Stimulation of superconductivity by a direct current in a superconductor-normal metal-superconductor junction

S. V. Lempitskiĭ

Moscow Institute of Steel and Alloys
(Submitted 17 March 1983)
Zh. Eksp. Teor. Fiz. **85**, 1072–1080 (September 1983)

The current-voltage characteristic of a superconductor-normal metal-superconductor junction is calculated. It is shown that the deviation from equilibrium of the electrons with energies of the order of the reciprocal diffusion time in the normal-metal bridge may stimulate superconductivity in the junction and give rise to a characteristic bend in the current-voltage characteristic.

PACS numbers: 74.50. + r

1. INTRODUCTION

The nonstationary Josephson effect in superconducting weak links is now being intensively investigated both experimentally and theoretically.[1–2] In the simplest resistance model it is assumed that the current through the junction is made up of ohmic and Josephson components, and the corresponding current-voltage characteristic (CVC) turns out to be hyperbolic.[3] Experimentally, however, deviations from this dependence due to a nonequilibrium electron energy distribution at the junction are frequently observed.[4–6]

This lack of equilibrium may be due to an external microwave field or to oscillations of the order parameter Δ in the vicinity of the weak link when the current exceeds the critical value I_c. In the second case an average voltage $\overline{V}$ appears across the junction, and this leads to periodic changes in the density of quasiparticle states at the Josephson frequency $2e\overline{V}/\hbar$. As a result, a nonequilibrium component appears in the current and the CVC of the junction may differ substantially from the hyperbola corresponding to the resistance model.

Nonequilibrium effects and their influence on the Josephson effect have been studied theoretically for superconducting systems having a constriction (bridges and point contacts). It turned out that whereas the nonequilibrium effects should tend to suppress superconductivity in the case of short bridges,[7] superconductivity should be stimulated by a direct current in the case of sufficiently long bridges.[8] In this case the superconducting current increases considerably even at low voltages, and a characteristic bend is observed on the CVC of the junction.[4]

In this paper we investigate the nonstationary Josephson effect at a superconductor-normal metal-superconductor (*S-N-S*) junction. Here, superconductivity is stimulated by a direct current provided the length d of the normal-metal bridge joining the superconducting films is large as compared with the coherence length $\xi_N \sim (\hbar D/T)^{1/2}$ in the normal metal (here D is the diffusion constant). We consider the case of a dirty metal in which the electron mean free path is short as compared with ξ_N. In this case the main contribution to the current comes from nonequilibrium electrons with energies $\varepsilon \sim \hbar D/d^2$ of the order of the reciprocal diffusion time along the bridge. The nonequilibrium current produced by quasiparticles with such energies decreases comparatively slowly as the length d of the bridge increases, and if the energy relaxation time τ_ε is long, it may considerably exceed both the ohmic and the Josephson components of the current.

As a result, the current through the junction increases considerably even at low voltages (and the effective resistance of the bridge in the resistive state correspondingly decreases). A bend appears on the CVC of the junction, analogous to the bend on the CVC of a superconducting bridge.[5] The effects under consideration of the stimulation of superconductivity in an *S-N-S* junction at a current above the critical value are analogous to those of the stimulation of superconductivity by an external microwave field, which have been investigated for such junctions in Ref. 9.

2. GREEN'S FUNCTIONS AND ELECTRON DISTRIBUTION FUNCTIONS FOR THE JUNCTION

The expression for the current density, written in terms of Green's functions integrated over the energy variable,[10] has the following form when none of the variables depends too strongly on time ($e\overline{V} \ll D/d^2$):

$$\mathbf{j}=\frac{\sigma}{2e}\int_{-\infty}^{\infty} d\varepsilon\left\{M_1\left(\nabla f_1-e\frac{\partial \mathbf{A}}{\partial t}\frac{\partial f}{\partial \varepsilon}\right)+\mathbf{J}f\right\}. \quad (1)$$

The quantities M_1 and $\mathbf{J}$ are related to the matrices G^R and G^A of the retarded and advanced Green's functions by the formulas

$$M_1=\frac{1}{4}\,\mathrm{Sp}\,(1-\tau_z G^R \tau_z G^A), \quad (2)$$

$$\mathbf{J}=\frac{1}{4}\,\mathrm{Sp}\,\tau_z\,(G^R\partial G^R-G^A\partial G^A),\quad \partial=\nabla-ie\mathbf{A}\tau_z. \quad (3)$$

In Eq. (1) f and f_1 are electron energy distribution functions (equations for them and for the functions G^R and G^A are given below), σ is the conductivity in the normal state, $\mathbf{A}$ is the vector potential, and τ_z is the appropriate Pauli matrix.

As a model of the junction we shall consider a bridge of variable thickness in which the connecting neck has constant transverse dimensions that are so small that all the quantities within the neck can be treated as functions of the longitudinal coordinate x alone, and we shall also set $\mathbf{A}=0$. We shall also assume that the electron-phonon interaction in the normal metal is weak, so that the order parameter Δ is zero.

 0038-5646/83/090624-05$04.00

The matrices G^R and G^A can be expressed as follows in terms of the ordinary Green's functions g and Gor'kov functions F:

$$G^R=-\tau_z G^{A+}\tau_z=\begin{pmatrix} g_1 & F_1 \\ -F_2 & g_2\end{pmatrix}=(\eta^2-1)^{-1/2}\begin{pmatrix} \eta & e^{i\chi} \\ -e^{-i\chi} & -\eta\end{pmatrix}. \quad (4)$$

As was shown in Ref. 9, the functions η and χ are determined by the equations

$$\left(\frac{\partial\eta}{\partial x}\right)^2-(\eta^2-1)^2 \times\left[\psi^2(\eta_0^2-\eta^2)+2iy\left(\frac{\eta_0}{(\eta_0^2-1)^{1/2}}-\frac{\eta}{(\eta^2-1)^{1/2}}\right)\right]=0, \quad (5)$$

$$\partial\chi/\partial x=\psi(1-\eta^2), \quad (6)$$

in which $y = 2\varepsilon d^2/D$ is a dimensionless parameter and the coordinate x is measured in units of the length d of the bridge and is reckoned from the center of the bridge. The coordinate-independent parameters $\eta_0 = \eta(0)$ and ψ (ψ is related to the parameter J by the formula $J = \mathrm{Im}\psi/d$) can be expressed in terms of the difference θ between the phases of the order parameter at the banks and the reduced energy y by means of the boundary conditions. At energies that are small as compared with the modulus of the order parameter Δ_0 at the banks, these conditions have the form[9]

$$\frac{\theta}{2}=\int_0^1 W(u)\,du, \qquad \frac{\psi}{2}=\int_0^1\frac{\zeta}{\zeta-u^2}W(u)\,du,$$

$$\varepsilon=\eta_0^{-2},$$

$$W(u)=\{1-u^2+(2iy\zeta/\psi^2)[(1-\zeta)^{-1/2}-u(u^2-\zeta)^{-1/2}]\}^{-1/2} \quad (7)$$

(the integration path in the complex plane of the variable $u = \eta/\eta_0$, as well as the branches of the root in Eqs. (4)–(7) are chosen in such a manner as to ensure the necessary analytic properties of the Green's functions). The parameters ζ and ψ describe the effect of the superconducting banks on the density of electron states in the junction, and as functions of energy their moduli are of the order of unity when $y\sim 1$ ($\varepsilon\sim D/d^2$) and decrease exponentially at higher energies.

The electron energy distribution functions $f=f_0+\delta f$ and f_1 are found from the kinetic equations,[10] which have the following form in a normal metal:

$$D\nabla(M\nabla f)+DJ\nabla f_1-\mathrm{Re}\,g_1\frac{\partial\delta f}{\partial t}=\mathrm{Re}\,g_1\frac{\delta f}{\tau_\varepsilon}, \quad (8)$$

$$D\nabla(M_1\nabla f_1)+DJ\nabla f-\mathrm{Re}\,g_1\frac{\partial}{\partial t}\left(f_1+e\varphi\frac{\partial f_0}{\partial\varepsilon}\right) =\mathrm{Re}\,g_1\frac{f_1+e\varphi\,\partial f_0/\partial\varepsilon}{\tau_\varepsilon}, \quad (9)$$

$$M={}^1/_4\,\mathrm{Sp}\,(1-G^RG^A), \quad (10)$$

where φ is the scalar potential. In equilibrium the distribution functions are

$$f=f_0=\mathrm{th}\,(\varepsilon/2T), \quad f_1=-e\varphi\partial f_0/\partial\varepsilon. \quad (11)$$

In connection with solving the set of equations (8) and (9), we note that M vanishes at the ends of the junction at the energies $\varepsilon<\Delta_0$ of interest to us. This corresponds to the fact that the excitations described by f cannot diffuse out of the junction because their state density vanishes on the banks. They are forbidden in the bridge and relax to equilibrium only by collisions with phonons (the right-hand sides of Eqs. (8) and (9) are the collision integrals). On the other hand, if the length d of the junction is small as compared with the diffusion length $l\sim(D\tau_\varepsilon)^{1/2}$ corresponding to these processes, δf can be treated as independent of the coordinates and can be found by averaging Eq. (8):

$$\frac{D}{d}J\left(f_1\left(\frac{d}{2}\right)-f_1\left(-\frac{d}{2}\right)\right)=\mathrm{Re}\langle g_1\rangle\left(\frac{\partial}{\partial t}+\frac{1}{\tau_\varepsilon}\right)\delta f \quad (12)$$

($\langle\ldots\rangle$ denotes averaging over the length of the bridge).

As regards the function f_1, the quasiparticle mode that it describes (an unbalance of electrons and holes) can diffuse out of the junction without hindrance (as follows from Eqs. (2) and (4), M_1 does not vanish on the banks). At the ends of the bridge it assumes the equilibrium values given by Eqs. (11). On substituting these values into Eq. (12) and solving it, we find the relation between the nonequilibrium addition δf to the distribution function f and the voltage $V(t)$ across the junction:

$$\delta f(\varepsilon,t)=\frac{D}{d}\int_{-\infty}^{t}\frac{\exp\{-(t-t')/\tau_\varepsilon\}}{\mathrm{Re}\langle g_1(t')\rangle}\frac{eV(t')}{2T}J(t')\,dt'. \quad (13)$$

As follows from Eq. (13), the nonequilibrium addition δf is proportional to $J(\varepsilon)$ and is therefore important only at energies $\varepsilon\sim D/d^2$.

Now we can calculate the current I using Eq. (1). In the first term, the energies $\varepsilon\sim T$ at which f_1 assumes equilibrium values are important. This term gives the normal current V/R, where R is the resistance of the bridge in the normal state (the deviations of f_1 from equilibrium when $\varepsilon\sim D/d^2$ can be neglected in comparison with δf because of the large value of τ_ε). In integrating the second term over the energy we express f as the sum of the equilibrium function $f_0=\tanh(\varepsilon/2T)$ and the nonequilibrium term δf given by Eq. (13). The integral containing f_0 is determined by the exponentially small values of J at the poles $\varepsilon=(2n+1)\pi Ti$ of the tangent (as is evident from Eq. (3), J consists of two terms, of which one is analytic in the upper half, and the other in the lower half, of the complex ε plane; this makes it possible to shift the integration path away from the real axis). As a result we obtain the equilibrium Josephson current $I_c\,\sin\theta$, where the critical current of the junction,

$$I_c\sim(\min\{T,\ \Delta_0^2/T\}/eR)\,(d/\xi_N)\exp(-d/\xi_N)$$

is exponentially small.[11] At the same time, as is evident from Eq. (13), δf is not analytic at low energies $\varepsilon\sim D/d^2$. The corresponding integral in Eq. (1) for the current therefore decreases only according to a power law as the length d of the bridge increases, and the nonequilibrium current given by the integral exceeds the Josephson current even at very low voltages.

We finally obtain the following expression for the current I:

$$I=\frac{V}{R}+I_c\sin\theta+\frac{D^2}{4R\,d^4T}\int_0^\infty dy\Big[\operatorname{Im}\psi(y,\theta(t))\times\int_{-\infty}^{t}\frac{\exp\{-(t-t')/\tau_\varepsilon\}V(t')\operatorname{Im}\psi(y,\theta(t'))}{\operatorname{Re}\langle g_1(y,\theta(t'))\rangle}dt'\Big]. \qquad (14)$$

With the aid of Eqs. (4)–(7) we can obtain the functions $\psi(\theta,y)$ and

$$\langle g_1(\theta,y)\rangle=\frac{2}{\psi}\int_0^1\frac{\zeta}{\zeta-u^2}W(u)\frac{u}{(u^2-\zeta)^{1/2}}du, \qquad (15)$$

which occur in Eq. (14). Then, assuming a fixed total current I and using Eq. (14) and the Josephson relation

$$\dot\theta=2eV \qquad (16)$$

we obtain the time dependence of θ and calculate the CVC of the junction from the formula

$$\bar V=\frac{\pi}{e}\Big(\int_{-\pi}^{\pi}\frac{d\theta}{\dot\theta}\Big)^{-1}. \qquad (17)$$

3. CURRENT-VOLTAGE CHARACTERISTIC OF THE JUNCTION

First we find the CVC of the bridge in the low-energy region where $e\bar V\ll\tau_\varepsilon^{-1}$. In this case the physical quantities change slowly as compared with the energy relaxation time τ_ε, and this permits us to take these quantities outside the integral with respect to time (replacing t' by t). Then the expression for the current becomes

$$I=\frac{\dot\theta}{2eR}\Big(1+\frac{D^2\tau_\varepsilon}{4d^4T}P(\theta)\Big)+I_c\sin\theta, \qquad (18)$$

$$P(\theta)=\int_0^\infty\frac{(\operatorname{Im}\psi(y,\theta))^2}{\operatorname{Re}\langle g_1(y,\theta)\rangle}dy. \qquad (19)$$

The function $P(\theta)$ as obtained by solving Eqs. (4)–(7) and (15) numerically is shown in Fig. 1. On substituting $\dot\theta$ from Eq. (18) into Eq. (17) we obtain the following expressions for the CVC:

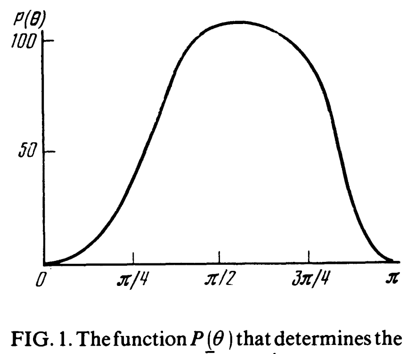

FIG. 1. The function $P(\theta)$ that determines the nonequilibrium current at low voltages ($e\bar V\ll\hbar\tau_\varepsilon^{-1}$).

a) a parabolic expression for the case $I-I_c\ll I_c$,

$$\bar V=R\Big(1+\frac{D^2\tau_\varepsilon}{4d^4T}\alpha_1\Big)^{-1}[2I_c(I-I_c)]^{1/2}, \qquad (20)$$

$$\alpha_1=P(\pi/2)=104;$$

b) a linear expression for the case $I\gg I_c$,

$$\bar V=R_{eff}I, \qquad (21)$$

$$R_{eff}=R\Big(1+\frac{D^2\tau_\varepsilon}{4d^4T}\alpha_2\Big)^{-1},\quad \alpha_2=\frac{1}{2\pi}\int_{-\pi}^{\pi}P(\theta)d\theta=57. \qquad (22)$$

As follows from Eqs. (20)–(22), the nonequilibrium effects influence the CVC of the bridge differently, depending on the length d of the bridge. If d is very large $(d\gg(\xi_N l_\varepsilon)^{1/2})$ the deviations from the resistance model are small. At intermediate bridge lengths

$$\xi_N\ll d\ll(\xi_N l_\varepsilon)^{1/2} \qquad (23)$$

the nonequilibrium effects considerably reduce the effective resistance of the bridge as compared with the ohmic resistance, and the superconductivity is strongly stimulated. We note that the order parameter at the banks, Δ_0, does not occur in the formula for the CVC, since the important energies $\varepsilon\sim D/d^2$ are assumed to be small as compared with it.

Thus, when the junction is not very long, virtually all the current of the initial section of the CVC is carried by the nonequilibrium component, and this leads to comparatively low energy dissipation. As is evident from Eq. (14), however, the nonequilibrium current cannot be arbitrarily strong. When the current I exceeds a certain value I_m, which will be estimated below, it can no longer be carried by the nonequilibrium component alone, and the ohmic component (the first term in Eq. (14)) becomes important. This leads to a rapid rise of the average voltage $\bar V$ across the junction from values of the order of $(e\tau_\varepsilon)^{-1}$, corresponding to currents $I<I_m$, to values of the order of I_mR, and a bend appears on the CVC of the bridge.

At voltages $\bar V\gg(e\tau_\varepsilon)^{-1}$, the phase difference θ and all the quantities associated with it change rapidly as compared with τ_ε. In that case the time integral in (14) can be transformed according to the formula

$$\int_{-\infty}^{t}\exp\Big(-\frac{t-t'}{\tau_\varepsilon}\Big)\frac{\dot\theta}{2e}\frac{\operatorname{Im}\psi(y,\theta(t'))}{\operatorname{Re}\langle g_1(y,\theta(t'))\rangle}dt'$$

$$=\frac{1}{2e}\Big\{\int_0^{\theta(t)}\frac{\operatorname{Im}\psi(y,\theta')d\theta'}{\operatorname{Re}\langle g_1(y,\theta')\rangle}-\overline{\int_0^{\theta}\frac{\operatorname{Im}\psi(y,\theta')}{\operatorname{Re}\langle g_1(y,\theta')\rangle}d\theta'}\Big\}. \qquad (24)$$

Here the superior bar denotes a time average of the type illustrated below for an arbitrary function $Z(\theta(t))$:

$$\bar Z=\frac{e\bar V}{\pi}\int_{-\pi}^{\pi}\frac{Z(\theta)d\theta}{\dot\theta}. \qquad (25)$$

Now Eq. (14) takes the form

$$I=\frac{\dot\theta}{2eR}+I_0Q(\theta),\qquad I_0=\frac{D^2}{8eR\,d^4T},\tag{26}$$

$$Q(\theta)=K(\theta,\ \theta)-\overline{K(\theta,\ \theta')},\tag{27}$$

$$K(\theta_1,\theta_2)=\int\limits_0^\infty \operatorname{Im}\psi(y,\theta_1)\int\limits_0^{\theta_2}\frac{\operatorname{Im}\psi(y,\theta_3)}{\operatorname{Re}\langle g_1(y,\theta_3)\rangle}d\theta_3\,dy\tag{28}$$

(the average in $\overline{K(\theta,\theta')}$ is taken over the time dependence of the second variable). Using Eq. (26) to express θ and employing formula (25), we obtain the following integral equation for $Q(\theta)$:

$$Q(\theta)=K(\theta,\theta)-\frac{\bar V}{2\pi R}\int\limits_{-\pi}^{\pi}\frac{K(\theta,\theta')\,d\theta'}{I-I_0Q(\theta')},\tag{29}$$

$$\bar V=2\pi R\left(\int\limits_{-\pi}^{\pi}\frac{d\theta'}{I-I_0Q(\theta')}\right)^{-1}.\tag{30}$$

As Eq. (29) shows, the function $Q(\theta)$, which determines the dependence of the nonequilibrium current on the phase difference, is determined by the parameter I/I_0. A search for an analytic expression for $Q(\theta)$ was unsuccessful. The form of the CVC of the junction can be estimated by considering Eqs. (29) and (30) in the limiting cases of low and high voltages.

At comparatively low voltages, such that $(e\tau_\varepsilon)^{-1}\ll\bar V\ll I_0R$, the phase θ is almost always close to one of the values θ_1 and θ_2 that correspond to the two maxima of the function $Q(\theta)$. The neighborhoods of these points, in which $Q(\theta)$ can be expressed in the form

$$Q=Q_{1,2}-{}^1/_2\gamma_{1,2}^2(\theta-\theta_{1,2})^2,$$

make the main contribution to the integral over θ' in Eqs. (29) and (30). In that case $Q(\theta)$ takes the form

$$Q(\theta)=K(\theta,\theta)-\left(\frac{K(\theta,\theta_1)}{\gamma_1[(I/I_0)-Q_1]^{1/2}}+\frac{K(\theta,\theta_2)}{\gamma_2[(I/I_0)-Q_2]^{1/2}}\right)$$

$$\times\left(\frac{1}{\gamma_1[(I/I_0)-Q_1]^{1/2}}+\frac{1}{\gamma_2[(I/I_0)-Q_2]^{1/2}}\right)^{-1}.\tag{31}$$

The self-consistency conditions

$$Q(\theta_{1,2})=Q_{1,2},\qquad Q'(\theta_{1,2})=0,\qquad Q''(\theta_{1,2})=-\gamma_{1,2}^2\tag{32}$$

provide a set of six equations from which one can determine the points $(\theta_{1,2})$ at which the function $Q(\theta)$ reaches its maxima, as well as its values $(Q_{1,2})$ and second derivatives $(-\gamma_{1,2}^2)$ at the maxima. That is enough to determine the I dependence of $\bar V$ from Eq. (30) in the limit under consideration. The rapid rise of the voltage on the CVC sets in at the current

$$I_m=Q_0I_0,\tag{33}$$

at which $Q_1=Q_2(=Q_0)$. Then Eq. (31) for $Q(\theta)$, and therefore Eqs. (32), become indeterminate. This indeterminacy can be lifted by considering small deviations of I from I_m. As a result, we obtain the parameters

$$c_{1,2}=\lim_{I\to I_m}\{(Q_{1,2}-Q_0)/[(I/I_0)-Q_0]\},$$

which determine the numerical coefficient in the parabolic dependence of the voltage $\bar V$ on the current I when $I-I_m\ll I_m$:

$$\bar V=\frac{\gamma_1\gamma_2[(1-c_1)(1-c_2)]^{1/2}}{\gamma_1(1-c_1)^{1/2}+\gamma_2(1-c_2)^{1/2}}R[2I_0(I-I_m)]^{1/2}.\tag{34}$$

An accurate determination of the numerical constants $c_{1,2}$ and Q_0 in Eqs. (33) and (34) would require a very complicated simultaneous solution of Eqs. (32) on a computer. The complexity of this calculation is due to the complicated form of the function $K(\theta,\theta')$, so we replace that function by the following approximate function that shares its basic properties:

$$\tilde K(\theta,\ \theta')=\alpha_2\sin\theta(1-\cos\theta')\tag{35}$$

(the θ dependence of ψ is approximated by the sine, and the coefficient α_2 is the average value of the function $P(\theta)$—Eq. (22)). Then we obtain the following expression for the CVC of the junction:

$$\bar V=R[2I_m(I-I_m)]^{1/2},\qquad I-I_m\ll I_m,\tag{36}$$

$$I_m=29\,I_0.\tag{37}$$

Now let us consider the CVC at very high voltages $\bar V\gg I_0R$ where almost all the current is carried by the ohmic component. Expanding Eqs. (29) and (30) in powers of the small quantity I_0/I and taking account of the fact that $Q(\theta)$ is an odd function, we obtain

$$\bar V=IR\left(1-\frac{I_0^2}{I^2}\frac{1}{2\pi}\int\limits_{-\pi}^{\pi}Q^2(\theta)\,d\theta\right),\tag{38}$$

$$Q(\theta)=K(\theta,\theta)-\frac{1}{2\pi}\int\limits_{-\pi}^{\pi}K(\theta,\theta')\,d\theta',\qquad \bar V\gg I_0R.$$

Using Eq. (35) we obtain

$$\bar V=IR(1-I_m^2/2I^2).\tag{39}$$

As is evident from Eqs. (38) and (39), the deviation of the CVC from Ohm's law at high voltages follows the $(I_0/I)^2$ law, as in the resistance model, while the current I_m plays the part of the critical current in order of magnitude.

4. CONCLUSION

Our results show that superconductivity may be stimulated in an *S-N-S* junction by currents exceeding the critical value. The effect is strongest when the length d of the junction satisfies condition (23). Then the CVC of the bridge becomes a curve having characteristic bends as shown in Fig. 2.

When the current I exceeds the critical current I_c, the average voltage $\bar V$ across the junction increases much less rapidly as I increases than would be required by Ohm's law (Eq. (22) for the effective resistance). This is associated with the fact that the current is carried not by the ohmic component, but by the nonequilibrium component. The latter is produced by electrons whose energies ε are of the order of the reciprocal time $\hbar D/d^2$ for the diffusion of electrons through the junction. Such electrons (unlike the electrons with energies $\varepsilon\sim T$ that carry the equilibrium current and

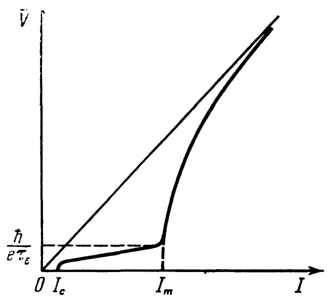

FIG. 2. Current-voltage characteristic of an S-N-S junction.

lose their coherence at distances of $\sim\xi_N$) do not lose their coherence when they diffuse through the layer of normal metal of thickness d, and their current gives rise to virtually no voltage drop.

The appearance of the nonequilibrium current is associated with the periodic variations of the density of quasiparticle states, which are responsible for the alternating voltage that appears when $I > I_c$. As long as the frequency $2e\bar{V}/\hbar$ of these variations is low as compared with the electron-phonon collision frequency τ_ε^{-1}, the nonequilibrium addition to the electron distribution function and the corresponding current are proportional to the voltage (Eq. (18)). When these frequencies become comparable, the nonequilibrium current reaches its maximum value of the order of I_0, which is large by the parameter (d/ξ_N) as compared with the critical current I_c (Eq. (26)). Then the voltage begins to rise sharply and approaches values corresponding to Ohm's law (Eqs. (33)–(39)). We note that here the voltage $\bar{V}$ is still small as compared with $\hbar D/d^2e$, and that is the condition for the applicability of the equations used in this study. Small deviations from Ohm's law (excess current) at higher voltages are discussed in Ref. 12.

The nonstationary Josephson effect in an S-N-S junction has been previously investigated in Ref. 13 on the basis of the nonstationary Ginzburg-Landau equations, which are valid for gapless superconductors. In addition, there is a recent paper[14] devoted mainly to clean junctions. Stimulation of superconductivity in the junction was predicted, in accordance with experiment,[5,6] in those papers as well as in the present work. The situation observed in Ref. 5 corresponds to the results of the present study. In our case the stimulation occurs at fairly high temperatures $T \gg \hbar D/d^2$. When this condition does not hold, the nonequilibrium current no longer exceeds the equilibrium current and the superconductivity may be suppressed rather than stimulated. (Thus, when $\Delta \ll T \ll \hbar D/d^2$ the result of Ref. 7, obtained for short superconducting bridges, are directly applicable.) Experimentally, the low-temperature CVC exhibits a sudden change in voltage at $I = I_c$, but at higher temperatures it behaves as described here: the CVC is a sharply bent curve, while the extent to which the superconductivity is stimulated is in accordance with Eqs. (20)–(22) (where $\tau_\varepsilon \propto T^{-3}$) and decreases with increasing T ($R_{\text{eff}} \propto T^4$).

The author thanks L. G. Aslamazov for formulating the problem and guiding the work, and A. A. Abrikosov and A. I. Larkin for discussing the results.

[1]K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).
[2]P. E. Lindelof, Rep. Prog. Phys. **44**, 949 (1981).
[3]L. G. Aslamazov and A. I. Larkin, Pis'ma v Zh. Eksp. Teor. Fiz. **9**, 150 (1969) [JETP Lett. **9**, 87 (1969)].
[4]a) V. N. Gubankov, V. P. Koshelets, and G. A. Ovsyannikov, Zh. Eksp. Teor. Fiz. **73**, 1435 (1977) [Sov. Phys. JETP **46**, 755 (1977)]; b) V. N. Gubankov, V. P. Koshelets, and G. A. Ovsyannikov Fiz. Nizk. Temp. **6**, 50 (1980) [Sov. J. Low Temp. Phys. **6**, 24 (1980)]; c) M. Octavio, W. J. Skocpol, and M. Tinkham, Phys. Rev. **17B**, 159 (1978); d) Yu. A. Divin and F. Ya. Nad', Fiz. Nizk. Temp. **4**, 1105 (1978) [Sov. J. Low Temp. Phys. **4**, 520 (1978)]; e) S. K. Zhilinskiĭ, Yu. I. Latyshev, and F. Ya. Nad', Fiz. Tverd. Tela **23**, 3710 (1981) [Sov. Phys. Solid State **23**, 2159 (1981)].
[5]J. M. Warlaumont, J. C. Brown, and R. A. Buhrman, Appl. Phys. Lett. **34**, 415 (1979).
[6]R. B. van Dover, A. de Lozanne, and M. R. Beasley, J. Appl. Phys. **52**, 7327 (1981).
[7]S. N. Artemenko, A. F. Volkov, and A. V. Zaĭtsev, Zh. Eksp. Teor. Fiz. **76**, 1816 (1979) [Sov. Phys. JETP **49**, 924 (1979)].
[8]L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **70**, 1340 (1976) [Sov. Phys. JETP **43**, 698 (1976)].
[9]L. G. Aslamazov and S. V. Lempitskiĭ, Zh. Eksp. Teor. Fiz. **82**, 1671 (1982) [Sov. Phys. JETP **82**, 967 (1982)].
[10]a) Albert Schmid and Gerd Schön, J. Low Temp. Phys. **20**, 207 (1975); b) A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)]; c) R. J. Watts-Tobin, Y. Krahenbül, and L. Kramer, J. Low Temp. Phys. **42**, 459 (1981).
[11]a) P. De Zhen, Sverkhprovodimost' metallov i splavov (Superconductivity of metals and alloys), Mir, Moscow, 1968; b) L. G. Aslamazov, A. I. Larkin, and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **55**, 323 (1968) [Sov. Phys. JETP **28**, 171 (1969)]; c) K. K. Likharev, Pis'ma v Zh. Tekh. Fiz. **2**, 29 (1976) [Sov. Tekh Fiz. Lett. **2**, 12 (1976)]; d) A. Barone and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **77**, 1463 (1979) [Sov. Phys. JETP **50**, 735 (1979)]; e) A. D. Zaikin and G. F. Zharkov, Fiz. Nizk. Temp. **7**, 375 (1981) [Sov. J. Low Temp. Phys. **7**, 184 (1981)].
[12]A. V. Zaĭtsev, Tezisy dokladov na 21-*m* Vsesoyuznom soveshchanii po FNT (Abstracts of the 21st All Union conference on low temperature physics), Khar'kov, 1980, Part 1, p. 132.
[13]K. K. Likharev and L. A. Yakobson, Zh. Eksp. Teor. Fiz. **68**, 1150 (1975) [Sov. Phys. JETP **41**, 570 (1975)].
[14]A. D. Zaikin, Zh. Eksp. Teor. Fiz. **84**, 1560 (1983) [Sov. Phys. JETP **57**, 910 (1983)].

Translated by E. Brunner