Secondary emission of a gaseous medium in the strong field of a standing electromagnetic wave

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Different variants are described of experiments in which one or two photodetectors are used to record the intensity fluctuation spectra of secondary emission by gas in the strong external electromagnetic field of a standing wave. It is shown that, at saturation, the signals recorded in these experiments are very different because of the contributions made by different correlation processes.

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INTRODUCTION

We shall consider the statistical properties of the radiation emitted by a gas by examining the average

$$G(\tau)=\langle E^+(\mathbf{r}_1t_1)E^+(\mathbf{r}_2t_2)E(\mathbf{r}_2t_2)E(\mathbf{r}_1t_1)\rangle, \qquad \tau=t_2-t_1>0$$

when the gas is placed in the field of a strong monochromatic standing wave. The quantity $G(\tau)$ is the signal received when the experiment is performed within the framework of intensity fluctuation spectroscopy (IFS). This signal may be qualitatively different from the signal received in traditional spectroscopy if the radiation noise is described by non-Gaussian statistics. The non-Gaussian effects originate in correlation processes that are different in different physical situations. When the external field consists of only one traveling wave, there is only one correlation process[1] of the form $2\mathbf{k}_f\to\mathbf{k}_1+\mathbf{k}_2$ in which two quanta of the original wave are transformed into two quanta of secondary emission. This process proceeds only in the direction of irradiation, and correlation occurs between the Fourier components of the secondary emission that are located symmetrically relative to the frequency of the original field ($2\omega_f=\omega_1+\omega_2$). In a standing wave (two traveling waves with wave vectors $\mathbf{k}_f$ and $\mathbf{k}'_f=-\mathbf{k}_f$), there are, in addition to this process, two further possibilities: $\mathbf{k}_f+\mathbf{k}'_f\to\mathbf{k}_1+\mathbf{k}'_1$ ($\mathbf{k}'_1=-\mathbf{k}_1$), $\mathbf{k}_f\to\mathbf{k}_2$ and $\mathbf{k}'_f\to\mathbf{k}'_1$ (the corresponding phase-locking conditions are $\mathbf{k}_f+\mathbf{k}'_f=\mathbf{k}_1+\mathbf{k}'_1$ and $\mathbf{k}_f-\mathbf{k}'_f=\mathbf{k}_2-\mathbf{k}'_1$).

In IFS, one can plan to observe the emission in such a way as to isolate particular correlation processes. We shall examine three variants of such experiments. In one of them (experiment I), we shall suppose that measurements are performed with a single photodetector facing the external beam along its path. The non-Gaussian component of the signal $G_{\rm I}(\tau)$ is then exclusively due to the process $2\mathbf{k}_f\to\mathbf{k}_1+\mathbf{k}_2$. In the second variant (experiment II), measurements are performed with two photodetectors facing one another along the incident beam in which the non-Gaussian component of the signal $G_{\rm II}(\tau)$ is due to two correlation processes that are new in comparison with the case of a traveling wave. Finally, when the line of observation does not coincide with the line of illumination in the case of two photodetectors facing one another, the only important process is $\mathbf{k}_f+\mathbf{k}'_f\to\mathbf{k}_1+\mathbf{k}'_1$ (experiment III). The figure illustrates all three variants of the experiment.

A situation analogous to our problem was discussed in Ref. 2, in which $G(\tau)$ was calculated within the framework of perturbation theory. We shall, in addition, consider saturation effects as well. Moreover, our calculations will refer to a somewhat different physical situation, namely, we shall examine the interaction between the standing wave and excited atoms (for example, in a gas discharge), whereas the authors of Ref. 2 investigated scattering by ground-state atoms. We shall also take into account the thermal motion of these atoms.

§1. BEATS BETWEEN SECONDARY EMISSION AND THE ORIGINAL FIELD

We shall write the Heisenberg positive-frequency operator $\hat{E}(\mathbf{r},t)$ in the form $\hat{E}(\mathbf{r},t)=E_0(\mathbf{r},t)+\hat{E}'(\mathbf{r},t)$, where $E_0(\mathbf{r},t)=E_f\exp(-i\omega_f t+i\mathbf{k}_f\mathbf{r})+E'_f\exp(-i\omega_f t+i\mathbf{k}'_f\mathbf{r})$ is the external field (c-number) in the form of two monochromatic plane waves traveling in opposite directions ($\mathbf{k}'_f=-\mathbf{k}_f$):

$$\hat{E}'(\mathbf{r},t)=\sum_{\mathbf{k}} i(k/2V)^{1/2}a_{\mathbf{k}}\exp(-i\omega_{\mathbf{k}}t+i\mathbf{kr}). \qquad (1)$$

The sum is evaluated over the wave eigenvectors in the auxiliary volume V, $a_{\mathbf{k}}$, $a^+_{\mathbf{k}}$ are the photon annihilation and creation operators, and $\omega_{\mathbf{k}}=|\mathbf{k}|$.

If we confine our attention to components of $G(\tau)$, that are due to beats with the participation of the external field E_0, we obtain

$$G(\tau)=E_0^*(\mathbf{r}_1,t_1)E_0^*(\mathbf{r}_2,t_2)\langle \hat{E}'(\mathbf{r}_2,t_2)\hat{E}'(\mathbf{r}_1,t_1)\rangle + E_0^*(\mathbf{r}_1,t_1)E_0(\mathbf{r}_2,t_2)\langle \hat{E}'^+(\mathbf{r}_2,t_2)\hat{E}'(\mathbf{r}_1,t_1)\rangle + \text{c.c.}$$

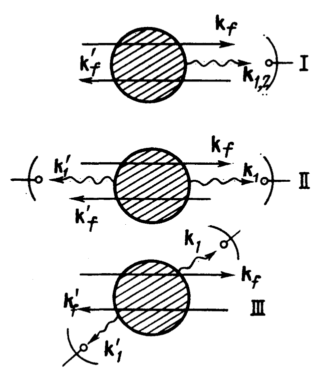

FIG. 1.

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We shall now express $G(\tau)$ in terms of averages of the form $\langle a_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle$, $\langle a_{\mathbf{k}_1}^+ a_{\mathbf{k}_2} \rangle$ and their complex conjugates by substituting (1) into the last expression. The dependence of these averages on τ need not be considered because this dependence determines the development (shift and broadening) of each mode separately, and will be of no interest in problems not involving a resonator as $V \rightarrow \infty$. The entire characteristic behavior of $G(\tau)$ will then be due to the exponential factors in the expansion given by (1).

A closed set of equations can be written down for averages of the form $\langle a_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle$, $\langle a_{\mathbf{k}_1}^+ a_{\mathbf{k}_2} \rangle$ that satisfy the phase-matching conditions. The medium is then assumed to fill uniformly the entire volume V of the auxiliary resonator and to consist of two-level atoms for which the widths of the working levels are mainly due to decays to extraneous levels. This set of equations can be obtained, for example, from the equation for the density matrix of a low-intensity secondary emission of the medium in the field of a strong standing wave, which can be derived by analogy with the case of the traveling wave (1).

We shall henceforth confine our attention to the case of an optically thin layer of the medium to be examined, which, in general, is in conflict with the previous assumption that the medium fills uniformly the entire volume. This contradiction may be obviated in a standard manner by introducing sufficiently rapid attenuation of the field within the volume V, which ensures weak enough secondary field, so that all processes induced by it can be neglected. When this is so, we can write down simple differential equations of the form

$$x_1 = \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle, \quad x_2 = \langle a_{\mathbf{k}} a_{2\mathbf{k}_f - \mathbf{k}} \rangle,$$

$$x_3 = \langle a_{\mathbf{k}}^+ a_{\mathbf{k} - 2\mathbf{k}_f} \rangle, \quad x_4 = \langle a_{\mathbf{k}} a_{-\mathbf{k}} \rangle$$

for the averages

$$\dot{x}_i = -Cx_i + \beta_i.$$

and other quantities of similar type. The appearance of the attenuation constant C in this theory has no effect on the final results because, in the final analysis, this constant can be incorporated in the normalization. Its physical significance is clear: C^{-1} is the lifetime of a photon within the volume of the radiating medium, i.e., it is of the order of the characteristic linear size L of the medium. Explicit expressions for $\beta_i(\omega)$ are given in the Appendix for the case where only the upper level of the two levels interacting with the external field is excited.

We shall now write down the expression for the received signal in the three typical experiments shown in the figure. By definition,

$$G(\omega) = \frac{1}{2\pi} \int G(\tau) e^{i\omega\tau} \, d\tau.$$

Using the phase-matching conditions for experiment I, we obtain

$$G_{\mathrm{I}}(\omega) = E_f^2 q [\beta_1(\omega_f - \omega) - \beta_2(\omega_f - \omega)] + (\text{c.c.}, \ \omega \rightarrow -\omega), \qquad (2)$$

where q is a proportionality coefficient, the explicit form of which is unimportant for the ensuing discussion.

By analogy, for experiment II, we have

$$G_{\mathrm{II}}(\omega) = E_f E_{f'} q [\beta_3(\omega_f - \omega) - \beta_4(\omega_f - \omega)] + (\text{c.c.}, \ \omega \rightarrow -\omega). \qquad (3)$$

We assume that the amplitudes E_f and E_f' are real.

In experiment III, we assume mixing of the fields E_f and E_f' on the photocathodes, which gives a signal of the form

$$G_{\mathrm{III}}(\omega) = G_{\mathrm{II}}(\omega) \big|_{\beta_3 = 0}. \qquad (4)$$

The angle θ between the line of observation and the line of illumination in experiment III must not, on the one hand, be too small, since it would then lie within the angular range of experiment II; on the other hand, it must not be too large, since the spatial dependence would then provide a substantial contribution to the signal, and the final result will vanish. The optimum angle θ for observations in experiment III must satisfy the condition

$$\frac{\gamma_{ab}}{\omega} < \sin\theta < \frac{\gamma_{ab}}{\omega} \frac{1}{\gamma_{ab} l},$$

where γ_{ab} is the lateral relaxation constant of the atomic system, ω is the optical frequency, and l is the separation between the photodetectors. This condition is readily satisfied in the optical range. Moreover, in order to observe the difference between the radiation statistics recorded in experiment III and the Gaussian statistics, a further condition must be satisfied by the angle, namely, $\sin\theta < \gamma_{ab}/ku$. Unless this condition is satisfied, lateral thermal motion relative to the line of excitation will ensure that the non-Gaussian noise component will vanish. However, this condition is very much less stringent than that given above.

In addition to the correlation spectroscopy signal $G(\omega)$, we also recall the traditional optical spectrum

$$I(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle E^+(t) E(t+\tau) \rangle e^{i\omega\tau} \, d\tau = q\beta_1(\omega). \qquad (5)$$

§2. WEAK NONLINEARITY

The quantities β_i, given in the Appendix, can be analyzed relatively easily within the framework of perturbation theory or in a strong field, provided we can neglect the spatial modulation of the population of the medium in the standing-wave field. Let us consider, to begin with, the lowest nonlinear order of perturbation theory. The signal generated in traditional spectroscopy is given by (5) and is determined by the quantity

$$q\beta_1(\omega) = I_0 W(\omega - \omega_0) \left\{ 1 + \frac{2R^2}{2\gamma_{ab} - \gamma} \left[\frac{2\gamma_{ab}}{4\gamma_{ab}^2 + (\omega - \omega_f)^2} - \frac{\gamma}{\gamma^2 + (\omega - \omega_f)^2} \right] - \frac{2R^2}{\gamma} \times \left[\frac{2\gamma_{ab}}{4\gamma_{ab}^2 + (\omega - \omega_f)^2} + \frac{2\gamma_{ab}}{4\gamma_{ab}^2 + (\omega + \omega_f - 2\omega_0)^2} \right] \right\}. \qquad (6)$$

Because of the presence of the correlation process of the form $2\mathbf{k}_f \rightarrow \mathbf{k}_1 + \mathbf{k}_2$, the signal $G_{\mathrm{I}}(\omega)$ differs in shape from (6) by a correction that is given by

$$q\beta_2(\omega)=I_0W(\omega)\frac{2R^2}{\gamma_{ab}-\gamma}\left\{\frac{\gamma}{\gamma^2+\omega^2}-\frac{\gamma_{ab}}{\gamma_{ab}^2+\omega^2}\right\}. \quad (7)$$

The signals recorded in experiments II and III are determined by (3) and (4):

$$q\beta_3(\omega_f-\omega)=-I_0W(\omega)\frac{R^2}{\gamma\gamma_{ab}}\frac{\gamma^2}{(\gamma-2i\omega)(\gamma-i\omega)}\frac{\gamma_{ab}}{\gamma_{ab}+i(\omega-\omega_{0f})}, \quad (8)$$

$$q\beta_4(\omega_f-\omega)$$

$$=I_0W(\omega)\frac{R^2}{\gamma\gamma_{ab}}\frac{\gamma^2}{(\gamma-2i\omega)(\gamma-i\omega)}\frac{\gamma_{ab}}{\gamma_{ab}-{}^3/_2i(\omega-{}^2/_3\omega_{0f})}. \quad (9)$$

The first of these is connected with the process $\mathbf{k}_f-\mathbf{k}_f'\to\mathbf{k}_2'-\mathbf{k}_1$ and the second with $\mathbf{k}_f+\mathbf{k}_f'\to\mathbf{k}_1+\mathbf{k}_1'$.

The notation used in the above formulas is as follows: $W(\omega)$—Doppler profile of width ku, normalized to unity and centered on zero frequency, ω_0—frequency of the atomic transition, ω_f—frequency of the strong field, $\omega_{0f}=\omega_0-\omega_f$, γ—longitudinal relaxation constant, I_0—light intensity due to the medium in the absence of the external field, and $R=\frac{1}{2}dE_f$—Rabi frequency. The expressions given by (6)–(9) were obtained in the zero-order approximation in the parameter ω_{ab}/ku, taking into account the noncoherent excitation of only the upper level (only the higher-order transitions were considered).

The field spectrum $I(\omega)$, observed in the direction defined by the external field, has the three different components given by (6). The unity in the expression in braces corresponds to the original Doppler profile of the spontaneous emission of the medium. The characteristic structure produced by two distinct mechanisms is superimposed on this background. One of them involves "hole-burning" by the external field in the upper-level population, which leads to the appearance of valleys on the Doppler profile at frequencies ω_f and $2\omega_0-\omega_f$ with widths $2\gamma_{ab}$. The other mechanism is connected with the fact that some of the atoms that interact actively with the external field have compensated Doppler shifts, so that these atoms radiate as if they were at rest, producing a structure of width γ.

We shall now confine our attention to the case $\gamma\ll\gamma_{ab}$, for which the signal in experiment I is given by

$$\frac{G_{\rm I}(\omega)}{E_f^2}=2I_0W(\omega)\left\{1-\frac{3R^2}{\gamma\gamma_{ab}}\frac{\gamma^2}{\gamma^2+\omega^2}-\frac{2R^2}{\gamma\gamma_{ab}}\left(\frac{2\gamma_{ab}^2}{4\gamma_{ab}^2+\omega^2}+\frac{2\gamma_{ab}^2}{4\gamma_{ab}^2+(\omega-2\omega_{0f})^2}\right)\right\}. \quad (10)$$

There is very little difference between this expression and $I(\omega)$: all that happens is that the entire spectrum is shifted from the optical frequency to the zero frequency, and there is an increase in the contrast of the narrow valley of width γ, as a consequence of the process $2\mathbf{k}_f\to\mathbf{k}_1+\mathbf{k}_2$.

The signal in experiment II has the following form:

$$\frac{G_{\rm II}(\omega)}{E_f^2}=2I_0W(\omega)\frac{2R^2}{\gamma\gamma_{ab}}\frac{\gamma_{ab}^2}{\gamma_{ab}^2+\omega_{0f}^2}\left(\frac{\gamma}{{}^1/_2\gamma-i\omega}-\frac{\gamma}{\gamma-i\omega}\right). \quad (11)$$

Here, the narrow structure appears in its pure form, but this does not as yet indicate the advantages of observing it in experiment II as compared with experiment I. The point is that the signals given by (10) and (11) are actually observed against the photodetection shot-noise background, and this noise limits the precision of measurement because it determines the signal-to-noise ratio in practise.

We note that the contribution of the process $\mathbf{k}_f-\mathbf{k}_f'\to\mathbf{k}_2-\mathbf{k}_1'$ to $G_{\rm II}(\omega)$ is exactly equal to the contribution of the process $\mathbf{k}_f+\mathbf{k}_f'\to\mathbf{k}_1+\mathbf{k}_1'$. This is the reason why the signal in experiment III is simply one-half of the signal in experiment II:

$$G_{\rm III}(\omega)={}^1/_2G_{\rm II}(\omega). \quad (12)$$

The phase-matching conditions $2\mathbf{k}_f=\mathbf{k}_1+\mathbf{k}_2$, $\mathbf{k}_f+\mathbf{k}_f'=\mathbf{k}_1+\mathbf{k}_1'$, $\mathbf{k}_f+\mathbf{k}_1'=\mathbf{k}_f'+\mathbf{k}_2$, that arise in the standing-wave field, and operate in the problem that we are considering, characterize the possible scattering processes in the nonlinear medium. It must not be thought, however, that these processes are wholly related to the spatial inhomogeneity that may be induced in the medium by the standing wave. The argument against this is that scattering of the form $2\mathbf{k}_f\to\mathbf{k}_1+\mathbf{k}_2$ occurs even in the case of the spatially homogeneous problem with a traveling wave. In this case, the entire scattering process is determined exclusively by fluctuations in the number density of the atoms. The results given by (10)–(12) must also be ascribed to fluctuations in the number of particles since, when $\gamma\ll\gamma_{ab}$, the effect associated with the standing-wave inhomogeneity must be of the order of R^2/γ_{ab}^2, whilst the structure obtained above is always of the order of $R^2/\gamma\gamma_{ab}$.

In all three experiments, the characteristic structure appears even in the lowest nonlinear orders of perturbation theory. All three experiments produce signals that are similar, both qualitatively and quantitatively, and they have no relative advantages. This result is different from that reported in Ref. 2, where signals in experiments II and III were of lower order in the Rabi frequency than the signal in experiment I (preferential observation of anomalous correlators). The reason for the difference is related to the fact that we have taken the scattering medium in the form of a set of atoms excited noncoherently to the upper working level, whereas the authors of Ref. 2 considered scattering by atoms in the ground state.

§3. SIGNALS IN SATURATING FIELDS

Part of the saturation effects is produced in the medium even in relatively weak fields, for which $\gamma\gamma_{ab}\ll R^2\ll\gamma_{ab}^2$. In this intensity range, the theoretical description of the phenomenon is still relatively simple. Using the expressions for β_i, given in the Appendix in zero order in the parameters R/γ_{ab} and $(\gamma\gamma_{ab})^{1/2}/R$, we obtain, for example, the spectrum of the field in the form

$$I(\omega)=I_0W(\omega-\omega_0)\left\{1-\frac{2\Gamma^2}{4\Gamma^2+(\omega-\omega_0)^2}+\left[\frac{\Gamma}{8\gamma_{ab}}\left(\frac{\gamma}{\gamma+i(\omega-\omega_f)}\right)^{1/2}+\text{c.c.}\right]\right\},\quad \Gamma=R\left(\frac{\gamma_{ab}}{\gamma}\right)^{1/2}.$$

Instead of the two valleys of width $2\gamma_{ab}$, at frequencies ω_f and $2\omega_0-\omega_f$ in the weak field (6), a valley of width 2Γ appears in the strong field due to the coalescence of the two weak-field valleys. A strong narrow peak of width $\sim\gamma$ and relative height $\Gamma/\gamma_{ab}=R/(\gamma\gamma_{ab})^{1/2}\gg 1$ appears at the bottom of the valley.

When experiment I is performed in weak fields, the presence of the correlation process $2\mathbf{k}_f\to\mathbf{k}_1+\mathbf{k}_2$ leads to an increase in the contrast of the narrow structure of width γ, as compared with the traditional experiment in which the spectrum $I(\omega)$ is recorded. At saturation, on the other hand, the correlation process has the opposite effect in that it compensates the narrow structure, which is then absent from $G_{\text{I}}(\omega)$. This compensation of strong and narrow peaks is also found to occur in experiment II: here, the correlation processes $\mathbf{k}_f-\mathbf{k}'_f\to\mathbf{k}_2-\mathbf{k}'_1$ and $\mathbf{k}_f+\mathbf{k}'_f\to\mathbf{k}_1+\mathbf{k}'_1$ act in opposite directions (again, in contrast to the case of a weak field where they enhance one another) in such a way that the structures produced by them compensate one another. However, the compensation is now incomplete because the structure with relative height $\sim\Gamma/\gamma_{ab}$ disappears, but that with relative height ~ 1 remains:

$$\frac{G_{\text{II}}(\omega)}{E_f^2}=I_0W(\omega)\frac{1}{4}\frac{\gamma}{\gamma-2i\omega}.$$

The only significant correlation process in experiment III is $\mathbf{k}_f+\mathbf{k}'_f\to\mathbf{k}_1+\mathbf{k}'_1$, so that the compensation of the strong narrow peak does not occur:

$$\frac{G_{\text{III}}(\omega)}{E_f^2}=I_0W(\omega)\frac{\Gamma}{\gamma_{ab}}\frac{\gamma}{\gamma-2i\omega}\left(\frac{\gamma}{\gamma-i\omega}\right)^{1/2}.$$

Thus, in saturated fields, the signals produced in the above experiments are very different: $G_{\text{I}}(\omega)$ does not contain the narrow structure, $G_{\text{II}}(\omega)$ contains a weak structure, and $I(\omega)$ and $G_{\text{III}}(\omega)$ contain a strong narrow structure.[1)]

In principle, experiment I can be planned in a somewhat different way. If we suppose that the photocathode is not a point-detector, as we have indeed assumed, but, on the contrary, is relatively large, then, as is well known,[3] the Gaussian component is averaged out in space and vanishes. As a result, $G_{\text{I}}(\omega)\sim-\beta_2(\omega-\omega_f)$, so that it turns out that the narrow structure is not canceled out in this signal, as was the case in our main discussion.

APPENDIX

The explicit expressions for the inhomogeneous term β_i in the set of equations for averages of the form $\langle aa^+\rangle$ are as follows:

$$\beta_1(\omega_1)=2\,\text{Re}\left\{|g|^2N_a\frac{A}{\gamma_{ab}-i\omega_{01}}-|g|^2N_a\frac{2\tilde{A}-1}{\gamma_{ab}-i\omega_{01}}\frac{R^2}{\gamma+i\omega_{1f}}\times\left(\frac{2A}{\gamma_{ab}-i\omega_{01}}+\frac{2A-1}{\gamma_{ab}+i\omega_{0f}}\right)+|g|^2N_a\frac{2\tilde{A}'-1}{\gamma_{ab}-i\omega_{01}}\frac{R^2}{\gamma+i\omega_{1f'}}\left(\frac{2A}{\gamma_{ab}-i\omega_{01}}+\frac{2A-1}{\gamma_{ab}+i\omega_{0f'}}\right)\right\},$$

$$\beta_2(\omega_1)=-2\,\text{Re}\left\{|g|^2N_a\frac{2\tilde{A}-1}{\gamma_{ab}-i\omega_{01}}\frac{R^2}{\gamma+i\omega_{1f}}\left(\frac{2A}{\gamma_{ab}+i(2\omega_{0f}-\omega_{1f})}+\frac{2A-1}{\gamma_{ab}-i\omega_{0f}}\right)\right\},$$

$$\beta_3(\omega_1)=-2|g|^2N_a\frac{\gamma}{\gamma-i\omega_{1f}}\frac{2\tilde{A}'-1}{\gamma_{ab}+i\omega_{01}}\frac{R^2}{\gamma-i\omega_{1f'}}\times\left(\frac{2A-1}{\gamma_{ab}-i(\omega_{0f}+\omega_{0f'}-\omega_{01})}+\frac{2A}{\gamma_{ab}-i(\omega_{0f'}-\omega_{01}-\omega_{01'})}\right)+(f\rightleftharpoons f'),$$

$$\beta_4(\omega_1)=-2|g|^2N_a\frac{\gamma}{\gamma-2i\omega_{1f}}\frac{2\tilde{A}-1}{\gamma_{ab}+i\omega_{01}}\frac{R^2}{\gamma-i\omega_{1f}}\times\left(\frac{2A-1}{\gamma_{ab}-i(\omega_{0f'}-2\omega_{1f})}+\frac{2A}{\gamma_{ab}-i(\omega_{0f}-\omega_{01}-\omega_{0f'})}\right).$$

where

$$A=\left[1+2R^2\frac{\gamma_{ab}}{\gamma}\left(\frac{1}{\gamma_{ab}^2+\omega_{0f}^2}+\frac{1}{\gamma_{ab}^2+\omega_{0f'}^2}\right)\right]\times\left[1+4R^2\frac{\gamma_{ab}}{\gamma}\left(\frac{1}{\gamma_{ab}^2+\omega_{0f}^2}+\frac{1}{\gamma_{ab}^2+\omega_{0f'}^2}\right)\right]^{-1}.$$

The quantity $\tilde{A}$ is obtained from A by introducing the substitution $\gamma_i\to\gamma_i+i\omega_{1f}$, and the quantity $\tilde{A}'$ is obtained by substituting $\gamma_i\to\gamma_i+\omega_{1f'}$. To take the Doppler shift into account, we must replace ω_{1f}, $\omega_{f'}$ with $\omega_{1f}-x$, $\omega_{f'}-x$, and average over x with the Maxwellian weight $W(x)$. We have used the following notation: $g=-i(k/2V)^{1/2}de^{-ikr}$—constant of interaction between the plane wave and the atom, d—dipole moment, $N_a=r_a/\gamma$—stationary population of the upper level in the absence of the external field, and $\omega_{01}=\omega_0-\omega_1$.

1)Strictly speaking, we have not described the frequency region $\omega\sim R^2/\gamma_{ab}^2$, at saturation. This requires numerical analysis, but estimates indicate that the appearance of any narrow structure is not expected in this case.

1. Yu. M. Golubev, Zh. Eksp. Teor. Fiz. **69**, 875 (1975) [Sov. Phys. JETP **42**, 447 (1975)].
2. A. P. Kazantsev, V. S. Smirnov, V. P. Sokolov, and A. N. Tumaĭkin, Zh. Eksp. Teor. Fiz. **81**, 889 (1981) [Sov. Phys. JETP **54**, 474 (1981)].
3. A. A. Kharkevich, Spektry i analiz (Spectra and Analysis) Fizmatgiz, Moscow, 1962 (English transl. published by Plenum Press, New York, 1960).

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