Exchange scattering of quasiparticles by a positive ion in ^3He

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The mobility difference of negative and positive ions in normal ^3He is discussed. The mobility mechanisms for ions of opposite signs are qualitatively different because the positive ion can exchange quasiparticles with the helium atoms from the ice-like shell surrounding the ion. A study of the mobility in a magnetic field may yield quantitative information on the magnitude of the exchange interaction. A calculation is carried out for the exchange scattering model and it is shown that a logarithmic contribution to the positive ion mobility $\mu_+(T)$ appears and is analogous to the Kondo effect.

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1. INTRODUCTION

In the study of ion mobility in normal liquid ^3He it is customarily assumed that the interaction of the quasiparticles with the ion is of the form of an impermeable sphere of radius a. It is known that at low temperatures the values of the mobility and of its temperature dependence differ greatly for positive and negative ions. The fact that at moderate pressures the mobility μ_+ of positive ions exceeds by approximately an order the mobility μ_- of negative ones[1] is usually attributed to the larger radius of a_- of the bubble around the electron[2] compared with the radius a_+ of the solidified helium around the positive ion.[3] However, the mobilities $\mu_+(T)$ and $\mu_-(T)$ are different also at high pressures $p \approx 30$ atm,[1,4] where estimates lead to $a_+ \approx a_-$. At not very low temperatures $T \approx 0.3$ K we have $\mu_+ \approx \mu_-$ and with decreasing temperaature $\mu_-(T)$ is almost constant whereas $\mu_+(T)$ increases logarithmically [we disregard the $\mu_+(T)$ jump usually attributed to settling of ^4He impurities on the "iceberg"]. If a potential of a hard sphere with the radius of either the bubble or the "iceberg" is used, and the cross section σ that determines the mobility is taken to be the cross section for scattering, by such a potential, of a particle with momentum close to k_F, then $\mu_-(T)$ should tend to a finite limit with decreasing temperature.[5] This agrees well with the behavior of $\mu_-(T)$, but contradicts the observed increase of $\mu_+(T)$. This circumstance, and namely the fact that at $a_+ \approx a_-$ the mobilities of ions of opposite sign have different temperature dependences, cannot be explained even qualitatively.

We propose here to regard exchange interaction as the scattering mechanism that distinguishes between ions of opposite sign. In the case of a negative ion the quasiparticle is scattered in fact by the electron density of the electron enclosed in a bubble. For a positive ion, on the contrary, the quasiparticle is scattered by helium atoms frozen on the iceberg surface. Besides the usual potential scattering that exists also in the case of a bubble, exchange scattering with spin flip is possible in a collision with an iceberg. Assuming elastic collisions and neglecting the ion recoil energy, exchange scattering is similar to electron scattering in a metal by a paramagnetic impurity—to the Kondo effect, i.e., it is temperature-sensitive at low temperatures.

The assumption that the collisions are elastic is not obvious. It follows from Boltzmann's equation that at $T < m\varepsilon_F/M$, where M is the effective mass of the ion, the mobility should increase like $\mu \propto T^{-2}$, contradicting the behavior of the ions of both signs. The validity of the single-particle Boltzmann equation, however is severely limited by the fact that the large radius of the ion causes it to interact simultaneously with many particles. Although there is still no complete answer to this question,[6,7] grounds for assuming that the collisions are elastic can be provided by experiments with negative ions,[8] where no deviation from the behavior that follows from the Fokker-Planck equation was found at any temperature.

2. EXCHANGE-SCATTERING CROSS SECTION

The exchange interaction between an atom of a liquid and a surface atom located at a point $\mathbf{R}$ is given by

$$V_{\alpha i|\beta j} = {}^1/_2 \Gamma(\mathbf{r}-\mathbf{R})\, \sigma_{\alpha\beta} \sigma_{ij}, \tag{1}$$

where Greek and Latin indices pertain respectively to the spin of an atom in the liquid and on the surface. The function Γ is concentrated in a region with linear dimension of the order of the diameter d of the ^3He atom and has a value V_0 of the order of the repulsion energy of two atoms separated by this distance, i.e., $V_0 \lesssim U_0$, where U_0 is the potential barrier produced by the entire ion. We assume that the exchange interaction of the quasiparticle with the surface is of the same form (1). The scattering amplitude is determined by the product of the probability of finding the particle at a location where the exchange potential differs substantially from zero, i.e., under a barrier of height V_0, by an amount equal to this potential. Although at large V_0 this probability decreases, the product indicated tends to a finite limit. To take better account of this circumstnce and to simplify the calculations, we make the substitution

$$\Gamma(\mathbf{r}-\mathbf{R}) \to (4\pi d^3/3)\, V_0 \delta(\mathbf{r}-\mathbf{R}), \tag{2}$$

and assume $\mathbf{R}$ to be arbitrarily located on the surface of an ideal sphere of radius a. To obtain the cross section for exchange scattering by the entire ion we multiply the cross section for scattering by the potential (2), averaged over the directions of $\mathbf{R}$, by the total number $N_s = 4\pi a^2 d n_s$ of surface

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atoms, where n_s is the density of solid ^{3}He.

In the first Born approximation in V_0, but taking exactly into account the potential U_0, the matrix element of the quasiparticle transition from the state **k** into **k'** is

$$M^{(1)}=\int d\mathbf{r}\psi_{\mathbf{k}'}^{*(-)}(\mathbf{r})\,V_{\alpha i|\beta j}(\mathbf{r}-\mathbf{R})\,\psi_{\mathbf{k}}^{(+)}(\mathbf{r}). \tag{3}$$

In the second-order approximation in V_0, the contribution to the matrix element is made by two graphs (Fig. 1)

$$M^{(2)}=-\int d\mathbf{r}\,d\mathbf{r}'\psi_{\mathbf{k}'}^{*(-)}(\mathbf{r}')\,T^2\sum_{\varepsilon,\omega}[V_{\alpha i|\gamma s}(\mathbf{r}-\mathbf{R})\,G_{\varepsilon_{\mathbf{k}}+\varepsilon-\omega}(\mathbf{r},\mathbf{r}')$$
$$\times V_{\gamma s|\beta j}(\mathbf{r}'-\mathbf{R})+V_{\gamma i|\beta s}(\mathbf{r}'-\mathbf{R})\,G_{\varepsilon_{\mathbf{k}}+\varepsilon+\omega}(\mathbf{r},\mathbf{r}')\,V_{\alpha s|\gamma i}(\mathbf{r}-\mathbf{R})]$$
$$\times\mathcal{G}_{\omega}\psi_{\mathbf{k}}^{(+)}(\mathbf{r}). \tag{4}$$

These expressions differ from the analogous expressions for the matrix elements of electron scattering by a paramagnetic impurity in a metal[9] that the quasiparticle moves between the two "exchange" collisions in the field of the repulsion potential of a solid ion of large radius $a\gg k_F^{-1}$, and consequently its propagator is of the form

$$G_{\varepsilon}(\mathbf{r},\mathbf{r}')=\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\psi_{\mathbf{k}}^{(+)}(r)\,\psi_{\mathbf{k}}^{*(+)}(r')}{i\varepsilon-\xi_{\mathbf{k}}}. \tag{5}$$

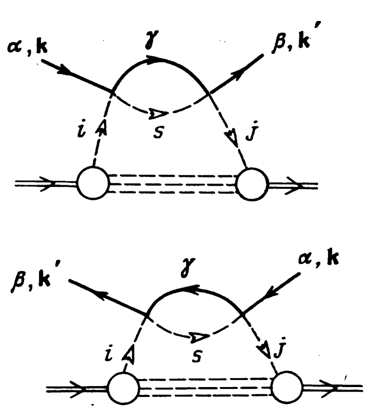
Since all the expressions contain a propagator with equal arguments, it does not matter whether it is expanded in terms of the set of functions $\psi_k^{(+)}$ or $\psi_k^{(-)}$. The designation and normalization of the scattering states $\psi_k^{(+)}$ coincide with those used in Ref. 10. If the frequency sums in (4) are calculated by the methods of Ref. 9, using (2) and substituting

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r})=\frac{1}{2k}\sum_l i^l(2l+1)\,e^{i\delta_l(k)}R_{kl}(r)\,P_l(\hat{\mathbf{k}}\hat{\mathbf{r}}), \tag{6}$$

from Ref. 10, it can be shown that

$$M=M^{(1)}+M^{(2)}=\sigma_{\alpha\beta}\sigma_{ij}\left(\frac{4\pi d^3}{3}V_0\right)\psi_{\mathbf{k}'}^{*(-)}(R)\,\psi_{\mathbf{k}}^{(+)}(R)$$
$$\times\left[\frac{1}{2}+\left(\frac{4\pi d^3}{3}V_0\right)\frac{m}{8\pi^2k_F}\sum_l(2l+1)R_{k_Fl}^2(a)\ln\frac{\varepsilon_F}{T}\right]. \tag{7}$$

The transport cross section for scattering by the whole ion



$$\sigma_{tr}=\frac{m^2}{4\pi^2}\int d\hat{\mathbf{k}}(1-\hat{\mathbf{k}}\hat{\mathbf{k}}')N_s\int\frac{d\hat{\mathbf{R}}}{4\pi}|M|^2 \tag{8}$$

can be reduced, after performing the necessary integrations, to the form

$$\sigma_{tr}=\frac{N_s}{16\pi^3}(\overline{\sigma\sigma'})^2\left(\frac{2\pi d^3}{3}V_0\right)^2\left[1+\frac{md^3V_0}{3\pi k_F}\sum_l(2l+1)\right.$$
$$\left.\times R_{k_Fl}^2(a)\ln\frac{\varepsilon_F}{T}\right]\left\{\left[\frac{\pi}{k_F^2}\sum_l(2l+1)R_{k_Fl}^2(a)\right]^2\right.$$
$$\left.+\left[\frac{\pi}{k_F^2}\sum_l 2lR_{k_Fl}^2(a)\sin(\delta_l^{(0)}-\delta_{l-1}^{(0)})\right]^2\right\}. \tag{9}$$

When the potential U_0 of the hard sphere is increased, the product of the potential by the wave function on the Fermi surface tends to a finite limit, namely, at $ka\gg1$ and $l\ll ka$ we have

$$U_0R_{kl}^2(a)\to\frac{2k^2}{ma^2}\left[1-\left(\frac{l}{ka}\right)^2\right]^{1/2}. \tag{10}$$

This equation, without derivation, was used in fact back in Ref. 11; the analogous equation cited in Ref. 6 contains in the right-hand side an extra factor $(2\pi)^{-1}$. The derivation of (9) is given in the Appendix. Using (10) and (A8), we obtain ultimately

$$\sigma_{tr}=\frac{25\pi}{108}\frac{N_s}{k_F^2}\left(k_F^3d^3\frac{V_0}{U_0}\right)^2\left[1+\left(k_F^3d^3\frac{V_0}{U_0}\right)\frac{16}{9\pi}\ln\frac{\varepsilon_F}{T}\right]. \tag{11}$$

3. THE RUDERMAN-KITTEL INTERACTION

We obtain within the framework of the described model the indirect interaction of the spins of two neighboring atoms on the ion surface. The molar volume of solid helium on the melting curve is $v=24\cdot10^{24}$ Å^3/mol, therefore the average distance R between the atoms of the solid helium, $(v/N_{Ab})^{1/3}\approx3.3$ Å, exceeds substantially the reciprocal Fermi momentum of the liquid $k_F=0.88$ Å^{-1}, namely: $Rk_F=2.95$. Bearing in mind this circumstance, we carry out all the calculations under the assumption $a\gg R\gg k_F^{-1}$.

Let $\vartheta=R/a\ll1$ be the angle between the two locations of the neighboring atoms on the sphere. It follows from (5) and (6) that

$$G_{i\varepsilon}(R)=\frac{mk_F}{2\pi^2}\int\frac{d\xi}{i\varepsilon-\xi}\frac{1}{4k^2}\sum_l(2l+1)R_{kl}^2(a)\,P_l(\cos\vartheta). \tag{12}$$

Using (10), the asymptotic equation[10]

$$P_l(\cos\vartheta)\approx J_0((l+1/2)\vartheta),\qquad \vartheta\ll1, \tag{13}$$

replacing the sum in (12) by an integral (the significant values are $l\gg1$), and taking into account Eq. (6.567.1) of Ref. 12

$$\int_0^1 x(1-x^2)^{1/2}J_0(bx)\,dx=(\pi/2)^{1/2}b^{-3/2}J_{3/2}(b) \tag{14}$$

and the asymptotic form of $J_{3/2}$, we obtain

$$G_{i\varepsilon}(R)\approx-\frac{k_F}{2\pi^2U_0R^2}\int\frac{d\xi}{i\varepsilon-\xi}\cos[(k_F+\xi/v_F)R]. \quad (15)$$

The indirect-exchange energy is expressed in standard fashion in terms of

$$T_{RK}=\left(V_0\frac{4\pi d^3}{3}\right)^2\int\frac{d\varepsilon}{2\pi}G_{i\varepsilon}^2(R) \quad (16)$$

and can be reduced to the form

$$T_{RK}=\varepsilon_F\frac{4}{9\pi}\left(k_F^3d^3\frac{V_0}{U_0}\right)^2\frac{\cos 2k_FR}{(k_FR)^5}. \quad (17)$$

4. CONCLUSION

Experiments[4,13] revealed a logarithmic increase of $\mu_+(T)$ starting with $T=50$ mK up to the superfluid transition temperature. The decrease of the exchange-scattering cross section means that the interaction has a ferromagnetic character, in agreement with the notion that ^{3}He is an almost-ferromagnetic Fermi liquid. Equation (11) is valid for a gas of interacting particles. In ^{3}He the interaction leads to damping of the quasiparticles far from the Fermi surface, and the upper limit in the logarithm should be taken to be a temperature $T\approx50$–100 mK, above which the Fermi-liquid theory does not hold. In the interval from 50 to 5 mK the mobility is approximately doubled. If all this increase is attributed to exchange interaction, it must be assumed that the effective coupling constant $g=16k_F^3d^3V_0/9\pi U_0$ is of the order of 1/3, i.e., quite large.

The contribution of the exchange scattering to the mobility can be quantitatively obtained by investigating the behavior of $\mu_+(T)$ and $\mu_-(T)$ at a pressure near the melting point in a magnetic field of strength such that the spin splitting $\Delta\varepsilon$ of the Fermi surface be larger than or of the order of the temperature. In ^{3}He we have $\Delta\varepsilon=2\beta(1+Z_0/4)^{-1}H$, where $Z_0=-3, 2\beta=0.16$; consequently the exchange part of the scattering, which exists only for $\mu_+(T)$, is suppressed at $T\lesssim50$ mK in fields $H\approx80$ kOe. A high pressure is necessary to bring a_+ as close as possible to a_- and actually study the scattering mechanisms that distinguish between these ions.

An estimate of the energy of indirect interaction with a maximum constant $g=1/3$ yields, according to (16) $4T_{RK}<1$ mK. This shows that the logarithmic correction to (11) can increase and become comparable with the first Born contribution even before the indirect interaction becomes substantial.

The observed vanishing of the logarithmic growth of $\mu_+(T)$ in a solution containing 1% ^{4}He (Ref. 13) can be interpreted as formation, around the ion, of a ^{4}He film that is stable at all temperatures and excludes exchange scattering.

We note that one more indication of the difference between the mechanisms of quasiparticle scattering by ions of opposite signs is the fact[14] that on going into the superfluid state $\mu_+(T)$ increases more slowly than $\mu_-(T)$ with decreaseing temperature.

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APPENDIX

A radial wave function in a potential $U(r)=U_0\Theta(a-r)$, finite at the origin and having at infinity the asymptotic form

$$R_{kl}(r)\approx\frac{2}{r}\sin\left(kr-\frac{l\pi}{2}+\delta_l\right) \quad (A1)$$

is of the form

$$R_{kl}(r)=\Theta(a-r)Br^{-1/2}I_{l+1/2}(k'r)$$
$$+\Theta(r-a)\left(\frac{2\pi k}{r}\right)^{1/2}[\cos\delta_lJ_{l+1/2}(kr)+(-1)^l\sin\delta_lJ_{-l-1/2}(kr)],$$

$$k'=(k_0^2-k^2)^{1/2},\qquad k_0^2=2mU_0/\hbar^2, \quad (A2)$$

where the notation for all the special functions is the same as in Ref. 12. The conditions that R_{kl} and all its derivatives be continuous at $r=a$ can be written in the form

$$BI_{l+1/2}(k'a)=(2\pi k)^{1/2}[\cos\delta_lJ_{l+1/2}(ka)-\sin\delta_lN_{l+1/2}(ka)], \quad (A3)$$

$$k'\frac{I_{l-1/2}(k'a)}{I_{l+1/2}(k'a)}=k\frac{\cos\delta_lJ_{l-1/2}(ka)-\sin\delta_lN_{l-1/2}(ka)}{\cos\delta_lJ_{l+1/2}(ka)-\sin\delta_lN_{l+1/2}(ka)}, \quad (A4)$$

where $J_{-n-1/2}=(-1)^{n-1}N_{n+1/2}$. We substitute in these expressions $J_{l+1/2}=\mathrm{Re}\,H^{(1)}_{l+1/2}$ and use for the function $H^{(1)}_\nu(x)$ the quasiclassical asymptotic expression at $ka\gg1$, $l\gg1$ and $l<ka$ in the form

$$H^{(1)}_{l+1/2}(x)\approx-i\left(\frac{2}{\pi}\right)^{1/2}\frac{\exp(i\alpha_l)}{(x^2-p^2)^{1/4}},$$
$$\alpha_l=(x^2-p^2)^{1/2}+p\,\mathrm{arctg}\frac{p}{x}-\frac{\pi l}{2}, \quad (A5)$$

where $p=l+1/2$. An impermeable sphere corresponds to the limit $k'\to\infty$, and then the wave function vanishes on the Fermi surface. The condition for this, as can be seen from (A3) and (A4), is

$$\delta_l^{(0)}=-\alpha_l. \quad (A6)$$

At finite k' such that $k_0\gg k$ it follows from (A3)–(A5) that

$$aR_{kl}(a)=(2\pi ka)^{1/2}\frac{\cos\delta_lJ_{l-1/2}(ka)-\sin\delta_lN_{l-1/2}(ka)}{k'I_{l-1/2}(k'a)/kI_{l+1/2}(k'a)}$$
$$\approx2\frac{k}{k'}\frac{\sin(\delta_l^{(0)}-\delta_{l-1}^{(0)})}{[1-(l/ka)^2]^{1/4}}. \quad (A7)$$

Taking (A6) into account, we have

$$\sin(\delta_l^{(0)}-\delta_{l-1}^{(0)})=[1-(l/ka)^2]^{1/2}, \quad (A8)$$

From which in fact follows Eq. (10) of the main text.

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