

# Suppression of mode interaction in gas lasers

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The interaction of two perpendicularly polarized modes with identical transverse structure, separated in space in a direction transverse to the laser axis, is investigated theoretically and experimentally for the first time. It is shown that the interaction between such modes can be arbitrarily weak by varying the indicated space shift. The proposed method of weakening the interaction of the mode is the most universal and can be used for all gas lasers.

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## INTRODUCTION

Two-mode gas lasers with adjustable coupling between the modes are gaining in use both in scientific research and in applications.<sup>1,2</sup> Two-mode lasers employed in precision laser spectroscopy,<sup>3</sup> in quantum frequency standards, in instruments for the measurements for the frequency characteristics of photodetectors, for plasma diagnostics, etc., has made it possible to decrease considerably the experimental apparatus size and increase the sensitivity and the resolution of the corresponding devices compared with the use of single-mode lasers.

Earlier investigations have shown that particular interest, among lasers with controlled coupling between the modes, attaches to lasers in which it is possible, by weakening the mode interaction, to tune continuously the intermode distance  $\omega_{12}$  in a range practically from zero to  $c/2L$  ( $c$  is the speed of light and  $L$  is the cavity length) with a large range of two-mode-lasing stability. However, the increase of the nonlinear interaction of the modes with decreasing  $\omega_{12}$  leads to a competing suppression of one of them if the frequency splitting of the natural modes of the cavity becomes less than a certain quantity, called the critical frequency  $\omega_{cr}$ . The intermode interaction is determined to a considerable degree both by the polarization of the generated modes and by the quantum numbers of the total angular momenta  $j_a$  and  $j_b$  of the lower and upper levels of the laser transition, respectively. By now there are two known methods of weakening the mode interaction: 1) the use of a transverse magnetic field<sup>4</sup>; 2) spatial relative shift of the standing waves of the generated modes<sup>5</sup>, in the region of the active medium along the laser axis.

Aimed at achieving one purpose, weakening the interaction of the modes, these methods have a substantial shortcoming. They can be used only for specific laser transitions and not to solve the problem for any type of gas laser.

In the present paper we report, for the first time ever, a new method of weakening intermode interaction. This method, in contrast to the previously known, can be used in any gas laser. The gist of the proposed method is to shift the generating-mode fields apart in a direction perpendicular to the laser axis. The method was experimentally verified in an

He-Ne laser on the  $3s_2-3p_4$  transition of Ne. The results agree well with the calculated data.

## 1. CALCULATION OF THE POLARIZATION OF THE MEDIUM

We write down the two-mode laser field in the form

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^{(1)}(\mathbf{r}, t) + \mathbf{E}^{(2)}(\mathbf{r}, t), \quad (1)$$

where  $\mathbf{E}^{(1)}(\mathbf{r}, t)$  and  $\mathbf{E}^{(2)}(\mathbf{r}, t)$  are the electric field intensities of the first and second modes, respectively. Since we are interested in the transverse distribution of the intensity of the generated mode, the plane-wave approximation customarily used for the calculations will not do in this case. We use the Gaussian-mode approximation, which reflects more correctly the distribution of the electromagnetic field of the fundamental mode in a gas laser.<sup>6</sup> Denoting by  $z$  the direction along the laser axis, we represent  $\mathbf{E}^{(1)}$  in the form

$$\mathbf{E}^{(1)}(\mathbf{r}, t) = \mathbf{e}_1 E_1 \left\{ \frac{\exp(ik_1 z)}{2il_+(z)} \exp\left[-\frac{x^2+y^2}{a_0^2 l_+(z)}\right] + \text{c.c.} \right\} \times \exp[i(\omega_1 t + \varphi_1)] + \text{c.c.}, \quad (2)$$

where  $\mathbf{E}_1$  and  $\varphi_1$  are the slowly varying amplitude and phase of the first mode and  $\mathbf{e}_1$  is the polarization unit vector. The functions  $l_{\pm}(z)$  characterize the change of the transverse dimensions of the mode and of the curvature of the wavefront along the laser axis.

$$l_{\pm}(z) = 1 \pm 2i(z - z_1)/k_1 a_0^2. \quad (3)$$

The coordinate  $z_1$  determines the position of the neck of the mode, where its transverse dimension is minimal and equal to  $a_0$ .

When writing down the expression for  $\mathbf{E}^{(2)}$  we assume that devices placed in the cavity can ensure, relative to  $\mathbf{E}^{(1)}$ , both a spatial shift along the laser axis and a transverse shift of the direction of the center of the Gaussian distribution

$$\mathbf{E}^{(2)}(\mathbf{r}, t) = \mathbf{e}_2 E_2 \left\{ \frac{\exp[i(k_2 z - \delta)]}{2il_+(z)} \exp\left[-\frac{(x-x_0)^2 + (y-y_0)^2}{a_0^2 l_+(z)}\right] + \text{c.c.} \right\} \exp[i(\omega_2 t + \varphi_2)] + \text{c.c.} \quad (4)$$

The quantity  $r_0 = (x_0^2 + y_0^2)^{1/2}$  characterizes the spatial transverse distribution of the modes. In (4) it was assumed for simplicity that the position of the neck of the second mode is also at  $z_1$  and its transverse dimension is determined by the same quantity  $a_0$ . In expression (3) for  $l_{\pm}(z)$  we must put  $k_1 \approx k_2 \approx k = \omega_0/c$ , where  $\omega_0$  is the central frequency of the considered transition and  $c$  is the speed of light. The plane-wave approximation in (2) and (4) is obtained by taking the limit as  $a_0 \rightarrow \infty$ .

Expressions (2) and (4) were used to calculate the polarization of the gas medium accurate to terms  $E^3$  by the method described in ref. 7. We disregarded in the calculation effects connected with the small parameters  $\lambda/a_0$ ,  $\lambda/l$ , and  $u/a_0\gamma$ , where  $\lambda$  is the radiation wavelength of the considered transition,  $l$  is the length of the active medium,  $u$  is the thermal velocity of the motion of the atoms, and  $\gamma$  is the homogeneous half-width of the radiation spectral line. These parameters are in fact small in all gas lasers. Finally, it was assumed that the diameter of the laser gas-discharge tube exceeds considerably the transverse dimension of the modes and the transverse shift  $r_0$ . The complex amplitude of the polarization of the medium at the frequency  $\omega_k$  ( $k = 1, 2$ ) in two-mode lasing can be represented in the form

$$P_k \sim (\sigma_k + i\alpha_k') E_k + (\rho_k - i\beta_k) E_k^3 + (\tau_{kl} - i\theta_{kl}) E_k E_l^2 \quad (5)$$

$(k, l = 1, 2; k \neq l)$ .

the coefficients introduced in (5) have the usual meaning<sup>8</sup>:  $\alpha_k'$  determines the linear gain of the medium;  $\beta_k$  and  $\theta_{kl}$  are the coefficients of the intrinsic and the crossover saturations of the gain of the  $k$ th mode;  $\sigma_k$  characterizes the decrease of the mode generation frequency at the center of the saturated gain curves;  $\rho_k E_k^2$  and  $\tau_{kl} E_l^2$  determine the generation frequency shift of the  $k$ th mode owing to the intrinsic and crossover saturation.

All these coefficients were calculated under the assumptions indicated above for transitions with arbitrary angular momenta of the working levels and different polarizations of the mode, with account taken of the depolarizing collisions. Allowance for the transverse distribution of the field was found to lead to the appearance in these coefficients of certain factors that take into account the geometry of the field and of the resonator with the active medium, compared with the case of plane waves.<sup>7</sup> By way of example we present the explicit form of  $\beta_k$  and  $\theta_{kl}$  for the limiting case of an inhomogeneously broadened line  $\gamma \ll ku$ . Apart from factors that are inessential in the subsequent discussion, we have

$$\begin{aligned} \beta_k &\propto J_1 \left( 1 + \frac{\gamma^2}{\gamma^2 + \omega_{k0}^2} \right) \sum_{j,x} \frac{a_j^{(x)}}{\gamma_j^{(x)}}, \\ \theta_{kl} &\propto J_2 \left[ \frac{4\gamma^2}{4\gamma^2 + \omega_{kl}^2} + \frac{4\gamma^2}{4\gamma^2 + (\omega_{k0} + \omega_{l0})^2} \right] \sum_{j,x} \frac{b_j}{\gamma_j^{(x)}} \quad (6) \\ &+ J_2 \frac{2\gamma}{4\gamma^2 + \omega_{kl}^2} \sum_{j,x} c_j^{(x)} \frac{2\gamma\gamma_j^{(x)} - \omega_{kl}^2}{(\gamma_j^{(x)})^2 + \omega_{kl}^2} \\ &+ J_3 \frac{\gamma}{\gamma^2 + \omega_{k0}^2} \sum_{j,x} c_j^{(x)} \frac{\gamma\gamma_j^{(x)} - \omega_{k0}\omega_{kl}}{(\gamma_j^{(x)})^2 + \omega_{kl}^2}, \end{aligned}$$

where  $\omega_{k0} = \omega_k - \omega_0$ ,  $\omega_{kl} = \omega_k - \omega_l$ ,  $\gamma_j^{(x)}$  ( $x = 0, 1, 2$ ;  $lj = a, b$ ) are the polarization characteristics of the working levels, which describe the relaxation of the population, the orientations, and the alignments of the considered excited states under the action of the least atomic collisions<sup>7</sup>;  $a_j^{(x)}$ ,  $b_j^{(x)}$ ,  $c_j^{(x)}$  are numerical coefficients that depend on the angular momenta of the working levels and the polarizations of the generated modes. For helium-neon lasers they are given in Ref. 7. The quantities  $J_1$ ,  $J_2$ , and  $J_3$  are determined by the geometric characteristics of the field and of the amplifying medium:

$$\begin{aligned} J_1 &= \frac{b}{2l} \operatorname{arctg} \frac{2l/b}{1 + 4(z_0 - z_1)(z_0 - z_1 + l)/b^2}, \\ J_2 &= \frac{b}{l} \int_{x_1}^{x_2} dx (1 + 4x^2)^{-1} \exp \left[ - \left( \frac{r_0}{a_0} \right)^2 (1 + 4x^2)^{-1} \right], \\ J_3 &= \exp \left[ - \left( \frac{r_0}{a_0} \right)^2 \right] \frac{b}{l} \int_{x_1}^{x_2} dx (1 + 4x^2)^{-1} \\ &\times \cos \left[ \frac{2l\omega_{12}l}{c} \left( x \frac{b}{l} + \frac{z_1}{l} \right) + 2\delta \right], \quad (7) \\ x_1 &= (z_0 - z_1)/b, \quad x_2 = (z_0 - z_1 + l)/b. \end{aligned}$$

It was assumed in (7) that the active medium is located in the resonator between  $z_0$  and  $z_0 + l$ , while the quantity  $b = ka_0^2$  has the meaning of the characteristic dimension, along the laser axis, over which a substantial change takes place in the transverse distribution of the field. In the limit of plane waves ( $a_0 \rightarrow \infty$ )  $J_1$  and  $J_2$  tend to unity, while  $J_3$  tends to the ratio  $n_2/n_-$  of the second harmonic of the inverted population to its mean value; expressions (6) go over into the corresponding results of Ref. 7.

In the case of an arbitrary ratio  $\gamma/ku$ , allowance for the transverse distribution of the field in the approximation indicated above leads to the following changes in the polarization of the medium compared with the case of plane waves<sup>7</sup>: the coefficients  $\alpha_k$  and  $\sigma_k$  remain unchanged;  $\beta_k$  and  $\theta_{kl}$  are multiplied by  $J_1$ ; in the coefficients  $\theta_{kl}$  and  $\tau_{kl}$  the factors  $n_2/n_-$  are replaced by  $J_3$ , while the remaining terms in them are multiplied by  $J_2$ .

The results allow us to investigate the stability of the two-mode lasing regime with transverse distribution of the modes in any gas laser.

Let us estimate the influence of the transverse distribution of the modes on the stability of the two-mode regime in the limit  $l \ll b$  and  $|z_0 - z_1| \ll b$ . Neglecting the change of the transverse dimension of the field over the length of the active medium, we obtain from (7)

$$\begin{aligned} J_1 &\approx 1, \quad J_2 \approx \exp[-(r_0/a_0)^2], \\ J_3 &\approx \exp \left[ - \left( \frac{r_0}{a_0} \right)^2 \right] \frac{\sin(\omega_{12}l/c)}{(\omega_{12}l/c)} \cos \left[ \frac{|\omega_{12}l|}{c} (l + 2z_0) + 2\delta \right]. \quad (8) \end{aligned}$$

Thus, the transverse distribution of the modes does not alter the coefficient  $\beta_k$  in this approximation, since  $J_1 \approx 1$  and the

coefficients  $\theta_{kl}$  which characterize the coupling of the modes, decrease by a factor  $\exp(r_0^2/a_0^2)$  compared with the case of plane waves. This result does not depend on the type of transition, on the character of its broadening, and on the polarizations of the generated modes, attesting thus to the universality of the weakening of the mode interaction in gas lasers when this method is used.

To analyze the stability of two-mode lasing when the modes are symmetrical, it is convenient to introduce a dimensionless quantity, namely the mode coupling factor  $S(\omega_{12}) = (\beta - \theta)/(\beta + \theta)$ , which characterizes the degree of the intermode interaction ( $\beta$  and  $\theta$  are the values of the coefficients  $\beta_k$  and  $\theta_{kl}$  when the modes  $\omega_{10} = \omega_{12}/2$  and  $\omega_{20} = -\omega_{12}/2$ ) are symmetric about  $\omega_0$ . Total absence of coupling between the modes corresponds to the value  $\theta = 0$  and consequently  $S = 1$ . Stable two-mode lasing is possible, as is well known, at  $S(\omega_{12}) > 0$ . The quantity  $S$  decreases with decreasing intermode distance, for in this case the crossover saturation of the mode increases. The condition  $S = 0$  determines the critical intermode distance  $\omega_{cr}$ , below which two-mode lasing becomes unstable. The requirement that there exist stable two-mode lasing at any distance  $\omega_{12}$  between the modes denotes vanishing of  $\omega_{cr}$ . Using expressions (6) for  $\beta$  and  $\theta$ , we investigate the possibility of satisfying the condition  $S > 0$  at  $\omega_{12} = 0$ . For  $\omega_{12} = 0$  we obtain from (6)

$$S = \frac{\sum_{j,x} \left\{ \frac{1}{\gamma_j^{(x)}} [2(J_1 a_j^{(x)} - J_2 b_j^{(x)}) - (J_2 + J_3) c_j^{(x)}] \right\}}{\sum_{j,x} \frac{1}{\nu_j^{(x)}} [2(J_1 a_j^{(x)} + J_2 b_j^{(x)}) + (J_2 + J_3) c_j^{(x)}]} \quad (9)$$

The quantity  $S$  determined by expression (9) depends on the angular momenta of the transition, on the polarizations of the generated modes, and on the spectroscopic characteristics of the level. The strongest interaction takes place for modes polarized in parallel. In this case  $a_j^{(x)} = b_j^{(x)} = c_j^{(x)}$  (Ref. 7), and the condition  $S(\omega_{12} = 0) > 0$  leads to an inequality that depends on neither the angular momenta of the levels nor on the spectroscopic characteristics of the levels of the working transition:

$$2J_1 > 3J_2 + J_3. \quad (10)$$

Using expressions (8) for estimates, we obtain from (10) the condition

$$\exp[-(r_0/a_0)^2] < 2/(3 + \cos 2\delta). \quad (11)$$

It follows from (11) that at  $r_0 = 0$  it is impossible to obtain stable two-mode lasing at small intermode distances by using the longitudinal shift  $\delta$  ( $0 \leq \delta \leq \pi/2$ ). At the same time, with transverse mode separation at

$$r_0/a_0 > (\ln 2)^{1/2} \approx 0.8 \quad (12)$$

Two-mode lasing is stable also without any longitudinal shift. At  $\delta = 0$ , two-mode lasing will be stable also in the limit of the homogeneously broadened line if the transverse mode separation satisfies the condition (12). Thus, the results point to the feasibility in principle of obtaining stable

two-mode lasing at any intermode distance and on arbitrary transitions. The results presented here, with account taken of the results of earlier work, make it possible to calculate all the characteristics of the two-mode lasing regime for a specific gas laser.

## 2. EXPERIMENTAL RESULTS AND THEIR DISCUSSION

The experiments were performed on a setup consisting of the investigated laser working on the Ne transition  $3s_2 - 3p_4$ , and recording apparatus (Fig. 1). The intermode distance  $\omega_{12}$  and the longitudinal spatial shift  $\delta$  were controlled with the aid of two oppositely tapered wedges of crystalline quartz, located on opposite sides of the active medium. It is known<sup>5</sup> that in this case the laser can generate two axial linear orthogonally polarized modes with independent variation of  $\omega_{12}$  and of  $\delta$  at a constant cavity length  $L$ . To separate the modes in the transverse direction we used birefringence. At normal incidence, the displacement  $r_0$  of the ordinary and extraordinary rays in a uniaxial crystal is a maximum when the angle  $\psi$  between the directions of the incident ray and the optical axis of the crystal satisfies the condition<sup>9</sup>

$$\psi = \arctg(n_o/n_e).$$

In this case the displacement is equal to

$$r_0 = d \operatorname{tg} [(n_o^2 - n_e^2)/2n_o n_e],$$

where  $d$  is the length of the crystal in the direction of light propagation;  $n_o$  and  $n_e$  are the refractive indices for the ordinary and extraordinary rays. For the employed  $\text{LiNbO}_3$  crystal we have  $\Delta n \ll n$  ( $\Delta n = |n_o - n_e|$ ,  $n \approx n_o \approx n_e$ ), so that the angle  $\psi \approx 45^\circ$  and  $r_0 = d \Delta n/n$ .

Thus, by using sets of plane-parallel plates of lithium niobate with different thicknesses  $d$  it is possible to vary discretely the ratio  $r_0/a_0$  that determines the intermode interaction.

The recording apparatus permitted observation of the change of the intensity of any of the two orthogonally polarized modes due to detuning of the resonator, and measurement of the frequency  $\omega_{12}$  of the intermode beats, thereby yielding complete information on the behavior of the region  $\Delta$  of the stable two-mode lasing in the vicinity of the symmetrical position of the modes and of the critical frequency  $\omega_{cr}$  when the laser parameters are varied.

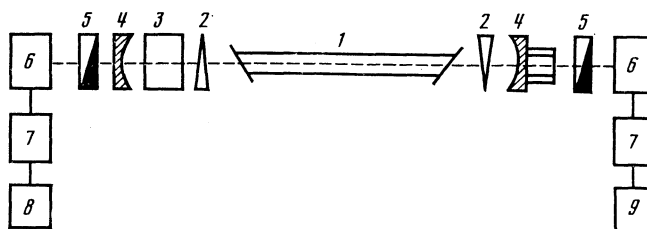


FIG. 1. Block diagram of experimental setup: 1—gas-discharge tube; 2—system of phase-anisotropic elements that ensure independent regulation of  $\omega_{12}$  and of  $\delta$ ; 3—uniaxial crystal that introduces a transverse spatial shift of the modes; 4—cavity mirrors; 5—polarizers; 6—photoreceivers; 7—amplifiers; 8—oscilloscope; 9—spectrum analyzer.

The parameters of the investigated laser were the following:  $l = 0.5$  m;  $z_0 = 0.2$  m;  $z_1 = 0.7$  m;  $b = 2$  m;  $a_0 = 10^{-3}$  m, with  $b$  and  $z_1$  calculated from formulas taken from Ref. 10, starting from the known curvature radii of the mirrors,  $R_1 = 5$  m and  $R_2 = 2$  m, and from the cavity length  $L = 0.9$  m.

For the parameters indicated, the stable two-mode-lasing region  $\Delta$  and the critical frequency  $\omega_{cr}$  were calculated in the limit of an inhomogeneously broadened line, since the gain line of the Ne transition  $3s_2-3p_4$  at mixture working pressures  $p = 1-3$  Torr, is closer to the inhomogeneously broadened one. In principle it is easy to perform similar calculations also for the real ratio  $\gamma/ku$ .

Figure 2a shows the results of the calculation of the stable two-mode-lasing region  $\Delta$  as a function of the intermode distance  $\omega_{12}$ . Curves 1-3 correspond to a longitudinal shift  $\delta = 0$ . They differ from each other in the values of the parameter  $r_0/a_0$ . Curve 1 corresponds to  $r_0 = 0$ . It can be seen that the region  $\Delta$  vanishes in this case at  $\omega_{12} \approx 22$  MHz, i.e., stable two-mode lasing is impossible at  $\omega_{12} < 22$  MHz =  $\omega_{cr}$ . Curve 2 corresponds to  $r_0/a_0 = 0.2$ . The small transverse mode separation led not only to a lowering of  $\omega_{cr}$  to 7 MHz, but also to an increase of the region of the two-mode lasing. At  $r_0/a_0 = 0.4$  (curve 3), the two-mode regime becomes stable at practically all intermode distances. For orthogonal polarizations of the modes, which are the ones considered here, it is possible to decrease  $\omega_{cr}$  and to extend the region of two-mode lasing also with the aid of a longitudinal shift  $\delta$ . Curve 4 illustrates this fact: it corresponds to  $r_0 = 0$  and to  $\delta = \pi/2$ , which is the optimal shift from the viewpoint of weakening of the intermode interaction. It can be seen that the two-mode regime does in fact become stable at any intermode distance, but the region  $\Delta$  is much smaller than in the case of curve 3.

Figure 2b shows analogous experimental results. It can be seen that the behavior of the curves and the character of their variation with increasing  $r_0/a_0$  agree with the results of the theoretical calculation.

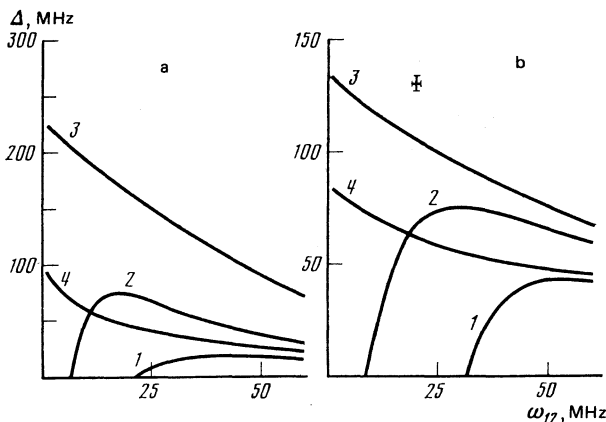


FIG. 2. Calculated (a) and experimental (b) plots of  $\Delta(\omega_{12})$  for  $p = 2$  Torr. 1— $r_0/a_0 = 0$ ; 2— $r_0/a_0 = 0.2$ ; 3— $r_0/a_0 = 0.4$ ; 4— $\delta = 90^\circ$ ,  $r_0/a_0 = 0$ . b)  $\delta = 0^\circ$ : 1— $r_0/a_0 = 0$ ; 2— $r_0/a_0 = 0.32$ ; 3— $r_0/a_0 = 0.46$ ; 4— $\delta = 90^\circ$ ,  $r_0/a_0 = 0$ .

We note the substantial differences in the influence of the parameters  $\delta$  and  $r_0/a_0$  on the mode interaction. The possibility of changing the coupling of the modes by varying  $\delta$  is limited. Thus, for  $p = 2$  Torr the mode coupling factor  $S$  at  $r_0 = 0$  is a small quantity, 0.02 even at  $\delta = \pi/2$ , i.e., the mode coupling remains strong as before. With increasing  $r_0/a_0$ , the factor  $S$  tends to unity and the intermode interaction can be made arbitrarily weak. In accordance with the different action of the longitudinal and transverse field separation on the intermode coupling, a region  $\Delta$  also changes. Thus, a change of  $\delta$  from zero to  $\pi/2$  increases the region of the two-mode lasing, but not to its maximum possible value. This value is determined by the condition that both modes be near frequencies where the gain exceeds the loss, and is equal to

$$\Delta_{\max} = 2ku(\ln \eta)^{1/2} - \omega_{12},$$

where  $\eta$  is the relative excess of the gain of the modes over the losses. With increasing  $r_0/a_0$ , the size of the stability region of two-mode lasing tends to its limiting value  $\Delta_{\max}$ . We note that in experiment  $\Delta_{\max}$  is limited to  $c/2L \approx 170$  MHz.

Since the transverse and longitudinal shifts act in the same direction and weaken the mode interaction, it is reasonable for practical purposes to use their simultaneous action to decrease the intermode coupling. To this end we investigate qualitatively the dependence of  $\omega_{cr}$  on  $\delta$  at different values of the transverse separation  $r_0/a_0$ . Replacing  $\gamma_j^{(x)}$  by a certain effective width  $\gamma_0$ , denoting

$$a = \sum_{j,x} a_j^{(x)}, \quad b = \sum_{j,x} b_j^{(x)}, \quad c = \sum_{j,x} c_j^{(x)},$$

$$F = \exp(-r_0^2/a_0^2), \quad N = \cos 2\delta$$

we obtain under the assumption  $\omega_{cr} \sim \gamma_0 \ll \gamma$ , using (6) and (8),

$$\omega_{cr}^2 \approx \gamma_0^2 [cF(1+N) - 2(a-bF)] / 2(a-bF). \quad (13)$$

For the considered transition and for orthogonal polarizations of the mode Ref. 7 gives  $a = 17/225$  and  $b = c = 23/450$ . It follows from (13) that  $\omega_{cr}$  is of the order of  $\gamma_0$  in the absence of shifts ( $r_0 = 0$  and  $\delta = 0$ ). At a specified transverse

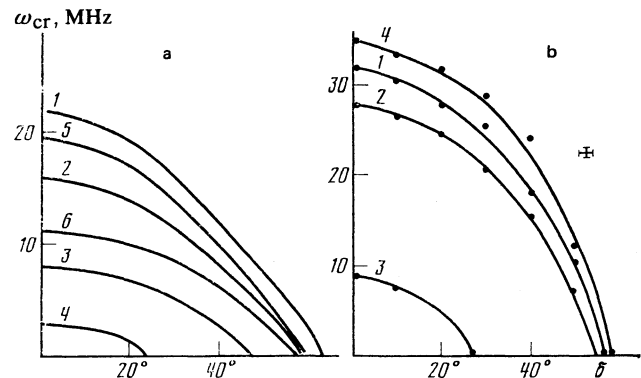


FIG. 3. Calculated (a) and experimental (b) plots of  $\omega_{cr}(\delta)$ . a)  $p = 2$  Torr: 1— $r_0/a_0 = 0$ ; 2— $r_0/a_0 = 0.1$ ; 3— $r_0/a_0 = 0.2$ ; 4— $r_0/a_0 = 0.3$ .  $r_0/a_0 = 0.1$ ; 5— $p = 3$  Torr; 6— $p = 1$  Torr. b)  $p = 2$  Torr 1— $r_0/a_0 = 0$ ; 2— $r_0/a_0 = 0.25$ ; 3— $r_0/a_0 = 0.32$ ; 4— $p = 2.5$  Torr,  $r_0/a_0 = 0$

shift  $r_0$ , it decreases monotonically with increasing  $\delta$ , vanishing at  $\delta^*$ , where

$$\delta^* = 0.5 \arccos \left[ \frac{2a}{c} \exp \left( \frac{r_0^2}{a_0^2} \right) - \frac{2b}{c} - 1 \right].$$

We note that  $\delta^*$  does not depend on the pressure of the active medium, in contrast to the critical frequency. These qualitative results agree with the numerical calculation.

Figure 3a shows the values of  $\omega_{cr}$  calculated under the same conditions as in the case of Fig. 2a. The corresponding experimental results, shown in Fig. 3b, agree well with the theoretical ones.

In conclusion, we consider the influence of the position of the tube with the amplifying medium relative to the neck of the mode and its length, on the degree of interaction of the modes. This can be understood qualitatively by writing down approximate expressions for the quantities  $J_1$ ,  $J_2$ , and  $J_3$  at  $\omega_{12} = 0$ , assuming  $l/b \ll 1$  and  $|z_0 - z_1|/b \ll 1$ , and retaining the first corrections in terms of these parameters. Denoting by  $m$ , to abbreviate the notation, the quantity

$$m = 4 \left[ (z_0 - z_1)(z_0 + z_1 + l) / b^2 + l^2 / 3b^2 \right] \ll 1,$$

we obtain from (7)

$$\begin{aligned} J_1 &\approx 1 - m, & J_2 &\approx \exp \left[ -(r_0/a_0)^2 \right] \left[ 1 - m(1 - r_0^2/a_0^2) \right], \\ J_3 &\approx \exp \left[ -(r_0/a_0)^2 \right] (1 - m) \cos 2\delta. \end{aligned} \quad (14)$$

The parameter  $m$  is determined by the geometric characteristics of the field and of the active medium. As follows from

(10), the conditions for the stability of the two-mode regime are satisfied better the smaller the ratio  $J_2/J_1$  or  $J_3/J_1$ . In the approximation (14), the ratio  $J_3/J_1$  is independent of the geometrical characteristics, and for  $J_2/J_1$  we obtain

$$J_2/J_1 \approx \exp \left[ -(r_0/a_0)^2 \right] \left[ 1 + m(r_0/a_0)^2 \right].$$

Since  $m > 0$ , the quantity  $J_2/J_1$  as a function of the position of the tube relative to the neck of the mode reaches a maximum value when  $m$  has the smallest value. This takes place when the tube is symmetrically located relative to the neck of the mode. In this case the coupling between the modes is a minimum. At any other position, owing to the mode divergence, their overlap decreases and the interaction increases. The results of a numerical calculation are shown in Fig. 4. Figure 4a shows the dependence of  $\omega_{cr}$  on  $z_1$ . It can be seen that the minimum of each of the curves corresponds to the position  $z_1 = z_0 + l/2$ . An increase of the parameter  $b$ , which characterizes the divergence of the mode fields, leads to a decrease of  $\omega_{cr}$  (Fig. 4b).

Thus, the investigations of the interaction of two Gaussian modes in a linear gas laser have made it possible to suggest an effective method of weakening their coupling by introducing a transverse shift. An advantage of this method of eliminating the competition of the mode is its applicability for arbitrary gas lasers, particularly on transitions where other methods of weakening the intermode interaction may turn out to be ineffective.

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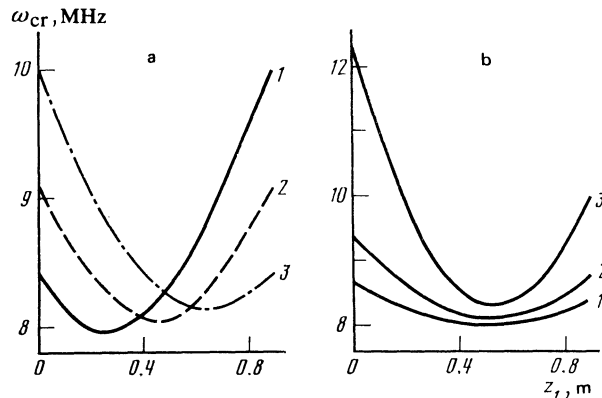


FIG. 4. Calculated plots of  $\omega_{cr}(z_1)$  for  $p = 2$  Torr;  $z_0 = 0.2$  m,  $r_0/a_0 = 0.2$ ,  $\delta = 0$ : a)  $b = 2$  m: 1— $l = 0.1$  m, 2— $l = 0.4$  m, 3— $l = 0.7$  m. b)  $l = 0.5$  m: 1— $b = 3$  m, 2— $b = 2$  m, 3— $b = 1$  m.

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