Enhancement of free-electron-laser gain by means of an external laser

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The feasibility of transforming the frequency of coherent radiation by means of high-energy electron beams is discussed. A free-electron laser whose principal cavity axis is collinear with the electron beam is considered. It is shown that the gain at the lasing frequency grows in proportion to the intensity of a laser beam having half the frequency and directed at an angle to the principal-cavity axis.

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1. INTRODUCTION

Following the experimental demonstration, by the Stanford group,[1,2] that coherent radiation can be generated by using high-energy electron beams and spatially periodic magnetic fields, many theoretical papers were published (see, e.g., the reviews[3,4]) devoted to free-electron lasers (FEL). An intensive search is under way for possibilities of enhancing the gain and the electron-energy conversion efficiency. Success in solving these concrete problems will determine whether the FEL will become as widely used as ordinary lasers or will be used only in those exceptional cases when the technique of ordinary lasers provides no alternative.

The gain of the previously considered FEL systems is limited by the energy scatter of the electron beam and cannot be increased above this limit. The known reason[3,4] is that the gain is determined by the electron-beam spontaneous linewidth, which cannot be less than the relative width of the electron energy spectrum. This pertains, in particular, to systems such as the optical klystron,[5,6] in which the gain over a given electron path length in the magnetic deflecting system can be increased, but not above the limit determined by the energy spread of the electron beam. In this paper is analyzed an FEL system that permits a substantial enhancement of the radiation gain of the same electron beam by additional bunching the electrons in the field of an external-laser beam.

2. SCHEMATIC DIAGRAM OF FEL SYSTEM WITH FORCED MODULATION OF THE ELECTRON BEAM

Just as in an ordinary FEL, the electron beam passes along an undulator axis (z axis in Fig. 1). The undulator is assumed to be three-dimensional, deflecting the electron along the x axis with a spatial period λ_{w2} and along the y axis with a period λ_{w1}. Under certain conditions it is possible to use also an ordinary helical undulator (cf. Ref. 3), for which $\lambda_{w1} = \lambda_{w2}$. A plane polarized generated-radiation beam is passed along the undulator axis. The length λ_s of this radiation should correspond to the conditions of resonant interaction with the electron oscillations along the y axis:

$$\lambda_s = \lambda_{w1}/2\gamma_z^2\beta_z, \qquad (1)$$

where β_z is the electron velocity component along the electron-beam axis in units of the speed of light c, and $\gamma_z = (1-\beta_z^2)^{-1/2}$ is the relativistic factor and determines the Doppler frequency shift of the radiation emitted along the undulator axis.

We note that condition (1) is the same as in an ordinary FEL for the resonant value γ_{rz}. It is assumed also that the energy spread is small enough and satisfies the lasing conditions at the wavelength λ_s, i.e.,

$$\Delta\gamma/\gamma < 1/2N_1, \qquad (2)$$

where γ is the relativistic factor for the total velocity and N_1 is the number of undulator periods over the total length of the undulator $L = N_1\lambda_{w1}$. This part of the system does not differ from the traditional FEL and, at sufficient gain, could ensure lasing at a wavelength λ_s in the cavity.

The new element is the presence of a second radiation beam propagating at a small angle α to the z axis (Fig. 1). We assume that the wavelength λ_α of this radiation is chosen such as to ensure resonant interaction with the electron oscillations along the x axis. At small values of the angle $\alpha \ll 1$

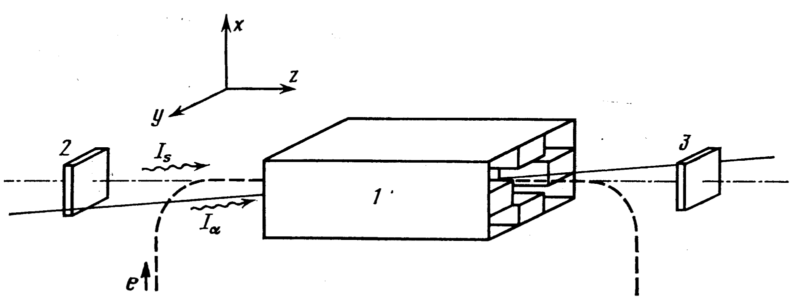

FIG. 1. Schematic diagram of free-electron laser frequency converter: 1—undulator, 2, 3—cavity mirrors, e—electron beam.

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the corresponding synchronism condition can be written in the form[4,7]

$$\lambda_\alpha=\lambda_{w2}(1+\alpha^2\gamma^2)/2\gamma_z\beta_z. \quad (3)$$

The presence of an intense radiation beam λ_α leads to modulation of the electron density with a spatial period λ_α by virtue of the usual mechanism of resonant (quasiresonant) interaction of the electrons with the field of the undulator λ_{w2} and with the field of the wave (see, e.g., Ref. 3). This modulation can affect also the generation of radiation λ_s at zero angle to the electron-beam axis, if the relation $\lambda_\alpha = k\lambda_s$ is maintained, where k is an integer. We can thus expect enhancement at the wavelength λ_s as a result of the coherence of the radiation of the density-modulated electron beam. We shall investigate below in detail the case of the harmonic $k = 2$, i.e., the case

$$\lambda_\alpha=2\lambda_s. \quad (4)$$

3. DETERMINATION OF THE GAIN FROM THE ELECTRON EQUATIONS OF MOTION

To determine the gain at the wavelength λ_s in the considered system it is necessary to solve the generally speaking complicated problem of self-consistent motion of electrons in the fields of two waves and of the undulator. For a qualitative demonstration of the feasibility, in principle, of the proposed frequency-conversion scheme we simplify the problem, assuming the wave intensity to be bounded. It is then correct to use a linear approximation in the field E_s and a quadratic approximation in the field E_α. It is assumed that these fields satisfy the inequality $I_\alpha \gg I_s$. We assume satisfaction of the condition for the applicability of the classical approach to the description of the fields and of their interaction with the electrons[3]:

$$I_s \gg \pi^{1/2}\hbar c^2/2\lambda_s^4\gamma^2.$$

We shall assume also that the intensity I_S exceeds the intensity of the electron-beam spontaneous emission in the undulator field, and the latter emission can be neglected when the electron motion is considered.

As already noted, both waves are plane-polarized and can be represented in the form

$$\mathbf{E}_s=-\mathbf{E}_{s0}\mathbf{e}_y\cos\left(\frac{2\pi}{\lambda_s}z-\omega_s t+\psi_0\right),\quad \mathbf{H}_s=[\mathbf{e}_z\times\mathbf{E}_s], \quad (5)$$

$$\mathbf{E}_\alpha=-\mathbf{E}_{\alpha0}\mathbf{e}_x\sin\left[\frac{2\pi}{\lambda_\alpha}(z\cos\alpha+y\sin\alpha)-\omega_\alpha t+\Phi_0\right], \quad (6)$$
$$\mathbf{H}_\alpha=[\mathbf{e}_z\times\mathbf{E}_\alpha]+[\mathbf{e}_y\times\mathbf{E}_\alpha]\sin\alpha=[(\mathbf{e}_z\cos\alpha+\mathbf{e}_y\sin\alpha)\times\mathbf{E}_\alpha],$$

where $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are unit vectors along the axes x, y, and z, respectively; $\omega_s = 2\pi c/\lambda_s$ and $\omega_\alpha = 2\pi c/\lambda_\alpha$ are the field frequencies; Φ_0 and ψ_0 are the initial phases.

It is known[3] that at the customarily employed undulator magnetic-field intensities and at limited radiation-field intensities the electron trajectory is determined mainly by the undulator field and depends little on the radiation fields. We assume on this basis that the velocity of the electrons passing through the undulator of the setup depends on their velocity as follows:

$$\beta(z)=\beta_z\mathbf{e}_z+\mathbf{e}_x\beta_{x0}\cos\frac{2\pi z}{\lambda_{w2}}+\mathbf{e}_y\beta_{y0}\sin\frac{2\pi z}{\lambda_{w1}}, \quad (7)$$

where $\beta_{x0} = K_2/\gamma$ and $\beta_{y0} = K_1/\gamma$ are the amplitudes of the transverse components of the electron velocity and are determined by the undulator fields. The constants K_1 and K_2 are the normalized amplitudes of the undulator magnetic field:

$$K_1=\frac{|e|H_{w1}\lambda_{w1}}{2\pi mc^2}.\quad K_2=\frac{|e|H_{w2}\lambda_{w2}}{2\pi mc^2}. \quad (8)$$

We note that the longitudinal velocity β_z depends in general on z. The mean longitudinal velocity at relatively small deviations $K_1 \ll 1$ and $K_2 \ll 1$ is equal to

$$\beta_z\approx\beta[1-(K_1^2+K_2^2)/4\gamma^2\beta^2].$$

Since the field E_s is assumed to be weaker than E_α, it can be disregarded when the modulation effect in the field E_α is determined. It can be assumed in this case that the electron motion is described by the synchrotron-oscillation equations, as in ordinary FEL (Ref. 3):

$$d\mu/d\tau=-\Omega_\tau^2\sin(\Phi_0+\Phi),\quad d\Phi/d\tau=\mu. \quad (9)$$

Here $\Phi_+\Phi_0$ is the relative phase shift of the wave E_α and the electron:

$$\Phi=2\pi z(\lambda_{w2}^{-1}+\lambda_\alpha^{-1}\cos\alpha)-\omega_\alpha t, \quad (10)$$
$$\Phi_0=\varphi_0+\varphi(y), \quad (11)$$

where $\varphi(y)$ is the relative initial phase and depends on the coordinate y (we recall that the electron beam and the radiation beam E_α are not collinear); τ is the relative time:

$$\tau=ct/N_2\lambda_{w2}, \quad (12)$$

N_2 is the number of periods of length λ_{w2} in the undulator; μ is the electron energy mismatch over the field E_α:

$$\mu=4\pi N_2\frac{\gamma-\gamma_{r2}}{\gamma}\left[1-\frac{\lambda_{w2}}{\lambda_\alpha}(1-\cos\alpha)\right]; \quad (13)$$

γ_{r2} is the resonant electron energy and satisfies the synchronism condition

$$\lambda_\alpha=\frac{\lambda_{w2}}{2\gamma_{r2}^2}\left[1+\frac{1}{2}(K_1^2+K_2^2)+2(1-\cos\alpha)\gamma_{r2}^2\right]; \quad (14)$$

Ω_τ is the synchrotron-oscillation frequency:

$$\Omega_\tau^2=2\pi\frac{|e|E_{\alpha0}\lambda_{w2}K_2N_2^2}{mc^2\gamma_{r2}\gamma}\left[1-\frac{\lambda_{w2}}{\lambda_\alpha}(1-\cos\alpha)\right]. \quad (15)$$

We note that expressions (9)–(15) differ from those given in Ref. 3, since they pertain to the case of noncollinearity of the electron-beam axis and the wave vector of the wave ($\alpha\neq0$).

To simplify the calculations we assume next that the field E_α and the mismatch μ are limited by the condition

$$(\Omega_\tau/2\pi N_2)^2 \ll \mu/4\pi N_2 \ll 1. \quad (16)$$

In this case the solution of the system (9) is represented in the form of a series in powers of Ω_τ^2:

$$\Phi = \Phi^{(1)} + \Phi^{(2)} + \Phi^{(3)} + \dots, \quad (17)$$

where

$$\Phi^{(1)} = \mu_0\tau,$$

$$\Phi^{(2)} = \frac{1}{\mu_0^3}\Omega_\tau^2\left[\tau\cos\Phi_0 - \frac{1}{\mu_0}\sin(\mu_0\tau+\Phi_0) + \frac{1}{\mu_0}\sin\Phi_0\right],$$

$$\Phi^{(3)} = \frac{1}{\mu_0^3}\Omega_\tau^4\left[-\tau\cos\Phi_0\cos(\mu_0\tau+\Phi_0) + \frac{2}{\mu_0}\cos\Phi_0\sin(\mu_0\tau+\Phi_0) - \frac{5}{8\mu_0}\sin 2\Phi_0 + \frac{1}{8\mu_0}\sin(2\mu_0\tau+2\Phi_0) - \frac{\tau}{4}\cos 2\Phi_0 - \frac{1}{\mu_0}\sin\Phi_0\cos(\mu_0\tau+\Phi_0) - \tau\right].$$

Here μ_0 is the initial value of the mismatch (at $\tau = 0$):

$$\mu_0 = \frac{4\pi N_2}{\gamma_0}(\gamma_0 - \gamma_{r2})\left[1 - \frac{\lambda_{w2}}{\lambda_\alpha}(1-\cos\alpha)\right]. \quad (18)$$

From the solution (17) and the definition (10) we easily find the $z(t)$ dependence, which is determined by the interaction with the field E_α:

$$z(t) = c\beta_z t + \frac{1}{2\pi}\left(\frac{1}{\lambda_{w2}} + \frac{\cos\alpha}{\lambda_\alpha}\right)^{-1}(\Phi^{(2)} + \Phi^{(3)}). \quad (19)$$

We now take into account the interaction of a phased beam of electrons with the generated wave (5). This interaction changes the wave intensity, which can be determined in terms of the work of the electron current in the field of the wave (5) with allowance for the phasing (19). For the contribution of one electron we can write

$$mc^2 d\gamma/dt = |e|c\beta E_s, \quad (20)$$

where β is determined by the function (7) with $z(t)$ in the form of the solution (19) in the assumed approximation. Retaining in (20) the terms that oscillate weakly under the synchronism condition, we rewrite this equation in the more explicit form

$$mc^2\frac{d\gamma}{dt} = \frac{c|e|E_{s0}K_1}{2\gamma}\sin\psi, \quad (21)$$

where ψ is the relative phase between the field E_s and the electron:

$$\psi = 2\pi z\left(\frac{1}{\lambda_s} + \frac{1}{\lambda_{w1}}\right) - \omega_s t + \psi_0, \quad (22)$$

and ψ_0 is the initial phase.

We use next the fact that the phase $\Phi^{(2)} + \Phi^{(3)}$ can be regarded as a correction to the principal phase [see (19)]. This gives grounds for reducing (21) to the form

$$mc^2\frac{d\gamma}{dt} = \frac{c|e|E_{s0}K_1}{2\gamma}[\sin(\psi_0+\psi^{(1)}) + (\psi^{(2)}+\psi^{(3)})\cos(\psi^{(1)}+\psi_0)]. \quad (23)$$

This equation can be integrated without additional physical assumption and in quite trivial a manner. After integration with respect to time from zero to $t_{\max} = N_1\lambda_{w1}/\beta_{z0}c$, the result should be averaged over the initial phases in accord with the fact that the electrons are uniformly distributed over the time of entry into the undulator (the beam is not bunched beforehand).

We recall that the employed solution (7) is valid under the initial conditions $t = 0$, $z = 0$, $\Phi = 0$, $\psi = \psi_0$. The solutions for different electrons that do not enter the undulator simultaneously will have different phases Φ_0 and ψ_0. Averaging over these phases will mean averaging over the time of entry of the electrons into the undulator. It must be taken into account here that the connection between the changes of the initial phases should be the same as between the frequencies of the considered waves, in accord with Eq. (4). Without loss of generality it can be assumed that the relative phase $\varphi(y)$ [see (11)] pertains to the plane $z = 0$, and this connection can be written in the form

$$\psi_0 = 2\Phi_0 - 2\varphi(y). \quad (24)$$

As a result, averaging over the electron entry time into the undulator reduces to integration with respect to one of the phases.

It can be seen from the structure of (23) that the first term in the square brackets vanishes on averaging over the phase ψ_0 between the limits 0 and 4π. The second term in the square brackets of (23) can be represented in the form

$$(\Phi^{(2)}+\Phi^{(3)})\cos[\psi^{(1)}+2\Phi_0-2\varphi(y)] \times (\lambda_{w1}^{-1}+\lambda_s^{-1})(\lambda_{w2}^{-1}+\lambda_\alpha^{-1}\cos\alpha)^{-1}.$$

Since the function $\Phi^{(2)}$ contains only odd powers of functions harmonic in Φ_0, the term containing $\Phi^{(2)}$ also vanishes after averaging over Φ_0. The problem reduces thus to averaging the function $2\Phi^{(3)}\cos[\psi^{(1)} + 2\Phi_0 - 2\varphi(y)]$. The concrete result of this averaging depends in the general case both on the initial mismatch (18) with respect to the field E_α and on the mismatch with respect to the field E_s, the latter being

$$\nu_0 = 4\pi N_1(\gamma_0 - \gamma_{r1})/\gamma_0. \quad (25)$$

Here γ_{r1} is the resonant electron energy that satisfies the synchronism condition (1), which can be represented in the form

$$\lambda_s = \frac{\lambda_{w1}}{2\gamma_{r1}^2}\left[1 + \frac{1}{2}(K_1^2+K_2^2)\right]. \quad (26)$$

Integration of (23) with respect to τ from 0 to 1 and averaging over Φ_0 from 0 to 2π leads to the following expression for the radiation-energy increment per electron at the wavelength λ_s:

$$mc^2\Delta\gamma = \frac{|e|E_{s0}K_1N_2\lambda_{w2}}{16\gamma\cos\alpha}\left(\frac{\Omega_\tau^4}{\mu_0^3}\right)\mathscr{F}(\mu_0,\nu_0,\varphi). \quad (27)$$

Here $\mu_0^{-3}\mathscr{F}(\mu_0,\nu_0,\varphi)$ is the gain function:

$$\mathscr{F}(\mu_0,\nu_0,\varphi)=\cos 2\varphi\left[\frac{\sin(\nu_0-\mu_0)}{\nu_0-\mu_0}-\frac{(\nu_0-2\mu_0)}{\mu_0}\frac{\cos(\nu_0-\mu_0)-1}{(\nu_0-\mu_0)^2}\right.$$
$$\left.+\left(\frac{5}{4\mu_0\nu_0}+\frac{1}{2\nu_0^2}\right)(\cos\nu_0-1)-\frac{\cos(\nu_0-2\mu_0)-1}{4\mu_0(\nu_0-2\mu_0)}+\frac{\sin\nu_0}{2\nu_0}\right]$$
$$+\sin 2\varphi\left[-\frac{\cos(\nu_0-\mu_0)}{\nu_0-\mu_0}-\frac{(\nu_0-2\mu_0)}{\mu_0}\sin(\nu_0-\mu_0)\right.$$
$$\left.+\left(\frac{5}{4\mu_0\nu_0}+\frac{1}{2\nu_0^2}\right)\sin\nu_0-\frac{\sin(\nu_0-2\mu_0)}{4\mu_0(\nu_0-2\mu_0)}-\frac{1}{2\nu_0}\cos\nu_0\right]. \tag{28}$$

We recall that Ω_τ is defined by (15), and the mismatches μ_0 and ν_0 by (18) and (25) respectively.

Actually (27) is the sought solution and yields the increment of the generated-wave intensity I_s within one pass through the undulator in the presence of a modulating field of intensity I_α and half the frequency of the generated radiation. Since the resultant expression is unwieldy, we continue the analysis for the particular case when the synchronism conditions are satisfied simultaneously for both fields, i.e., $\gamma_{r1}=\gamma_{r2}=\gamma_r$.

4. ANALYSIS OF THE GAIN FUNCTION

We consider the particular case when the mismatches are connected by the relation

$$\nu_0=2\mu_0. \tag{29}$$

In this case (13) and (15) cease to depend on the angle α, in view of the equality

$$1-\lambda_{w2}/\lambda_\alpha(1-\cos\alpha)=\lambda_{w2}/2\lambda_{w1}.$$

As a result, (28) becomes a function of one variable

$$\mathscr{F}(\mu_0,\nu_0,\varphi)/\mu_0^3=F(\mu_0,\varphi)/\mu_0^5,$$
$$F(\mu_0,\varphi)=2\sin(\mu_0-2\varphi)(3\sin\mu_0-\mu_0\cos 2\mu_0-2\mu_0). \tag{30}$$

Normalizing the wave-energy increment to the wave flux sI_s and summing the contribution $i(y)/e$ from the beam electrons, we obtain the gain G_s, equal to the relative increment of the wave intensity per pass through the undulator:

$$G_s=\pi^4\frac{I_\alpha\lambda_{w2}}{(I_sW_0)^{1/2}}\frac{\lambda_{w1}^2 i(y)K_1K_2^2}{si_A\gamma^5}N_1^4N_2\frac{1}{\mu_0^5}F(\mu_0,\varphi), \tag{31}$$

where I_α is the intensity of the wave of the modulating field at the wavelength $\lambda_\alpha=2\lambda_s$; λ_s, is the wavelength of the generated field; $i(y)$ is the current of a beam of sufficiently small size along the y axis (see below); $s=\max\{s_e,s\}$,s_e is the maximum electron-beam cross-section area; s is the cross section area of the generated-radiation beam; $i_A=mc^3/e$ is the Alfven current; K_1 and K_2 are the undulator constants (8) with respective periods λ_{w1} and λ_{w2} and with N_1 and N_2 periods, respectively, over the undulator length; $W_0=(c/8\pi)(2\pi mc^2/e)^2=1.19\cdot10^{10}$ W is a numerical constant equal to the intensity of a field whose intensity is equal to the amplitude of the undulator field at $K_2=1$. We note that the gain function (30) depends on the relative phase between the fields (5) and (6), which depends in turn on y. To determine the effective gain of the wave in an electron beam of finite size along the y axis, expression (31) must be averaged over the coordinate y with allowance for the corresponding dependences of the electron current density, of the wave intensity I_s, and of the gain function (30) on y.

The increase of the gain on account of the modulation of the electron density in the field E_α can be described by the ratio of the gain (31) to the gain of the wave E_s in the usual FEL regime with the same undulator and at the same current. According to Ref. 3 the gain of a plane-parallel wave in a planar undulator with reduced field amplitude K_1 is given by

$$G=4\pi^2\frac{\lambda_{w1}^2 i}{si_A}\frac{K_1^2}{\gamma^3}N_1^3\frac{1}{\mu_0^3}f(\mu_0), \tag{32}$$

where $f(\mu_0)/\mu_0^3$ is the normalized gain function of the FEL:

$$f(\mu_0)=\cos\mu_0-1+\frac{\mu_0}{2}\sin\mu_0. \tag{33}$$

It can be seen from a comparison of (31) and (32) that the field E_α increases the gain by $\varkappa$ times, where

$$\varkappa=\frac{\pi^2}{4}\frac{I_\alpha\lambda_{w2}K_2}{(I_sW_0)^{1/2}K_1\gamma^2}N_1N_2\frac{F(\mu_0,\varphi)/\mu_0^5}{f(\mu_0)/\mu_0^3}. \tag{34}$$

The unusual dependence, $\propto I_s^{-1/2}$, of the wave gain (31) on its intensity should be no surprise. In contrast to the ordinary FEL, here the electron beam is modulated by an external field, so that the absolute increment of the generated-radiation intensity differs from zero even in first order in the field E_s (27). I_s appears in the denominator of the derived relative gain (31) only as a result of the normalization of this absolute increment. At the same time, such a dependence is evidence of the possibility of obtaining large gains (31) at $I_\alpha \gg I_s$. The lower bound of I_s is imposed only by the condition considered above that the classical description of the field E_s be applicable. According to (31), therefore, it is possible formally to obtain values $G>1$, and this result is not contradictory. A large gain leads, naturally, to a broadening of the emission spectral line, but this broadening will not decrease the gain. In contrast to the ordinary FEL, the gain (31) does not depend on the electron emission spectrum in the region of the wavelength λ_s, since the electron density is modulated by the second field. At large values of the gain, the result (31) ceases to be valid for another reason: it was derived in the given-field approximation. At large gains it would be necessary to take into account the change of the field E_s upon integration of the expressions (21) and (22). This, however, would lead generally speaking to even larger gains per undulator pass.

A general idea of the gain function (30) can be obtained from the curves in Fig. 2. It can be seen that, just as in the case of the FEL gain function (33) (see Ref. 3), the dependence on the mismatch is resonant. The gain curves (30) are not much narrower than the curves (33), and it can therefore be assumed that the requirements that the electron beam be monochromatic remain practically the same as in the case of ordinary FEL. Without going into a detailed analysis, we note that in the general case the gain function (28) contains an additional degree of freedom connected with the parameter ν_0. This makes possible the choice of even more conven-

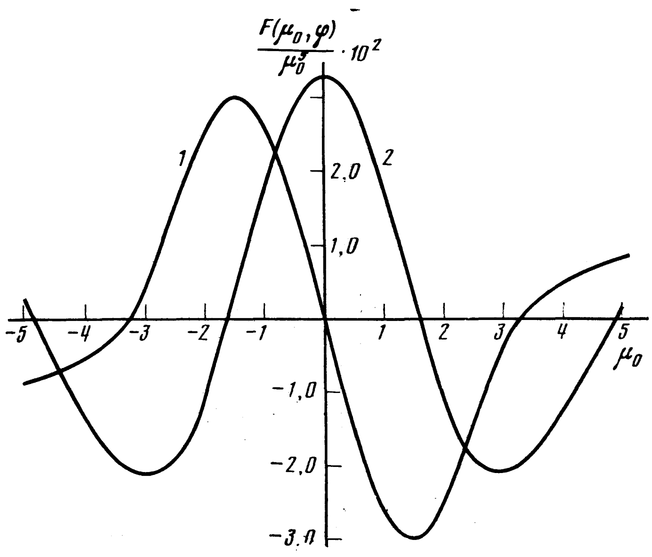

FIG. 2. The gain function $F(\mu_0,\varphi)/\mu_0^5$ [see (30) vs the mismatch μ_0 (18) at various values of the phase shift between the modulating and the generating waves]: $1 - \varphi = 0$; $2 - \varphi = \pi/4$.

ient regimes. In particular, a variant is possible with zero mismatch of the field E_s ($\mu_0 = 0$). In this case the mean value of the field intensity I_α should not be changed by interaction with the electron beam.

The curves of Fig. 2 illustrate also the dependence of the gain of the phase difference $\varphi(y)$ between the waves. In the scheme considered, the phase shift is not strictly fixed, so that when the wave E_s is generated by a sufficiently narrow electron beam the field phase will assume the value corresponding to the maximum gain. From the form of the function (30) and from Fig. 2 it follows that the condition (31) hardly requires averaging over the coordinate y if the beam size in the y direction is small enough and satisfies the inequality

$$\Delta y_e < (\pi/2)(\lambda_\alpha/\alpha). \tag{35}$$

For relatively wide beams, the requirement (35) necessitates a choice of small values of the angle α. We note that it is also possible to set formally $\alpha = 0$. The ensuing difficulties are purely practical and due to the need for establishing collinearity of the generated-radiation cavity and the intense wave of half the frequency.

The maximum value of the gain function (30) (see Fig. 2) differs from the maximum value of the gain function (33) by a coefficient ≈ 2. To estimate $\varkappa$ we can therefore use the simplified expression

$$\varkappa = \frac{\pi^2}{2}\frac{I_\alpha\lambda_w}{(I_s W_0)^{1/2}}\frac{N^2}{\gamma^2}, \tag{36}$$

in which we put for simplicity also $K_1 = K_2$ and $N_1 = N_2 = N$. As typical values (cf. Ref. 8) one can assume $\gamma/N = 2\lambda_w = 6$ cm and $I_s = 10^{-3}$ W/cm^2. Substitution of these values in (36) leads to $\varkappa > 2$ in the case $I_\alpha = 10^3$ W/cm^2. At $I_\alpha = 10^8$ W/cm^2 we have $\varkappa > 20$ all the way to values of I_s comparable with $I_\alpha \cdot 10^{-3}$. Obviously, the considered gain-enhancement mechanism can be particularly effective in pulsed FEL (cf. Ref. 8), for in this case the time of amplification of individual radiation pulsed is limited and during a considerable part of this time the intensity I_s remains at a low level. It was shown that it is precisely under these conditions that large enhancement of the gain ($\varkappa \gg 1$) can be achieved. We recall (see Ref. 8) that in the pulsed regime the FEL emission spectrum constitutes a set of individual lines with widths of the order of $\delta\omega_s = 2\pi/\tau$ in a lasing band $\Delta\omega_s = 2\pi/T_{\min}$, where τ is the duration of the current macropulse ($\sim$msec), and $T_{\min}$ is the duration of the electron-current micropulses ($\sim 10^{-10}$–10^{-11} sec). It is assumed here that the width of the cavity modes is less than $\delta\omega_s$. In the considered pulsed frequency-conversion regime the spectrum of the generated radiation retains the same character if the following two additional conditions are satisfied: the modulating field E_α must be monochromatic enough (with linewidth not larger than $\delta\omega_s$); the configurations of the fields E_α and E_s inside the electron beam must be similar enough for the phase difference of these waves to be determined only by the corresponding geometric ray path difference, accurate to $\varphi(x,y,z) < \pi/2$. When account is taken of the temporal structure of the electron beam, the last condition is equivalent to the condition that the radiation be generated in the form of individual spectral lines of width $\delta\omega_s$ (see above) in the lasing band $\Delta\omega_s$. This width of the lasing band remains unchanged up to gain values Gs of the order of $2L_{\text{cav}}/cT_{\min} \gg 1$, where L_{cav} is the cavity length.

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