Induced transition radiation in the field of a strong electromagnetic wave

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Induced transition radiation produced by relativistic electrons crossing the interface between two different dielectric media irradiated by a strong electromagnetic wave is considered. Expressions for the probability of emission (absorption) of an arbitrary number of field quanta are obtained within the framework of a quantum-mechanical description of the behavior of an electron in the field of a strong wave. Numerical estimates that demonstrate the feasibility of light-wave amplification are presented.

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1. INTRODUCTION

Transition radiation arises when a uniformly traveling charged particle crosses the interface between optically different media. The phenomenon has been extensively studied both theoretically and experimentally.[1-3] Whenever the interface between two transparent media is irradiated by an external electromagnetic wave, the transition-radiation characteristics change. First, as has been previously demonstrated,[4] when a particle emits an electromagnetic-field quantum not identical to the wave quanta the angular and spectral distributions of the radiation are changed. In particular, the radiation becomes polarized owing to the polarization of the external electromagnetic wave. Second, the transition radiation may become induced radiation if the photon emitted or absorbed by the particle is identical to the external wave quanta. This phenomenon is of interest both for acceleration of charged particles by means of an electromagnetic field as well as for amplification of a light wave.

Induced single-photon processes with participation of nonrelativistic particles have been considered[5] in first-order perturbation theory in terms of the external electromagnetic field. Multiphoton induced transition radiation of relativistic particles has also been considered[6] without making use of perturbation theory. However, the wave function derived in that reference for an electron in an electromagnetic-wave field, with account taken of the interface, does not go over in the limit of optically identical media to the well-known solution.[7]

In the present article we obtain expressions for the probabilities of induced transition radiation (absorption) in a strong electromagnetic-wave field for relativistic charged particles (for the sake of definiteness, we consider electrons). These probabilities are computed within the framework of a quantum-mechanical description of the behavior of electrons in a given classical field. The wave field is taken into account in all orders of perturbation theory when finding the Ψ-function of the electrons (we assume satisfaction of the strong-field condition

$$e(\mathbf{A}_0\mathbf{p})/\varepsilon\omega \gtrsim 1,$$

where $\mathbf{A}_0$ is the maximum amplitude of the vector potential of the wave field; ω is the wave frequency; and ε and $\mathbf{p}$ are the energy and momentum of the electrons incident on the interface between the media: $\hbar = c = 1$).

2. BASIC EQUATIONS

The interface between two transparent dielectric media with refractive indices n_1 and n_2 is aligned with the plane $z = 0$, the z-axis being directed towards the dielectric with the refractive index n_2. Let this interface be irradiated by a plane monochromatic electromagnetic wave incident in the xz-plane at an angle θ_0 to the normal to the interface. Using the classical description of the wave field, we write the components of the vector potential of the wave field in the form

$$A_j = {}^1/_2 A_{0j}\{\Theta(-z)[(e^{-i\varphi_0}+e^{+i\varphi_0}) +\alpha_j(e^{-i\varphi_1}+e^{+i\varphi_1})]+\beta_j\Theta(z)(e^{-i\varphi_2}+e^{+i\varphi_2})\}. \quad (1)$$

In (1) we have introduced the following notation: A_0 is the amplitude of the vector potential of the incident wave:

$$\varphi_i = \omega t - \mathbf{k}_i\mathbf{r} \quad (i=0,\ 1,\ 2)$$

the alternating part of the phase of the corresponding waves; ω is the wave frequency; $\mathbf{k}_0$, $\mathbf{k}_1$, and $\mathbf{k}_2$ are the wave vectors of the incident and reflected waves and of the wave that crossed the interface, respectively;

$$\Theta(z)=1 \ \text{if} \ z>0, \ \Theta(z)=0 \ \text{if} \ z<0,$$

α_j and β_j are the proportionality factors between the wave amplitudes given by Fresnel's formulas,[8] and $j = x, y, z$.

We direct the electron flux at an arbitrary angle to the interface, aligning its incidence with the xz plane. We assume, as usual, that the particles themselves do not interact with the medium, i.e., move uniformly in the absence of an electromagnetic wave. Neglecting small spin corrections, we may use the Klein-Gordon equation as our basic equation, in which we incorporate the wave field (1):

$$\left[-\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\mu} - 2ie\left(A^\mu\frac{\partial}{\partial x^\mu}\right) + (eA)^2 - m^2\right]\Psi = 0, \quad (2)$$

where $A^\mu = (0,\mathbf{A})$ is the four-potential of the wave field.

In accordance with the nature of the electromagnetic field, we seek the solutions of Eqs. (2) in the form

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$$\Psi(z<0)e^{-ipx}F_p(A_<)+\sum_{n=-\infty}^{+\infty}B_n e^{-ip_{1n}x}F_{p_{1n}}(A_<),$$
$$\Psi(z>0)=\sum_{n=-\infty}^{\infty}C_n e^{-ip_n x}F_{p_n}(A_>),\tag{3}$$

where $A_<$ and $A_>$ are the vector potentials of the field for $z<0$ and $z>0$, respectively; in (3), we have used the standard notation for the scalar product of four-vectors: $px\equiv(px)=\varepsilon t-\mathbf{p}\cdot\mathbf{r}$.

Because of the space-time periodicity of the interaction term in (2), the state of the incident particles with the four-momentum $p=(\varepsilon,\mathbf{p})$ is related in (3) to the states with the four-momenta p_{1n} (reflected electron wave) and p_n (wave which has crossed the interface). Here

$$p_{1n}=(\varepsilon_n,\ p_{nx},\ 0,\ -p_{nz}),\quad p_n=(\varepsilon_n,\ p_{nx},\ 0,\ p_{nz}),$$
$$\varepsilon_n=\varepsilon-n\omega,\quad p_{nx}=p_x-nk_x,\quad p_{nz}=(\varepsilon_n^2-p_{nx}^2-m^2)^{1/2}.$$

Below, we will use for the functions F introduced in (3) the notation $F_p(A_<)=F_p(\varphi_0)F_p(\varphi_1)$ and $F_{p_n}(A_>)=F_{p_n}(\varphi_2)$, the meaning of which is self-evident.

Substituting (3) in (2), we obtain an equation for $F(\varphi)$:

$$2i(pk)F'(\varphi)=2e(Ap)F(\varphi)\tag{4}$$

[in (4), we have omitted the inessential indices for the functions F as well as for the four-vectors p, k, and A]. In deriving equations (4), we made a number of approximations, for example, we have omitted small terms such as $e(Ak)F'$, terms containing second derivatives of F, as well as terms $\sim(eA)^2$ quadratic in the field. These terms are small whenever

$$\left|\frac{eA_0}{\varepsilon}\frac{1}{1-n_{1,2}(p/\varepsilon)\cos\measuredangle\mathbf{pk}}\right|=\frac{\xi/\gamma}{|1-n_{1,2}(p/\varepsilon)\cos\measuredangle\mathbf{pk}|}\ll1,\tag{5}$$

where the dimensionless parameter $\xi=eA_0/m$ characterizes the intensity of the interaction between the electron and wave, and $\gamma=\varepsilon/m$. Obviously, condition (5) imposes constraints on the feasible angles between the directions of propagation of the electrons and the electromagnetic waves.

We assume that (5) holds. Then the solutions of (4) may be written in the form

$$F_p(A_<)=\exp\left[-i\int\frac{e(Ap)}{(pk)}d\varphi_0-i\alpha\int\frac{e(A(\varphi_1)p)}{(pk_1)}d\varphi_1\right],$$
$$F_p(A_>)=\exp\left[-i\beta\int\frac{e(A(\varphi_2)p)}{(pk_2)}d\varphi_2\right].\tag{6}$$

The coefficients B_n and C_n of the expansions in (3) will be found from the continuity and smoothness conditions of the Ψ-function in the plane $z=0$. From these conditions, we obtain the system of equations

$$F_p(A_<)+\sum_n B_n\exp\{in(\omega t-k_x x)\}F_{p_{1n}}(A_<)$$
$$=\sum_n C_n\exp\{i(\omega t-k_x x)n\}F_{p_n}(A_>),$$
$$p_zF_p(A_<)-\sum_n B_n p_{nz}\exp\{in(\omega t-k_x x)\}$$
$$\times F_{p_{1n}}(A_<)=\sum_n C_n p_{nz}\exp\{in(\omega t-k_x x)\}F_{p_n}(A_>).\tag{7}$$

In deriving (7), we used the condition $n\varepsilon\omega\ll p_z^2$, as well as the inequality

$$\left|\frac{eA_0}{\omega}\frac{k_z/p_z}{1-n_{1,2}(p/\varepsilon)\cos\measuredangle\mathbf{pk}}\right|\ll1,$$

which in the general case coincides with condition (5).

The solution of the system of equations (7) may be greatly simplified if the criterion

$$\left|\frac{n\omega}{\varepsilon}\frac{1}{1-n_{1,2}(p/\varepsilon)\cos\measuredangle\mathbf{pk}}\right|\ll1$$

and the condition $p_z\sim|p_{1nz}|\sim p_{nz}$ are satisfied, since in this case it is possible to disregard the dependence on n in the functions $F_{p_{1n}}$ and F_{p_n}.

Assuming that all the above conditions hold, we present the explicit form for the functions F:

$$F_p(A_<)=\exp\left[ie\left(\frac{\mathbf{A}_0\mathbf{p}}{(pk_0)}\sin\varphi_0+\frac{\alpha_xA_{0x}p_x+\alpha_zA_{0z}p_z}{(pk_1)}\sin\varphi_1\right)\right]$$
$$F_{p_{1n}}(A_<)\approx F_{p_1}(A_<)$$
$$=\exp\left[ie\left(\frac{\mathbf{A}_0\mathbf{p}_1}{(p_1k_0)}\sin\varphi_0+\frac{\alpha_xA_{0x}p_x-\alpha_zA_{0z}p_z}{(p_1k_1)}\sin\varphi_1\right)\right],\tag{8}$$
$$F_{p_n}(A_>)\approx F_p(A_>)=\exp\left[ie\frac{\beta_xA_{0x}p_x+\beta_zA_{0z}p_z}{(pk_2)}\sin\varphi_2\right].$$

In (8), we have introduced the notation

$$\alpha_x=-\frac{n_2\cos\theta_0-n_1\cos\theta}{n_2\cos\theta_0+n_1\cos\theta},\quad\alpha_z=-\alpha_x,$$
$$\beta_x=\frac{2n_1\cos\theta}{n_2\cos\theta_0+n_1\cos\theta},\quad\beta_z=\frac{2n_1^2\cos\theta_0}{n_2(n_2\cos\theta_0+n_1\cos\theta)},\tag{9}$$

where θ is the angle of refraction, related to the angle of incidence of the wave by the well-known law $n_1\sin\theta_0=n_2\sin\theta$.

Solving the system of equations (7) under our assumptions we find

$$\sum_{n=-\infty}^{\infty}B_n\exp\{in(\omega t-k_x x)\}\approx0,$$
$$\sum_{n=-\infty}^{\infty}C_n\exp\{in(\omega t-k_x x)\}$$
$$=F_p(A_<)/F_p(A_>)=\sum_{n=-\infty}^{\infty}J_n(\Delta)\exp\{in(\omega t-k_x x)\},\tag{10}$$

where J_n are Bessel functions of order n of the argument

$$\Delta=e\left[\frac{A_{0x}p_x+A_{0z}p_z}{(pk_0)}+\frac{\alpha_x A_{0x}p_x+\alpha_z A_{0z}p_z}{(pk_1)}-\frac{\beta_x A_{0x}p_x+\beta_z A_{0z}p_z}{(pk_2)}\right]. \quad (11)$$

Finally, from (3), (6), and (10) we obtain

$$\Psi(x)=e^{-ipx}\left\{\Theta(-z)\exp i\left[e\frac{\mathbf{A}_0\mathbf{p}}{(pk_0)}\sin(\omega t-k_x x-k_{0z}z)+e\frac{\alpha_x A_{0x}p_x+\alpha_z A_{0z}p_z}{(pk_1)}\sin(\omega t-k_x x+k_{0z}z)\right]+\Theta(z)\exp i\left[e\frac{\beta_x A_{0x}p_x+\beta_z A_{0z}p_z}{(pk_2)}\sin(\omega t-k_x x-k_{2z}z)\right]\times\sum_n J_n(\Delta)\exp[in(\omega t-k_x x)]\exp[-i(p_z-p_{nz})z]\right\}. \quad (12)$$

It is easily verified that the function (12) turns into the well-known solution for the wave function of an electron in an electromagnetic wave field propagating in a homogeneous medium (for the limiting case of optically identical media, i.e., $n_1=n_2$).[7]

We expand the electron Ψ-function (12) in series in four-momentum eigenfunctions. The coefficients of the expansion are determined by the expression

$$a_{p'p}=\int e^{ip'x}\Psi(x)\,d^4x. \quad (13)$$

The integrations in (13) with respect to time as well as in the interface plane between the media may be performed within infinite limits, and in computing the integrals with respect to z we use the well-known formula

$$\int_{-\infty}^{\infty}\Theta(\mp z)e^{-iqz}\,dz=\frac{i}{\pm q+i\gamma},\quad \gamma\to+0.$$

The integration with respect to z in (13) is actually cut off at the length L_e of attenuation of the electron wave in the matter or at the layer thickness l (in the case of layered media). The amplitudes of the emission (absorption) processes of quanta of light waves incident on, reflected by, or transmitted by the interface interfere whenever $|\Delta k_z|L\ll 1$, where L is the path of integration (the smaller of L_e or l may be taken as L), and Δk_z is the spread (with respect to the z-component) of the wave vector in the incident wave, due, for example, to the finiteness of the wave train.

Let us assume that the condition of interference of the amplitudes of the processes holds true. We then find for the coefficients $a_{p'p}$

$$a_{p'p}=\frac{(2\pi)^3}{2(\varepsilon\varepsilon')^{1/2}V}\sum_{nn'}W_{nn'}(p_z')\,\delta(\varepsilon'-\varepsilon+(n+n')\omega)\,\delta^{(2)}\times[\mathbf{p}_\tau'-\mathbf{p}_\tau+(n+n')\mathbf{k}_\tau], \quad (14)$$

$$W_{nn'}(p_z')=\left[J_n(\Delta_0)J_{n'}(\Delta_\alpha)\frac{i\mathscr{P}}{p_z'-p_z+(n-n')k_{0z}}-J_n(\Delta)J_{n'}(\Delta_\beta)\frac{i\mathscr{P}}{p_z'-p_{nz}+n'k_{2z}}\right]; \quad (15)$$

where $\mathbf{p}_\tau$ and $\mathbf{p}_\tau'$ are the tangential components of the momentum of the initial and final state of the particle, respectively, and $\mathbf{k}_\tau$ is the tangential component of the wave vectors. In (14) we have used the usual normalization of the wave function of a free particle relative to a single particle in a volume V and introduced the notation

$$\Delta_0=e\frac{\mathbf{A}_0\mathbf{p}}{(pk_0)},\quad \Delta_\alpha=e\frac{\alpha_x A_{0x}p_x+\alpha_z A_{0z}p_z}{(pk_1)},\quad \Delta_\beta=e\frac{\beta_x A_{0x}p_x+\beta_z A_{0z}p_z}{(pk_2)},\quad \Delta=\Delta_0+\Delta_\alpha-\Delta_\beta. \quad (16)$$

Note that terms proportional to $\delta(p_z'-p_z+(n-n')k_{0z})$ and $\delta(p_{z'}-p_{nz}+n'k_{2z})$, which lead[2] to Cherenkov emission processes in the first and second media, have been omitted from (15). In the present article, these effects will not be considered, and we accordingly assume that $(pk_0)\approx(pk_1)\approx(pk_2)\approx\varepsilon\omega$.

Comparison of (14) with the S-matrix representation[7] leads to a correspondence between the amplitudes $a_{p'p}$ and the elements of the S-matrix $S_{p'p}$:

$$S_{p'p}=-i(pk)a_{p'p}\approx-i\varepsilon\omega a_{p'p}. \quad (17)$$

The latter equation holds, as can be easily verified by perturbation theory, for the case of a weak enough external electromagnetic wave:

$$e\mathbf{Ap}/(pk)\ll 1.$$

According to the usual procedure of S-matrix theory, the probability that induced emission (absorption) of an arbitrary number s of photons will occur when a particle crosses the interface between two media is given by the expression

$$dW_s=\frac{(\varepsilon\omega)^2}{4v\varepsilon\varepsilon'}\delta(\varepsilon'-\varepsilon+s\omega)\,\delta^{(2)}\times(\mathbf{p}_\tau'-\mathbf{p}_\tau+s\mathbf{k}_\tau)\left|\sum_n W_{n,s-n}\right|^2 d\mathbf{p}', \quad (18)$$

where $n+n'=s$ and v is the initial velocity of the particles. As a result of integration with respect to the tangential component of the momentum of the scattered electron, we find from (18) that

$$dW_s=\frac{\varepsilon\omega^2}{4v\varepsilon'}\delta(\varepsilon'-\varepsilon+s\omega)\left|\sum_n W_{n,s-n}\right|^2 dp_z'. \quad (19)$$

To carry out the succeeding integration in (19) with respect to dp_z', it is necessary to introduce explicitly into the argument a delta function that yields conservation laws for the system energy and for the z-component of the particle momentum. This may be done by means of the known procedure

$$\delta(\varepsilon'-\varepsilon+s\omega)=\frac{\delta(p_z'-p_0)}{|\partial\varepsilon'/\partial p_z'|}=\frac{\varepsilon'\delta(p_z'-p_0)}{|p_0|},$$
$$p_0=\pm[p_z^2-2s\omega\varepsilon+2s(\mathbf{p}_\tau\mathbf{k}_\tau)+s^2(\omega^2-\mathbf{k}_\tau^2)]^{1/2}. \quad (20)$$

Substituting (20) in (19) and integrating next with respect to dp_z' we find for the total probability of emission ($s>0$) or absorption ($s<0$) the following:

$$W_s=\frac{\varepsilon\omega^2}{4v|p_0|}\left|\sum_n W_{n,s-n}(p_0)\right|^2. \tag{21}$$

It can be easily shown that in the weak-external-field limit ($\Delta_0, \Delta_\alpha, \Delta_\beta \ll 1$), by expanding $W_{n,s-n}$ in a series in the small parameters Δ_i and limiting ourselves to single-photon approximation, we may obtain from (21) the corresponding expression obtain in Ref. 5 in first-order perturbation theory. For this reason, the case of a weak field will not be considered here.

The statement of the problem can be given a more physical interpretation if we consider instead of the emission or absorption probability the total increment of the electron energy

$$\Delta\varepsilon=\varepsilon'-\varepsilon=-\omega\langle s\rangle=-\omega\sum_{s=-\infty}^{\infty} sW_s. \tag{22}$$

Substituting (21) and (15) in (22), we find that the mean number of emitted quanta is given as follows (when $n\omega/\varepsilon\ll 1$):

$$\langle s\rangle=\frac{\varepsilon\omega^2}{4v}\sum_{s=-\infty}^{+\infty}\frac{s}{|p_0|}\left|\sum_{n=-\infty}^{\infty}\frac{J_n(\Delta_0)J_{s-n}(\Delta_\alpha)}{fn+As+Bs^2}-\frac{J_n(\Delta)J_{s-n}(\Delta_\beta)}{\varphi n+Cs+B(s^2-n^2)}\right|^2. \tag{23}$$

In (23) we have introduced the notation

$$\begin{aligned}
&f=2k_{0z},\quad \varphi=-k_{2z}+\frac{\omega\varepsilon-(\mathbf{p}_\tau\mathbf{k}_\tau)}{p_z},\\
&A=-k_{0z}-\frac{\omega\varepsilon-(\mathbf{p}_\tau\mathbf{k}_\tau)}{p_z},\\
&C=k_{2z}-\frac{\omega\varepsilon-(\mathbf{p}_\tau\mathbf{k}_\tau)}{p_z},\\
&B=\frac{p_z^2(\omega^2-k_\tau^2)-[\omega\varepsilon-(\mathbf{p}_\tau\mathbf{k}_\tau)]^2}{2p_z^3}.
\end{aligned} \tag{24}$$

The summation in (23) may be performed analytically, using the fact that terms quadratic in s are small (a comparative estimate of the terms is $Bs/A\sim n\omega/\varepsilon\ll 1$). As a result of the computations we find from (23) that

$$\begin{aligned}
\langle s\rangle=\frac{\varepsilon\omega^2}{4vp_z}\Bigg\{&-2B\sum_{s=1}^{\infty}s\sum_{n=-\infty}^{\infty}\left[\frac{J_n(\Delta_0)J_{s-n}(\Delta_\alpha)}{fn+As}-\frac{J_n(\Delta)J_{s-n}(\Delta_\beta)}{\varphi n+Cs}\right]\\
&\times\sum_{n=-\infty}^{\infty}\left[\frac{J_n(\Delta_0)J_{s-n}(\Delta_\alpha)}{(fn+As)^2}s^2-\frac{J_n(\Delta)J_{s-n}(\Delta_\beta)}{(\varphi n+Cs)^2}(s^2-n^2)\right]\\
&+\sum_{s=1}^{\infty}s^2\frac{\omega\varepsilon-(\mathbf{p}_\tau\mathbf{k}_\tau)}{p_z^2}\\
&\times\left[\sum_{n=-\infty}^{\infty}\left(\frac{J_n(\Delta_0)J_{s-n}(\Delta_\alpha)}{fn+As}-\frac{J_n(\Delta)J_{s-n}(\Delta_\beta)}{\varphi n+Cs}\right)\right]^2\Bigg\}.
\end{aligned} \tag{25}$$

In the case of a strong electromagnetic field, there are two special cases of interest: (1) all the parameters Δ_i are large ($\Delta_0, \Delta_\alpha, \Delta_\beta, \Delta\gg 1$); (2) two of the parameters are large, and the other two are small (for example, $\Delta_\alpha, \Delta<1$ and Δ_0, $\Delta_\beta\gg 1$). Let us consider both cases in turn, starting with the comparatively simpler case (2).

If $\Delta_\alpha, \Delta<1$, we may limit the summations in (25) with respect to n to terms with $n=s, s\pm 1$ in the terms containing $J_{s-n}(\Delta_\alpha)$, and to $n=0, \pm 1$ in the terms containing $J_n(\Delta)$. As a result, we have after simple algebra, using a well-known formula[9]

$$\sum_{k=-\infty}^{\infty}J_k(z)J_{p+k}(t)=J_p(t+z),$$

and obtain (the summation includes an approximation that uses the condition $s\gg 1$)

$$\langle s\rangle=\frac{\varepsilon\omega^2}{2vp_z}\left[\frac{\omega\varepsilon-(\mathbf{p}_\tau\mathbf{k}_\tau)}{p_z^2}\Phi-2BD\right]. \tag{26}$$

In (26) we have used the notation

$$\begin{aligned}
&\Phi=\frac{y(\Delta_\alpha)}{(f+A)^2}+\frac{y(\Delta)}{C^2}-\frac{2y(|\Delta_0-\Delta_\beta|)}{(f+A)C},\\
&D=\frac{y(\Delta_\alpha)}{(f+A)^3}+\frac{y(\Delta)}{C^3}-\frac{y(|\Delta_0-\Delta_\beta|)}{(f+A)C}\left(\frac{1}{f+A}+\frac{1}{C}\right),
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
&y(\Delta_\alpha)=\frac{1}{2}[2J_1^2(\Delta_\alpha)+J_0^2(\Delta_\alpha)],\\
&y(\Delta)=\frac{1}{2}[2J_1^2(\Delta)+J_0^2(\Delta)],\\
&y(|\Delta_0-\Delta_\beta|)=\frac{1}{2}J_0(\Delta_\alpha)J_0(\Delta)J_0(|\Delta_0-\Delta_\beta|)\\
&\quad-[J_0(\Delta_\alpha)J_1(\Delta)+J_1(\Delta_\alpha)J_0(\Delta_0)]J_1(|\Delta_0-\Delta_\beta|)\\
&\quad+J_1(\Delta_\alpha)J_1(\Delta)J_0(|\Delta_0-\Delta_\beta|).
\end{aligned} \tag{28}$$

From the standpoint of experimental measurements, it is also of interest to study the energy spread of the electron beam, which is characterized by the mean square variation of the number of quanta of the external electromagnetic wave:

$$\langle s^2\rangle=\sum_{s=-\infty}^{\infty}s^2W_s. \tag{29}$$

After substituting (21) and (15) in (29) and carrying out the indicated transformations, we obtain for the particular case considered (Δ_α, $\Delta < 1$ and Δ_0, $\Delta_\beta \gg 1$) the expression

$$\langle s^2\rangle = (\varepsilon\omega^2/2vp_z)\,\Phi, \qquad (30)$$

in which the function Φ is given by (27) and (28).

In the other particular case of a strong external wave (Δ_0, Δ_α, Δ_β and $\Delta \gg 1$), summation in (25), using the completeness condition for the Bessel functions, leads to the limiting value

$$\lim\langle s\rangle \simeq \max\left\{-\frac{\varepsilon\omega^2}{vp_z}B\left[\frac{1}{(f+A)^3},\frac{1}{C^3}\right];\right.$$

$$\left.\frac{\varepsilon\omega^2}{2vp_z}\frac{\omega\varepsilon-(\mathbf{p}_\tau\mathbf{k}_\tau)}{p_z^2}\left[\frac{1}{(f+A)^2},\frac{1}{C^2}\right]\right\}. \qquad (31)$$

The dispersion of the electron energy in the field of a strong wave will be analogously determined by the limiting value

$$\lim\langle s^2\rangle \approx \frac{\varepsilon\omega^2}{2vp_z}\max\left\{\frac{1}{(f+A)^2},\frac{1}{C^2}\right\}. \qquad (32)$$

3. CONCLUSION. DISCUSSION OF APPROXIMATIONS AND RESULTS

In this section we will discuss in more detail the physical interpretation of the approximations made in the paper. The external electromagnetic field (1) is treated classically, since it is assumed that the number of quanta in the given field mode is much greater than 1. For the usual field strengths $E_0 = 10^4$ W/cm, the mean number of quanta in a single laser mode with energy $\omega = 1$ eV is about 10^8. Thus, the probability of induced transition radiation into this field mode with this number of quanta is greater than that of non-induced (spontaneous) radiation and the latter can therefore be ignored. If the strength of the external electromagnetic field decreases to such an extent that the number of quanta in the mode turns out to be on the order of 1, the classical expression (1) for the vector potential must be replaced by a quantum expression similar to that given in Ref. 2. In this case, we obtain well-known results for the probability of spontaneous transition radiation.[2]

The solution (6) of the Klein-Gordon equation is found in an approximation given in the form of an inequality (5). From the physical standpoint, this inequality asserts that conditions for Cherenkov emission are not satisfied in media in the presence of an external electromagnetic field:

$$\cos\sphericalangle\mathbf{pk} \ll \frac{1}{vn_{1,2}}(1-\xi/\gamma).$$

Note that satisfaction of condition (5) does not mean that the external field is weak (i.e., is comparable with the spontaneous field). As was shown above, the strength of an electromagnetic field is determined by the values of the parameters Δ_i in (16). It follows from (16) that the strong field conditions $\Delta_i \gg 1$ or $\xi m/\omega \gg 1$ do not contradict (5) ($\cos < \mathbf{pk} \ll 1, \xi/\gamma \ll 1$). Thus, the amplitude of the vector potential of the external electromagnetic field may vary in the range

$$\omega/m \ll eA_0/m \ll \varepsilon/m,$$

which is a rather broad range of variation of A_0 for the case of relativistic electrons.

In the expressions (14) and (15) for the matrix element we have omitted terms proportional to the δ-functions of the z-components of the particle momenta. Therefore, in the limiting case, when there is no interface between the media, i.e., $n_1 = n_2$ ($\Delta_\alpha = \Delta = 0$, $\Delta_\beta = \Delta_0$) and the transition radiation vanishes,[1] there is in general no radiation, since the conditions under which Cherenkov radiation may appear are not satisfied in our statement of the problem.

In this paper we considered the influence of the medium on the electromagnetic-field quanta (in non-absorbing media this leads to a change of the momentum and does not alter the photon energy[1]). The presence of two media with different optical properties leads to inhomogeneity of the external wave and, consequently, to the appearance of radiation.[1-3] The basic results of the paper were obtained under the usual assumption that the electrons themselves do not interact with the medium. In fact, because of the existence of an interface the electron wave experiences in general reflection and refraction. For relativistic particles, these effects are negligibly small and, if taken into account, lead to corrections on the order $U_0/\varepsilon \sim 10^{-7}$, where U_0 is the difference between the potential energies of the electrons in the two media. Note that the presence of an external wave likewise does not cause the relativistic electrons to be reflected from the boundary, since the amplitude of the reflected electron wave is proportional to the parameter $\Delta\omega/p_z \sim \xi/\gamma \ll 1$.

We have not taken into account in the field of the strong electromagnetic wave the electron-mass renormalization[7] that can be as great as $\xi^2 = (eA_0/m)^2$. For the fields that we have considered ($E_0 = 10^2$ W/cm and $\omega = 0.1$ eV), this correction is on the order of 10^{-11}.

Now let us discuss the basic results. As follows from (22), the possibility of amplifying an external electromagnetic wave through induced electron transition radiation is determined by the sign of the mean number of emitted quanta $\langle s\rangle$. To estimate $\langle s\rangle$, we limit ourselves to the simplest case of normal incidence of electrons on the interface between the media: $\mathbf{p}_\tau = 0$, $p_z = p$. The angle of incidence of the wave here must be great enough so that the condition $pk \sim \varepsilon\omega$ used in deriving the formulas be satisfied. In the case of two large and two small parameters Δ_i, that is, Δ, $\Delta_\alpha < 1$ and Δ_0, $\Delta_\beta \gg 1$, we obtain from (26) for this geometry the expression

$$\langle s\rangle \approx \frac{(k_{2z}-k_{0z})^2}{4\omega\varepsilon} = \frac{\omega}{4\varepsilon}[(n_2^2-n_1^2\sin^2\theta_0)^{1/2}-n_1\cos\theta_0]^2. \qquad (33)$$

It is of interest to compare the total energy emitted by the electron as a result of a single pass across the interface between two media in the limiting cases of weak ($\Delta\varepsilon_{\rm w.f}$) and strong ($\Delta\varepsilon_{\rm s.f}$) external fields. A comparative estimate of these energies is given by the ratio $\Delta\varepsilon_{\rm w.f}/\Delta\varepsilon_{\rm s.f} \sim \Delta^2_{\rm w.f} \ll 1$, where $\Delta_{\rm w.f} = e(\mathbf{A}_{\rm w.f}\cdot\mathbf{p})/(pk)$ is the small parameter of perturbation theory for the case of a weak wave with vector potential $\mathbf{A}_{\rm w.f}$. This estimate shows that a strong light wave stimulates transition-radiation processes, and thereby seems to effectively increase the inhomogeneity of the optical properties of the media.

In the other even case of high values of all the parameters (Δ, Δ_α, Δ_β, $\Delta_0 > 1$), Eq. (31) leads to

$$\lim \langle s\rangle \approx \omega/2\varepsilon, \qquad (34)$$

which in the general case leads to an estimate of the same order as (33). Note that in Eqs. (33) and (34) for $\langle s\rangle$, saturation in the external wave occurs relative to the field strength.

Using (34), we now present an expression for the density of the power lost by an electron beam upon crossing the interface between two media (in standard units):

$$P_1 = \frac{(\hbar\omega)^2}{2\varepsilon}\frac{j}{e}, \qquad (35)$$

where j is the current density of the electrons. A numerical estimate of P_1 for the following parameters of the electron beam and light wave ($j = 5$ kA/cm^2; $\varepsilon = 1$ MeV; $n_1 = 1.0$ and $n_2 = 1.2$; $\hbar\omega = 0.12$ eV; $E_0 = 10^4$ W/cm; $\theta_0 = 80°$) leads to the result $P_1 = 0.4\cdot 10^{-4}$ W/cm^2.

Though the effect of amplification by one interface is small, it can be greatly increased (by several orders of magnitude) by using a transparent layered dielectric. Computations[6] have shown that the power lost by electrons increases in proportion to the number N of interfaces they have crossed. The thickness l of an individual layer must satisfy the condition $1/\Delta k_z \gg l \gtrsim L_t$, where L_t is the path of the transition radiation. The symbolic expression that follows from the integrals in (13) may serve as an estimate of L_t:

$$L_t \sim [p_0 - p_z + (2n-s)k_{0z}]^{-1} \sim p\lambda/s\varepsilon,$$

where λ is the wavelength of the radiation and

$$s \sim \Delta = \frac{e\mathbf{A}_0\mathbf{p}}{pk}$$

is the effective number of emitted quanta. For the parameters we have used, $L_t \sim \omega/eE_0 \sim 0.1\ \mu$m.

To obtain amplification of an external light wave, the number N of layers in the dielectric must be such that the total power emitted by the electron beam exceeds the dissipated power in the cavity.

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