

# Interaction of helicons and electromagnetic drift waves in semimetals

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A new mechanism is proposed for an instability of electromagnetic waves in multivalley conductors. This instability consists of a spatial separation of the density fluctuations of carriers of different type by the Lorentz-force component longitudinal with respect to  $\mathbf{k}$ . This force is caused by the alternating magnetic field of the wave. This instability mechanism operates when the transverse components of the drift velocities of the carriers of different types are unequal. The propagation of helicons and intervalley drift waves in a conductor with two types of carrier is analyzed for the case in which the conductor is in crossed electric and magnetic fields. The growth rates are derived. The nature of the instabilities and the conditions for their occurrence are studied.

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In multivalley semimetals and semiconductors at sufficiently low temperatures, the frequency  $\nu_B$  of intragroup carrier scattering, may be much higher than the frequency  $\nu_M$  of scattering with umklapp ( $\nu_B \gg \nu_M$ ). The electrons of different groups, at thermodynamic equilibrium within their own valleys, may have densities that do not correspond to an equilibrium of the overall system, which remains electrically neutral on the whole. Such a redistribution of electrons among groups has important consequences for the electrical conductivity of semiconducting and semimetal films<sup>1-5</sup> and for the propagation of acoustic<sup>6-10</sup> and electromagnetic<sup>11,12</sup> waves.

In this paper we show that the nonequilibrium filling of the valleys in a multicomponent solid-state plasma with a drift may cause a specific electromagnetic-wave instability associated with the alternating magnetic field of the wave. The amplification of electromagnetic waves in solids has been studied by several investigators (see Refs. 13–16, for example). It has generally been assumed that the light carriers of a bipolar conductor are magnetized by the external magnetic field  $\mathbf{H}_0$ , while the heavy carriers are not.<sup>14-16</sup> It has been the relative motion of the carriers along the direction of the wave vector  $\mathbf{k}$  which has caused the waves to become unstable. Furthermore, most of the studies (Ref. 14 is an exception) have dealt with cases in which the carriers drift along the direction of  $\mathbf{H}_0$ . In Ref. 14 the amplification resulted from a Hall drift, but the holes were assumed immobile (this was an extremely important assumption).

We derive the wave spectrum here for external fields in the Hall configuration under the assumption that the carriers of both types are magnetized. The amplification is caused by either a dissipative drift perpendicular to  $\mathbf{k}$  or by the difference between the nondissipative components of the drift velocity if there is a complicated carrier dispersion law. The conductor may be either bipolar or monopolar with several groups of electrons. The instability mechanism has no threshold, in contrast with the Čerenkov mechanism. The instability consists of a spatial separation of the fluctuations of the densities of the different types of carrier, caused by the longitudinal ( $\parallel \mathbf{k}$ ) Lorentz force resulting from the alternat-

ing magnetic field of the wave. The electrostatic fields which arise interact with the longitudinal field of the wave and cause it to grow. To the best of our knowledge, this instability mechanism has not been discussed previously in the literature.

## 1. DISPERSION RELATION FOR THE COUPLED WAVES

We consider an uncompensated semimetal or semiconductor with two groups of carrier, in uniform, static, and mutually perpendicular electric and magnetic fields (Fig. 1). For each group we assume

$$\omega, \nu_M \ll \nu_B \ll \Omega, \quad kl \ll 1, \quad (1)$$

where  $\omega$  is the wave frequency,  $\Omega$  is the cyclotron frequency, and  $l$  is the mean free path.

The system of equations comprises Maxwell's equations (without a displacement current), the continuity equations for the two types of carrier,

$$\frac{\partial n_\alpha}{\partial t} - \frac{1}{e} \operatorname{div} \mathbf{j}_\alpha + \nu_M n_\alpha = 0, \quad (2)$$

and material equations for the partial currents,

$$\mathbf{j}_\alpha = -eN_\alpha \mathbf{V}^\alpha + \hat{\sigma}^\alpha (\mathbf{E} + c^{-1} [\mathbf{V}^\alpha \times \mathbf{H}]) + e\hat{D}^\alpha \nabla n_\alpha. \quad (3)$$

Here  $\mathbf{E}$  and  $\mathbf{H}$  are the alternating fields;  $\mathbf{j}_\alpha$ ,  $\mathbf{V}^\alpha$ ,  $n_\alpha$ , and  $N_\alpha$  are respectively the current, drift velocity, nonequilibrium density, and total density of the carriers of type  $\alpha$ ;  $\hat{\sigma}^\alpha$  and  $\hat{D}^\alpha$

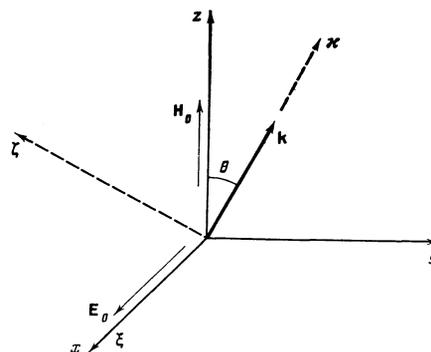


FIG. 1.

are the conductivity and diffusion tensors;  $e$  is the modulus of the electron charge; and  $c$  is the speed of light. System (2), (3) is written in a fixed coordinate system, since the Hall drift velocities of the carriers are generally different in the case of an anisotropic dispersion law.

We consider waves which are propagating in a plane perpendicular to  $\mathbf{E}_0$  at an arbitrary angle  $\theta$  to the magnetic field  $\mathbf{H}_0$  (Fig. 1); we do not consider the case of nearly perpendicular propagation, in which helicons cannot exist:  $|\pi/2 - \theta| \gg \gamma = v_B/\Omega$ . Assuming that all the alternating quantities in (2) and (3) are proportional to  $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ , we can reduce the system of equations to

$$[-i(\omega - kV_d) + \Gamma_d^0]n_\alpha - \frac{ik}{\omega} [(\omega - kV_d)s_{\alpha\nu} + k\tilde{W}_\nu\Sigma_{\alpha\nu}] \mathcal{E}_\nu = 0, \\ \left( \frac{ik}{e^2 Q} s_{\mu\alpha} - W_\mu \right) n_\alpha + \frac{1}{\omega} \left[ \omega \tilde{\sigma}_{\mu\nu} - k(\tilde{\sigma}_{\mu\nu}^\alpha V_\nu^\alpha + \tilde{\sigma}_{\mu\nu}^\beta V_\nu^\beta - s_{\mu\alpha} \Delta V_\nu) - \delta_{\mu\nu} \frac{c^2 k^2}{4\pi i} \right] \mathcal{E}_\nu = 0. \quad (4)$$

Here  $\Gamma_d^0 = \nu_M + k^2 D_{\chi\chi}$  is the damping rate of the noninteracting drift wave, whose existence in a two-component solid-state plasma was first pointed out in Ref. 11. The indices  $\mu$  and  $\nu$  run over the values of  $\xi$  and  $\zeta$ ;

$$V_d = \sigma_{xx}^{-1} (V_x^\alpha \sigma_{xx}^\beta + V_x^\beta \sigma_{xx}^\alpha), \\ D_{xx} = \sigma_{xx}^{-1} (D_{xx}^\alpha \sigma_{xx}^\beta + D_{xx}^\beta \sigma_{xx}^\alpha), \\ \Sigma_{xx} = \sigma_{xx}^\alpha \sigma_{xx}^\beta / \sigma_{xx}, \\ s_{ik} = \sigma_{xx}^{-1} (\sigma_{ik}^\alpha \sigma_{xx}^\beta - \sigma_{ik}^\beta \sigma_{xx}^\alpha), \\ W_\mu = \Delta V_\mu - \Delta V_x \sigma_{\mu x} / \sigma_{xx}, \quad \tilde{W}_\nu = \Delta V_\nu - \Delta V_x \sigma_{\nu x} / \sigma_{xx}, \\ \Delta V_i = V_i^\alpha - V_i^\beta, \quad \tilde{\sigma}_{\mu\nu}^\alpha = \sigma_{\mu\nu}^\alpha - \sigma_{\mu\alpha} \sigma_{\nu\alpha} / \sigma_{xx}, \\ \tilde{\sigma}_{\mu\nu} = \sum_\alpha \tilde{\sigma}_{\mu\nu}^\alpha, \quad Q^{-1} = \sum_\alpha Q_\alpha^{-1}, \quad \mathcal{E}_\nu = E_\nu / e.$$

Here  $\sigma_{ik} = \sum_\alpha \sigma_{ik}^\alpha$  is the static conductivity,  $Q_\alpha$  is the state density of the carriers of type  $\alpha$ , and  $\delta_{\mu\nu}$  is the Kronecker delta.

The meaning of some of the quantities in Eqs. (4) deserves comment. The quantity  $V_d$  is the propagation velocity of intervalley perturbations along  $\mathbf{k}$  in a system of carriers which are drifting with different velocities  $V_x^\alpha$  and  $V_x^\beta$ , while  $D_{xx}$  is the ambipolar diffusion coefficient which determines the diffusive "dissipation" of such perturbations. The reason for the appearance of these quantities as well as for the renormalization of the conductivities  $\sigma_{ik}$  is the requirement of quasineutrality. The differences  $\Delta V_i$ , their combinations  $W_\mu$  and  $\tilde{W}_\nu$ , and the "conductivities"  $s_{ik}$  are nonzero when the corresponding components of the conductivity and mobility tensors are not identical for different carrier groups. The alternating transverse currents  $j_\xi$  and  $j_\zeta$ , which are proportional to these quantities, couple the helicons with each other and with drift waves. The degree of wave coupling is determined by the anisotropy of the carrier energy spectrum and by the orientation of the magnetic field with respect to the crystallographic axes. The two cases (1)  $\sigma_{xz} = \sigma_{yz} = 0$  and (2)  $\sigma_{xz} \sim \sigma_{yz} \sim \sigma_{xy} \neq 0$  are substantially different: In the

first case, there is only a weak coupling of waves, and only the damping of the waves is changed substantially in almost all real materials; in the second, the waves are strongly coupled.

## 2. WEAK INTERACTION OF WAVES

Here we consider the case  $\sigma_{xz} = \sigma_{yz} = 0$ . The Hall velocities of the carriers,  $C_x^\alpha$  and  $V_x^\beta$ , are identical in this case and equal to  $V_x$ ;  $\Delta V_\xi = 0$ ;  $W_\xi = \tilde{W}_\xi = \Delta V_\xi$ ; and  $s_{\xi\chi} \gg s_{\zeta\chi}$ . The dispersion relation can be written

$$(\omega'^2 - \omega_h^2 + 2i\omega'\Gamma_h^0)(\omega' + i\Gamma_d^0) = \Phi(k, \omega'). \quad (5)$$

Here  $\omega_h = \pm k^2 c^2 / 4\pi |\tilde{\sigma}_{\xi\xi}^0|$  and  $\Gamma_h^0$  are the spectrum and damping rate of the helicons in the absence of a drift, and  $\omega' = \omega - kV_\chi$ . The function  $\Phi(k, \omega')$  on the right side of (5) is a polynomial in  $k$  and  $\omega'$  with a degree lower than that of the expression on the left side. To save space, we omit the lengthy complete expression for  $\Phi(k, \omega')$ .

There is a well-developed procedure for analyzing dispersion relations of this sort (see Refs. 17 and 18, for example). We omit the straightforward but extremely lengthy detailed calculations and proceed immediately to the final results, which we will briefly analyze. In the region of a substantial coupling of the unperturbed solutions of Eq. (5) (estimates show that the condition  $|\omega'| \ll |\omega_h|/\gamma$  holds in this case), the helicon damping rate is

$$\Gamma_h^{(\pm)} = \Gamma_h^0 + \Delta\Gamma_h^{(1)} \pm \Delta\Gamma_h^{(2)}, \quad (6) \\ \Delta\Gamma_h^{(1)} = \frac{|\omega_h|}{2|\tilde{\sigma}_{\xi\xi}^0|(\omega_h^2 + \Gamma_d^{02})} \left[ \Gamma_d^0 \frac{k^2 s_{\xi\xi}^2}{e^2 Q} + (k\Delta V_\xi)^2 \Sigma_{xx} \right], \\ \Delta\Gamma_h^{(2)} = \frac{\nu_M \Gamma_d^0 k \Delta V_\xi r_\xi}{2(\omega_h^2 + \Gamma_d^{02})}, \quad r_\nu = s_{\nu\alpha} / |\tilde{\sigma}_{\xi\xi}^0|,$$

where the plus and minus signs correspond to fast and slow helicons, respectively. Equations (6) are applicable under the inequalities

$$\frac{\nu_M |k\Delta V_\xi|}{\omega_h^2 + \Gamma_d^{02}} \ll M(k), \quad \frac{(k\Delta V_\xi)^2}{\omega_h^2 + \Gamma_d^{02}} \ll \gamma M(k), \quad (7) \\ M(k) = \min(1, |\omega_h|/\Gamma_d^0).$$

In general, these requirements are more stringent than the condition that the wave coupling be weak. They mean that the correction to the helicon spectrum is small in comparison with  $|\omega_h|$ , while in the case of weak coupling the correction would have to be small in comparison with  $|kV_\chi \pm |\omega_h||$ .

It can be seen from (6) that one of the helicons may be unstable. This possibility arises under the inequality

$$|\omega'| > \frac{4}{\nu_M \Gamma_d^0} [2\Gamma_h^0(\omega_h^2 + \Gamma_d^{02}) + \Gamma_d^0 k^2 |\tilde{D}_{\xi\xi} \omega_h| r_\xi^2], \quad (8) \\ \omega' = \frac{\nu_M \Gamma_d^0 |\tilde{\sigma}_{\xi\xi}^0| r_\xi^2}{\omega_h \Sigma_{xx}}, \quad \tilde{D}_{\xi\xi} = \frac{\tilde{\sigma}_{\xi\xi}}{e^2 Q}.$$

In this case, for parameter values satisfying

$$R_- < k\Delta V_\xi r_\xi < R_+, \quad (9)$$

where

$$R_{\pm} = \frac{\omega^*}{2} \left\{ -1 \pm \operatorname{sgn} \omega_h \left[ 1 - \frac{4}{v_M \Gamma_d^0 |\omega^*|} \right. \right. \\ \left. \left. \times [2\Gamma_h^0 (\omega_h^2 + \Gamma_d^{02}) + \Gamma_d^0 k^2 |D_{\xi\xi}(\omega_h) r_{\xi}^2] \right]^{1/2} \right\}$$

the damping rate becomes negative for one of the helicons. Just which helicon this is depends on the sign of the product  $\Delta V_{\xi} r_{\xi}$ . We cannot say anything definite about the sign of this quantity in general, since it depends on both the particular material and on the orientation of the field  $\mathbf{E}_0$ . If we are dealing with a bipolar crystal, however, the sign of the term  $\Delta \Gamma_h^{(2)}$  is determined unambiguously:  $\operatorname{sgn}(\Delta V_{\xi} r_{\xi}) = \operatorname{sgn} E_0$ . For definiteness we assume  $V_{\chi} > 0$ . Since  $V_{\chi} \sim -\operatorname{sgn} E_0$ , it is clear that it is the fast helicon which is unstable in bipolar crystals.

Turning now to the drift wave, we assume that the unperturbed drift wave is weakly damped:  $kV_{\chi} \gg \Gamma_d^0$ . Its damping rate in the case of a weak coupling with a helicon is

$$\Gamma_d = \Gamma_d^0 - \frac{(k\Delta V_{\xi})^2 |\omega_h / \sigma_{\xi\xi} \Sigma_{xx}|}{\omega_h^2 + \Gamma_d^{02}}. \quad (10)$$

Expression (10) was derived under the assumption that the second term on the right side is small in comparison with  $(\omega_h^2 + \Gamma_d^{02})^{1/2}$ . It can be seen that under the conditions

$$|\omega_h| \gg \Gamma_d^0, \quad \frac{4\pi}{c^2} (\Delta V_{\xi})^2 \Sigma_{xx} > \Gamma_d^0$$

the drift wave also becomes unstable.

Analysis of the nature of the instability reveals the following: The interaction of helicons with the drift wave can drive only a convective instability. As for the interaction of the fast and slow helicons, we note that it leads to both convective instability and—under certain conditions—absolute instability. Only a slow helicon ( $\omega_h < 0$ ) can be absolutely unstable. A necessary condition for this instability is that the branch point in the helicon spectrum,  $k_0 = 2\pi V_{\chi} |\sigma_{\xi\xi}| / c^2$ , must fall in the instability region for the slow helicon, found from (9).

### 3. STRONG INTERACTION OF WAVES

In the extremely anisotropic case with  $\sigma_{xz} \sim \sigma_{yz} \sim \sigma_{xy}$ , the coupling of the unperturbed solutions of Eq. (5) becomes strong, and the wave spectra are greatly deformed. The dispersion relation can nevertheless be written in a form similar to (5), where the coupling of the unrenormalized solutions corresponding to the vanishing of the left side is weak. This dispersion relation is

$$(\omega - \Omega_h^+ + i\Gamma_h^0) (\omega - \Omega_h^- + i\Gamma_h^0) (\omega - kV_d + i\Gamma_d^0) = \Phi(k, \omega), \quad (11) \\ \Omega_h^{\pm} = kV_h \pm [\omega_h^2 + (k\Delta V_{\xi} r_{\xi})^2 - i|\omega_h| k\Delta V_{\xi} r_{\xi}]^{1/2}, \\ V_h = \sigma_{\xi\xi}^{-1} (\sigma_{\xi\xi}^{\alpha} V_{\alpha}^{\alpha} + \sigma_{\xi\xi}^{\beta} V_{\alpha}^{\beta} - s_{\xi\xi} \Delta V_{\xi} / 2).$$

We omit the extremely lengthy expression for the function  $\Phi(k, \omega)$ .

Analysis of this case is complicated because all the velocities ( $V_{\alpha}^{\alpha}$ ,  $V_{\alpha}^{\beta}$ ,  $V_d$ ,  $V_h$ ,  $\Delta V_{\xi}$ ) are of the same order of magnitude. In other words, there is no drifting coordinate

system which is common to the helicons and the electrostatic intervalley perturbations of the electron density. In fact, we do not, in general, find even the helicons with the customary spectrum. The solutions

$$\omega = \Omega_h^{\pm} - i\Gamma_h^0 \quad (12)$$

correspond to ordinary helicon solutions only in the limit  $|\omega_h| \gg |k\Delta V_{\xi}|$ . We might note that one of solutions (12) may be unstable even if we ignore the right side of (11).

As examples we consider two limiting cases. We first assume  $|\omega_h| \gg |k\Delta V_{\xi}|$ . In this case the spectra of the unrenormalized helicons are

$$\omega = kV_h + \omega_h - i(\Gamma_h^0 + 1/2 k\Delta V_{\xi} r_{\xi} \operatorname{sgn} \omega_h). \quad (13)$$

The damping rate of one of the helicons becomes negative as soon as  $|k\Delta V_{\xi}|$  exceeds a value on the order of  $\Gamma_h^0$ . The right side of (11) gives rise to additional imaginary terms in (13), with a modulus less than or on the order of  $\Gamma_h^0$ . Although these terms are clearly important for an accurate determination of the instability threshold, we will not discuss them here.

Now assuming  $|\omega_h| \ll |k\Delta V_{\xi}|$ , we find from (12)

$$\omega = kV_h + \frac{1}{2} |k\Delta V_{\xi} r_{\xi}| \operatorname{sgn} \omega_h - i\omega_h \frac{\Delta V_{\xi} r_{\xi}}{|k\Delta V_{\xi} r_{\xi}|}. \quad (14)$$

In this limit the helicon spectra are linear with different phase velocities, and one of the helicons is clearly unstable, since the function  $\Phi(k, \omega)$  adds terms  $\sim \gamma\omega_h$  to the imaginary part of (14).

The drift-wave spectrum could also be found by perturbation theory from (11). Quite naturally, this spectrum may also be unstable if  $\Gamma_d^0$  is small in comparison with a quantity on the order of  $\Gamma_h^0$ . However, we will omit the lengthy expression for the damping rate of the drift wave.

### 4. INSTABILITY MECHANISM

To understand the instability mechanism, we consider for simplicity the weak-coupling case. In the limit of sufficiently long waves the dissipative processes (the scattering and diffusion) become inconsequential in  $\Gamma_h^{(\pm)}$ , and we have  $\Gamma_h^{(\pm)} \approx \pm k\Delta V_{\xi} r_{\xi} / 2$ . The same damping rate can be found by the following arguments: A drift wave is an electrostatic wave. In the presence of drift, helicons also have a large electrostatic component (on the order of  $kV_{\chi} / |\omega_h|$ ). If we consider the coupling of the  $n_{\alpha}$  and  $n_{\beta}$  oscillations with the electromagnetic wave only through the longitudinal field  $E_{\chi}$ , we can write the helicon spectrum as follows (we are ignoring collisions):

$$\omega = kV_{\chi} + \omega_h - 1/2 i k\Delta V_{\xi} r_{\xi} \operatorname{sgn} \omega_h \quad (15)$$

[cf. (13)].

A more detailed analysis shows that  $\operatorname{Im} \omega$  in (15) stems from the longitudinal component of the Lorentz force caused by the alternating magnetic field of the wave. If the drift velocities of the two types of carrier are identical ( $\Delta V_{\xi} = 0$ ), this force will also be the same for the two types of carrier. We can eliminate this force by transforming to a drifting coordinate system, and there will of course be no

oscillation growth. If, on the other hand, the drift velocities are different ( $\Delta V_\xi \neq 0$ ), the longitudinal ( $\parallel \mathbf{k}$ ) components of the Lorentz force will lead to a spatial separation of the fluctuations, associated with the quasineutrality condition, of the densities  $n_\alpha$  and  $n_\beta$ . This effect will in turn give rise to longitudinal electric fields which will oppose the process.

This picture of the instability is confirmed by the following calculations. If we ignore intergroup relaxation in the continuity equations, and if we retain in expression (3) for the current only the first term and that associated with the longitudinal Lorentz force, we find the following equation for  $n_\alpha$ :

$$\frac{\partial n_\alpha}{\partial t} - \frac{4\pi}{c^2} V_\xi^\alpha \sigma_{xx} \Delta V_\xi n_\alpha = 0. \quad (16)$$

The substitution  $\alpha \rightarrow \beta$  in (16) changes also the sign of the second term. We see that in a coordinate system drifting with the carriers of one type the density of these carriers does not depend on the time, while that of the carriers of the other type increases exponentially. The electric fields of course stop this growth but they drive an instability of the electromagnetic wave.

The vortical fields  $E_\xi$  and  $E_\zeta$  give rise to an additional damping of the helicons when there is wave coupling. Some of the energy is lost by diffusion, while some is pumped into the drift wave through its small transverse components. This process corresponds to the  $\Delta \Gamma_h^{(1)}$  and  $\Gamma_d$  terms quadratic in  $\Delta V_\xi$  [see (6) and (10)]. If the pumping of the drift wave exceeds its natural losses the wave will start to grow.

This instability mechanism can apparently operate also in several other cases, according to Refs. 16 and 19. Those papers, however, dealt with a situation in which the mechanism proposed above is not the fundamental one. Furthermore, in those studies this mechanism could be manifested only when the self-magnetic field of the current is taken into account, while this is not necessary for the instabilities discussed there.

We conclude with a brief discussion of the role played by the self-magnetic field of the Hall current in our problem. The sample should be regarded as bounded, at least along the  $E_0$  direction. It is not difficult to show that the self-magnetic field in a plate of this type is oriented, just as the external field, along the  $z$  axis and its  $x$  profile is

$$(H_0 + H_c)^2 = -8\pi \Delta N E_0 x + H^2(0),$$

where  $H_c$  is the self-field,  $\Delta N$  is the difference between the carrier densities, and  $H(0)$  is an integration constant. The

Hall velocity of the carriers also begins to depend on  $x$ . These results are valid when the drift velocity changes only slowly over the plate thickness  $\mathcal{L}$ , i.e., under the condition

$$4\pi \Delta N E_0 \mathcal{L} \ll H_0^2.$$

If this condition is not satisfied, a packet of spatial harmonics corresponding to the interval  $\Delta V_d$  should be excited at the given frequency. It would apparently be possible to achieve amplification of only some of these harmonics by suitable choice of parameters.

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