\Phi_n(k)$, and so on. We shall not give the expressions for them here, since these calculations do not present any difficulties. We shall assume that the conditions of the anomalous skin effect are satisfied:

$$\frac{\omega\delta}{\bar V_{H\max}}\ll1,\quad\frac{\Omega\delta}{\bar V_{H\max}}\ll1,\quad\frac{\delta}{\bar V_{H\max}\tau}\ll1,$$

where $\bar V_{H\max}$ is the maximum value of the velocity $\bar V_H$. Taking Eqs. (5) and (8) into account, we can verify that the basic contribution to the nonlinear current and amplitude of the reflected second harmonic will be made by the Fourier components of the electromagnetic fields with wave vectors $|k|\sim|k_1|\sim|k_2|\sim\delta^{-1}$. Thus, we must investigate the obtained expressions (5) and (8) in the case $|k|\sim|k_1|\sim|k_2|\sim\delta^{-1}$.

It is seen from Eqs. (5) and (8) that the components of the tensors $\sigma^{(h)}$ and $\sigma^{(E)}$ can experience resonance upon satisfaction of the conditions $\omega = n\Omega$ (by virtue of (9) we have $\Omega(\varepsilon, p_y) = \mathrm{const}$), if $\Omega\tau\gg1$. Mathematically, this is connected with the fact that $kk_1<0$ poles relative to the variable p_y appear in the integrands of Eqs. (5) and (8); these poles lie on opposite sides of the real axis and approaching one another when the ratio ω/Ω approaches an integer. Of course, the resonance takes place only in the case of an anisotropic dispersion law.

It is clear that the principal contribution to the nonlinear conductivity at the resonance $\omega = n\Omega$ as $\tau\to\infty$ is made by electrons with velocities $\overline{V}_H \lesssim \delta/\tau$. These electrons can, moving out of the skin layer, return to it frequently (the dispersion law is anisotropic), and it is this which leads to resonance. If they do not emerge from the skin layer within the time $2\pi/\Omega$, however, the resonance is due to the inhomogeneity of the field in the skin layer.

In the linear approximation, a resonant increase in the distribution function of the electrons in the region of velocities $\overline{V}_H \lesssim \delta/\tau$, which takes place if $\omega = \Omega$, is compensated in the calculation of the current by the smallness of this region. As a result, the contribution to the current from the electrons taking part in the cyclotron resonance turns out, generally speaking, to be of the same order as the contribution of the nonresonant electrons with velocities $\overline{V}_H \lesssim \delta\omega$. In other words, the linear conductivity in the anomalous skin effect, as is well known, has no singularities at $\omega = \Omega$ (and also at $\omega = n\Omega$). The increment of the distribution function is quadratic in the electromagnetic field at $\omega = n\Omega$, $\tau\to\infty$ in the region $\overline{V}_H \lesssim \delta/\tau$ experiences a much greater increase than the linear increment, which is not compensated for by the smallness of the given region. Therefore, cyclotron resonance can appear in the nonlinear conductivity at $\omega = n\Omega$.

It is seen from Eqs. (5) and (8) that the components of the tensors $\sigma^{(E)}$ and $\sigma^{(h)}$ contain both contributions that increase as $\omega = n\Omega$, $\tau\to\infty$ in proportion to τ, and contributions that increase in proportion to τ^2. In the tensor $\sigma^{(E)}$ an increase that is proportional to τ^2 can be had only by those terms with indices $\alpha\beta y$. The corresponding resonance contribution (we denote it by $\bar{\sigma}^{(E)}_{\alpha\beta y}$) is connected with the penultimate term in the curly brackets of Eq. (5). It is not difficult to calculate it:

$$\bar{\sigma}^{(E)}_{\alpha\beta y}(k_1,k_2)=i\frac{e^3}{\pi}\frac{k_1}{|k_1|}\theta(-kk_1)\frac{m^*}{kk_1}\times\sum_n V_{2n}^{\alpha^*}(k)J_n\left(\frac{k_2\tilde{V}}{\Omega}\right)V_n^{\beta}(k_1)|_{\varepsilon=\varepsilon_F,p_y=0}\times\left[\frac{n(\Omega-\omega)-i\tau^{-1}}{k_1}-\frac{2n(\Omega-\omega)-i\tau^{-1}}{k}\right]^{-2}. \quad (13)$$

Upon satisfaction of (13), we have shown that $\tilde{V}$, and also the quantities $V_{2n}{}^{\alpha}(k)$, $V_n{}^{\beta}(k_1)$, $J_n(k_2\tilde{V}/\Omega)$ are practically independent of p_y if

$$|p_y|\lesssim\delta\tau^{-1}\left(\frac{\partial V_H}{\partial p_y}\right)^{-1}\Big|_{p_y=0}.$$

Similarly, we can calculate $\sigma^{(h)}_{\alpha\beta\gamma}$—the resonance term in (8) that increases most rapidly with increase in τ:

$$\bar{\sigma}^{(h)}_{\alpha\beta\gamma}(k_1,k_2)=i\frac{e^3}{\pi}\frac{k_1}{|k_1|}\theta(-kk_1)\frac{m^*}{kk_1}\times\frac{k_2(1-\delta_{\gamma y})}{\omega}\sum_n V_{2n}^{\alpha^*}(k)V_n^{\gamma}(k_2)V_n^{\beta}(k_1)|_{\varepsilon=\varepsilon_F,p_y=0}\times\left[\frac{n(\Omega-\omega)-i\tau^{-1}}{k_1}-\frac{2n(\Omega-\omega)-i\tau^{-1}}{k}\right]^{-2}. \quad (14)$$

We now proceed to the consideration of the nonlinearity in bismuth. We shall assume that the C_1 axis is identical with the y axis and the C_3 axis with the z axis. Also, we shall assume that $\omega = 2\pi\cdot 9.3\cdot 10^9$, while the parameter $\omega\tau = 10$–30. At this frequency, the depth of the skin layer in bismuth $\delta\approx 1.5\cdot 10^{-4}$ cm.

We now estimate the contribution to the nonlinearity of the electrons of the α ellipsoid. For the α ellipsoid, we have

$$\delta\omega/\tilde{V}_{max}=\delta\omega/\tilde{V}(p_y=0)\sim 1. \quad (15)$$

Consequently, the given situation is very favorable for the observation of the resonance $\omega = \Omega^{(\alpha)}$, where $\Omega^{(\alpha)}$ is the cyclotron frequency of the ellipsoid α. Actually, in this case,

$$J_1\left(\frac{k_{1,2}\tilde{V}}{\Omega^{(\alpha)}}\right)\Big|_{y=0}\sim 1;\quad V_2{}^{\alpha}(k)|_{p_y=0}\sim V_1{}^{\alpha}(k_{1,2})|_{p_y=0}\sim V_{F max}, \quad (16)$$

where $V_{F\max}$ is the maximum value of the velocity of the electrons on the Fermi surface. It follows from (15) that the electrons close to the central cross section move into the skin layer for about one-half of the cyclotron period. It should be noted that the skin effect for the electrons of the α ellipsoid is not extremely anomalous, since

$$\omega\delta/\overline{V}_{H\,max}\sim 1. \quad (17)$$

Nevertheless, the asymptotic formulas (13) and (14) "work" well near the resonance $\omega = \Omega^{(\alpha)}$, since the condition for their applicability is the fulfillment of the inequality

$$|\omega-\Omega+i\tau^{-1}|\delta/\overline{V}_{H\,max}\ll 1, \quad (18)$$

while in our case $\delta/\overline{V}_{H\max}\tau\sim 0.1$–$0.03$. By virtue of (15), the principal contribution to Eqs. (5) and (8) at $\omega\sim\Omega^{(\alpha)}$ will be made by the terms with n, $l\sim 1$.

As can be demonstrated, for the electrons of the α ellipsoid at $\omega\sim\Omega^{(\alpha)}$, far from resonance, the components of the tensors $\sigma^{(E)}$, $\sigma^{(h)}$ with indices $\alpha,\beta,\gamma\neq y$, are dominant, while

$$|\sigma^{(E)}_{\alpha\beta\gamma}|\sim|\sigma^{(h)}_{\alpha\beta\gamma}|\sim|\sigma^{(E)}_{zzz}(H=0)|\sim|\sigma^{(h)}_{zzz}(H=0)|,\quad \alpha,\beta,\gamma=x,z; \quad (19)$$

$$|\sigma^{(h)}_{zzz}(H=0)|\sim\frac{e^3}{2\pi}\frac{1}{\omega^2}\frac{p_1p_2}{m_1}\sin\beta_0\frac{\theta(-k,k_1)}{[(m_3/m_1)\cos^2\beta_0+\sin^2\beta_0]^{1/2}},$$

where p_1, p_2, p_3 are the semiaxes of the ellipsoid; m_1, m_2, m_3 are the effective masses, corresponding to the principal axes 1, 2, 3 (Ref. 7). Near the resonance $\omega = \Omega^{(\alpha)}$ at $\omega\tau\gg 1$, the components $\sigma^{(h)}_{\alpha\beta\gamma}$ with α, β, $\gamma = x$, z become the principal terms. Directly at the resonance $\omega = \Omega^{(\alpha)}$, we have

$$|\sigma^{(h)}_{\alpha\beta\gamma}|\sim|\sigma^{(h)}_{zzz}(H=0)|(\omega\tau)^2. \quad (20)$$

We now use the quadratic model of the electron spectrum (9), which is quite sufficient for the estimate of the nonlinearity. However, we note that account of the nonquadratic spectrum, for example, in the model of McClure and Choi,[8] leads [as can be established from (5)] to the appearance at resonance of components of the same order as (20) in $\sigma^{(E)}_{\alpha\beta\gamma}$. This circumstance is connected on the one hand with the fact that because of the nonquadratic nature of the dispersion law the cyclotron frequency $\Omega^{(\alpha)}$ depends significantly on the energy, and on the other hand, with the fact that the skin effect for electrons of the α ellipsoid is not strongly anomalous.

The cyclotron mass of the β and γ ellipsoids is twice as large as that for the α ellipsoid. Therefore, at $\omega = \Omega^{(\alpha)}$ the electrons of the β and γ ellipsoids are also located at resonance $\omega = 2\Omega^{(\beta)}$.

For electrons of the β and γ ellipsoids,

$$\delta\Omega/\bar{V}_{max} \sim 0{,}1. \tag{21}$$

Taking into account (21) Eqs. (5) and (8), and also (13) and (14), we can establish the fact that the resonant contribution of the β and γ ellipsoids to the nonlinearity at $\omega = 2\Omega^{(\beta)}$ is much smaller than the contribution of the α ellipsoid. This is connected with the fact that the electrons of the β and γ ellipsoids, even close to the central cross section, stay in the skin layer a time that is much shorter than the cyclotron period. Estimates show also that the nonresonant contribution of the nonlinearity of the β and γ ellipsoids is also much smaller than the resonant contribution of the ellipsoid.

In normal incidence of an electromagnetic wave on the bismuth sample, longitudinal fields are excited in it due to the anisotropy. As a result, even in our case, when the C_1 axis is directed along the normal to the surface, holes can make a contribution to the amplitude of the reflected second harmonic. However, the contribution turns out to be insignificant.

We see thus that at the resonance $\omega = \Omega^{(\alpha)}$ the nonlinear conductivity increases strongly and turns out to be due to the electrons of the α ellipsoid. In zero magnetic field, as can be shown, in the case in which the electric field of the incident wave is parallel to the z axis (the C_3 axis of bismuth), the contribution to the transverse nonlinear current of the β and γ ellipsoids is approximately an order of magnitude larger than the contribution of the α ellipsoid (we note that the nonlinear transverse current in the given case is also directed along the z axis). Taking this into account, and also the estimate (20), we see that the transverse nonlinear current at the resonance $\omega = \Omega^{(\alpha)}$ is about an order greater than in zero magnetic field. Inasmuch as the surface impedance and the skin depth in the region of magnetic fields at which $\Omega^{(\alpha)} \lesssim \omega$ are practically constant, it follows that the amplitude of the reflected second harmonic at the resonance $\omega = \Omega^{(\alpha)}$ is also about an order of magnitude greater than in zero magnetic field.

It is seen from Eq. (14) that the contribution to the nonlinear current made by an electron in one revolution in the magnetic field depends on the value of the field and is not a maximum at $\omega = \Omega^{(\alpha)}$, since $V_1^{\gamma}(k_1, k_2)$, $V_2^{\alpha}(k)$ depends on the magnetic field. It must therefore be expected that the amplitude of the reflected second harmonic as a function of the magnetic field will have its maximum somewhat shifted (by about the width of the resonance line) relative to the position determined by the equation $\omega = \Omega^{(\alpha)}$.

It should be noted that in spite of the closeness of the observed resonance to the resonance from the point of the almost vertical ellipsoid, it must be assumed that the observed resonance singularity in the generation is connected with the electrons of the central cross section of the Fermi surface, since these electrons interact much more effectively with the electromagnetic field in the skin layer than the electrons of the limiting point, which depart from the region of interaction after less than the period of the UHF field.

CONCLUSION

The resonance in the nonlinear response observed in the research is completely described within the framework of the theory developed. However, the exact location of the resonance and also the line shape require numerical calculation. The strong increase in generation of the second harmonic in the region of cyclotron resonance allows us to hope that the nonlinear cyclotron resonance can be observed not only in metals of the bismuth type, but also in metals with a large number of carriers.

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