## Fluorescence spectrum of a two-level atom under conditions of partial suppression of phase relaxation by a strong resonance field

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(Submitted 28 October 1982; resubmitted 27 December 1982) Zh. Eksp. Teor. Fiz. **85**, 447–455 (August 1983)

The fluorescence spectrum of a two-level atom is calculated in the diffusion Markov approximation by taking into account the relaxation-constant change due to the decrease of the effectiveness of the phase noise when a strong monochromatic field acts on the atom. The results describe the suppression of the diffusion component of the collisional broadening of the fluorescence lines of two-level atoms with static dipole moments. The suppression mechanism reduces to a decrease, in a strong field, of the upper cutoff radius of the broadening particles.

PACS numbers: 32.50. + d

### **1. INTRODUCTION**

In addition to the well-known mechanism of the change of the fluorescence spectrum of a two-level atom in a strong field, <sup>1,2</sup> on account of the difference between the damping rates of the natural oscillations of the excited atoms and the initial relaxation constants that remain unchanged, theoretical predictions<sup>3-5</sup> and experimental data<sup>6,7</sup> point to the possibility of a mechanism capable of a more drastic rearrangement of the spectrum. This mechanism operates because the atom's relaxation constants themselves are altered by the change of the effectiveness of the noise that causes the relaxation and acts on an atom with a vibration spectrum substantially changed by a strong external field.

The corresponding calculations were performed in Refs. 8 and 9 for two-level-atom spectra broadened by shortrange collisions that give rise to a jumplike random process in the atom. If noise is considered against the background of the motion of the atom + field compound system and appropriate temporal relations are satisfied, this is a Markov process for arbitrarily strong external fields (see also Sec. 2). In the present paper is considered another, "diffusion" type of Markov process in a two-level atom, a process characterized by a continuous change of the state under the action of the noise. The evolution operator for such processes is calculated using a perturbation-theory series, so that explicit analytic expressions can be obtained for the spectral characteristics. These results describe the suppression, by a strong field, of the collisional broadening of the atom, but for only longrange and not for short-range interaction potentials, when the broadening is due mainly to accumulation of a large number of collisions with a small loss of the phase coherence. This takes place, for example, for hydrogen atoms in a plasma.<sup>10</sup> The results of the present article, which pertain to twolevel atoms, are directly applicable to noncentrosymmetric atoms with nonzero difference of the dipole-moment diagonal matrix elements broadened by charged particles. It must be noted, to be sure, that in such a system there is a nonzero contribution of the jumplike component from close collisions, a contribution suppressed ultimately by the strong field. The results presented here describe only the diffusion component and are therefore valid for not too strong fields, when the diffusion component of the broadening predominates strongly even when account is taken of the partial suppression.

The relaxation of a two-level atom is described by the Hamiltonian

$$\hat{\mathscr{H}}_{t} = \hbar \hat{\boldsymbol{\xi}}(t) \hat{\boldsymbol{\sigma}}(t), \quad \hat{\boldsymbol{\sigma}} = (\hat{\boldsymbol{\sigma}}_{1}, \hat{\boldsymbol{\sigma}}_{2}, \hat{\boldsymbol{\sigma}}_{3}), \quad (1)$$

where  $\hat{\xi}(t) = (\hat{\xi}_3, \hat{\xi}_1, \hat{\xi}_2)$  is the external noise. As applied to optics,  $\hat{\xi}_{1,2}$  represent the noise of the electromagnetic field, which takes into account in the general case also the interaction with other atoms (the Weisskopf-Fursov-Vlasov mechanism),<sup>11</sup> while  $\hat{\xi}_3$  describes the stochastic action, due to collisions, on the phase (more accurately, on the frequency) by the atomic vibrations. By virtue of the relatively low-frequency character of  $\hat{\sigma}_3$ , the quantum oscillations of the population can lead to a substantial change of the noise  $\xi_3$  at the relaxation matrix on account of its phase component. In the limit, this mechanism can lead to a complete suppression of the phase relaxation, i.e., in particular, to a suppression of the collisional broadening.<sup>9</sup>

We calculate in this paper all the relaxation constants and the fluorescence spectrum of a two-atom level, with allowance for the described mechanism of suppression of the phase relaxation, within the framework of the validity of the diffusion Markov description of the dynamics of the atom, and of the possibility of neglecting the changes of the atom velocity by collisions during the relaxation times. The latter means the possibility of considering for each atom a timeinvariant Doppler frequency and a corresponding detuning  $\delta = \omega_L - \omega_{12}$ , i.e., the absence of a Doppler contribution to the homogeneous line width.

### 2. CALCULATION OF THE RELAXATION MATRIX AND OF THE EVOLUTION OPERATOR

The total Hamiltonian for the considered model is of the form

 $\hat{\mathscr{H}}_{a}+\hat{\mathscr{H}}_{I}+\hat{\mathscr{H}}_{f}; \hat{\mathscr{H}}_{a}=-\hbar\omega_{12}\sigma_{3}/2,$ 

where  $\hat{\mathcal{H}}_a$  is the free-atom Hamiltonian. The Hamiltonian  $\hat{\mathcal{H}}_I = \hat{\mathbf{V}}(t)\hat{\mathbf{\sigma}}$  with the classical vector  $\hat{\mathbf{V}}(t)$  of the the external fields, which take here the form of sinusoids with frequency  $\omega_L$ , describes the interaction with the external field. It is

assumed that  $\hat{\xi}(t)$  in the noise Hamiltonian (1) are stationary (in the broad sense) processes.

To calculate the relaxation with allowance for the strong-field effects it is necessary in the general case to consider the relaxation against the background of the unperturbed motion, which should include the motion corresponding not only to  $\hat{\mathcal{H}}_a$  but also to  $\hat{\mathcal{H}}_I$ .

The analysis here is based on the fact that in this case a Markov description of the relaxation evolution is also possible, since it is possible to distinguish in the intensities of interest a scale of slow times  $\Delta > \tau_c$ , for which, on the one hand, the relaxation motion is still of no importance, while the number of field-induced quantum oscillations is large enough, i.e.,  $\Omega \Delta > 1$ . As a result, their action is taken into account by smoothening the mean values with frequency  $\Omega$  over the fast oscillations, which are of no interest. Thus, the conditions for stationary Markov dynamics are the relations

$$\Omega \gg \Gamma_0, \ \tau_c \Gamma_0 \ll 1, \tag{2}$$

where  $\Omega = (\vartheta^2 + \delta^2)^{1/2}$  is the effective frequency of the quantum oscillations and takes into account the detuning  $\delta$ , whereas the Rabi frequency is  $\vartheta = |dE|/\hbar$ ;  $\Gamma_0$  denotes the rate of transverse relaxation in the zero-field limit, and  $\tau_c$  is the effective correlation time of the noise  $\hat{\xi}_3$ . The first relation in (2) simultaneously determines at  $\delta = 0$  the intensities of the external field, for which one can really expect changes of the relaxation constants, while the second is the standard condition for Markov behavior in the case of a weak or moderately strong  $(\vartheta \sim \Gamma_0)$  field. The quantity  $\Omega \tau_c$  can have here any value, and at  $\Omega \tau_c \ll 1$  we have a transition to the theory of Refs.2 and 3, in which the relaxation constants are invariant. We note that in the Markov theory the small parameter  $\tau_c \Gamma_0$  corresponding to (2) is assumed to be zero, while effects produced when it is not zero can be considered consistently only in a non-Markov theory.

As regards the properties of the considered random processes  $\hat{\xi}(t)$ , it is assumed that the higher kinetic coefficients

$$\int \dots \int d\tau_{1} d\tau_{n} \langle \hat{\xi}_{i}(\tau_{1}) \dots \hat{\xi}_{i} \langle \tau_{n} \rangle \rangle$$

$$= \chi \exp(i\omega_{1}\tau_{1} + \dots + i\omega_{n}\tau_{n}) = o(\Delta)$$

are insignificant for all  $t, \omega_1, \ldots, \omega_n$  and n > 2. This condition makes it possible, when calculating the relaxation matrix, to take into account only correlations of higher order, corresponding to n = 2, and separate in the atom quantum random processes of "quasidiffusion" type, which include, in particular, processes with Gaussian noise  $\hat{\xi}$ . The process  $\hat{\xi}_3$ , which describes the collision-broadening mechanism, satisfies the foregoing conditions, provided only that the change of the phase per collision is small.

The initial expression that determines the evolutional operator  $S(t,\tau)$  that describes the change of the atomic operators from the instant t to  $\tau$ , takes in the notation of Ref. 12 the form

$$S(t,\tau) = \left\langle T \exp\left\{\frac{i}{\hbar} \int_{t}^{\tau} \left[ \left(\hat{\mathscr{H}}_{a} + \hat{\mathscr{H}}_{I} + \hat{\mathscr{H}}_{J}\right), \odot \right] d\tau \right\} \right\rangle , \quad (3)$$

where the symbol  $[\hat{A}, \odot]$  denotes the operation of commutation with  $\hat{A}$ , i.e.,  $[\hat{A}, \odot]$   $\hat{B} = [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ . Separating first in (3) the free precession of the atom having the frequency  $\omega_L$  of the external field, with account taken of the detuning, and averaging this operator over the high-frequency oscillations, we represent (3) in the form

$$S(t, \tau) = \exp [W_r(\tau - t)] S_0(t, \tau).$$
 (4)

Here

$$S_0(t, \tau) = \exp \left[ \widehat{W}_I(\tau - t) \right] \exp \left[ W_L(\tau - t) \right]$$
(5)

is the evolution operator of the atom that is not perturbed by the noise;

 $W_{L} = -(i\omega_{L}/2) [\hat{\sigma}_{3}, \odot]$ 

is the free precession operator;  $\tilde{W}_I$  has in the standard basis  $\{\hat{I}, \hat{\sigma}\}$  of the atomic operator the form

$$\mathcal{W}_{I} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \vartheta_{c} & \vartheta_{s} \\ 0 & -\vartheta_{c} & 0 & \delta \\ 0 & -\vartheta_{s} & -\delta & 0 \end{pmatrix}$$
(6)

and describes the averaged evolution under the action of the external field with allowance for the detuning;  $w_r$  is the sought relaxation matrix, defined by the equation

$$W_{r} = \lim_{\Delta \to 0} \frac{1}{\Delta} \int_{t}^{t+\Delta} \int_{t}^{\tau_{1}} \langle [\hat{\mathscr{H}}_{f}(\tau_{1}), [\hat{\mathscr{H}}_{f}(\tau_{2}), \odot]] \rangle_{f} d\tau_{1} d\tau_{2}, \quad (7)$$

in which  $\Delta$  is chosen, with allowance for (2), in accord with the relations

$$\mathbf{r}_c, \quad \Omega^{-1} \ll \Delta \ll \Gamma^{-1}, \tag{8}$$

and the formal limit  $\Delta \rightarrow 0$  is understood in the usual manner.<sup>13</sup> The  $\mathscr{H}_{f}(\tau)$  dependence in (7) takes into account the unperturbed motion (5) of the atomic operators  $\hat{\sigma}(\tau) = S_{0\sigma}^{T}(\tau)\hat{\sigma}$ ;  $\hat{\sigma} = \hat{\sigma}(0)$  are Pauli matrices in a canonical basis, and  $S_{0\sigma}$  is the projection of  $S_{0}$  on the three-dimensional basis  $\hat{\sigma}$ . In the case of the standard problem of one-photon resonance in a non-ultrastrong field  $(Ed/\hbar\omega_L \leq 1)$ ,  $\delta$  in (6) has the meaning of detuning, and  $\vartheta_c$  and  $\vartheta_s$  are the components of the complex amplitude  $(Ed/\hbar) \exp(i\varphi)$ ; in other cases,  $\tilde{w}_I$  has the same form but the matrix elements have a different meaning.

The relaxation operator (7) is the derivative, with respect to time, of the operator of the quantum transition distribution probability<sup>12</sup>  $S_r$  in the compound system,<sup>4,5</sup> the definite part of the proper evolution of which is described by the operator (5). Expression (7) is obtained by expanding in a perturbation-theory series the exact representation

$$S_r(t,t+\Delta) = \left\langle T \exp\left\{ (i/\hbar) \int_t^{t+\Delta} \left[ \hat{\mathscr{H}}_t(\tau), \odot \right] d\tau \right\} \right\rangle_t.$$

The crucial factor of the described method is the independence of the conditions of applicability of this expansion of the quantity  $\Omega$ , which determines the frequency of the quantum oscillations of the operators  $\hat{\sigma}(\tau) = S_{0\sigma}(\tau)\hat{\sigma}$  in  $\hat{\mathcal{H}}_f(\tau) = \hbar \xi(\tau)\hat{\sigma}(\tau)$ , since these oscillations do not increase  $\hat{\mathcal{H}}_f$  in the numerical expression. By virtue of (8) these oscillations do not exist at all in the described time scale  $\Delta t \gtrsim \Delta$ . and determine only the form of the relaxation operation  $W_r$ that is formed within the "elementary" time intervals  $\sim \Delta$ . The same applies also to the conditions of applicability of the Markov approximation, represented by the relation  $\tau_c \ll \Gamma_0^{-1}$ , since the oscillations do not increase the correlation time  $\tau_c$ : conversely, when the relaxation is substantially suppressed,  $\Gamma_0$  in this inequality can be replaced by the corresponding longer relaxation time, and the condition  $\Omega \gg \Gamma_0$ simplifies the analysis, making the operator  $W_r$  stationary. Under this conditions, in the case of collision broadening, a contribution to the  $W_r$  component connected with the oscillations is made only by passages over impact distances  $\rho \gtrsim \rho_{\Omega}, \rho_{\Omega} = v/\Omega$ , so that for this component the upper cutoff radius is  $\rho_{\alpha}$  as against  $\rho_m$  (Ref. 10) for a weak field. The jumplike contribution from the region  $\rho \leq \rho_W$ , corresponding to the lower cutoff radius (the Weisskopf radius  $\rho_W$ ), should be treated by the methods of Ref. 9, since perturbation theory cannot be used for such distances. The relative fractions of the jumplike and diffusion components, as can be readily shown, are 1 and  $\ln(\rho_{\Omega}/\rho_{W})$ , therefore when the intensity is increased the only component that is weakened is the diffusion one, which predominates only at  $\ln(\rho_{\Omega}/\rho_{W}) \ge 1$ .

Expanding the commutators in (7) and averaging, we obtain

$$W_{r} = \operatorname{Tr} \left[ (Q+Q^{+}) \hat{\boldsymbol{\sigma}} \odot \hat{\boldsymbol{\sigma}}^{T} - (Q \hat{\boldsymbol{\sigma}} \hat{\boldsymbol{\sigma}}^{T} \odot + Q^{+} \odot \hat{\boldsymbol{\sigma}} \hat{\boldsymbol{\sigma}}^{T}) \right];$$

$$Q = \lim_{\Delta \to 0} \frac{1}{\Delta} \int_{t}^{t+\Delta} \int_{t}^{\tau_{2}} d\tau_{2} d\tau_{1} S_{0\sigma}(\tau_{2}) K^{T}(\tau_{1}, \tau_{2}) S_{0\sigma}^{-T}(\tau_{1}),$$
(9)

where  $K(\tau_1, \tau_2) = K(\tau_2 - \tau_1)$  is the correlation matrix of the noise. When account is taken of the temporal relations (8), the limiting Markov expression for Q is simpler:

$$Q = \int_{\bullet}^{\bullet} d\tau \langle S_{\sigma\sigma}(t+\tau/2) K^{\tau}(\tau) S_{\sigma\sigma}^{\tau}(t-\tau/2) \rangle, \qquad (10)$$

where the angle brackets denote smoothing over t (over scales  $\sim \Delta$  in the initial expression). Representing the operators  $\sigma_i \odot \hat{\sigma}_j$  and their like by matrices, we obtain (10) in matrix form:

$$W_{r} = \begin{pmatrix} 0 & | & 8Q_{12}^{ai} - 8Q_{13}^{ai} & 8Q_{32}^{ai} \\ 0 & & \\ 0 & & \\ 0 & W_{r\sigma} & \\ 0 & & \\ \end{pmatrix},$$
(11)

$$W_{ro} = 2(Q+Q^{+r}) - 2[\operatorname{Tr}(Q+Q^{+})]I;$$
 (12)

 $Q^{ai}$  is the imaginary part of the antisymmetric part of the matrix Q.

Calculation of the matrix Q in accord with (10) with allowance for the form (5) of the unperturbed evolution operator and for the left-hand relation in (8), using the spectral expansion of the matrices  $\bar{W}_{I\sigma}$  and  $W_{L\sigma}$ , leads after smoothing over t to

$$Q = \sum_{k=1}^{n} \lambda_k |\Omega_k\rangle \langle \Omega_k|, \qquad (13)$$

where  $|\Omega_k\rangle$ ,  $\langle \Omega_k |$  denote respectively the right-hand eigenvector column and the left-hand eigenvector row of the matrix  $W_{I\sigma}$  with eigenvalues  $i\Omega_3 = 0$ ,  $i\Omega_{1,2} = \pm i\Omega$ ,  $\Omega = (\delta^2 \vartheta_s^2 + \vartheta_c^2)^{1/2}$ ;

$$\lambda_{k} = \sum_{l=1}^{3} |\langle \Omega_{k} | \omega_{l} \rangle|^{2} \langle \omega_{l} | \tilde{K}^{T}(\omega_{l} + \Omega_{k}) | \omega_{l} \rangle, \qquad (14)$$

where  $\omega_3 = 0$ ,  $i\omega_{1,2} = \pm uw_L$ , and  $\tilde{K}(\omega) = \int_{0}^{\infty} K(\tau) \exp(i\omega\tau) d\tau.$  (15)

The matrix Q, and in accord with (12) also the relaxation submatrix  $W_{r\sigma}$ , have thus according to (14) the proper ortho-projectors  $|\Omega_k\rangle\langle\Omega_k|$ .

Expression (14) for  $\lambda_k$  can be simplified with allowance for

$$K_{ij} = K_{ii} \delta_{ij}$$

i.e., with allowance for the linear independence of the phase and radiation noise, and also of the cosine and sine components of the latter. Taking into account the form  $\langle \omega_3 | = (1,0,0), \langle \omega_{1,2} | = (0, 1/2^{1/2}, \mp i/2^{1/2}), \text{ Eq. (14) will}$ contain the combinations  $\tilde{K}_1(\omega) = [\tilde{K}_{11}(\omega) + \tilde{K}_{22}(\omega)]/2$  and  $K_{11}(\omega) = K_{33}(\omega)$ , and it suffices to assume that  $\tilde{K}_1(\pm \omega_L \pm \Omega) = \tilde{K}_1(\pm \omega_L)$ . Substituting  $|\omega_k\rangle = \langle \omega_k |^+$ , and also

$$\begin{split} &\langle \Omega_{\mathfrak{s}} | = (\delta/\Omega, -\vartheta_{\mathfrak{s}}/\Omega, \vartheta_{\mathfrak{c}}/\Omega), \\ &\langle \Omega_{\mathfrak{s},\mathfrak{s}} | = (\overline{\sqrt{2}}\vartheta\Omega)^{-\mathfrak{s}} (\vartheta, \mp i\Omega\vartheta_{\mathfrak{c}} + \delta\vartheta_{\mathfrak{s}}, \mp i\Omega\vartheta_{\mathfrak{s}} - \delta\vartheta_{\mathfrak{c}}), \end{split}$$

we obtain ultimately

$$\lambda_{3} = (\delta^{2}/\Omega^{2}) \tilde{K}_{\parallel}(0) + (\vartheta^{2}/\Omega^{2}) [\tilde{K}_{\perp}(\omega_{L}) + \tilde{K}_{\perp}(-\omega_{L})]/2,$$

$$\lambda_{4} = \frac{\vartheta^{2}}{2\Omega^{2}} \tilde{K}_{\parallel}(\Omega) + \frac{(\Omega + \delta)^{2}}{4\Omega^{2}} \tilde{K}_{\perp}(\omega_{L}) + \frac{(\Omega - \delta)^{2}}{4\Omega^{2}} \tilde{K}_{\perp}(-\omega_{L}),$$

$$\lambda_{2} = \frac{\vartheta^{2}}{2\Omega^{2}} \tilde{K}_{\parallel}(-\Omega) + \frac{(\Omega - \delta)^{2}}{4\Omega^{2}} \tilde{K}_{\perp}(\omega_{L}) + \frac{(\Omega + \delta)^{2}}{4\Omega^{2}} \tilde{K}_{\perp}(-\omega_{L}).$$
(16)

On the basis of the form (13) of the matrix Q and of Eqs. (11) and (12) we obtain the relaxation matrix in the form

$$W_{r} = \begin{pmatrix} 0 \mid -w \langle \Omega_{3} \mid \\ 0 \mid \end{pmatrix}^{*}$$
(17)

where

$$W_{r_{0}} = -\gamma |\Omega_{3}\rangle \langle \Omega_{3}| - (\Gamma - i\Lambda) |\Omega_{1}\rangle \langle \Omega_{1}| - (\Gamma + i\Lambda) |\Omega_{2}\rangle \langle \Omega_{2}|,$$
  

$$w = 4 \operatorname{Re} (\lambda_{1} - \lambda_{2}), \quad \gamma = 4 \operatorname{Re} (\lambda_{1} + \lambda_{2}), \quad (18)$$
  

$$\Gamma = 4 \operatorname{Re} \left(\lambda_{3} + \frac{\lambda_{1} + \lambda_{2}}{2}\right), \quad \Lambda = 2 \operatorname{Im} (\lambda_{1} - \lambda_{2})$$

are the rates of the decay of the excitation, and coincide in the limit  $|\delta| \rightarrow \infty$  with the constants that describe respectively the noise-radiation pumping, the rate of the longitudinal damping, the rate of the transverse damping, and the Lamb shift of the atom in the absence of a strong field. At  $|\delta| \neq \infty$  there is no such coincidence even in the absence of suppression of the phase relaxation.

# 3. FLUORESCENCE SPECTRUM AND CONNECTION OF RELAXATION CONSTANTS IN A STRONG AND A WEAK FIELD

We present for simplicity expressions only for forward scattering. The fluorescence spectrum is determined by the Fourier transform of the normally ordered correlation fuction of the dipole moment of the atom, which has, when account is taken of the representation (10), the form

$$K(\tau) = \operatorname{const} |d_{12}|^2 \langle 0| \langle \hat{\sigma}^{(-)}(t) [e^{w_r \tau} \hat{\sigma}^{(+)}(t+\tau) \rangle] \rangle, \quad (19)$$

where  $\langle 0 |$  denotes the left-hand eigenvector of the matrix  $W_r$  and corresponds to  $\lambda_0 = 0$ ;  $\hat{\sigma}^{(\pm)}(s) = \exp(\hat{W}_I s) \times \exp(W_L s) |\omega_{1,2}\rangle$ , and the product  $\hat{\sigma}^{(-)}(t) [\ldots]$  is understood as the product of atomic operators. For the calculations we need the eigenvalues  $\lambda_n$  and the eigenvectors  $\langle \lambda_n |$ ,  $|\lambda_n \rangle$  of the matrix  $W_r$ :

$$\lambda_{0} = 0, \quad \lambda_{3} = -\gamma, \quad \lambda_{1} = \lambda_{2}^{*} = -\Gamma + i\Lambda;$$
  
$$\langle 0 | = (1, -(w/\gamma) \langle \Omega_{3} |), \quad |0\rangle = (1, 0, 0, 0)^{T};$$
  
$$\langle \lambda_{k} | = (0, \langle \Omega_{k} |), \quad |\lambda_{k}\rangle = \begin{pmatrix} (w/\gamma) \delta_{k3} \\ |\Omega_{k}\rangle \end{pmatrix},$$

k=3, 1, 2.

As a result we obtain

$$K(\tau) = \operatorname{const} \exp \left(-i\omega_{L}\tau\right) \left[ \left(\vartheta^{2}/2\Omega^{2}\right) \left(w^{2}/\gamma^{2}\right) + \left(\vartheta^{2}/2\Omega^{2}\right) \left(1-w^{2}/\gamma^{2}\right) \exp \left(-\gamma\tau\right) + \left[ \left(\Omega+\delta\right)^{2}/4\Omega^{2} \right] \left(1-w/\gamma\right) \exp \left[-\Gamma\tau+i(\Omega+\Lambda)\tau\right] + \left[ \left(\Omega-\delta\right)^{2}/4\Omega^{2} \right] \left(1+w/\gamma\right) \exp \left[-\Gamma\tau-i(\Omega+\Lambda)\tau\right].$$
(20)

The first term describes the coherent response at the frequency  $\omega_L$ , the second describes the unshifted spontaneous-scattering line with width  $\gamma$ , while the third and fourth describe the shifted spontaneous scattering lines with frequencies  $\omega_L \mp (\Omega + \Lambda)$  with identical width  $\Gamma$ . The coefficients preceding the exponentials characterize the total intensity of each component. The main difference between this structure and the corresponding structure of Refs. 1–3 is that the parameters  $\gamma$ ,  $\Gamma$ , and  $\Lambda$  described by (18) differ from the decay rates and the Lamb shift expressed in terms of  $\gamma_0$ ,  $w_0$ ,  $\Gamma_0$ ,  $\Lambda_0$  without allowance for their change. Taking into account (18) and (16) and using the fact that as  $\delta \to \pm \infty$  and at  $\hat{K}_{\parallel}(\pm \Omega) = \hat{K}_{\parallel}(0)$  we have  $\gamma \to \gamma_0$ ,  $\Gamma \to \Gamma_0$ ,  $w \to w_0$  sign  $\delta$ ,  $\Lambda \to \Lambda_0$  sign  $\delta$ , we obtain for these parameters the expressions

$$\gamma = \gamma_0 \left[ \delta^2 (1+\chi) + \Omega^2 (1-\chi) \right] / 2\Omega^2 + \Gamma_0 (\vartheta^2 / \Omega^2) \chi,$$
  

$$\Gamma = \Gamma_0 \left[ \Omega^2 \chi + \delta^2 (2-\chi) \right] / 2\Omega^2 + \gamma_0 (\vartheta^2 / 4\Omega^2) (3-\chi), \qquad (21)$$
  

$$w = w_0 \delta / \Omega + (\Gamma_0 - \gamma_0 / 2) (\vartheta^2 / \Omega^2) \mu,$$
  

$$\Lambda \approx \Lambda_0 \delta / \Omega.$$

(In the last formula we have discarded the contribution of the low-frequency noise, which does not permit  $\Lambda$  to be expressed directly in terms of  $\Lambda_{0}$ .) Here  $\chi$  and  $\mu$  are parameters that describe the change of the relaxation constants on ac-

count of suppression of the phase noise:

$$\chi = \operatorname{Re} \left[ \tilde{K}_{\parallel}(\Omega) + \tilde{K}_{\parallel}(-\Omega) \right] / 2 \tilde{K}_{\parallel}(0),$$

$$\mu = \operatorname{Re} \left[ \tilde{K}_{\parallel}(\Omega) - \tilde{K}_{\parallel}(-\Omega) \right] / \tilde{K}_{\parallel}(0).$$
(22)

It is not difficult to check on the agreement of the results obtained at  $\chi = 1$  and  $\mu = 0$  with the corresponding results<sup>2</sup> as  $\Omega \rightarrow \infty$  in particular, the agreement between  $W_r$  (17) and the effective relaxation matrix obtained by averaging the unperturbed one over the oscillations).

For two-level atoms with  $d_{11} - d_{22} \neq 0$ , broadened by collisions with classically charged particles for fields satisfying the relation  $\Omega_0 \leq \Omega \leq \Omega_W$ , where  $\Omega_0$  and  $\Omega_W$  correspond respectively to the upper and lower cutoff radii  $\rho_0$  and  $\rho_W$ , we obtain from (22) using formula  $\Omega_{\rho} = v/\rho$ 

$$\chi = \ln \left( \boldsymbol{\varOmega}_{\boldsymbol{W}} / \Omega \right) / \ln \left( \boldsymbol{\varOmega}_{\boldsymbol{W}} / \Omega_0 \right), \, \mu = 0.$$

In the limit of weak fields  $\Omega \leq \Omega_0$  we have  $\chi \approx 1$  and  $\chi = 0$  at  $\Omega \sim \Omega_W$ . Comparison of this result with Eq. (5.13) of Ref. 10 for the linewidth in the linear theory of hydrogen lines shows that the considered mechanism of nonlinear suppression of the broadening reduces simply to a decrease of the upper cutoff radius  $\rho_{\omega} = v/\delta$  (which appears in place of  $\rho_0$  at large  $\delta$ ) as a result of replacing  $\delta$  by the total frequency  $\Omega$  of the quantum oscillations. When  $\Omega$  increases to values of the order of  $\Omega_W \approx \tau_c^{-1}$  the fraction of the diffusion broadening drops to zero, and the suppression, with further increase of the field, of the jumplike component of the broadening by close collisions is described outside the framework of perturbation theory.<sup>9</sup>

Expressions (22) show that the effective intensity of the phase noise is determined not directly by the intensity of the external field, i.e., not by the Rabi frequency  $\vartheta$ , but by the frequency  $\Omega$  of the quantum oscillations. It can decrease not only on account of the increase of the field intensity, but also on account of the large detuning. The latter possibility, however, is quite limited because according to (21) as  $\delta \rightarrow \infty$  we have  $\Gamma \rightarrow \Gamma^0$ , which does not depend on  $\chi$  and  $\mu$ , just as all the remaining parameters. The reason is that as  $\delta \rightarrow \infty$  the relaxation parameters, as follows from (18) and (16), are influenced only by the spectral density  $K_{\parallel}(0)$  of the phase noise at zero frequency, which leads to a difference between  $\Gamma_0$ and  $\gamma_0/2$ : the quantum oscillations with frequency  $\Omega$ , on the other hand, are inessential because their amplitude vanishes. Therefore the suppression effects are most strongly pronounced at small detunings  $\delta \leq \vartheta$ .

#### 4. SOME PHYSICAL CONSEQUENCES

The frequency spectrum corresponding to the correlation function (20) at zero detuning  $\delta = 0$  in a weak field has an unshifted peak width  $\gamma_1 = \Gamma_0$  and side band widths  $\Gamma_1 = (\Gamma_0 + \gamma_0)/2$ . In a field strong enough to have  $\chi = 0$ , however, we have

$$\gamma_2 = \gamma_0/2, \quad \Gamma_2 = (3/4) \gamma_0.$$

This means total suppression of the phase relaxation at zero detuning: it is possible because at  $\delta = 0$  the quantity  $K_{\parallel}(0)$  does not enter in the expression for  $\Gamma$ , i.e., at exact resonance

the quasistatic phase breakdown does not lead to damping, and only phase changes having the Rabi frequency  $\vartheta = \Omega$ are significant. If  $\Gamma_0 > \gamma_0/2$ , the lines become drastically narrower in order of magnitude. These conclusions and the structure of the dependence of the relaxation constants on  $\Omega$ are analogous in general outline to the case of the jumplike relaxation,<sup>9</sup> for which, however, since perturbation theory does not hold, it is impossible to obtain general analytic solutions in the general case.

At  $\mu = 0$  and  $\delta = 0$  we have in the considered approximation w = 0, i.e., the intensity of the coherent component is zero and it is strongly suppressed: its residual intensity at large  $\Omega / \Gamma_0$  is not taken into account here, and the saturation is complete. At  $\mu \neq 0$  we have  $w = (\Gamma_0 - \gamma_0/2)\mu$ : a nonzero intensity  $\sim w^2/2\gamma^2$  appears even at exact resonance. This leads simultaneously to a difference by an amount  $\sim w/2\gamma$  in the intensities of the sideband components. The physical meaning of this effect consists in a quantum character of the interaction of the atom with the low-frequency noise at the frequency  $\Omega$ . The difference between  $\tilde{K}_{\parallel}(\Omega)$  and  $\tilde{K}_{\parallel}(-\Omega)$  in the expression for  $\mu$  is connected with the sustatial value of the quantum energy relative to the spectral noise energy  $\hbar\Omega$ at this frequency: for classical  $\xi_3$  we always have  $\mu = 0$ . The difference between  $\tilde{K}_{\parallel}(\Omega)$  and  $\tilde{K}_{\parallel}(-\Omega)$  is due to the fact that the absorption and quantum loss are not on a par. The natural estimate  $\mu \sim \hbar \Omega / kT$  leads to the estimate  $w/\gamma \sim (\Gamma_0/$  $\gamma_0 (\hbar \Omega / kT)$  for the asymmetry of the intensity of the sideband components and  $w^2/\gamma^2$  for the intensity of the coherent component. For sufficiently strong fields  $(\Omega \sim 10^{10} \text{ sec}^{-1})$ , even at room temperatures, one can expect already discernible manifestations of this effect.

The author takes the opportunity to thank S. I. Yakovlenko for a discussion of the work and for a number of helpful remarks.

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Translated by J. G. Adashko