Spin Green functions and the problem of summation over physical states

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A new representation is proposed for spin operators and is used to develop a method for evaluating spin Green functions. The method is based on the standard diagram technique for interacting Bose and Fermi particles. The method is valid for systems with arbitrary spin Hamiltonians and can be used to exclude in a simple manner the contribution of nonphysical states at arbitrary temperatures. The concept of the mass operator of the spin Green function is introduced rigorously for a Heisenberg ferromagnet and is expressed, for the first time, in a closed form in terms of Bose and Fermi Green functions and effective vertex parts. Expressions are obtained for the kinematic frequency shift and the kinematic damping of magnons. The basic difference between the structure of the high-frequency magnetic susceptibility tensor of a Heisenberg ferromagnet and that ordinarily used is demonstrated.

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1. INTRODUCTION

The determination of the high-frequency, thermodynamic, and kinetic properties of magnetically ordered crystals is reduced in microscopic theory to the evaluation of spin correlation functions. The methods available for determining these functions can be divided into three groups. The first includes calculations based on different algebraic realizations of spin operators in terms of Bose operators, followed by the application of the standard diagram technique for Bose systems. The $(2S+1)$-dimensional physical space of states of spin operators (S is the spin of the atom) is replaced by an infinite-dimensional space of Bose-particle occupation numbers. Since the projection operator onto physical space is not taken into account, the range of validity of these theories is confined to low temperatures. The second group includes numerous methods based on uncoupling the chains of equations for spin Green functions (see, for example, Refs. 1–3). Finally, the last group includes the method for calculating spin Green functions based on the diagram technique of Vaks, Larkin, and Pikin[4,5] (see also Ref. 6). Although a series of important results has been obtained within the framework of the formalism described in Refs. 4–6, it is, nevertheless, important to note that this formalism is relatively complicated and nonstandard as compared with the technique for Bose and Fermi system. This complexity is connected, in the first instance, with the fact that the diagram series of perturbation theory does not have the principal advantage of the theory of Bose and Fermi systems, namely, ease of interpretation which ensures that each diagram describes certain scattering of quasiparticles by one another. In the procedure used in Refs. 4, 5, and 6, even the introduction of the quasiparticles (magnons) requires preliminary selective summation of diagrams, whilst the determination of the amplitudes for the interaction between them remains an unsolved problem. On the other hand, it is precisely its physical clarity that has made the diagram technique for Bose and Fermi systems[7] a uiversal and effective mathematical formalism in physics. It is therefore natural to try to formulate the problem of interacting spins in the language of interacting Bose and Fermi particles.

In this paper, we put forward a method for evaluating the spin Green functions, which is based on a new representation for the spin operators that has a simple projection operator. The corresponding diagram technique is identical with the standard diagram technique for Bose and Fermi particles but, at the same time, it enables us to perform a rigorous summation over the physical states of the interacting spins at arbitrary temperature.

We shall evaluate the high-frequency magnetic susceptibility tensor $\chi^{+-}(\mathbf{k},\omega)$ of a Heisenberg ferromagnet and analyse its properties. We shall show that the form of $\chi^{+-}(\mathbf{k},\omega)$ at high tempratures in the ordered state is substantially different from the Lorentz shape.

2. REPRESENTATION FOR SPIN OPERATORS

We shall take the spin operators in the following form:

$$\tilde{S}_l^- = (2S)^{1/2}\left(a_l^+ - \frac{1}{2S}a_l^+ a_l^+ a_l\right) - 2\frac{2S+1}{(2S)^{1/2}}a_l^+ b_l^+ b_l,$$

$$\tilde{S}_l^+ = (2S)^{1/2} a_l, \quad \tilde{S}_l^z = S - a_l^+ a_l - (2S+1) b_l^+ b_l. \qquad (1)$$

where a_l^+ and a_l are the Bose and b_l^+ and b_l the Fermi creation and annihilation operators satisfying the usual commutation relations and l labels the lattice sites ($l = 1, 2, \ldots, N$). The operator $\tilde{S}_l^z$ is self-adjoint and $\tilde{S}_l^+$ and $\tilde{S}_l^-$ are conjugate Hermitian operators, as in the Dyson-Maleev representation.[8,9] It is readily verified that the operators $\tilde{S}_l^{\pm,z}$ satisfy the same commutation relations as the operators $S_l^{\pm,z}$ and $\tilde{S}_l^2 = S(S+1)$.

The space of states in which the $\tilde{S}_l^{\pm,z}$ operate is an infinite-dimensional Hilbert space of the occupation numbers with basis vectors

$$\varphi_{n,0}(l) = \frac{1}{(n!)^{1/2}}(a_l^+)^n|0\rangle, \quad \varphi_{n,1}(l) = \frac{1}{(n!)^{1/2}}(a_l^+)^n b_l^+|0\rangle, \qquad (2)$$

where $n = 0, 1, \ldots, \infty$.

By applying the operator $\tilde{S}_l^z$ to (2), we can verify that the

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states $\varphi_{n,0}$ with $n>2S$ and all the states $\varphi_{n,1}$ are nonphysical. It is clear that the quasiparticles created by the operator band b_l^+ have no real physical meaning because their occupation numbers are nonzero only for nonphysical states.

We shall show that the mean of the product of an arbitrary number of spin operators $S^{\pm,z}(\tau)=\exp(\mathscr{H}\tau)S^{\pm,z}\exp(-\mathscr{H}\tau)$, over physical space can be expressed in terms of the mean of the operators $\tilde{S}^{\pm,z}(\tau)=\exp(\tilde{\mathscr{H}}\tau)\tilde{S}^{\pm,z}\exp(-\tilde{\mathscr{H}}\tau)$, evaluated in the space (2), using the following relation:

$$\langle\hat{T}_\tau\ldots S_l^+(\tau)\ldots S_{l'}^-(\tau')\ldots S_{l''}^z(\tau'')\ldots\rangle_{\mathscr{H}}=\langle\hat{P}\hat{T}_\tau\ldots\tilde{S}_l^+(\tau)\ldots\tilde{S}_{l'}^-(\tau')\ldots\tilde{S}_{l''}^z(\tau'')\ldots\rangle_{\tilde{\mathscr{H}}},\qquad(3)$$

where

$$\hat{P}=\Pi_l\hat{P}_l,\qquad \hat{P}_l=\exp(i\pi b_l^+b_l).$$

In these expressions, $\hat{P}$ is the projection operator onto physical space, $\hat{T}_\tau$ is the chronological ordering operator

$$\langle\ldots\rangle_{\mathscr{H}}=\mathrm{Sp}\,\{\exp(-\beta\mathscr{H})\ldots\}/\mathrm{Sp}\,\{\exp(-\beta\mathscr{H})\},$$
$$\langle\hat{P}\ldots\rangle_{\tilde{\mathscr{H}}}=\mathrm{Sp}\{\hat{P}\exp(-\beta\tilde{\mathscr{H}})\ldots\}/\mathrm{Sp}\{\hat{P}\exp(-\beta\tilde{\mathscr{H}})\}$$

$\tilde{\mathscr{H}}$ is the Hamiltonian of the spin system in which the spin operators are described in accordance with (1), and $\beta^{-1}=T$ is the temperature in energy units. The projection operator $\hat{P}_l$ is diagonal in the basis defined by (2):

$$\hat{P}_l\varphi_{n,0}(l)=\varphi_{n,0}(l);\qquad \hat{P}_l\varphi_{n,1}(l)=-\varphi_{n,1}(l),\qquad(4)$$

and, as we shall see below, the application of this operator leads to the mutual cancelation of contributions due to the nonphysical states $\varphi_{n,0}$ with $n>2S$ and $\varphi_{n,1}$ with $n\geqslant0$. It is precisely in this sense that we shall use the phrase, "projection operator" for $\hat{P}$, although it does not, in fact, satisfy the standard relation $\hat{P}^2=\hat{P}$ for a projection operator.

It is well known that a mean of the form given by (3) can be expressed in the interaction representation by writing the Hamiltonian in the form $\mathscr{H}=\mathscr{H}_0+\mathscr{H}_{\rm int}$. The evaluation of any spin correlation function then reduces to the evaluation of the sum of means in which the time dependence of the operators and the statistical averaging are determined by the Hamiltonian $\mathscr{H}_0$. It will therefore be sufficient to prove (3) for $\mathscr{H}=\mathscr{H}_0$ and $\tilde{\mathscr{H}}=\mathscr{H}_0$.

We shall use the method put forward in Ref. 4 for the evaluation of the spin correlation functions. We shall take $\mathscr{H}_0$ in the form

$$\mathscr{H}_0=-g\mu_BH\sum_l S_l^z\equiv-y_0\sum_l S_l^z.\qquad(5)$$

The evaluation of any spin correlation function with $\mathscr{H}_0$ given by (5) is performed in two stages.[4-6] The first step is to use the commutation relations for the spin operators and then evaluate all the possible convolutions for the operators S^+ with the operators S^- and S^z. This procedure gets rid of all the operators S^+ and S^- in the mean given by (3). (By virtue of the structure of the Hamiltonian $\mathscr{H}_0$, the only nonzero means are those containing an equal number of operators S^+ and S^-.) The final result is that the original mean divides into the sum of terms, each of which consists of the product of elementary spin Green functions $K_{12}^0(\tau_1-\tau_2)$, multiplied by the mean of the operators S^z of the form $\langle S_1^zS_2^z\ldots S_p^z\rangle_0$.

The second step is to write down the mean of the S^z operators. The means of an arbitrary power of the operator S^z are expressed in terms of the function $b(\beta t_0)=SB_S(\beta Sy_0)$ and its derivatives,[4-6] where $B_s(x)$ is the Brillouin function for spin S.

Let us now consider the mean of the product of an arbitrary number of spin operators, in which each spin operator is given by (1) and

$$\tilde{\mathscr{H}}_0=-Sy_0N+y_0\sum_l a_l^+a_l+x_0\sum_l b_l^+b_l;\quad x_0=(2S+1)y_0.\qquad(6)$$

Repeating the discussion used in Ref. 10 in connection with the Dyson-Maleev representation, it can be shown that, at the stage at which the operators S^+ and S^- are excluded from the initial mean, we obtain a result identical with that obtained in the technique described in Refs. 4–6. This is due to the fact that, during the first stage, one uses only for $\tilde{S}^{\pm,z}$ commutation relations that are identical with the commutation relations for $S^{\pm,z}$, and the time dependence of $S^{\pm,z}(\tau)$ and $\tilde{S}^{\pm,z}(\tau)$ also turns out to be the same. The presence of the projection operator in (3) is unimportant because it commutes with $\tilde{S}^{\pm,z}$. Since the operators $\tilde{S}_l^z$ are self-adjoint, we arrive at the following important conclusion: the non-Hermitian property of (1) has no ifluence on the evaluated correlation functions.

Let us now consider the mean of the product of an arbitrary number of $\tilde{S}^z$ operators. Since this product can be obtained in a standard manner by differentiating the partition function

$$\tilde{Z}=\Pi_l\tilde{Z}_l,\qquad \tilde{Z}_l=\mathrm{Sp}\,\exp\,\{\beta y_0\tilde{S}_l^z+i\pi b_l^+b_l\}.$$

It will be sufficient to confine our attention to the evaluation of $\tilde{Z}_l$. In the basis defined by (2), we have

$$\tilde{Z}_l=\sum_{n=0}^{\infty}\exp\{\beta y_0(S-n)\}+\sum_{n=0}^{\infty}\exp\{\beta y_0[S-n-(2S+1)]\}\exp i\pi$$
$$=\sum_{n=0}^{2S}\exp\{\beta y_0(S-n)\}=\mathrm{Sp}\,\exp\{\beta y_0S_l^z\}=Z_l.\qquad(7)$$

It is clear from (7) that the operator $\hat{P}_l$ is, in fact, a projection operator. Thus, the results obtained for the mean of an arbitrary power of $\tilde{S}_l^z$ are exactly the same as in Refs. 4–6.

Consider the functions

$$G_{12}(\tau_1-\tau_2)=\langle\hat{P}\hat{T}_\tau a_1(\tau_1)a_2^+(\tau_2)\rangle_{\tilde{\mathscr{H}}},\qquad(8)$$
$$F_{12}(\tau_1-\tau_2)=\langle\hat{P}\hat{T}_\tau b_1(\tau_1)b_2^+(\tau_2)\rangle_{\tilde{\mathscr{H}}}.\qquad(9)$$

When the means of only the operators a_l^+, a_l are evaluated, the projection operator need not be taken into account because it commutes with them. For the Green function (8) with $\tilde{\mathscr{H}}=\tilde{\mathscr{H}}_0$ (6), we obtain

$$G_{12}^0(\tau_1-\tau_2)=\delta_{12}\exp\{-y_0(\tau_1-\tau_2)\}\begin{cases}n_{y_0} & \tau_1<\tau_2\\ 1+n_{y_0} & \tau_1>\tau_2\end{cases},\qquad(10)$$

where δ_{12} is the Kronecker symbol in the site indices and $n_{y_0}=[\exp\beta y_0-1]^{-1}$. This expression is identical with the elementary spin Green function $K_{12}^0(\tau_1-\tau_2)$.

When $\tilde{\mathscr{H}}=\tilde{\mathscr{H}}_0$, the Green function (9) is given by

$$F_{12}^0(\tau_1-\tau_2)=\delta_{12}\exp\{-x_0(\tau_1-\tau_2)\}\begin{cases}n_{x_0}, & \tau_1<\tau_2\\ 1+n_{x_0}, & \tau_1>\tau_2\end{cases},\qquad(11)$$

where $n_{x_0} = [\exp \beta x_0 - 1]^{-1}$. Because of the presence of the projection operator $\hat{P}_l$, the mean $\langle \hat{P}_l b_l + b_l \rangle_0$ is determined not by the Fermi but by the Bose distribution function. This is a consequence of the fact that the projection operator influences the analytic properties of Green's function (9). It may be shown that the latter satisfies Bose-type parity condition

$$F_{ll'}(\tau<0) = +F_{ll'}(\tau+\beta). \tag{12}$$

We also note that

$$F_{ll'}(\tau) = \delta_{ll'} F(\tau). \tag{13}$$

This important property is connected with the fact that the spin operators (1) contain Fermi operators only in the combination b^+b. None of the spin Hamiltonians will therefore have matrix elements describing the transitions of a fermion from one site to another.

The representartion given by (1) will thus enable us to evaluate any spin correlation functions by standard diagram techniques for interacting Bose and Fermi particles. Actually, the resulting diagram technique is equivalent to that for interacting boson fields. The only outstanding feature is that, with each closed loop on the $F_{ll'}(\tau)$ lines, we must associate the additional factor (-1) (this is a consequence of the anticommuting property of the operators b^+ and b). The formalism developed in this paper leads to a rigorous solution of the problem of summation over physical states and, consequently, to a description of the spin system in the entire range of temperatures.

3. DYNAMIC AND KINEMATIC INTERACTION BETWEEN SPIN WAVES IN THE HEISENBERG FERROMAGNET

Complete information on magnon interactions can be obtained by evaluating Green's function

$$G_{12}(\tau_1-\tau_2) = \langle \hat{P} \hat{T}_\tau a_1(\tau_1) a_2^+(\tau_2) \rangle. \tag{14}$$

Consider the Heisenberg ferromagnet with

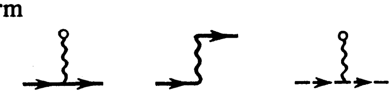
$$\mathscr{H} = -g\mu_B H \sum_l S_l^z - \frac{1}{2} \sum_{l \neq l'} J_{ll'} \mathbf{S}_l \mathbf{S}_{l'},$$

where $J_{ll'}$ is the exchange interaction between the spins on sites l and l'. Substituting (1) in the Hamiltonian, we obtain[1)]

$$\mathscr{H} = E_0 + \mathscr{H}_0 + \mathscr{H}_{int};$$
$$E_0 = -Ng\mu_B HS - {}^1/_2 N J_0 S^2,$$
$$\mathscr{H}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} a_{\mathbf{k}}^+ a_{\mathbf{k}} + x \sum_{\mathbf{k}} b_{\mathbf{k}}^+ b_{\mathbf{k}},$$
$$\varepsilon_{\mathbf{k}} = g\mu_B H + S(J_0 - J_{\mathbf{k}}), \quad x = (2S+1)(g\mu_B H + SJ_0), \tag{15}$$

where $J_{\mathbf{k}}, a_{\mathbf{k}}, b_{\mathbf{k}}$ are the Fourier components of the exchange integral and, correspondingly, of the operators a_l, b_l.

The Hamiltonian given by (6), used to derive the result in the last section, differs from (15) in that the latter includes all quadratic terms in the operators that are due to the exchange interaction. This regrouping within the Hamiltonian means that lines corresponding to Green's functions $G^0(\mathbf{k},\omega_n)$ (10) and $F^0(\mathbf{k},\omega_n)$ (11) are dressed in diagrams of the form



The way line in these diagrams represents the interactions $\beta J_{\mathbf{k}}$. We shall assume that this dressing has been carried out, and associate the function $G^0(\mathbf{k},\omega_n) = (\varepsilon_{\mathbf{k}} - i\omega_n)^{-1}$ with the solid line and $F^0(k,\omega_n) = (x - i\omega_n)^{-1}$ with the broken line, in accordance with the choice of $\mathscr{H}_0$ (15).

It is clear from (15) that the energy of the fermions does not depend on the wave vector. This property remains in any order of perturbation theory and is connected with the condition given by (13).

The interaction Hamiltonian is now given by

$$\mathscr{H}_{int} = N^{-1} \sum_{1234} \{\varphi_1(12;34)\, a_1^+ a_2^+ a_3 a_4 + \varphi_2(1;3)\, a_1^+ b_2^+ a_3 b_4 + \varphi_3(12;34)\, b_1^+ b_2^+ b_3 b_4\} \Delta(1+2-3-4), \tag{16}$$

where $1 \equiv \mathbf{k}_1$, $2 \equiv \mathbf{k}_2$, and so on, and

$$\begin{aligned} \varphi_1(12;34) &= {}^1/_4 (J_3 + J_4 - J_{1-3} - J_{1-4}), \\ \varphi_2(1;3) &= (2S+1)(J_3 - J_{1-3}), \\ \varphi_3(12;34) &= {}^1/_4 (2S+1)^2 (J_{1-3} - J_{1-4}). \end{aligned} \tag{17}$$

The first term in $\mathscr{H}_{int}$ describes the dynamic interaction between magnons, and is identical with the Dyson scattering amplitude.[8] The second and third terms describe effects connected with the finite dimensionality of physical space (according to Dyson, this is the kinematic interaction between magnons). Thus, in the representation defined by (1), effects connected with the finite dimensionality of physical space are transferred directly to the Hamiltonian and in the form of an addition to the dynamic interaction.

We shall now use Dyson's equaton and express G in terms of the mass operator Σ:

$$G(\mathbf{k}, \omega_n) = \{\varepsilon_{\mathbf{k}} - \Sigma(\mathbf{k}, \omega_n) - i\omega_n\}^{-1}. \tag{18}$$

The series for the mass operator can be represented graphically in a standard fashion[7] in terms of the complete Green functions (8) and (9) and the effective vertex parts, as shown in Fig. 1. This expression includes the Green function F (9), which we shall also write in terms of the mass operator Σ_F:

$$F(\mathbf{k}, \omega_n) = \{x - \Sigma_F(\mathbf{k}, \omega_n) - i\omega_n\}^{-1}. \tag{19}$$

In its turn, Σ_F can be represented by the diagram series shown in Fig. 2.

Let us first evaluate the contribution of the diagrams in Fig. 1 to Σ, assuming that all the lines and vertices are elementary. The first two graphs then describe the dynamic interaction between the magnons, and the last two the kinematic interaction:

$$\begin{aligned} \Sigma_{dyn}(\mathbf{k}, \omega_n) = {} & N^{-1} \sum_{\mathbf{q}} (J_0 + J_{\mathbf{k}+\mathbf{q}} - J_{\mathbf{k}} - J_{\mathbf{q}}) n_{\mathbf{q}} \\ & + \frac{1}{2N^2} \sum_{\mathbf{pq}} (J_{\mathbf{k}-\mathbf{p}} + J_{\mathbf{k}-\mathbf{q}} - J_{\mathbf{q}} - J_{\mathbf{p}}) \\ & \times (J_{\mathbf{k}-\mathbf{p}} + J_{\mathbf{k}-\mathbf{q}} - J_{\mathbf{k}} - J_{\mathbf{p}+\mathbf{q}-\mathbf{k}}) \frac{(1+n_{\mathbf{p}}+n_{\mathbf{q}}) n_{\mathbf{p}+\mathbf{q}-\mathbf{k}} - n_{\mathbf{p}} n_{\mathbf{q}}}{\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{q}} - \varepsilon_{\mathbf{p}+\mathbf{q}-\mathbf{k}} - i\omega_n} \end{aligned} \tag{20}$$

$$\begin{aligned} \Sigma_{\mathrm{kin}}(\mathbf{k}, \omega_n) = {} & -(2S+1)(J_0 - J_{\mathbf{k}}) n_x - (2S+1)^2 (1+n_x) n_x \frac{1}{N} \\ & \times \sum_{\mathbf{q}} \frac{(J_{\mathbf{q}} - J_{\mathbf{k}-\mathbf{q}})(J_{\mathbf{k}} - J_{\mathbf{k}-\mathbf{q}})}{\varepsilon_{\mathbf{q}} - i\omega_n}. \end{aligned} \tag{21}$$

We note that $\Sigma(0,\omega_n) = 0$, in accordance with the requirement of the nonactivation nature of the spin-wave spectrum

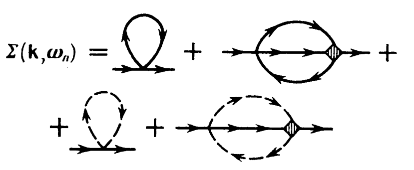


FIG. 1.

of the Heisenberg ferromagnet.

The mass operator, evaluated with the Dyson scattering amplitude φ_1 (12; 34), provides us with a precise description of the renormalization of the spinb-wave spectrukm and the attenuation of these waves when all thermodynamic means are evaluated in physical space. However, both physical and nonphysical states contribute to the evaluation of $\Sigma_{\rm dyn}$ in the basis of the Bose operators. To obtain correct results, it is essential to exclude the contribution of nonphysical states form $\Sigma_{\rm dyn}$. This exclusion procedure can be implemented because of the presence of $\Sigma_{\rm kin}$ in the mass operator Σ.

It is clear from (21) that $\Sigma_{\rm kin}$ contains real and imaginary parts that determine the kinematic frequency shift and the kinematic attenuation of magnons. To estimate the magnitude of this shift, let us take into account the temperature renormalization of x. The principal contribution at low temperatures is provided by diagram a of Fig. 2, namely.

$$\Sigma_F^a(\mathbf{k},\omega_n)=(2S+1)N^{-1}\sum_{\mathbf{q}}(J_0-J_{\mathbf{q}})n_{\mathbf{q}}. \qquad (22)$$

It is clear from (22) that essential renormalization of x occurs for $n_q \sim S$, i.e., for $T/SJ_0 \sim S$, which corresponds to $T \sim T_c \sim S^2 J_0$. Consequently, when $T \ll T_c$, we have $x(T) \approx x(0)$, and the formulas given by (20) and (21) provide the correct description of the principal contribution of the kinematic interction to the mass operator. This contribution turns out to be of the order of $\exp\{-\beta(2S+1)SJ_0\}$, which is negligible in this temperature range.

The kinematic contribution at $T \lesssim T_c$ can be estimated by using perturbation theory[4–5] in the reciprocal interaction radius R_0^{-1}.. To classify the diagrams in accordance with this parameter, we must divide each sum over the momentum $\mathbf{q}$ into two intervals in the corresponding analytic expressions: the first of these is $q > q_0 \sim R_0^{-1}$, for which we must set $J_q \sim 0$ and $\varepsilon_q \sim y = g\mu_B H + SJ_0$, and the second is $q < q_0$ for which $J_q \sim J_0$. In general, each diagram will then provide a contribution of different order. By performing selective summation, we can obtain the required quantity in any approximation.

It is shown in Ref. 11 that the kinematic interaction becomes important for $\langle S^z\rangle/S \lesssim T/S(2S+1)J_0$. If we take for the Curie temperature the value predicted by the molecular field theory, namely, $T_c = S(S+1)J_0/3$ (which corresponds to the leading approximation in R_0^{-1}), we obtain the following estimate: for $S=1/2$ we have $\langle S^z\rangle/S \lesssim 0.25$, and for $S=5/2$ we have $\langle S^z\rangle/S \lesssim 0.2$. Consequently, the kinematic interaction plays an important role in the interval $0.75-0.8T_c \lesssim T \leqslant T_c$.

Let us now consider the mass operator in this temperature range. In the leading approximation in R_0^{-1}, we have

$$\varepsilon_{\mathbf{k}}-\Sigma(\mathbf{k},\omega_n)=g\mu_B H+\langle S^z\rangle(J_0-J_{\mathbf{k}}),$$

where S^z is given by the molecular filed equation. This result is identical with that reported in Ref. 5. The quantity $\Sigma(\mathbf{k},\omega_n)$ turns out to be real; the imaginary part appears in the mass operator only in the next order of perturbation theory. Retaining first-order terms in (20) and (21), we obtain

$$\Sigma^{(1)}(\mathbf{k},\omega_n)=-\{(1+n_y)n_y-(2S+1)^2(1+n_x)n_x\}N^{-1}\times\sum_{\mathbf{q}}\frac{(J_{\mathbf{k}-\mathbf{q}}-J_{\mathbf{q}})(J_{\mathbf{k}-\mathbf{q}}-J_{\mathbf{k}})}{\varepsilon_{\mathbf{q}}-i\omega_n}. \qquad (23)$$

The factor in braces in (23) is given by

$$b'(\beta y)=\beta^{-1}\frac{\partial}{\partial y}b(\beta y).$$

The expression (23) corresponds to the case where all the lines and vertices in the diagrams of Fig. 1 are assumed to be elementary. The results reported in Ref. 5 can be obtained be dressing one of the elementary vertices in the way indicated in Fig. 3. The infinite diagram series shown in Fig. 3 also have first-order terms. When these are taken into account,(23) assumes the form

$$\Sigma^{(1)}(\mathbf{k},\omega_n)=-b'N^{-1}\sum_{\mathbf{q}}\frac{(J_{\mathbf{k}-\mathbf{q}}-J_{\mathbf{q}})(J_{\mathbf{k}-\mathbf{q}}-J_{\mathbf{k}})}{(1-b'\beta J_{\mathbf{k}-\mathbf{q}})(\varepsilon_{\mathbf{q}}-i\omega_n)}. \qquad (24)$$

Using (24), we obtain an explicit expression for the attentuationof the spin wiave:

$$\gamma_{\mathbf{k}}(\omega)=\pi b'N'\sum_{\mathbf{q}}\frac{(J_{\mathbf{k}-\mathbf{q}}-J_{\mathbf{q}})(J_{\mathbf{k}-\mathbf{q}}-J_{\mathbf{k}})}{1-b'\beta J_{\mathbf{k}-\mathbf{q}}}\delta(\varepsilon_{\mathbf{q}}-\omega). \qquad (25)$$

The attenuation described by this expression is proportional to the derivatives of the function $b(\beta y)$, and was referred to as "fluctuational" in Ref. 5.[2)] It is clear from the foregoing discussion that the attenuation is due to the scattering of the spin wave by thermal magnons in the short-wave part of the spectrum, which was obtained with allowance for the kinematic interaction. The attenuation of the spin waves was discussed in detail in Ref. 5 for $T \lesssim T_c$ and small $\mathbf{k}$.

4. HIGH-FREQUENCY MAGNETIC SUSCEPTIBILITY TENSOR OF THE HEISENBERG FERROMAGNET

Let us evaluate the spin Green function

$$K^{+-}(1-1';\ \tau)={}^1/_2\langle \hat{T}_\tau S_1^+(\tau)S_{1'}^-(0)\rangle, \qquad (26)$$

whose Fourier transform is known to be equal (apart from a

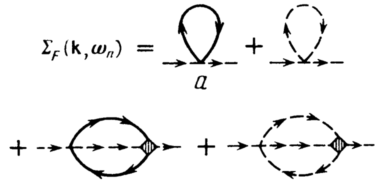


FIG. 2.

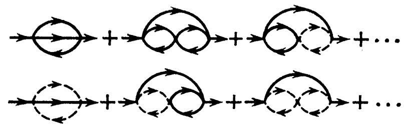


FIG. 3.

constant factor) to the component $X^{+-}(\mathbf{k},\omega)$ of the high-frequency magnetic susceptibility tensor. In the representation defined by (1), we have

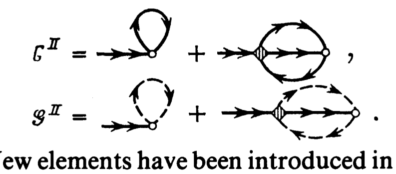
$$K^{+-}(1-1';\,\tau)=S\langle \hat P\hat T_\tau a_1(\tau)a_{1'}^+(0)\rangle$$
$$-{}^1/_2\langle \hat P\hat T_\tau a_1(\tau)a_{1'}^+(0)a_{1'}^+(0)a_{1'}(0)\rangle$$
$$-(2S+1)\langle \hat P\hat T_\tau a_1(\tau)a_{1'}^+(0)b_{1'}^+(0)b_{1'}(0)\rangle. \qquad (27)$$

Consequently, we have reduced the evaluation (26) to the evaluation of the single-particle Green function $G_{11'}(\tau)$ (14) and two two-particle Green functions

$$G_{11'}^{\mathrm{II}}(\tau)=-{}^1/_2\langle \hat P\hat T_\tau a_1(\tau)a_{1'}^+(0)a_{1'}^+(0)a_{1'}(0)\rangle,$$

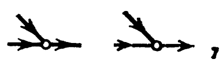
$$\mathcal{G}_{11'}^{\mathrm{II}}(\tau)=-(2S+1)\langle \hat P\hat T_\tau a_1(\tau)a_{1'}^+(0)b_{1'}^+(0)b_{1'}(0)\rangle. \qquad (28)$$

Dzyaloshinskiĭ[12] has developed the diagram technique for many-particle Green functions of this type. For the functions given by (28), the diagram series can be written in the following effective graphical form:

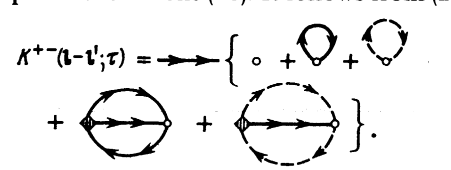


(29)

New elements have been introduced in (29), namely, the vertices



with which we must associate (-1) and $[-(2S+1)]$, respectively. Their origin is connected with the presence of three operators referring to the same instant of time in the two-particle functions (28). It follows from (29) that



(30)

The first three terms in (30) correspond to $\langle S^z\rangle$. Taking the Fourier transform of (30), and representing the Fourier transform of the last two terms in braces by $A(\mathbf{k},\omega_n)$, we obtain

$$K^{+-}(\mathbf{k},\omega_n)=\frac{\langle S^z\rangle+A(\mathbf{k},\omega_n)}{\varepsilon_\mathbf{k}-\Sigma(\mathbf{k},\omega_n)-i\omega_n}. \qquad (31)$$

In deriving this expression, we used equation (18) for $G(\mathbf{k},\omega_n)$. It is clear from (31) that the quantitty $\langle S^z\rangle+A(\mathbf{k},\omega_n)$ is the residue or the oscillator strength. We shall refer to it as the strength operator. Equation (31) establishes the relation between the spin Green function $K^{+-}(\mathbf{k},\omega_n)$ and its strength and mass operators. We note that an expression for $K^{+-}(\mathbf{k},\omega_n)$ that was of this type and was valid at low temperatures was first obtained in Ref. 13 with the Dyson-Maleev representation.[8,9]

Using the properties of the interaction amplitude, it can be shown that $A(0,w_n)=0$. Hence, it follows that $K^{+-}(0,\omega_n)=\langle S^z\rangle(\varepsilon_0-\omega-i\delta)^{-1}$, which is identical with the exact result for $X^{+-}(0,\omega)$.

It is clear from (31) that the mass operator of the spin Green function is identical with the mass operator of the single-particle Bose Green function (14). Its evaluation was discussed in the last section. Let us consider the strength operator and evaluate $A(\mathbf{k},\omega_n)$. Suppose that the diagrams for $A(\mathbf{k},\omega_n)$ in (29) have been constructed from elementary vertices and zero-order Green functions. Simple calculations then yield

$$A(\mathbf{k},\omega_n)=-\frac{1}{N^2}\sum_{\mathbf{pq}}(J_{\mathbf{k-p}}+J_{\mathbf{k-q}}-J_\mathbf{p}-J_\mathbf{q})\times\frac{(1+n_\mathbf{p}+n_\mathbf{q})n_{\mathbf{p+q-k}}-n_\mathbf{p}n_\mathbf{q}}{\varepsilon_\mathbf{p}+\varepsilon_\mathbf{q}-\varepsilon_{\mathbf{p+q-k}}-i\omega_n}$$
$$-(2S+1)^2(1+n_x)n_x\frac{1}{N}\sum_\mathbf{q}\frac{J_\mathbf{q}-J_{\mathbf{k-q}}}{\varepsilon-i\omega_n}. \qquad (32)$$

The first term in (32) is due to the dynamic interaction between the magnon and the second is due to kinematic interactions. As in the case of the mass operator, the role of the kinematic term reduces to the exclusion of the contribution due to nonphysical states.

Let us now investigate different limiting cases of (32). When $T\ll SJ_0$ the contribution of the kinematic term is exponentially small and is of the order of $\exp\{-\beta(2S+1)SJ_0\}$, so that it can be neglected. We present explicit expressions for $A''(\mathbf{k},\omega)=\operatorname{Im}A(\mathbf{k},\omega)$ as functions of temperature and wave vector. We examine the region of wve vectors and fields for which $J_k=J_0[1-\nu(ak)^2]$, $\varepsilon_k\approx SJ_0\nu(ak)^2$. The result for $\varepsilon_k\ll T$ is

$$A''(\mathbf{k},\varepsilon_\mathbf{k})=\frac{(ak)^2}{2^6\pi^4}\left(\frac{T}{SJ_0\nu}\right)^2\left[\frac{\varepsilon_k}{3T}\ln\frac{T}{\varepsilon_k}+I_1\right], \qquad (33)$$

where

$$I_1=\int_0^1\frac{(1+y)\,dy}{1+y^2}\int_{-1}^1\frac{z\,dz}{[1+y^2+2zy]^{1/2}}\ln\frac{1+y^2-[(1+y^2)^2-4y^2z^2]^{1/2}}{1+y^2+[(1+y^2)^2-4y^2z^2]^{1/2}}$$
$$\approx 0.217.$$

When $T\ll\varepsilon_k$, we have

$$A''(\mathbf{k},\varepsilon_\mathbf{k})=\frac{1}{(2\pi)^4}\frac{\varepsilon_kT^{5/2}}{24S(ak)^2(SJ_0\nu)^{7/2}}\Gamma\left(\frac{5}{2}\right). \qquad (34)$$

It is shown in Ref. 5 that, in the case of high spin for which $S\gg 1$, there is one further characteristic temperature interval, namely, $SJ_0\ll T\ll S^2J_0$. In accordance with the estimates given in Ref. 11, the contribution of the kinematic term can, as before be neglected in this temperature interval, and we have

$$A''(\mathbf{k},\varepsilon_k)=\frac{\varepsilon_kT^2}{2^5\pi^4S(SJ_0\nu)^3}I_2(p_0/k), \qquad (35)$$

where p_0 is the momentum on the boundary of the Brillouin zone and

$$I_2(\alpha)=\int_0^\alpha\frac{y\,dy}{1+y^2}\int_{-1}^1\frac{z\,dz}{[1+y^2+2zy]^{1/2}}\ln\frac{1+y^2-[(1+y^2)^2-4y^2z^2]^{1/2}}{1+y^2+[(1+y^2)^2-4y^2z^2]^{1/2}},$$

$$I_2(1)\approx 0.084;\quad I_2(5)\approx 0.232;\quad I_2(8)\approx 0.236.$$

The temperature behavior of the real part of the strength operator for $T\ll SJ_0, S^2J_0$ is determined by $\langle S^2\rangle$ in the leading approximation.

It is clear from (32) that the strength boundary of the spin Green function $K^{+-}(\mathbf{k},\omega)$ contains real and imaginary parts, both of which depend on ω and $\mathbf{k}$. Let us consider the influence of this structure of the numerator $K^{+-}(\mathbf{k},\omega)$ on the behavior of $\mathrm{Im}K^{+-}(\mathbf{k},\omega)$ and $\mathrm{Re}\,K^{+-}(\mathbf{k},\omega)$ as functions of ω. In accordance with (31), we have

$$K''(\mathbf{k},\omega)=\frac{(\langle S^z\rangle+A_{\mathbf{k}}')\gamma_{\mathbf{k}}}{(\tilde{\varepsilon}_{\mathbf{k}}-\omega)^2+\gamma_{\mathbf{k}}^2}+\frac{A_{\mathbf{k}}''(\tilde{\varepsilon}_{\mathbf{k}}-\omega)}{(\tilde{\varepsilon}_{\mathbf{k}}-\omega)^2+\gamma_{\mathbf{k}}^2}, \quad (36)$$

$$K'(\mathbf{k},\omega)=\frac{(\langle S^z\rangle+A_{\mathbf{k}}')(\tilde{\varepsilon}_{\mathbf{k}}-\omega)}{(\tilde{\varepsilon}_{\mathbf{k}}-\omega)^2+\gamma_{\mathbf{k}}^2}-\frac{A_{\mathbf{k}}''\gamma_{\mathbf{k}}}{(\tilde{\varepsilon}_{\mathbf{k}}-\omega)^2+\gamma_{\mathbf{k}}^2}, \quad (37)$$

where

$$\tilde{\varepsilon}_{\mathbf{k}}=\varepsilon_{\mathbf{k}}-\mathrm{Re}\,\Sigma(\mathbf{k},\varepsilon_{\mathbf{k}}),\quad \gamma_{\mathbf{k}}=\mathrm{Im}\,\Sigma(\mathbf{k},\varepsilon_{\mathbf{k}})$$

are, respectively, the energy and attenuation of the spin waves at the given temperature, and $A_{\mathbf{k}}'=A'(\mathbf{k},\varepsilon_{\mathbf{k}})$, $A_{\mathbf{k}}''=A''(\mathbf{k},\varepsilon_{\mathbf{k}})$. We shall use the values of $\Sigma(\mathbf{k},\omega)$ and $A(\mathbf{k},\omega)$ for $\omega=\varepsilon_{\mathbf{k}}$, since we shall be interested in the shape of the curves corresponding to (36) and (37) near resonance.

Consider the shape of the curve representing $K''(\mathbf{k},\omega)$ as a function of ω. It has a maximum at

$$\omega=\tilde{\varepsilon}_{\mathbf{k}}+\gamma_{\mathbf{k}}A_{\mathbf{k}}''/2\langle S^z\rangle.$$

Since, at low temperatures,

$$J_0A_{\mathbf{k}}''\sim\gamma_{\mathbf{k}}\ll\varepsilon_{\mathbf{k}},$$

the position of the maximum of $K''(\mathbf{k},\omega)$ departs from $\omega=\tilde{\varepsilon}_{\mathbf{k}}$ by an amount of the second order of small quantitites in the parameter γ_k/ε_k. It is readily seen that, in the leading approximation in this parameter, the half-width of the curve is $\gamma_{\mathbf{k}}$, and the shape is that of a Lorentzian. However, even for $\varepsilon_k\gg|\tilde{\varepsilon}_k-\omega|\gg\gamma_k$, the presence of the second term in (36) ensures that the shape of the wings of the $K''(\mathbf{k},\omega)$ curve is asymmetric relative to the position of the maximum.

An increase in the temperature is accompanied by an increase in γ_k and A_k', A_k'', whereas $\langle S^z\rangle$ decreases. For $T'\lesssim T_c$ and large enough $\mathbf{k}$, the shape of the line $K''(\mathbf{k},\omega)$ is appreciably non-Lorentzian and asymmetric even for $|\tilde{\varepsilon}_k-\omega|\lesssim\gamma_k$.

Consider the real part of $X^{+-}(\mathbf{k},\omega)$. The curve representing $K'(\mathbf{k},\omega)$ as a function of ω has a maximum at that point

$$\omega=\tilde{\varepsilon}_k-\gamma_k-\gamma_kA_k''/\langle S^z\rangle$$

and a minimum at

$$\omega=\tilde{\varepsilon}_k+\gamma_k-\gamma_kA_k''/\langle S^z\rangle.$$

At low temperatures, the halfwidth of the curve in the leading approximation in γ_k/ε_k is equal to $\omega_{\mathbf{k}}$ and the shape of the line is Lorentzian. However, the wings of the curve are asymmetric relative to the position of the maximum and minimum. As the temperature increases, and when $\mathbf{k}$ is not too low, the shape of the $K'(\mathbf{k},\omega)$ curve becomes essentially non-Lorentzian even for $|\tilde{\varepsilon}_k-\omega|\lesssim\gamma_k$.

Neutron scattering is an effective method for the experimental investigation of the dependence of the spin correlation functions on $\mathbf{k}$ and ω. The shape of the peaks on the energy distribution of inelastically scattered neutrons is determined by the imaginary part of the tensor $X^{+-}(\mathbf{k},\omega)$ and, according to (36), should be non-Lorentzian. The non-Lorentzian shape of the peak corresponding to the magnetic scattering of neutrons by EuO and EuS was observed in Ref. 14 and, apparently, in Refs. 15 and 16.

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[1)]We have omitted the tilde from the Hamiltonian because this should not lead to any misunderstanding.

[2)]The expression given by (25) is identical with the corresponding expression in Ref. 5 only for $\omega=\varepsilon_{\mathbf{k}}$. The reasons for this discrepancy are explained in Ref. 11.

[1]S. V. Tyablikov, Metody kvantovoĭ teorii magnetizma (Methods in the Quantum Theory of Magnetism), Nauka, Moscow, 1975.
[2]V. P. Kalashnikov, Dokl. Akad. Nauk SSSR **219**, 59 (1974) [Sov. Phys. Dokl. 19, 760 (1974)].
[3]D. Forster, Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions, Benjamin, Reading, Mass. (Russ. transl. by Atomizdat, Moscow, 1980), Chap. 5.
[4]V. G. Vaks, A. I. Larkin, and S. A. Pikin, Zh. Eksp. Teor. Fiz. **53**, 281 (1967) [Sov. Fiz. JETPP **26**, 188 (1968)].
[5]V. G. Vaks, A. I. Larkin, and S. A. Pikin, Zh. Eksp. Teor. Fiz. **53**, 1089 (1967) [Sov. Phys. JETP **26**, 647 (1968)].
[6]Yu. A. Izyumov, F. A.Kassan-ogly, and Yu. N. Skryabin, Polevye metody v teorii ferromagnetizma (Field Methods in the Theory of Feromagnetism), Nauka, Moscow, 1974, Chaps. 1, 2.
[7]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, Moscow, 1962 (English translation published by Prentice-hall, Englewood Cliffs, 1963).
[8]F. Dyson, Phys. Rev. **102**, 1217, 1230 (1956).
[9]S. V. Maleev, Zh. Eksp. Teor. Fiz. **33**, 1010 (1957) [Sov. Phys. JETP **6**, 776 (1958)].
[10]V. G. Bar'yakhtar, V. N. Krivoruchko, and D. A. Yablonskiĭ, Teor. Mat. Fiz. **53**, 156 (1982).
[11]V. G. Bar'yakhtar, V. N. Krivoruchko, and D. A. Yablonskiĭ, Preprint ITF-82-148R, 1982.
[12]I. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. **42**, 1126 (1962) [Sov. Phys. JETP **15**, 778 (1962)].
[13]S. V. Peletminskiĭ and V. G. Bar'yakhtar, Fiz. Tverd. Tela (Leningrad **6**, 219 (1964) [Sov. Phys. Solid State **6**, 174 (1964)].
[14]H. A. Mook, Phys. Rev. Lett. **46**, 508 (1981).
[15]H. G. Bohn, W. Zinn, B. Dorner, and A. Kollmar, J. Appl. Phys. **52**, 2228 (1981).
[16]L. Passell, O. W. Dietrich, and J. Als-Nielsen, Phys. Rev. B **14**, 4897 (1976).

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