Bound state formation and resonance electron scattering by atoms in an external electromagnetic field in the presence of an ionization decay channel

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Resonance scattering of electrons by atoms in the presence of an external electromagnetic field is discussed. For the case of an isolated resonance, both the probability for bound-state (negative ion) production and the resonance scattering cross section are calculated. The photon energy of the external field appears in the resonance denominators of the resulting equations, and this makes it possible to use tunable lasers in order to approach closer to the resonance and to increase both the probability for bound-state production and the resonance scattering cross section. The width and shift of the resonance depend linearly on the strength of the external field. The resonance scattering peak is independent of the external field, while the resonance peak for bound state production decreases linearly with increasing strength of the external field. The case of two levels of which the upper one is an autoionizing state and is in two-photon resonance with the lower one is also examined. The elastic and inelastic (in the ionization channel) resonance scattering cross sections are calculated using the Fano method. It is shown that, owing to the presence of the external electromagnetic field, scattering takes place not only from the upper quasienergy level, but also from the lower one. Cases are discussed in which coincidence of poles and overlapping of resonances are possible.

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1. INTRODUCTION

The investigation of the effect of an external electromagnetic field on electron scattering is not only of theoretical but also of practical interest. The elastic scattering of electrons in an external electromagnetic field, induced bremsstrahlung, and absorption have been the subjects of many studies (see, e.g., Refs. 1 and 2). The inelastic collision of electrons with atoms in the presence of an electromagnetic field has also been intensively studied in recent years (see, e.g., Refs. 3 and 4). These studies may open up new possibilities for obtaining sources of short-wavelength light, for heating plasmas, for investigating highly excited and autoionizing states of atoms and the breakdown of gases, and so on. The capture of neutrons by nuclei and the formation of mesic atoms and mesic molecules in the field of a strong electromagnetic wave has been discussed in Refs. 5 and 6.

In the present work we investigate the resonance scattering of an electron with the formation of a bound state. In the first part of the paper we examine the case of an isolated resonance level. The scattering wave function is obtained in the adiabatic approximation, and the probability for the production of a bound state and the resonance scattering cross section are calculated. A similar result for the resonance scattering was obtained in Ref. 7 by perturbation theory, the width and shift of the resonance being taken into account phenomenologically. In the second part of the paper we consider the case of two levels that are coupled by resonance radiation. The well-known Fano method[9] was first generalized to the case of a periodic perturbation in Ref. 8. This method was used to calculate the wave functions and to obtain the cross sections for elastic scattering and for inelastic scattering in the ionization channel. The case of overlapping resonances is investigated.

2. THE CASE OF AN ISOLATED LEVEL

We shall consider the scattering of an electron and a photon by an atom when the incident electron can be captured, i.e., when the unperturbed system consisting of the atom and the electron has bound states that include the absorbed particle.[10] The presence of such bound states of the unperturbed system substantially affects the scattering in the perturbed system. As stationary states we choose the states

$$|\alpha, \mathbf{p}\rangle \exp\left(-\frac{i}{\hbar}E_\alpha(p)t\right), \quad E_\alpha(p)=E_\alpha+\frac{\mathbf{p}^2}{2m}, \tag{1}$$

in which the electron has a definite momentum $\mathbf{p}$ and the atom is in a definite state α, together with a separate discrete set of bound states

$$|k\rangle \exp\left(-\frac{i}{\hbar}E_k t\right),$$

in which the incident particle is bound. We shall assume that all these states are independent and orthogonal, although if the states $|k\rangle$ are metastable they will satisfy the orthogonality relations only approximately.

In the case of scattering in an external field the electron is induced to emit a photon with the frequency of the external field and forms a discrete state, after which it absorbs a photon from the external field and is ejected from the atom. We shall regard a resonance level as isolated provided

$$\Gamma_k/\Delta E_k \sim \Delta_k/\Delta E_k \ll 1,$$

where Γ_k and Δ_k are the width and shift of the resonance level, and ΔE_k is the separation of the resonance from the nearest neighbor level.

The Schrödinger equation for a system consisting of an

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atom and an electron in an external electromagnetic field has the form

$$i\hbar\partial\psi(t)/\partial t=[H_0+V(t)+U]\psi(t),\quad V(t)=V^+e^{i\omega t}+V^-e^{-i\omega t},\tag{2}$$

where H_0 is the free Hamiltonian for the atom and the electron, $V(t)$ is the dipole interaction with the electromagnetic radiation field, and U is the interaction of the electron with the atom.

We express the solution of Eq. (2) as an expansion in unperturbed wave functions:

$$\psi(t)=\sum_k c_k(t)\exp\left[-\frac{i}{\hbar}E_kt\right]|k\rangle+\sum_{\alpha,\mathbf{p}}c_\alpha(\mathbf{p},t)\exp\left[-\frac{i}{\hbar}E_\alpha(p)t\right]|\alpha,\mathbf{p}\rangle.\tag{3}$$

On substituting the expansion (3) into the Schrödinger equation (2) we obtain a set of equations for the coefficients $c_k(t)$ and $c_\alpha(\mathbf{p},t)$ from which, with the aid of the Fourier transformations

$$c_k(t)=\int d\lambda c_k(\lambda+E_k+\hbar\omega)\exp\left(-\frac{i}{\hbar}\lambda t\right),$$
$$c_\alpha(\mathbf{p},t)=\int d\lambda c_\alpha(\mathbf{p},\lambda+E_\alpha(p))\exp\left(-\frac{i}{\hbar}\lambda t\right)\tag{4}$$

we obtain the following set of algebraic equations for the functions $c_k(\lambda)$ and $c_\alpha(\mathbf{p},\lambda)$:

$$(\lambda-E_k-\hbar\omega)c_k(\lambda)=\sum_{k'}[V^+_{kk'}c_{k'}(\lambda+\hbar\omega)+V^-_{kk'}c_{k'}(\lambda-\hbar\omega)+U_{kk'}c_{k'}(\lambda)]$$
$$+\sum_{\alpha,\mathbf{p}}[V^+_{k,\alpha\mathbf{p}}c_\alpha(\mathbf{p},\lambda)+V^-_{k,\alpha\mathbf{p}}c_\alpha(\mathbf{p},\lambda-2\hbar\omega)+U_{k,\alpha\mathbf{p}}c_\alpha(\mathbf{p},\lambda-\hbar\omega)],$$
$$(\lambda-E_\alpha(p))c_\alpha(\mathbf{p},\lambda)$$
$$=\sum_k[V^+_{\alpha\mathbf{p},k}c_k(\lambda+2\hbar\omega)+V^-_{\alpha\mathbf{p},k}c_k(\lambda)+U_{\alpha\mathbf{p},k}c_k(\lambda+\hbar\omega)]\tag{5}$$
$$+\sum_{\alpha',\mathbf{p}'}[V^+_{\alpha\mathbf{p},\alpha'\mathbf{p}'}c_{\alpha'}(\mathbf{p}',\lambda+\hbar\omega)+V^-_{\alpha\mathbf{p},\alpha'\mathbf{p}'}c_{\alpha'}(\mathbf{p}',\lambda-\hbar\omega)+U_{\alpha\mathbf{p},\alpha'\mathbf{p}'}c_{\alpha'}(\mathbf{p}',\lambda)].$$

In order to consider the simultaneous interaction of the photon and electron with the atom we must iterate Eq. (5) and retain only the terms that contain V^+ or V^- and U. It is assumed that an isolated resonance state of energy E_k is formed. In that case we retain only the terms that involve resonance with the level k, i.e., terms containing matrix elements for the transition $k\leftrightarrow(\alpha,\mathbf{p})$. All the other terms obtained from the iteration, which contain matrix elements of the type $k\leftrightarrow k'$ and $(\alpha\mathbf{p})\leftrightarrow(\alpha'\mathbf{p}')$, make only negligible higher-order contributions. Then, introducing the reduced matrix element

$$W_{k,\alpha\mathbf{p}}(\lambda)=\sum_k\left(\frac{V^+_{k,k'}U_{k',\alpha\mathbf{p}}}{\lambda-E_{k'}}+\frac{U_{kk'}V^+_{k',\alpha\mathbf{p}}}{\lambda-E_{k'}-\hbar\omega}\right)$$
$$+\sum_{\alpha',\mathbf{p}'}\left(\frac{V^+_{k,\alpha'\mathbf{p}'}U_{\alpha'\mathbf{p}',\alpha\mathbf{p}}}{\lambda-E_{\alpha'}(p')}+\frac{U_{k,\alpha'\mathbf{p}'}V^+_{\alpha'\mathbf{p}'\alpha\mathbf{p}}}{\lambda-E_{\alpha'}(p')-\hbar\omega}\right),\tag{6}$$

we obtain the following set of equations:

$$(\lambda-E_k-\hbar\omega)c_k(\lambda)=\sum_{\alpha,\mathbf{p}}W_{k,\alpha\mathbf{p}}(\lambda)c_\alpha(\mathbf{p},\lambda),\tag{7a}$$

$$(\lambda-E_\alpha(p))c_\alpha(\mathbf{p},\lambda)=W^*_{k,\alpha\mathbf{p}}(\lambda)c_k(\lambda).\tag{7b}$$

If the field is turned on adiabatically at $t\to-\infty$, the initial free state is

$$|\alpha_i\mathbf{p}_i\rangle\exp\left(-\frac{i}{\hbar}E_{\alpha_i}(p_i)t\right),$$

and the solution of Eq. (7b) can be expressed in the form

$$c_\alpha(\mathbf{p},\lambda)=\langle\mathbf{p},\alpha|\mathbf{p}_i,\alpha_i\rangle\delta(\lambda-E_\alpha(p))+W^*_{k,\alpha\mathbf{p}}(\lambda)c_k(\lambda)\zeta(\lambda-E_\alpha(p)),\tag{8}$$

where

$$\zeta(x)=\lim_{\varepsilon\to0}\frac{1}{x+i\varepsilon}=\frac{P}{x}-i\pi\delta(x).\tag{9}$$

On substituting (8) into (7a) we obtain

$$c_k(\lambda)=\frac{W_{k,\alpha_i\mathbf{p}_i}(\lambda)\,\delta(\lambda-E_{\alpha_i}(p_i))}{\lambda-E_k-\hbar\omega-\Delta_k(\lambda)+i\Gamma_k(\lambda)/2},\tag{10}$$

where

$$\Delta_k(\lambda)=P\sum_{\alpha,\mathbf{p}}\frac{|W_{k,\alpha\mathbf{p}}(\lambda)|^2}{\lambda-E_\alpha(p)},$$
$$\Gamma_k(\lambda)=2\pi\sum_{\alpha,\mathbf{p}}|W_{k,\alpha\mathbf{p}}(\lambda)|^2\delta(\lambda-E_\alpha(p)).\tag{11}$$

Then the solution (8) of Eq. (7b) takes the form

$$c_\alpha(\mathbf{p},\lambda)=\langle\mathbf{p},\alpha|\mathbf{p}_i,\alpha_i\rangle\,\delta(\lambda-E_\alpha(p))+\frac{W^+_{\alpha\mathbf{p},k}(\lambda)\,W_{k,\alpha_i\mathbf{p}_i}(\lambda)}{\lambda-E_k-\hbar\omega-\Delta_k(\lambda)+i\Gamma_k(\lambda)/2}\times\delta(\lambda-E_{\alpha_i}(p_i))\,\zeta(\lambda-E_\alpha(p)).\tag{12}$$

On substituting (10) and (12) into (4) we obtain the following expression for the complete adiabatic wave function (3) in the presence of an isolated resonance:

$$\psi(t)=|\alpha_i\mathbf{p}_i\rangle\exp\left[-\frac{i}{\hbar}E_{\alpha_i}(p_i)t\right]+\frac{W_{k,\alpha_i\mathbf{p}_i}\exp\left[-\frac{i}{\hbar}(E_{\alpha_i}(p_i)-\hbar\omega)t\right]}{E_{\alpha_i}(p_i)-E_k-\hbar\omega-\Delta_k+i\Gamma_k/2}|k\rangle$$
$$+\sum_{\alpha,\mathbf{p}}\frac{W^+_{\alpha\mathbf{p},k}W_{k,\alpha_i\mathbf{p}_i}}{E_{\alpha_i}(p_i)-E_k-\hbar\omega-\Delta_k+i\Gamma_k/2}\zeta(E_{\alpha_i}(p_i)-E_\alpha(p))\exp\left[-\frac{i}{\hbar}E_{\alpha_i}(p_i)t\right]|\alpha,\mathbf{p}\rangle,\tag{13}$$

where $W_{k,\alpha\mathbf{p}}(\lambda)$, $\Delta_k(\lambda)$, and $\Gamma_k(\lambda)$ are taken at $\lambda=E_{\alpha_i}(p_i)$.

It is evident that in Eq. (13) for the wave function, the first term corresponds to the initial state $|\alpha_i\rangle$ of the atom together with an incident particle of momentum $\mathbf{p}_i$, and the second term corresponds to the capture of the incident particle with the formation of a discrete state $|k\rangle$ whose width Γ_k and shift Δ_k depend substantially on the strength of the external field [Eqs. (6) and (11)]. The third term describes the resonance scattering.

From Eq. (13) we obtain the following expression for the probability amplitude for capture of the incident electron with the formation of a bound state (negative ion):

$$A_k(t)=\frac{W_{k,\,\alpha_i\mathbf{p}_i}}{E_{\alpha_i}(p_i)-E_k-\hbar\omega-\Delta_k+i\Gamma_k/2}\times\exp\left[-\frac{i}{\hbar}(E_{\alpha_i}(p_i)-E_k-\hbar\omega)\,t\right] \tag{14}$$

from which we obtain the capture probability

$$w_k=\frac{|W_{k,\,\alpha_i\mathbf{p}_i}|^2}{[E_{\alpha_i}(p_i)-E_k-\hbar\omega-\Delta_k]^2+\Gamma_k^2/4}\,. \tag{15}$$

The probability amplitude in the limit $t\to+\infty$ for resonance scattering from the initial state $|\alpha_i\mathbf{p}_i\rangle$ to the final state $|\alpha_f\mathbf{p}_f\rangle$ will be

$$A_{\text{scat}}=-2\pi i\,\frac{W^+_{\alpha_f\mathbf{p}_f,\,k}W_{k,\,\alpha_i\mathbf{p}_i}}{E_{\alpha_i}(p_i)-E_k-\hbar\omega-\Delta_k+i\Gamma_k/2}\times\delta\,(E_{\alpha_i}(p_i)-E_{\alpha_f}(p_f)). \tag{16}$$

Then for the differential cross section, integrated over the energy of the ejected electron, we have

$$\frac{d\sigma}{do}=\frac{m^2}{4\pi^2\hbar^4}\,\frac{p_f}{p_i}\,\frac{|W_{k,\,\alpha_f\mathbf{p}_f}|^2\,|W_{k,\,\alpha_i\mathbf{p}_i}|^2}{[E_{\alpha_i}(p_i)-E_k-\hbar\omega-\Delta_k]^2+\Gamma_k^2/4}\,. \tag{17}$$

In the calculations we have limited ourselves to the resonance approximation. Actually, the scattering amplitude (16) should also include the amplitude for potential (nonresonant) scattering, but this may be neglected in the presence of a resonance since

$$|A_{\text{pot}}/A_{\text{res}}|\sim|(U_{\alpha_f\mathbf{p}_f,\alpha_i\mathbf{p}_i}/W^+_{\alpha_f\mathbf{p}_f,k}W_{k,\alpha_i\mathbf{p}_i})\Gamma_k|\ll 1.$$

In the present treatment, the formation of a discrete state and the resonance scattering are due to the presence of an external electromagnetic field. If the external field vanishes, the interaction ceases and so both the probability for the formation of a bound state and the resonance-scattering cross section vanish. The important point here is that the resonance denominator in Eqs. (15) and (17) also contains the photon energy, and this makes it possible to use tuneable lasers to approach closer to the resonance and thereby substantially to increase both the probability for the formation of a bound state and the resonance-scattering cross section. It should be noted that according to the formulas obtained, the width and shift of the resonance depend on the strength $\mathscr{E}$ of the external electromagnetic field as $\mathscr{E}^2$. It is evident from Eqs. (15) and (17) that the resonance peak for scattering is independent of the strength $\mathscr{E}$ of the external field, while the resonance peak for the production of a bound state decreases as $\mathscr{E}^2$ with increasing $\mathscr{E}$. In treating the formation of a negative ion, the photodetachment width is[8]

$$\Gamma\sim E_a(\mathscr{E}/\mathscr{E}_{\text{cr}})^2,\quad \mathscr{E}_{\text{cr}}\sim E_a/e\langle r\rangle.$$

In the case of a negative hydrogen ion $\langle r\rangle\sim 5$ a. u. and $E_a\sim 0.03$ a. u. (Ref. 11). If the energy spread of the incident electron beam of energy $E\sim 10$ eV amounts to $\Delta E/E\sim 10^{-3}$, for an external field of strength $\mathscr{E}\gtrsim(E_a\Delta E)^{1/2}/(E\langle r\rangle)\sim 10^6$ V/cm we have $\Gamma\gtrsim\Delta E$.

3. THE TWO-LEVEL CASE

Now let us consider the resonance scattering of an electron by an atom in the case of two levels of which one is an autoionization level in two-photon resonance with a low-lying discrete level (Fig. 1). The calculations will be carried through by the Fano method[9] as generalized in Ref. 8 to the case of a periodic perturbation. On the one hand, the two-photon resonance between discrete levels makes it possible to investigate the dependence of the dynamics of the levels (and in particular, of the coincidence of poles) on the strength of the external field, and on the other hand, it allows one to take into account one-photon ionization from the upper level alone. In the case of a one-photon resonance, the ionization channel can be neglected and the elastic scattering cross section is given by Eq. (35a) with $\tilde{E}_{1,2}=E_1+\hbar\omega+\Omega_{1,2}$ and ε' replaced by ε. As the basis wave functions for the discrete spectrum we choose the quasienergy wave functions obtained in the two-photon resonance approximation with the periodic perturbation

$$V(t)=V^+e^{i\omega t}+V^-e^{-i\omega t} \tag{18}$$

turned on adiabatically. If the discrete levels are not degenerate, these wave functions have the form

$$\begin{aligned}\Phi_1(t)&=\exp\left[-\frac{i}{\hbar}\lambda_1 t\right](a_1\psi_1+b_1\psi_2e^{-2i\omega t}),\\ \Phi_2(t)&=\exp\left[-\frac{i}{\hbar}(\lambda_2-2\hbar\omega)\,t\right](a_2\psi_2+b_2\psi_2e^{-2i\omega t}),\end{aligned} \tag{19}$$

where $\psi_{1,2}(r)$ and $E_{1,2}$ are the wave functions and energies of the unperturbed levels, $\lambda_1=E_1+\delta_1$, $\lambda_2=E_1+\delta_2+2\hbar\omega$, and

$$\begin{gathered}a_{1,2}=\left(\frac{\Delta_{2,1}}{\Delta_{2,1}-\Delta_{1,2}}\right)^{1/2},\quad b_{1,2}=\frac{f_{12}^*}{f_{12}}\left(\frac{\Delta_{1,2}}{\Delta_{1,2}-\Delta_{2,1}}\right)^{1/2},\\ W_i=\sum_k\left(\frac{|V_{ik}^+|^2}{E_i-E_k+\hbar\omega}+\frac{|V_{ik}^-|^2}{E_i-E_k-\hbar\omega}\right)\qquad(i=1,2),\\ \delta_{1,2}=\Delta_{1,2}+W_1,\quad \Delta_{1,2}={}^1/_2[\varepsilon'\mp(\varepsilon'^2+\Gamma_f^2/4)^{1/2}],\quad \Gamma_f=4|f_{12}|^2,\\ \varepsilon'=\varepsilon-(W_1-W_2),\quad \varepsilon=E_2-E_1-2\hbar\omega,\quad f_{12}=\sum_k\frac{V_{1k}^+V_{k2}^+}{E_i-E_k+\hbar\omega}.\end{gathered} \tag{20}$$

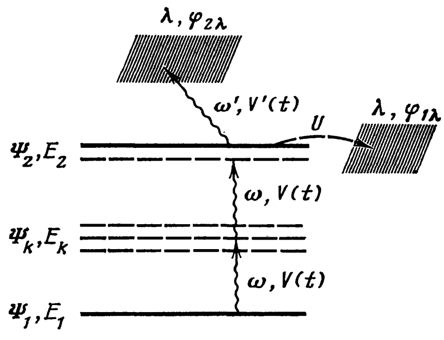

FIG. 1.

We neglect the effect of the external electromagnetic field on the continuous spectrum and choose the unperturbed wave functions

$$\varphi_{1,2}(\mathbf{r},t)=\varphi_{1,2}(\mathbf{r})\exp\left(-\frac{i}{\hbar}\lambda t\right) \tag{21}$$

as the basis functions for the continuum.

The Schrödinger equation for this problem has the form

$$i\hbar\frac{\partial\Psi(t)}{\partial t}=(H_0+V(t)+V'(t)+U)\Psi(t),$$
$$V'(t)=V'^{+}e^{i\omega' t}+V'^{-}e^{-i\omega' t}, \tag{22}$$

where H_0 is the free Hamiltonian, which has both discrete and continuous spectra, $V(t)$ is the interaction of the external electromagnetic field (18) with the two resonance levels of the atom, $V'(t)$ is the interaction with the electromagnetic field causing the ionization (at the upper level) which, in particular, may be the same as $V(t)$ as given in Eq. (18), and U is Fano's[9] "configuration" interaction.

We express the solution of Eq. (22) as an expansion in the quasienergy states (19) of the discrete spectrum of the atom and in the wave functions of the continuous spectrum (neglecting transitions between the states of the continuum):

$$\Psi(t)=c_1(t)\Phi_1(t)+c_2(t)\Phi_2(t)+\int d\lambda[h_{1\lambda}(t)\varphi_{1\lambda}(t)+h_{2\lambda}(t)\varphi_{2\lambda}(t)]. \tag{23}$$

On substituting (23) into (22), we obtain a set of equations for the coefficients $c_{1,2}(t)$ and $h_{1,2}(t)$, which, as a result of the Fourier transformations

$$c_1(t)=\int dE c_1(E+\lambda_1+2\hbar\omega)\exp\left(-\frac{i}{\hbar}Et\right)$$
$$c_2(t)=\int dE c_2(E+\lambda_2)\exp\left(-\frac{i}{\hbar}Et\right),$$
$$h_{1\lambda}(t)=\int dE h_{1\lambda}(E+\lambda)\exp\left(-\frac{i}{\hbar}Et\right), \tag{24}$$
$$h_{2\lambda}(t)=\int dE h_{2\lambda}(E+\lambda+\hbar\omega')\exp\left(-\frac{i}{\hbar}Et\right)$$

leads to a set of algebraic equations for the Fourier transforms:

$$(E-\lambda-2\hbar\omega)c_1(E)=b_1^*\int d\lambda[U_\lambda h_{1\lambda}(E)+V_\lambda' h_{2\lambda}(E)],$$
$$(E-\lambda)c_2(E)=b_2^*\int d\lambda[U_\lambda h_{1\lambda}(E)+V_\lambda' h_{2\lambda}(E)], \tag{25}$$
$$(E-\lambda)h_{1\lambda}(E)=U_\lambda^*[b_1c_1(E)+b_2c_2(E)],$$
$$(E-\lambda+\hbar\omega')h_{2\lambda}(E)=V_\lambda'^*[b_1c_1(E)+b_2c_2(E)],$$

where

$$U_\lambda\equiv\langle\psi_2|U|\varphi_{1\lambda}\rangle,\quad V_\lambda'\equiv\langle\psi_2|V'^{+}|\varphi_{2\lambda}\rangle.$$

Equations (25) have two linearly independent solutions, the first being

$$c_2(E)=\frac{b_2^*}{b_1^*}\frac{E-\lambda_1-2\hbar\omega}{E-\lambda_2}c_1(E), \tag{26a}$$

$$h_{1\lambda}(E)=b_1\left[\frac{P}{E-\lambda}+z(E)\delta(E-\lambda)\right]\times U_\lambda^*\left(1+\frac{|b_2|^2}{|b_1|^2}\frac{E-\lambda_1-2\hbar\omega}{E-\lambda_2}\right)c_1(E),$$
$$h_{2\lambda}(E)=b_1\left[\frac{P}{E-\lambda+\hbar\omega'}+z(E)\delta(E-\lambda+\hbar\omega')\right]\times V_\lambda'^*\left(1+\frac{|b_2|^2}{|b_1|^2}\frac{E-\lambda_1-2\hbar\omega}{E-\lambda_2}\right)c_1(E)$$

and the second,

$$c_1(E)=c_2(E)=0,\ h_{1\lambda}(E)=f(E)\delta(E-\lambda),$$
$$h_{2\lambda}(E)=-f(E)\,(U_E/V'_{E+\hbar\omega'})\,\delta(E-\lambda+\hbar\omega'), \tag{26b}$$

where $c_1(E)$ and $f(E)$ are two arbitrary functions that are determined by the condition for the normalization of the wave function (23). On substituting (26a) and (26b) into (24) we obtain the following complete orthonormal set of the quasienergy wave functions (23):

$$\Psi_{1,2E}(t)=X_{1,2E}(t)\exp\left(-\frac{i}{\hbar}Et\right). \tag{27}$$

Here

$$X_{1E}(t)=\frac{1}{\eta(E)}\left(\frac{2\pi}{\Gamma(E)[z^2(E)+\pi^2]}\right)^{1/2}\times\left\{\exp\left[\frac{i}{\hbar}(\lambda_1+2\hbar\omega)t\right]\Phi_1(t)\right.$$
$$+\exp\left(\frac{i}{\hbar}\lambda_2 t\right)\left(-\frac{\Delta_2}{\Delta_1}\right)^{1/2}\frac{E-\lambda_1-2\hbar\omega}{E-\lambda_2}\Phi_2(t)+\frac{f_{12}^*}{|f_{12}|}\eta(E)$$
$$\times\left[\int d\lambda\exp\left(\frac{i}{\hbar}\lambda t\right)\varphi_{1\lambda}(t)U_\lambda^*\left[\frac{P}{E-\lambda}+z(E)\delta(E-\lambda)\right]\right. \tag{28a}$$
$$+\int d\lambda\exp\left[\frac{i}{\hbar}(\lambda-\hbar\omega')t\right]\varphi_{2\lambda}(t)V_\lambda'^*$$
$$\left.\left.\times\left(\frac{P}{E-\lambda+\hbar\omega'}+z(E)\delta(E-\lambda+\hbar\omega')\right)\right]\right\}$$

and

$$X_{2E}(t)=(|U_E|^2+|V'_{E+\hbar\omega'}|^2)^{-1/2}(V'_{E+\hbar\omega'}\varphi_{1E}-U_Ee^{-i\omega' t}\varphi_{2E+\hbar\omega'}), \tag{28b}$$

where

$$\eta(E)=\left(\frac{\Delta_1}{\Delta_1-\Delta_2}\right)^{1/2}\left(1-\frac{\Delta_2}{\Delta_1}\frac{E-\lambda_1-2\hbar\omega}{E-\lambda_2}\right)$$
$$z(E)=\frac{2\pi}{\Gamma(E)}\left[\frac{(E-\lambda_1-2\hbar\omega)(E-\lambda_2)}{E-\lambda_2+\Delta_2}-\Delta(E)\right],$$
$$\Delta(E)=\Delta_1(E)+\Delta_2(E),\ \Gamma(E)=\Gamma_1(E)+\Gamma_2(E), \tag{29}$$
$$\Delta_1(E)=P\int d\lambda\frac{|U_\lambda|^2}{E-\lambda},\quad \Delta_2(E)=P\int d\lambda\frac{|V_\lambda'|^2}{E-\lambda+\hbar\omega'},$$
$$\Gamma_1(E)=2\pi|U_E|^2,\quad \Gamma_2(E)=2\pi|V_{E+\hbar\omega'}|^2.$$

As was shown in Ref. 12, the expression for the shift $\Delta(E)$ should also contain a term with the opposite sign for the frequency of the external field as well as the contribution from the remaining nonresonant levels, which are not taken into account in the present theory. In what follows we must regard Δ as representing the exact expression for the shift in order to take these terms correctly into account.

Now let us consider the scattering process in which the incident particle forms a bound state with the scatterer and this state subsequently decays, emitting the particle either in the initial channel (elastic scattering) or in some other (ionization) channel (inelastic scattering). This process is represented by the eigenvector $\Psi(1\to 1,2)$, which at large distances represents the sum of the incoming wave and two outgoing waves. In order to obtain an expression of the required form we express the eigenvector $\Psi(1\to 1,2)$ as a superposition of the eigenstates (28a) and (28b):

$$\Psi_E(1\to 1,2)=\sum_n B_n X_{nE}(t), \qquad (30)$$

where the coefficients B_n are determined by the normalization condition and the requirement that the second (ionization) channel not be present in the incident wave at asymptotically large distances. Then we obtain the following expressions for the coefficients $B_{1,2}$:

$$B_1=U_E(|U_E|^2+|V'_{E+\hbar\omega'}|^2)^{-1/2},$$
$$B_2=-V'^{*}_{E+\hbar\omega'}e^{-i\delta}(|U_E|^2+|V'_{E+\hbar\omega'}|^2)^{-1/2}, \qquad (31)$$

where δ is defined by the equation $\tan\delta = -\pi/z(E)$.

Specifying the asymptotic expressions for the continuum wave functions (21) at large distances,

$$\psi_{1,2\lambda}\infty k_{1,2}^{-1/2}(\lambda)\sin[k_{1,2}(\lambda)r_{1,2}+\delta_{1,2}^{(0)}-{}^1/_2\pi l_{1,2}]Y_{l_{1,2}m_{1,2}}(\theta_{1,2},\varphi_{1,2}), \qquad (32)$$

we obtain the following expression for the scattering eigenvector (30):

$$\Psi_E(1\to 1,2)\infty\frac{\exp[-i(\delta_1^{(0)}+\delta)]}{k_1^{1/2}(E)}\Big\{\sin[k_1(E)r_1-{}^1/_2\pi l_1]$$
$$+\frac{\exp(2i\delta_1^{(0)})}{2i}\left[\frac{|U_E|^2}{|U_E|^2+|V'_{E+\hbar\omega'}|^2}(1-e^{2i\delta})+\exp(-2i\delta_1^{(0)})-1\right]$$
$$\times\exp[i(k_1(E)r_1-{}^1/_2\pi l_1)]\Big\}Y_{l_1m_1}(\theta_1,\varphi_1)+\frac{\exp(-i\omega' t)\sin\delta}{k_2^{1/2}(E+\hbar\omega')}$$
$$\times\exp(i\delta_2^{(0)})\frac{U_E V'_{E+\hbar\omega'}}{|U_E|^2+|V'_{E+\hbar\omega'}|^2} \qquad (33)$$
$$\times\exp[i(k_2(E+\hbar\omega')r_2-{}^1/_2\pi l_2)]Y_{l_2m_2}(\theta_2,\varphi_2).$$

From this we obtain the following expressions for the elastic- and inelastic-scattering cross sections:

$$\sigma(1\to 1)=\frac{\pi(2l+1)}{k^2(E)}\left|\exp(2i\delta_1^{(0)})-1+\frac{\Gamma_1(E)}{\Gamma(E)}(1-e^{2i\delta})\right|^2, \qquad (33a)$$

$$\sigma(1\to 2)=\frac{\pi(2l+1)}{k^2(E+\hbar\omega')}\frac{\Gamma_1(E)\Gamma_2(E)}{\Gamma^2(E)}\sin^2\delta. \qquad (33b)$$

If we remove the potential-scattering part

$$\sigma_{\rm pot}=\frac{\pi(2l+1)}{k^2(E)}|\exp(2i\delta_1^{(0)})-1|^2, \qquad (34)$$

from Eq. (33a) we can express the cross sections (33a) and (33b) in the form

$$\sigma(1\to 1)=\frac{\pi(2l+1)}{k^2(E)}\frac{\Gamma_1^2(E)}{|\Omega_1-\Omega_2|^2}\left|\frac{\Omega_2}{E-\tilde E_2}-\frac{\Omega_1}{E-\tilde E_1}\right|^2, \qquad (35a)$$

$$\sigma(1\to 2)=\frac{\pi(2l+1)}{k^2(E+\hbar\omega')}\frac{\Gamma_1(E)\Gamma_2(E)'}{|\Omega_1-\Omega_2|^2}\left|\frac{\Omega_2}{E-\tilde E_2}-\frac{\Omega_1}{E-\tilde E_1}\right|^2, \qquad (35b)$$

where

$$\tilde E_{1,2}=E_1+W_1+2\hbar\omega+\Omega_{1,2}, \qquad (36)$$

$$\Omega_{1,2}={}^1/_2\{\varepsilon'+\Delta(E)-i\Gamma(E)/2 \mp[(\varepsilon'+\Delta(E)-i\Gamma(E)/2)^2+\Gamma_f^2/4]^{1/2}\}. \qquad (37)$$

It is evident from the expressions (35a) and (35b) for the cross sections that the scattering takes place not only from the upper quasienergy level, but also from the lower one. The appearance of two peaks in the resonance scattering cross section is associated with splitting of the quasienergy levels in the two-level system. This effect is analogous to the Autler-Townes effect for resonance scattering of photons. Thus, the presence of an external electromagnetic field leads to the appearance of a second term that has a pole at $E = E_1$, and consequently to the overlapping of the resonances. When the external electromagnetic field is turned off the second term in each of Eqs. (35a) and (35b) disappears and we have the well-known formulas for elastic and inelastic resonance scattering in the case of an isolated level.

The values of the poles obtained in (36) and (37) agree with the expressions obtained in Ref. 13 in a treatment of the decay of a bound state in an external electromagnetic field with a two-photon resonance. The limiting cases of narrow and broad resonances were treated in Ref. 13. A substantial asymmetry depending on the sign of the two-photon detuning (an effect of self induced adiabatic passage of the resonance) was found in sufficiently strong external resonance fields. There are no analogous effects in the case of resonance scattering, as the derived formulas show. This is evidently associated with the assumption that the interaction is turned on adiabatically.

The expressions obtained show that the real parts of the quasienergies cross when $\Gamma_f \ll \Gamma$: $\mathrm{Re}\,\tilde E_1 = \mathrm{Re}\,\tilde E_2$. The pattern of the interaction therefore changes substantially at the point $\Gamma_f = \Gamma$ since the branches of the quasienergies always cross when $\Gamma_f \ll \Gamma$ and always anticross when $\Gamma_f > \Gamma$. The resonance poles coincide ($\tilde E_1 = \tilde E_2$) when $\Gamma_f = \Gamma$ and $\varepsilon' + \Delta = 0$. Then we have a resonance with a double pole.

On the basis of Eq. (35a), (35b), and (37), in order to detect the second peak in the resonance scattering cross section it is necessary that $\Gamma_f \gtrsim \Gamma$. In the case of a two-photon resonance, $\Gamma \sim \Gamma_f \sim \mathrm{Ry}(\mathscr{E}/\mathscr{E}_{\rm at})^2$ (Ref. 8), and in an external field of strength $\mathscr{E} \sim 5\times 10^5$ V/cm we have $\Gamma^{-1} \sim \Gamma_f^{-1} \sim 10^{-8}$ sec. However, an intermediate real level E_k (outside the resonance), if present, may increase Γ_f by two or three orders of magnitude. In the case of a one-photon resonance the quasienergy levels are more widely split and the condition $\Gamma_f > \Gamma$ is satisfied in comparatively weak fields. Another, more serious, difficulty in detecting the resonances is the high degree of monochromaticity required of the electron beam: the energy spread of the beam electrons must be smaller than the widths of the resonances.

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